# experiments\_8v3

# June 9, 2023

 $source: \ https://medium.com/dataseries/convolutional-autoencoder-in-pytorch-on-mnist-dataset-d65145c132ac\#63b2 \ source: \ https://stackoverflow.com/questions/37228371/visualize-mnist-dataset-using-opency-or-matplotlib-pyplot$ 

 $color source: https://matplotlib.org/stable/tutorials/colors/colors.html palette source: https://seaborn.pydata.org/tutorial/color_palettes.html \\$ 

```
[]: import torch
     import torch.nn as nn
     import torch.nn.functional as F
     from torch.utils.data import DataLoader, random_split
     import torch.optim as optim
     import torchvision
     from torchvision import transforms
     import matplotlib.pyplot as plt
     import seaborn as sns
     import pandas as pd
     import numpy as np
     from sklearn.metrics.pairwise import pairwise_kernels
     import random
     import tqdm
     import time
     from permutation_testing import *
     from model import *
     plt.rcParams['patch.edgecolor'] = 'none' # remove edges from histogram bars
```

```
[]: # setting random seed
    # https://pytorch.org/docs/stable/notes/randomness.html
    torch.manual_seed(0)
    np.random.seed(0)
    random.seed(0)

torch.use_deterministic_algorithms(True)
```

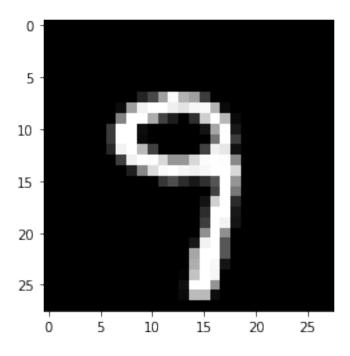
```
torch.backends.cudnn.deterministic = True
torch.backends.cudnn.benchmark = False
```

[]: %load\_ext autoreload %autoreload 2

The autoreload extension is already loaded. To reload it, use: %reload\_ext\_autoreload

```
[]: data dir = 'dataset'
     train_dataset = torchvision.datasets.MNIST(data_dir, train=True, download=True)
     test_dataset = torchvision.datasets.MNIST(data_dir, train=False, download=True)
     train_transform = transforms.Compose([transforms.ToTensor(),])
     test_transform = transforms.Compose([transforms.ToTensor(),])
     train_dataset.transform = train_transform
     test_dataset.transform = test_transform
     m = len(train_dataset)
     batch size = 256
     train_data, val_data = random_split(train_dataset, [int(0.8 * m), int(0.2 * m)])
     train_loader = DataLoader(train_data, batch_size=batch_size)
     val_loader = DataLoader(val_data, batch_size=batch_size)
     test_loader = DataLoader(test_dataset, batch_size=batch_size,shuffle=False) #_J
      \hookrightarrow dangit
     for data in train_loader:
         print(data[0].shape)
         break
     plt.imshow(data[0][0].squeeze(), cmap='gray')
     print('Number of training samples:', len(train_dataset))
     print('Number of test samples:', len(test_dataset))
```

torch.Size([256, 1, 28, 28])
Number of training samples: 60000
Number of test samples: 10000



# 0.0.1 keeping track of model weights

- 05232023a: epochs=50, lr=1e-3, d=4, inplace=T, ReLU
- 05302023a: epochs=50, lr=1e-3, d=4, inplace=F, ReLU
- 05302023b: epochs=50, lr=1e-3, d=4, inplace=F, ELU
- 06072023a: epochs=50, lr=1e-3, d=4, inplace=F, ELU; used deterministic things from Py-Torch
  - epoch 50: train loss: 0.02580, val loss: 0.02636
- 06072023b: epochs=50, lr=1e-3, d=4, inplace=F, ELU; additionally used torch.backends.cudnn.benchmark = False
  - epoch 50: train loss: 0.02580, val loss: 0.02636
- 06072023c: same settings as 06072023b except d=6
  - epoch 50: train loss: 0.01868, val loss: 0.01931

```
device = torch.device("cuda") if torch.cuda.is_available() else torch.
    device("cpu")
    print(f'Selected device: {device}')

loss_fn = torch.nn.MSELoss()
lr = 0.001
num_epochs = 50
d = 6
```

Selected device: cpu

```
[]: |%%time
     train = False
     version = '06072023c' # NOTE: MAKE SURE TO UPDATE THIS EVERY TIME
     encoder = Encoder(encoded_space_dim=d,fc2_input_dim=128)
     decoder = Decoder(encoded_space_dim=d,fc2_input_dim=128)
     if train:
         params_to_optimize = [
             {'params': encoder.parameters()},
             {'params': decoder.parameters()}
         ]
         optimizer = optim.Adam(params_to_optimize, lr=lr)
         print_every = 10
         encoder.to(device)
         decoder.to(device)
         plot_ae_outputs(encoder, decoder, test_dataset, device, n=10)
         losses = {'train_loss': [], 'val_loss': []}
         for epoch in range(num_epochs):
             train_loss = train_epoch(encoder,decoder,device, train_loader,loss_fn,u
      →optimizer)
             val_loss = test_epoch(encoder, decoder, device, val_loader, loss_fn) #__
      ⇒better for this to be val_loader?
             print('EPOCH {}/{}: train loss: {:.5f}, val loss: {:.5f}'.format(epoch⊔
      →+ 1, num_epochs, train_loss, val_loss))
             losses['train_loss'].append(train_loss)
             losses['val_loss'].append(val_loss)
             if (epoch + 1) % print_every == 0:
                 plot_ae_outputs(encoder, decoder, test_dataset, device, n=10)
         # save the model
         torch.save(encoder.state_dict(), 'weights/encoder_weights_' + version + '.
         torch.save(decoder.state_dict(), 'weights/decoder_weights_' + version + '.
      ⇔pth')
     else:
         encoder.load state_dict(torch.load('weights/encoder_weights_' + version + '.
      ⇔pth'))
         decoder.load state dict(torch.load('weights/decoder weights ' + version + '.
      ⇔pth'))
     encoder.eval(), decoder.eval()
```

