Homework 12

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Proposition. *If p is prime and $p \mid ab$, then $p \mid a$ or $p \mid b$.*

Proof. We know that $\gcd(p, a)$ must be equal to 1 or p. If $\gcd(p, a) = 1$, then our helpful theorem gives $p \mid b$. If $\gcd(p, a) = p$, then we know that $p \mid a$. Hence either $p \mid a$ or $p \mid b$, as desired. $\square$

Proposition. *The first p probed locations are unique.*

Proof. Assume for the sake of contradiction that the first p probed locations are not unique. Then there are $0 \leq i < j < p$ such that $h + ik \equiv h + jk \mod p$. Algebraic manipulation gives $(j - i)k \equiv 0 \mod p$, hence $p \mid (j - i)k$. From the previous exercise, this implies $p \mid (j - i)$ or $p \mid k$. But both $j - i$ and k are strictly less than p! Thus we have contradiction. $\square$