Exercises 6, 12, pp. 52-53.

6. (a) If H is a subgroup of G, then for any $h, h' \in H$, we have $h^{-1}h'h \in H$. Hence $h^{-1}Hh = H$, and $h \in N_G(H)$. Therefore $H \leq N_G(H)$.

   If H is not a subgroup of G, then multiplication fails so we have no reason to expect $h^{-1}h'h \in H$. For example, let

$$H = \left\{ \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} \right\}.$$

   Then

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & -5 \\ 0 & 3 \end{pmatrix} \notin H.$$

   Hence $H \nleq N_G(H)$.

   (b) If $H \leq C_G(H)$, then for any $h, h' \in H$, we have $h^{-1}h'h = h' \Rightarrow h'h = hh'$. Hence H is abelian, as desired.

12. Too much work for now.

Exercises 16, 17, pp. 65-66.

16. (a) Since G is finite there can only be a finite amount of subgroups. In particular, there are only a finite amount of subgroups $\{H_i\}_{i=1}^n$ containing H. Then any chain $H \leq H_{i_1} \leq H_{i_2} \leq \cdots \leq H_{i_k} \leq G$ is finite, and we may prescribe H_{i_k} as the maximal subgroup containing H.

   (b) Suppose $\langle r \rangle \leq K$. Then $|\langle r \rangle| \leq |K|$ while $|K| \mid |G|$. But $\langle r \rangle$ has order n and G has order $2n$. Hence $|K|$ can only be n, in which case $H = K$, or $2n$, in which case $K = G$. This is exactly the definition of H being maximal, as desired.

   (c)

17.

Exercises 1, 18, 24, 40, 41 pp. 85-89.

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18.

24.

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41.

Exercise 4, pp. 111.

4.