CPU times: user  $3.74~\mathrm{ms}$ , sys:  $1.36~\mathrm{ms}$ , total:  $5.09~\mathrm{ms}$  Wall time:  $3.92~\mathrm{ms}$ 

```
[]: (Encoder(
        (encoder_cnn): Sequential(
          (0): Conv2d(1, 8, kernel_size=(3, 3), stride=(2, 2), padding=(1, 1))
          (1): ELU(alpha=1.0)
          (2): Conv2d(8, 16, kernel size=(3, 3), stride=(2, 2), padding=(1, 1))
          (3): ELU(alpha=1.0)
          (4): Conv2d(16, 32, kernel size=(3, 3), stride=(2, 2))
          (5): ELU(alpha=1.0)
        (flatten): Flatten(start_dim=1, end_dim=-1)
        (encoder_lin): Sequential(
          (0): Linear(in_features=288, out_features=128, bias=True)
          (1): ELU(alpha=1.0)
          (2): Linear(in_features=128, out_features=6, bias=True)
        )
      ),
      Decoder(
        (decoder_lin): Sequential(
          (0): Linear(in_features=6, out_features=128, bias=True)
          (1): ELU(alpha=1.0)
          (2): Linear(in_features=128, out_features=288, bias=True)
          (3): ELU(alpha=1.0)
        (unflatten): Unflatten(dim=1, unflattened_size=(32, 3, 3))
        (decoder_conv): Sequential(
          (0): ConvTranspose2d(32, 16, kernel_size=(3, 3), stride=(2, 2))
          (1): ELU(alpha=1.0)
          (2): ConvTranspose2d(16, 8, kernel_size=(3, 3), stride=(2, 2), padding=(1,
     1), output_padding=(1, 1))
          (3): ELU(alpha=1.0)
          (4): ConvTranspose2d(8, 1, kernel_size=(3, 3), stride=(2, 2), padding=(1,
     1), output_padding=(1, 1))
     ))
[]: test_error_on_classes = True
     losses = {}
     centers = {}
     if test_error_on_classes:
         for label in data[1].unique():
             losses[label.item()] = 0
             centers[label.item()] = torch.zeros(6)
             num_digits = 0
             for ind, data in enumerate(test loader):
                 idx = data[1] == label.item()
                 num_digits += idx.sum().item()
                 original_images = data[0][idx]
```

```
latent_representations = encoder(original_images)
                 recon_images = decoder(latent_representations)
                 losses[label.item()] += loss_fn(original_images, recon_images).
      →item()
                 centers[label.item()] += latent_representations.mean(dim=0)
             losses[label.item()] /= num_digits
             losses[label.item()] = np.round(losses[label.item()], 7)
             centers[label.item()] /= num_digits
         pd.Series(losses).sort_values()
[]: pd.Series(losses).sort_values(ascending=False) # 8 has the greatest_
      ⇔reconstruction error
         0.001194
[]:8
     5
         0.001158
     2
         0.001020
     3
         0.000971
     6
         0.000748
     0
         0.000733
         0.000728
     4
     9
         0.000646
     7
         0.000541
         0.000144
     dtype: float64
[]: pairwise_center_distances = torch.zeros(10, 10)
     for num1 in range(0, 10):
         for num2 in range(0, 10):
             pairwise_center_distances[num1, num2] = ((centers[num1] -__
      ⇔centers[num2]) ** 2).sum()
[]: # see which digits have closest embeddings on average
     torch.topk(pairwise_center_distances, k=2, dim=1, largest=False)
     # so 8 and 3 are the digits with closest latent embeddings, on average
[]: torch.return_types.topk(
     values=tensor([[0.0000, 0.5765],
             [0.0000, 0.9230],
             [0.0000, 0.6556],
             [0.0000, 0.3532],
             [0.0000, 0.5178],
             [0.0000, 0.3877],
             [0.0000, 0.5765],
```

# 0.1 Convenience Functions

```
[]: def p_val(simulated_stats, observed_stat):
    return (np.array([stat.item() for stat in simulated_stats]) > observed_stat.
    item()).mean()#i
```

```
[]: def plot_permutation(
             simulated stats,
             observed_stat,
             bins=50,
             filepath='',
             hist_color='tab:blue',
             line_color='tab.red'
         ):
         print('p_value:', p_val(simulated_stats, observed_stat))
         sns.displot([mmd.item() for mmd in simulated_stats], bins=bins,__

¬color=hist_color)
         plt.axvline(observed_stat.item(), color=line_color)
         if filepath != '':
             print('Image saved to ' + str(filepath))
             plt.savefig(filepath + '.jpg', bbox_inches='tight')
         else:
             print('No image saved.')
         return
```

# 0.2 Creating X and Y samples

# 0.2.1 $X = \{\text{images of 8's}\}, Y = \{\text{images of 3's}\}$

```
[]: # generate x_batch, y_batch
x_batch = []
y_batch = []
for ind, data in enumerate(test_loader): # filter batches based on digit
```

```
if ind == 5:
    r_batch = data[0][(data[1] == 8) | (data[1] == 3)]
    elif 5 < ind and ind < 15:
        x_batch.append(data[0][data[1] == 8])
        y_batch.append(data[0][data[1] == 3])
    elif ind == 15:
        break

x_batch = torch.cat(x_batch, dim=0)
y_batch = torch.cat(y_batch, dim=0)
print('x_batch shape:', x_batch.shape)
print('y_batch shape:', y_batch.shape)
print('y_batch shape:', r_batch.shape)

n_perms = 250</pre>
```

x\_batch shape: torch.Size([224, 1, 28, 28])
y\_batch shape: torch.Size([237, 1, 28, 28])
r\_batch shape: torch.Size([51, 1, 28, 28])

### 0.2.2 Statistics for Anisotropic Kernel

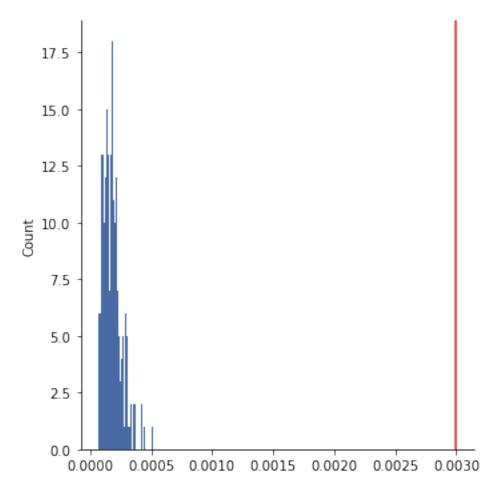
```
[]: %%time
     # anisotropic mmd
     encoder.eval()
     diff_anisotropic_stats = {}
     for val in [-3, -2, -1, 0, 1, 2, 3]:
         start = time.time()
         observed_mmd_anisotropic8v3, simulated_mmds_anisotropic8v3 =__
      permutation_test_anisotropic_mmd(
             x_batch, y_batch, r_batch, encoder, n_perms=n_perms, sigma_squared=(2_
      →** val)
         end = time.time()
         p = p val(simulated mmds anisotropic8v3, observed mmd anisotropic8v3)
         diff_anisotropic_stats[val] = {'p': p, 'time': end - start}
         hist_color = '#093885'
         line_color = 'tab:red'
         filepath = 'images_latentdim6_8v3v8/diff_anisotropic_s' + str(val) + '_8v3'
         print('bandwidth:', str(np.round(2 ** val, 3)))
         plot_permutation(
             simulated_mmds_anisotropic8v3,
             observed_mmd_anisotropic8v3,
             hist_color = hist_color,
```

```
line_color = line_color,
  filepath = filepath
)
plt.show()
```

100% | 250/250 [00:14<00:00, 17.14it/s]

bandwidth: 0.125
p\_value: 0.0

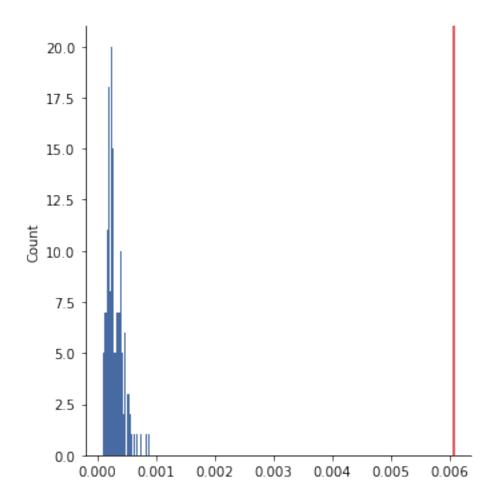
Image saved to images\_latentdim6\_8v3v8/diff\_anisotropic\_s-3\_8v3



