

## 2. Resolution for First Order Logic - 20 points

Consider the following knowledge base: (a) All programmers who do not have bugs in their code and comment their code write high-quality code. (b) Programmers who use best coding practices comment their code. (c) Programmers who write high-quality code solve programming problems quickly. (d) Amanda uses best coding practices and (e) does not have bugs in her code. (f) Bill does not comment his code.

1. You want to find out whether the knowledge base entails that Amanda solves programming problems quickly. Use resolution for a proof by contradiction.
  - (a) Write down the entailment that you want to prove, expressing the English statements in first-order logic. Use the predicates  $\text{HasBugs}(x)$ ,  $\text{Comments}(x)$ ,  $\text{HighQuality}(x)$ ,  $\text{QuickSolve}(x)$ , and  $\text{BestPractice}(x)$ , where  $x$  is a programmer.
  - (b) Do the transformations as explained in class.
  - (c) Use resolution to try to obtain false (that is, the empty clause).
  - (d) Interpret the result of the previous step in plain English.
2. You want to find out whether the knowledge base entails that Bill solves programming problems quickly. Proceed using the steps given above.

1. We start with the knowledge base

- (1)  $\forall x, \neg \text{HasBugs}(x) \wedge \text{Comment}(x) \implies \text{HighQuality}(x)$
- (2)  $\forall x, \text{BestPractice}(x) \implies \text{Comment}(x)$
- (3)  $\forall x, \text{HighQuality}(x) \implies \text{QuickSolve}(x)$
- (4)  $\text{BestPractice}(A)$
- (5)  $\neg \text{HasBugs}(A)$
- (6)  $\neg \text{Comment}(B)$

By applying rewrite rules for implies, we can equivalently say

- (1)  $\forall x, \text{HasBugs}(x) \vee \neg \text{Comment}(x) \vee \text{HighQuality}(x)$
- (2)  $\forall x, \neg \text{BestPractice}(x) \vee \text{Comment}(x)$
- (3)  $\forall x, \neg \text{HighQuality}(x) \vee \text{QuickSolve}(x)$
- (4)  $\text{BestPractice}(A)$
- (5)  $\neg \text{HasBugs}(A)$
- (6)  $\neg \text{Comment}(B)$

To show  $\text{QuickSolve}(A)$ , we do the transformation of adding: (7)  $\neg \text{QuickSolve}(A)$ , and do resolution to try to prove false. Indeed, we have:

- (2) + (5)  $\Rightarrow$  (8)  $\text{Comment}(A)$
- (1) + (8)  $\Rightarrow$  (9)  $\text{HasBugs}(A) \vee \text{HighQuality}(A)$
- (3) + (9)  $\Rightarrow$  (10)  $\text{HasBugs}(A) \vee \text{QuickSolve}(A)$
- (5) + (10)  $\Rightarrow$  (11)  $\text{QuickSolve}(A)$
- (7) + (11)  $\Rightarrow$  (12)  $\perp$

Since we have false, resolution succeeds. In plain english, this means that Alice can solve problems quickly.

2. Start with the same 6 rules as we did for part 1 and add (7)  $\neg\text{QuickSolve}(B)$ . We try to use resolution to prove false:

$$(2) + (6) \Rightarrow (8) \neg\text{BestPractice}(B)$$

$$(3) + (7) \Rightarrow (9) \neg\text{HighQuality}(B)$$

$$(1) + (10) \Rightarrow (7) \text{HasBugs}(B) \vee \neg\text{Comment}(B)$$

No other resolutions can be made.

Since no other resolutions can be made, we can only conclude that the knowledge base does not entail that Bob solves problems quickly.