Chocolate 3

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Problem 3. [0]

Chocolate Problem: 2 chocolate bars

Reminder: If you solve a chocolate problem (which you can do in groups of size up to 3), please e-mail David with the solution — do not submit it on Gradescope. Also, feel free to list preferences or dietary restrictions for/against particular types of chocolate.

Exercise 4.31 in the textbook. Notice that Part (b) is really the interesting thing here — Part (a) is basically a slightly harder regular problem.

Proposition 1. *Prove that for every pair of nodes $u, v \in V$, the length of the shortest $u - v$ path in H is at most 3 times the length of the shortest $u - v$ path in G.*

Proof. Denote the length of the shortest $u - v$ path in H and G by $d_H(u, v)$ and $d_G(u, v)$, respectively. Suppose for the sake of contradiction that there exists some $u, v \in V$ such that $d_H(u, v) > 3d_G(u, v)$. Furthermore set u, v to be the vertices such that $d_H(u, v)$ is minimum with repsect to $d_H(u, v) > 3d_G(u, v)$. Note that such a minimum exists since edges have positive side lengths. (And the graph must be finite?) Consider the $u - v$ path in G made up of the sequence of vertices $u = v_1, v_2, \ldots, v_k = v$. $\square$