100%| | 250/250 [00:16<00:00, 14.92it/s]

bandwidth: 0.25
p\_value: 0.0

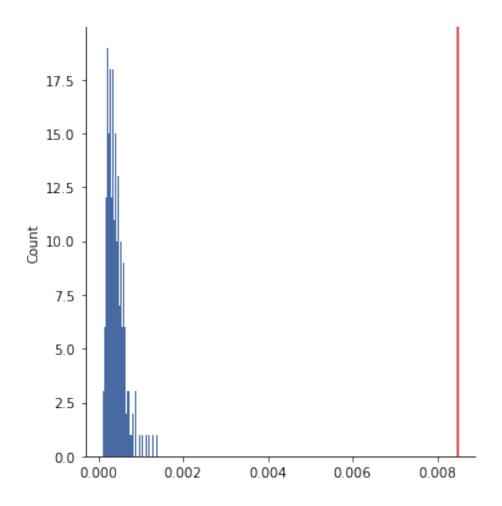
 ${\tt Image \ saved \ to \ images\_latentdim6\_8v3v8/diff\_anisotropic\_s-2\_8v3}$ 



100%| | 250/250 [00:13<00:00, 18.09it/s]

bandwidth: 0.5
p\_value: 0.0

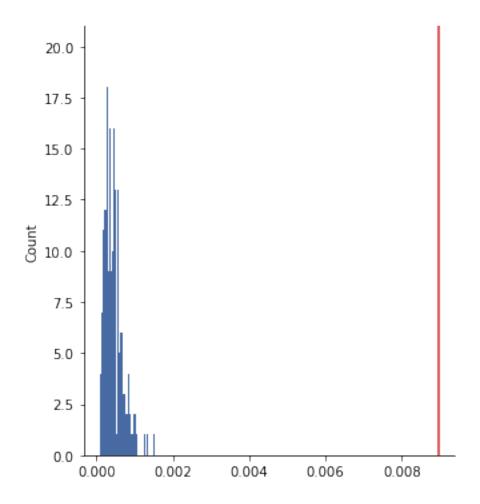
Image saved to images\_latentdim6\_8v3v8/diff\_anisotropic\_s-1\_8v3



100%| | 250/250 [00:14<00:00, 17.84it/s]

bandwidth: 1
p\_value: 0.0

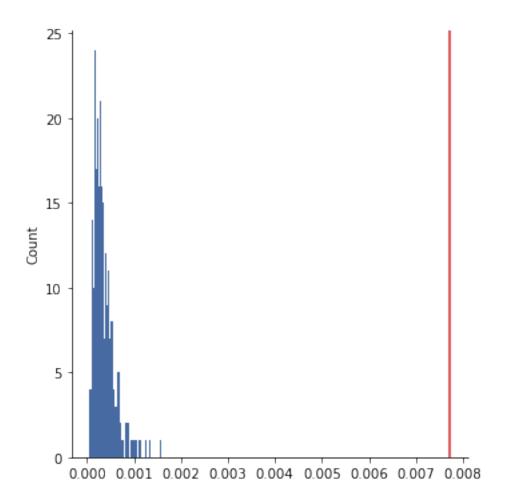
Image saved to images\_latentdim6\_8v3v8/diff\_anisotropic\_s0\_8v3



100%| | 250/250 [00:14<00:00, 17.79it/s]

bandwidth: 2
p\_value: 0.0

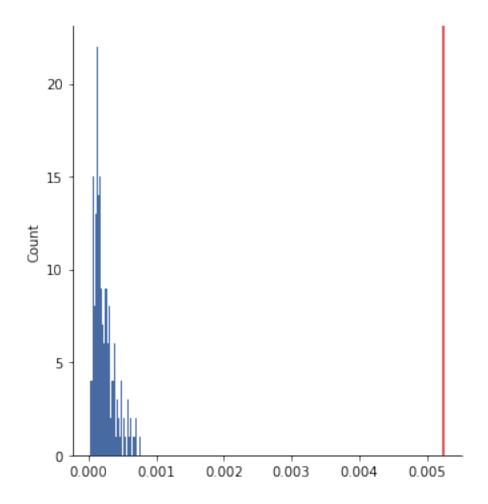
Image saved to images\_latentdim6\_8v3v8/diff\_anisotropic\_s1\_8v3



100%| | 250/250 [00:14<00:00, 17.25it/s]

bandwidth: 4
p\_value: 0.0

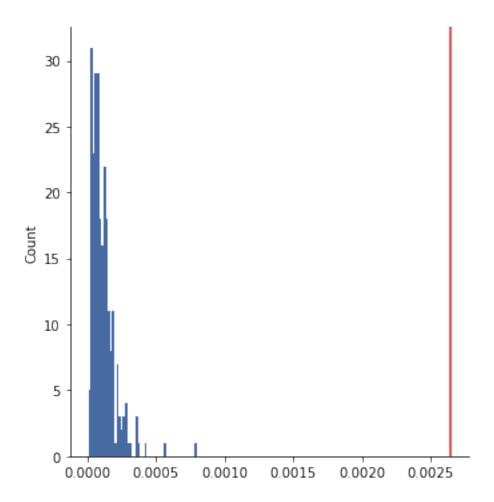
Image saved to images\_latentdim6\_8v3v8/diff\_anisotropic\_s2\_8v3



100%| | 250/250 [00:13<00:00, 17.97it/s]

bandwidth: 8
p\_value: 0.0

Image saved to images\_latentdim6\_8v3v8/diff\_anisotropic\_s3\_8v3



```
CPU times: user 2min 53s, sys: 6min 5s, total: 8min 58s Wall time: 1min 55s
```

# []: diff\_anisotropic\_stats

```
[]: {-3: {'p': 0.0, 'time': 16.131669998168945},

-2: {'p': 0.0, 'time': 19.450505018234253},

-1: {'p': 0.0, 'time': 16.172513961791992},

0: {'p': 0.0, 'time': 15.594956159591675},

1: {'p': 0.0, 'time': 15.692456007003784},

2: {'p': 0.0, 'time': 15.834607124328613},

3: {'p': 0.0, 'time': 15.415678024291992}}
```

# 0.2.3 Statistics for Encoder Kernel

```
[]: # encoder mmd
# suspect this is less robust to variance since this is not linear
encoder.eval()
```

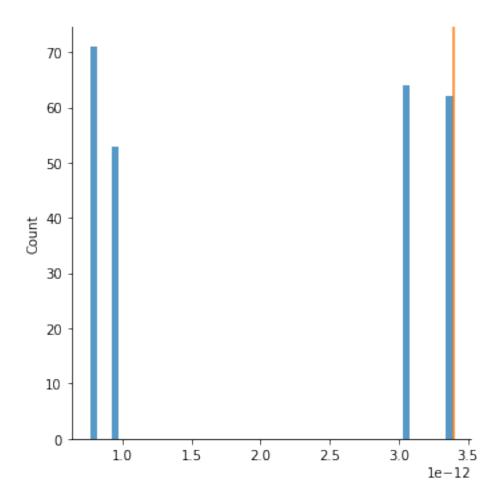
```
diff_encoder_stats = {}
for val in [-3, -2, -1, 0, 1, 2, 3]:
    start = time.time()
    observed_mmd_encoder8v3, simulated_mmds_encoder8v3 =__

¬permutation_test_encoder_mmd(
        x_batch, y_batch, r_batch, encoder, n_perms=n_perms, sigma_squared=(2_
 →** val)
    )
    end = time.time()
    p = p_val(simulated_mmds_encoder8v3, observed_mmd_encoder8v3)
    diff_encoder_stats[val] = {'p': p, 'time': end - start}
    hist_color = 'tab:blue'
    line_color = 'tab:orange'
    filepath = 'images_latentdim6_8v3v8/diff_encoder_s' + str(val) + '_8v3'
    print('bandwidth:', str(np.round(2 ** val, 3)))
    plot_permutation(
        simulated_mmds_encoder8v3,
        observed_mmd_encoder8v3,
        hist_color = hist_color,
        line_color = line_color,
        filepath = filepath
    )
    plt.show()
```

bandwidth: 0.125
p\_value: 0.0
Image saved to images\_latentdim6\_8v3v8/diff\_encoder\_s-3\_8v3

| 250/250 [00:26<00:00, 9.57it/s]

