Homework 11

Hanting Zhang

Chapter 7; # 4 and 8 (pg. 175)

Problem 4. Consider

$$f(x) = \sum_{n=1}^{\infty} \frac{1}{1+n^2x}.$$

For what values of x does the series converge absolutely? On what interval does it converge uniformly? On what interval does it fail to converge uniformly? Is f continuous whenever it converges? If f bounded?

Problem 8. if

$$I(x) = \begin{cases} 0 & (x \le 0), \\ 1 & (x > 0), \end{cases}$$

if $\{x_n\}$ is a sequence of distinct points of (a, b), and if $\sum |c_n|$ converges, prove that the series

$$f(x) = \sum_{n=1}^{\infty} c_n I(x - x_n) \quad (a \le x \le b)$$

converges uniformly, and that f is continuous for every $x \neq x_n$.