Homework 5

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Problem 1. Prove that convergence of $\{s_n\}$ implies convergence of $\{|s_n|\}$. Is the converse true?

Problem 2. Calculate $\lim_{n\to\infty}(\sqrt{n^2+n}-n)$.

Problem 3. If $s_1 = \sqrt{2}$, and

$$s_{n+1} = \sqrt{2+\sqrt{s_n}} \qquad (n = 1, 2, 3, \dots),$$

prove that $\{s_n\}$ converges, and that $s_n < 2$ for $n = 1, 2, 3, \dots$.

Problem 4. Find the upper and lower limits of the sequence $\{s_n\}$ defined by

$$s_1 = 0; \qquad s_{2m} = \frac{s_{2m-1}}{2}; \qquad s_{2m+1} = \frac{1}{2} + s_{2m}.$$