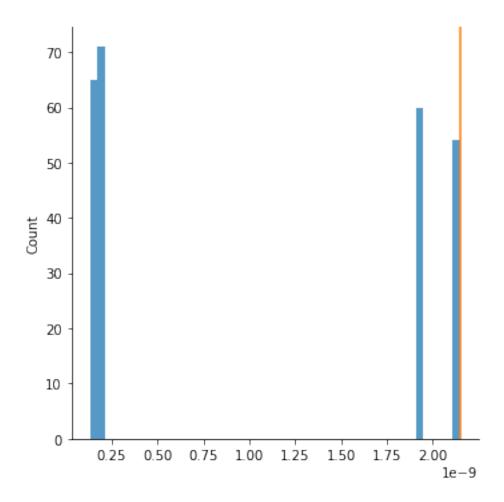
100%|



100%| | 250/250 [00:26<00:00, 9.61it/s]

bandwidth: 0.25
p\_value: 0.0

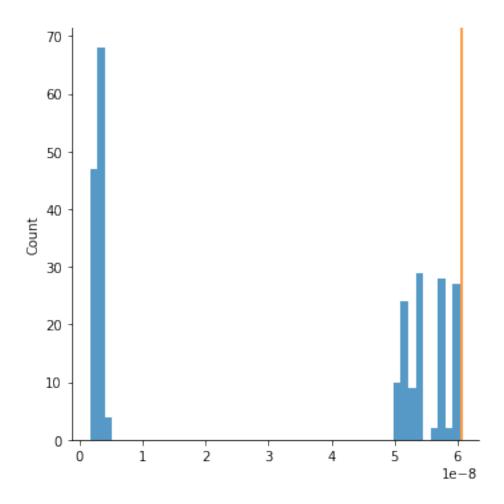
 $Image \ saved \ to \ images\_latentdim6\_8v3v8/diff\_encoder\_s-2\_8v3$ 



100%| | 250/250 [00:25<00:00, 9.87it/s]

bandwidth: 0.5
p\_value: 0.0

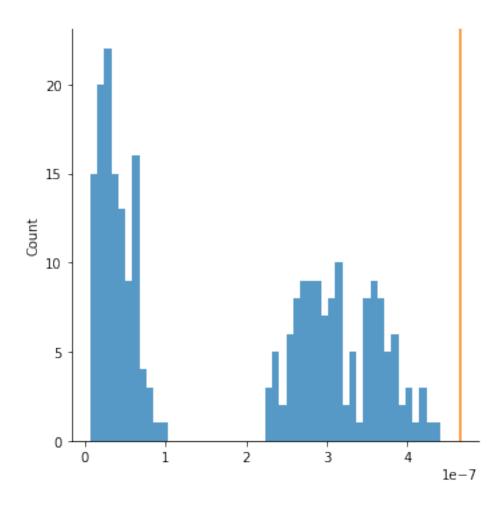
 $Image \ saved \ to \ images\_latentdim6\_8v3v8/diff\_encoder\_s-1\_8v3$ 



100%| | 250/250 [00:25<00:00, 9.96it/s]

bandwidth: 1
p\_value: 0.0

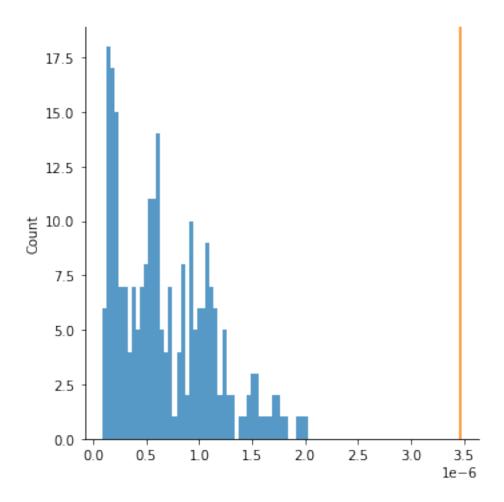
 ${\tt Image \ saved \ to \ images\_latentdim6\_8v3v8/diff\_encoder\_s0\_8v3}$ 



100%| | 250/250 [00:25<00:00, 9.96it/s]

bandwidth: 2
p\_value: 0.0

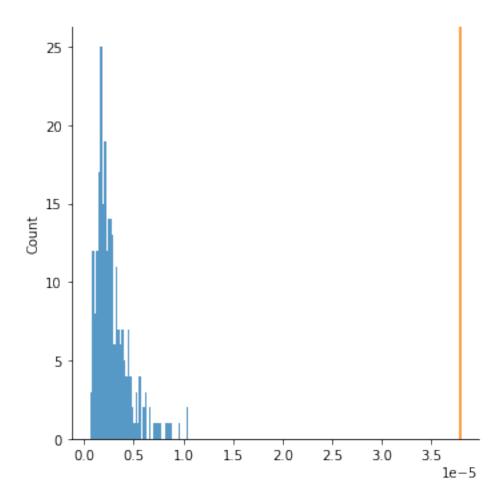
 ${\tt Image \ saved \ to \ images\_latentdim6\_8v3v8/diff\_encoder\_s1\_8v3}$ 



100%| | 250/250 [00:26<00:00, 9.57it/s]

bandwidth: 4
p\_value: 0.0

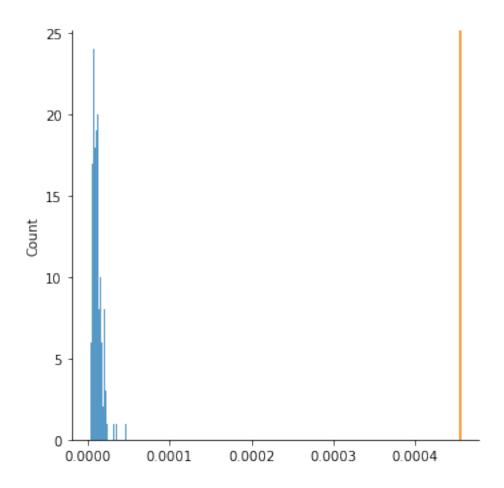
Image saved to images\_latentdim6\_8v3v8/diff\_encoder\_s2\_8v3



