Exercises 16, 17, pp. 138.

Problem 16. Prove that $(\mathbb{Z}/24\mathbb{Z})^\times$ is an elementary abelian group of order 8.

Problem 17. Let $\rangle G\langle$ be a cyclic group of order n. For $n = 2, 3, 4, 5, 6$, write out the elements of $\mathrm{Aut}(G)$ explicitly.

Exercises 3, 5, 6, 7, 8, 14 pp. 184-187.

Problem 3. Continue for Example 1. Prove that every element of $G-H$ has order 2. Prove that G is abelian if and only if $h^2 = 1$ for all $h \in H$.

Problem 5. Let $G = \mathrm{Hol}(\mathbb{Z}_2 \times \mathbb{Z}_2)$.

(a) Prove that $G = H \rtimes K$ where $H = \mathbb{Z}_2 \times \mathbb{Z}_2$ and $K \cong S_3$. Deduce that $|G| = 24$.

(b) Prove that G is isomorphic to S_4.

Problem 6.

Problem 7.

Problem 8. Construct an non-abelian group of order 75. Classify all groups of order 75.

Problem 14.

Exercises 2, 5 pp. 165-167.

Problem 5. Let G be a finite abelian group of type $(n_1, n_2, \ldots, n_t)$. Prove that G contains an element of order m if and only if $m \div n_1$. Deduce that G is of exponent n_1.

Exercise 15 p. 174.

Problem 15. If A and B are normal subgroups of G such that G/A and G/B is both abelian, prove that $G/A \cap B$ is abelian.