100%| | 250/250 [00:25<00:00, 9.71it/s]

bandwidth: 8
p\_value: 0.0

Image saved to images\_latentdim6\_8v3v8/diff\_encoder\_s3\_8v3

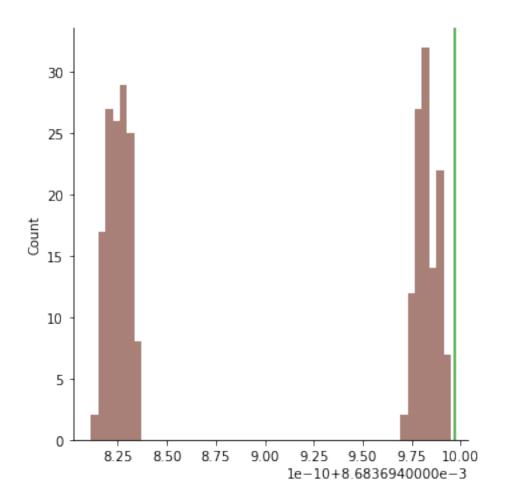


print('xy\_batch sample variance:', xy\_batch\_sample\_var)

```
num_features = 28 * 28
     print('number of features:', num_features)
     scale_sigma_squared = 1 / (xy_batch_sample_var * num_features)
     auto_sigma_squared = 1 / num_features
     print('bandwidth selected by sklearn\'s scale method:', scale_sigma_squared)
     print('bandwidth selected by sklearn\'s auto method:', auto_sigma_squared)
    xy_batch sample variance: 44.21445846557617
    number of features: 784
    bandwidth selected by sklearn's scale method: 2.8848260237648284e-05
    bandwidth selected by sklearn's auto method: 0.0012755102040816326
[]: |%%time
     # qaussian mmd
     diff_gaussian_stats = {}
     for val in [scale_sigma_squared, auto_sigma_squared, -3, -2, -1, 0, 1, 2, 3]:
         start = time.time()
         observed_mmd_gaussian8v3, simulated_mmds_gaussian8v3 =__

-permutation_test_gaussian_mmd(
             x_batch, y_batch, n_perms=n_perms, sigma_squared=(2 ** val)
         )
         end = time.time()
         p = p_val(simulated_mmds_gaussian8v3, observed_mmd_gaussian8v3)
         diff_gaussian_stats[val] = {'p': p, 'time': end - start}
         hist_color = 'tab:brown'
         line_color = 'tab:green'
         filepath = 'images_latentdim6_8v3v8/diff_gaussian_s' + str(val) + '_8v3'
         print('bandwidth:', str(np.round(2 ** val, 3)))
         plot_permutation(
             simulated_mmds_gaussian8v3,
             observed_mmd_gaussian8v3,
             hist_color = hist_color,
             line_color = line_color,
             filepath = filepath
         )
         plt.show()
    100%|
               | 250/250 [00:05<00:00, 48.38it/s]
    bandwidth: 1.0
    p value: 0.0
```

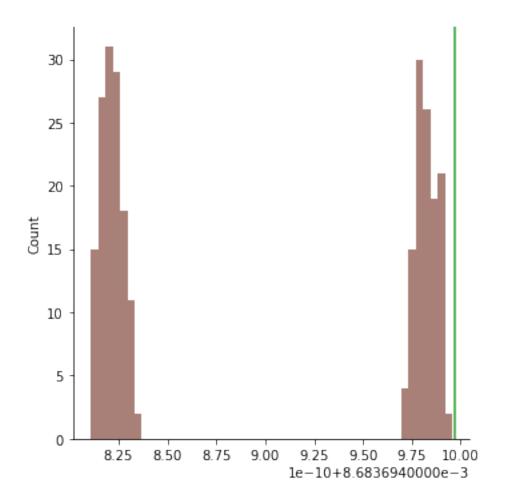
Image saved to images\_latentdim6\_8v3v8/diff\_gaussian\_s2.8848260237648284e-05\_8v3



100%| | 250/250 [00:03<00:00, 68.57it/s]

bandwidth: 1.001
p\_value: 0.0

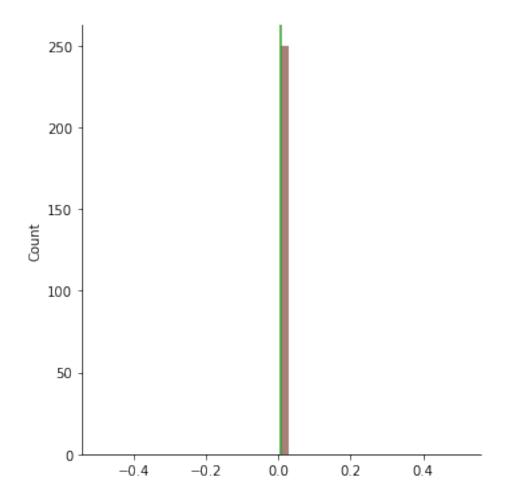
Image saved to images\_latentdim6\_8v3v8/diff\_gaussian\_s0.0012755102040816326\_8v3



100%| | 250/250 [00:03<00:00, 81.08it/s]

bandwidth: 0.125
p\_value: 0.0

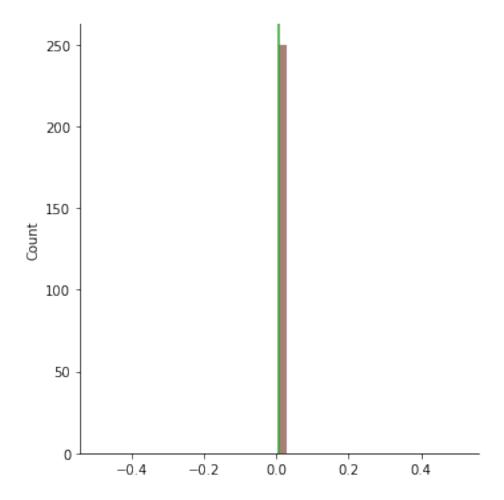
Image saved to images\_latentdim6\_8v3v8/diff\_gaussian\_s-3\_8v3



100%| | 250/250 [00:02<00:00, 89.47it/s]

bandwidth: 0.25
p\_value: 0.0

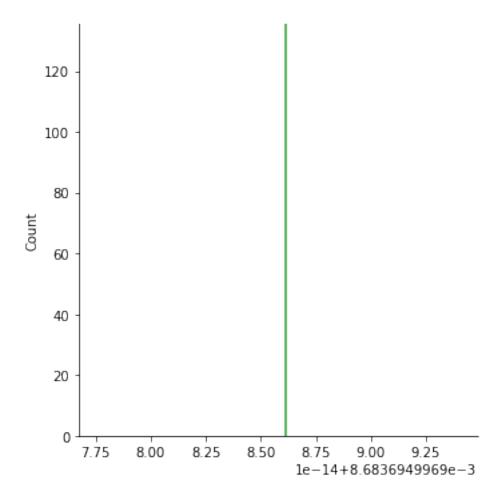
 $Image \ saved \ to \ images\_latentdim6\_8v3v8/diff\_gaussian\_s-2\_8v3$ 



100%| | 250/250 [00:03<00:00, 69.37it/s]

bandwidth: 0.5
p\_value: 0.0

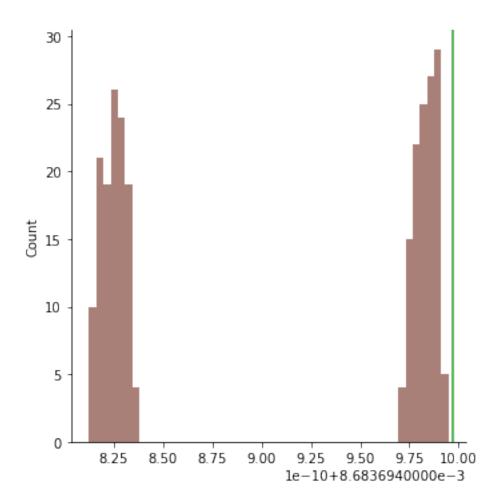
Image saved to images\_latentdim6\_8v3v8/diff\_gaussian\_s-1\_8v3



100%| | 250/250 [00:03<00:00, 79.94it/s]

bandwidth: 1
p\_value: 0.0

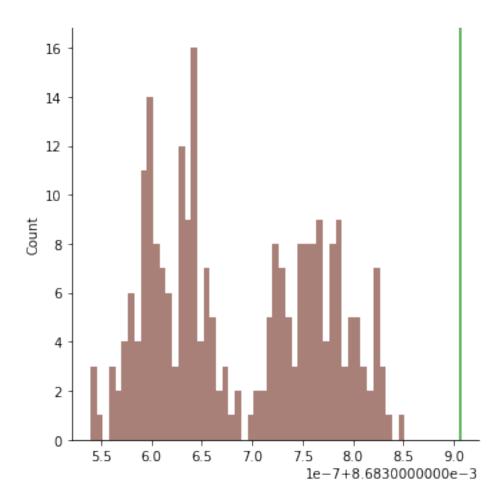
Image saved to images\_latentdim6\_8v3v8/diff\_gaussian\_s0\_8v3



100%| | 250/250 [00:02<00:00, 89.96it/s]

bandwidth: 2
p\_value: 0.0

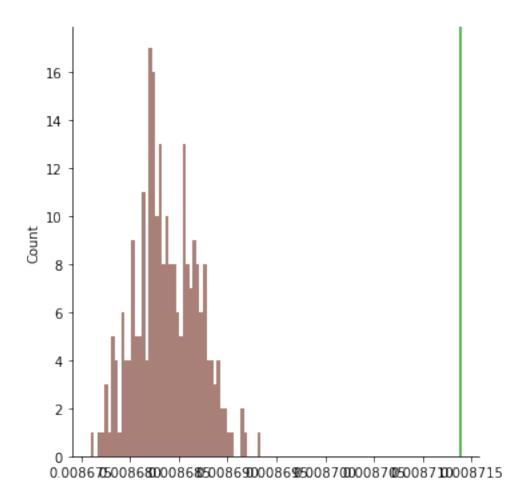
Image saved to images\_latentdim6\_8v3v8/diff\_gaussian\_s1\_8v3



100%| | 250/250 [00:03<00:00, 66.38it/s]

bandwidth: 4
p\_value: 0.0

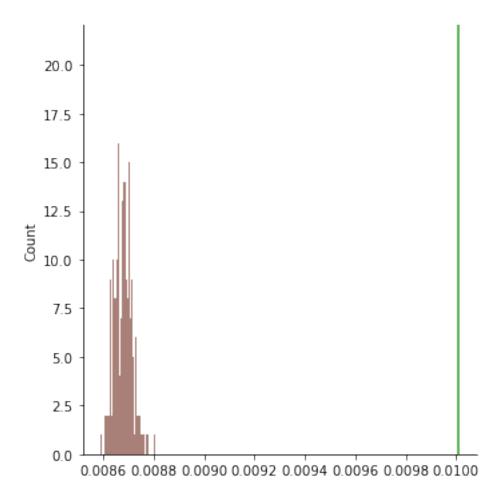
Image saved to images\_latentdim6\_8v3v8/diff\_gaussian\_s2\_8v3



100%| | 250/250 [00:03<00:00, 68.47it/s]

bandwidth: 8
p\_value: 0.0

Image saved to images\_latentdim6\_8v3v8/diff\_gaussian\_s3\_8v3



CPU times: user 2min 25s, sys: 59.6 s, total: 3min 25s Wall time: 33 s

# []: diff\_gaussian\_stats

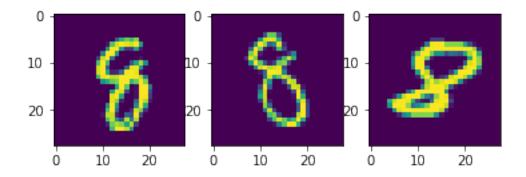
```
[]: {2.8848260237648284e-05: {'p': 0.0, 'time': 5.196414947509766}, 0.0012755102040816326: {'p': 0.0, 'time': 3.683488130569458}, -3: {'p': 0.0, 'time': 3.092616319656372}, -2: {'p': 0.0, 'time': 2.801586866378784}, -1: {'p': 0.0, 'time': 3.6118781566619873}, 0: {'p': 0.0, 'time': 3.1359469890594482}, 1: {'p': 0.0, 'time': 2.788443088531494}, 2: {'p': 0.0, 'time': 3.7773919105529785}, 3: {'p': 0.0, 'time': 3.6625001430511475}}
```

# 0.2.5 $X_1 = \{\text{images of 8's}\}, X_2 = \{\text{images of 8's}\}$

```
[]: # testing when x1_batch and x2_batch are from the same distribution
     x1 batch = []
     x2_batch = []
     r_batch = []
     for ind, data in enumerate(test_loader): # filter batches based on digit
         if ind < 3:
             r_batch.append(data[0][data[1] == 8])
         elif 3 \le ind and ind % 2 == 0:
             x1_batch.append(data[0][data[1] == 8])
         elif 3 <= ind and ind \% 2 == 1:
             x2_batch.append(data[0][data[1] == 8])
         if ind == 25: break
     x1_batch = torch.cat(x1_batch, dim=0)
     x2_batch = torch.cat(x2_batch, dim=0)
     r_batch = torch.cat(r_batch, dim=0)
     print('x1_batch shape:', x1_batch.shape)
     print('x2_batch shape:', x2_batch.shape)
     print('r_batch shape:', r_batch.shape)
    x1_batch shape: torch.Size([299, 1, 28, 28])
    x2_batch shape: torch.Size([289, 1, 28, 28])
    r_batch shape: torch.Size([67, 1, 28, 28])
[]: plt.subplot(1,3,1)
    plt.imshow(x1_batch[0].squeeze())
```

# []: plt.subplot(1,3,1) plt.imshow(x1\_batch[0].squeeze()) plt.subplot(1,3,2) plt.imshow(x2\_batch[0].squeeze()) plt.subplot(1,3,3) plt.imshow(r\_batch[0].squeeze())

# []: <matplotlib.image.AxesImage at 0x2a8476a30>

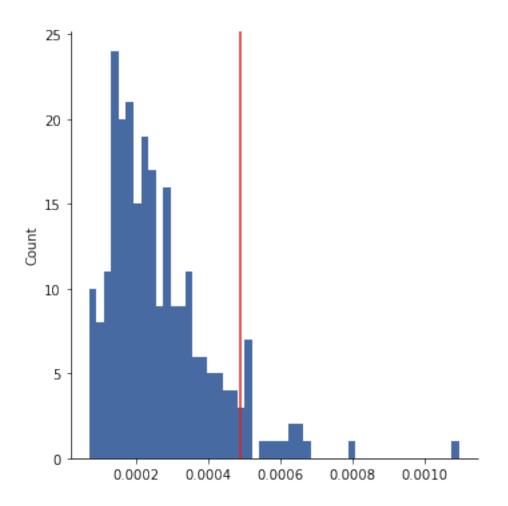


# 0.2.6 Statistics for Anisotropic Kernel

```
[]: |%%time
     # anisotropic mmd
     same_anisotropic_stats = {}
     for val in [-3, -2, -1, 0, 1, 2, 3]:
         start = time.time()
         observed_mmd_anisotropic8v8, simulated_mmds_anisotropic8v8 =__

¬permutation_test_anisotropic_mmd(
             x1_batch, x2_batch, r_batch, encoder, n_perms=n_perms, sigma_squared =_u
      →(2 ** val)
         )
         end = time.time()
         p = p_val(simulated_mmds_anisotropic8v8, observed_mmd_anisotropic8v8)
         same_anisotropic_stats[val] = {'p': p, 'time': end - time}
         hist_color = '#093885'
         line_color = 'tab:red'
         filepath = 'images_latentdim6_8v3v8/same_anisotropic_s' + str(val) + '_8v8'
         print('bandwidth:', str(np.round(2 ** val, 3)))
         plot_permutation(
             simulated_mmds_anisotropic8v8,
             observed_mmd_anisotropic8v8,
             hist_color = hist_color,
             line_color = line_color,
             filepath = filepath
         plt.show()
    100%|
              | 250/250 [00:18<00:00, 13.69it/s]
```

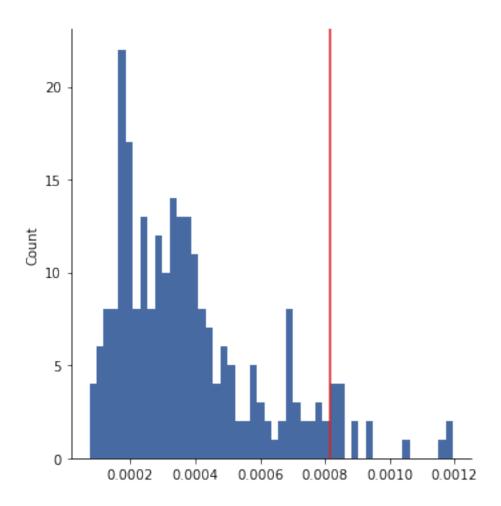
```
bandwidth: 0.125
p_value: 0.076
Image saved to images_latentdim6_8v3v8/same_anisotropic_s-3_8v8
```



100%| | 250/250 [00:19<00:00, 13.04it/s]

bandwidth: 0.25 p\_value: 0.064

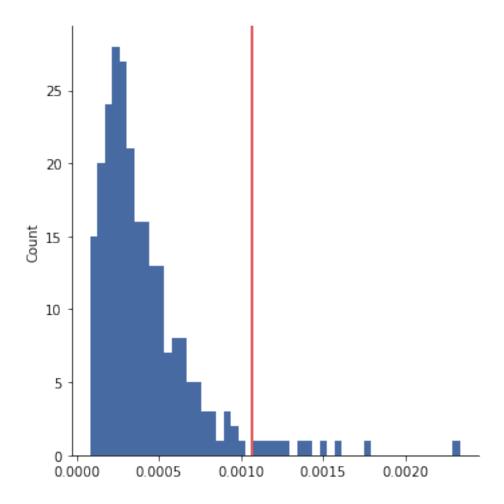
Image saved to images\_latentdim6\_8v3v8/same\_anisotropic\_s-2\_8v8



100%| | 250/250 [00:21<00:00, 11.49it/s]

bandwidth: 0.5
p\_value: 0.044

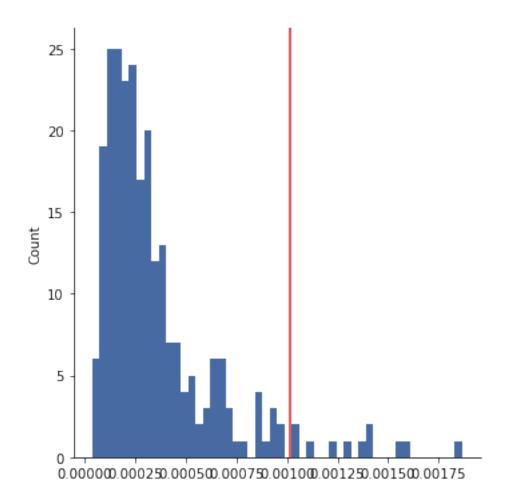
Image saved to images\_latentdim6\_8v3v8/same\_anisotropic\_s-1\_8v8



100%| | 250/250 [00:19<00:00, 13.14it/s]

bandwidth: 1
p\_value: 0.044

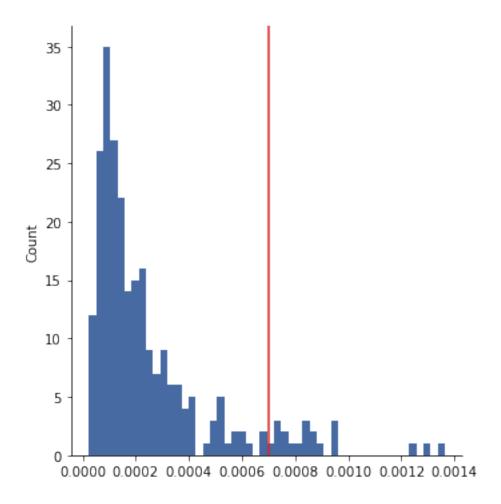
 ${\tt Image \ saved \ to \ images\_latentdim6\_8v3v8/same\_anisotropic\_s0\_8v8}$ 



100%| | 250/250 [00:17<00:00, 13.93it/s]

bandwidth: 2
p\_value: 0.08

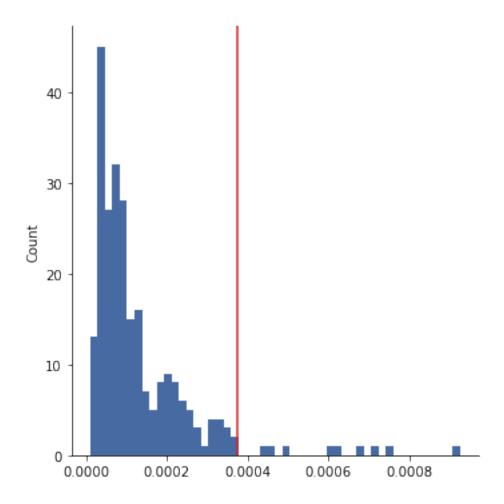
Image saved to images\_latentdim6\_8v3v8/same\_anisotropic\_s1\_8v8



100%| | 250/250 [00:19<00:00, 13.05it/s]

bandwidth: 4
p\_value: 0.04

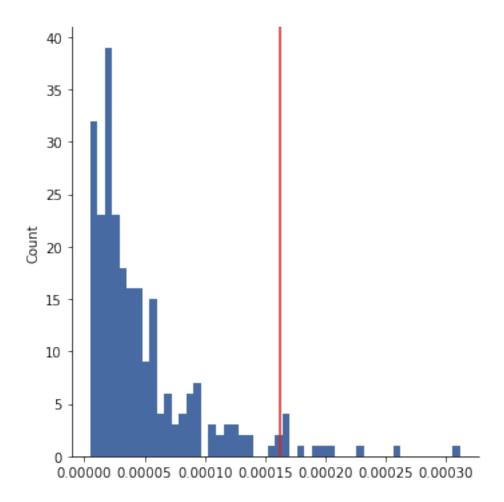
Image saved to images\_latentdim6\_8v3v8/same\_anisotropic\_s2\_8v8



100%| | 250/250 [00:19<00:00, 12.84it/s]

bandwidth: 8
p\_value: 0.048

Image saved to images\_latentdim6\_8v3v8/same\_anisotropic\_s3\_8v8



```
CPU times: user 4min 3s, sys: 8min 16s, total: 12min 19s Wall time: 2min 32s
```

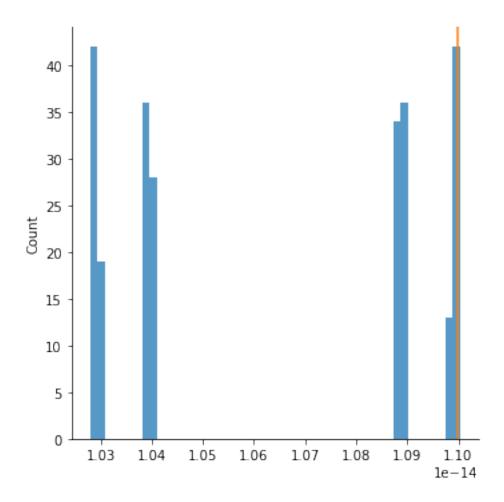
```
[]: same_anisotropic_stats
```

## 0.2.7 Statistics for Encoder Kernel

```
[]: %%time
encoder.eval()
same_encoder_stats = {}
```

```
for val in [-3, -2, -1, 0, 1, 2, 3]:
    start = time.time()
    observed_mmd_encoder8v8, simulated_mmds_encoder8v8 =__
 →permutation_test_encoder_mmd(
        x1_batch, x2_batch, r_batch, encoder, n_perms=n_perms, sigma_squared=(2_
 →** val)
    end = time.time()
    p = p_val(simulated_mmds_encoder8v8, observed_mmd_encoder8v8)
    same_encoder_stats[val] = {'p': p, 'time': end - start}
    hist_color = 'tab:blue'
    line_color = 'tab:orange'
    filepath = 'images_latentdim6_8v3v8/same_encoder_s' + str(val) + '_8v8'
    print('bandwidth:', str(np.round(2 ** val, 3)))
    plot_permutation(
        simulated_mmds_encoder8v8,
        observed_mmd_encoder8v8,
        hist_color = hist_color,
        line_color = line_color,
        filepath = filepath
    )
    plt.show()
```

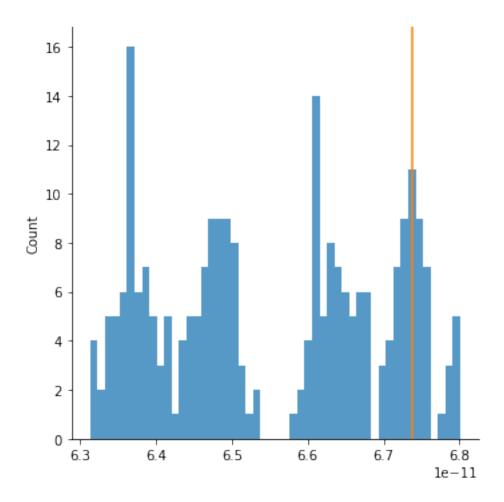
100%| | 250/250 [00:34<00:00, 7.23it/s]
bandwidth: 0.125
p\_value: 0.1
Image saved to images\_latentdim6\_8v3v8/same\_encoder\_s-3\_8v8



100%| | 250/250 [00:34<00:00, 7.28it/s]

bandwidth: 0.25 p\_value: 0.124

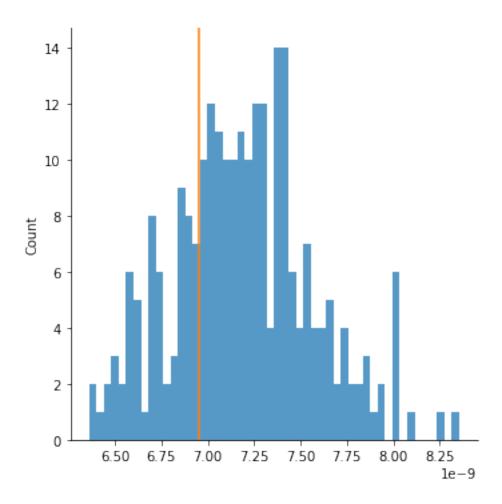
Image saved to images\_latentdim6\_8v3v8/same\_encoder\_s-2\_8v8



100%| | 250/250 [00:36<00:00, 6.91it/s]

bandwidth: 0.5
p\_value: 0.744

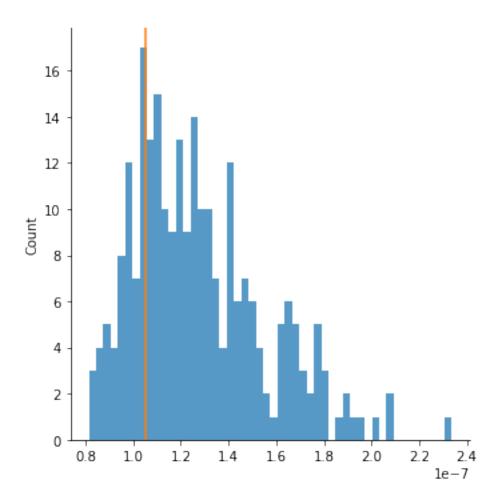
Image saved to images\_latentdim6\_8v3v8/same\_encoder\_s-1\_8v8



100%| | 250/250 [00:32<00:00, 7.58it/s]

bandwidth: 1
p\_value: 0.768

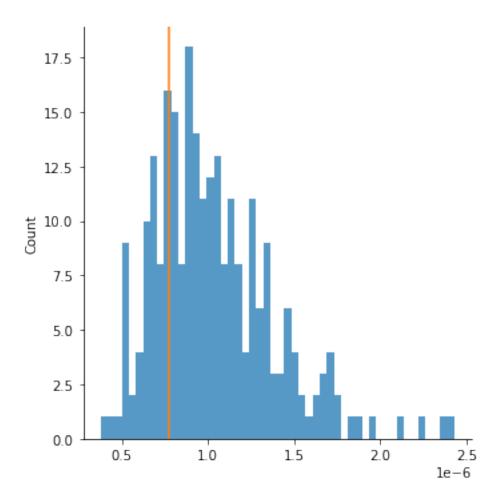
Image saved to images\_latentdim6\_8v3v8/same\_encoder\_s0\_8v8



100%| | 250/250 [00:32<00:00, 7.62it/s]

bandwidth: 2
p\_value: 0.76

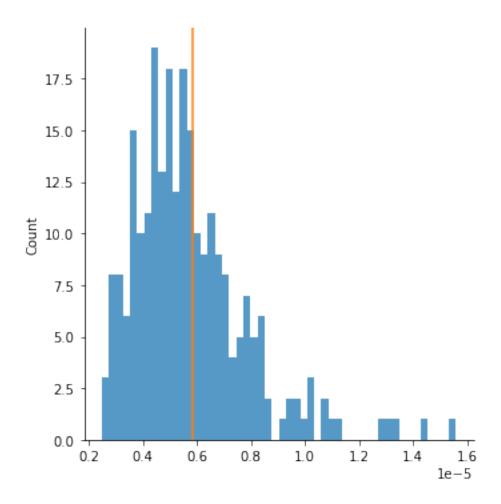
Image saved to images\_latentdim6\_8v3v8/same\_encoder\_s1\_8v8



100%| | 250/250 [00:33<00:00, 7.36it/s]

bandwidth: 4
p\_value: 0.376

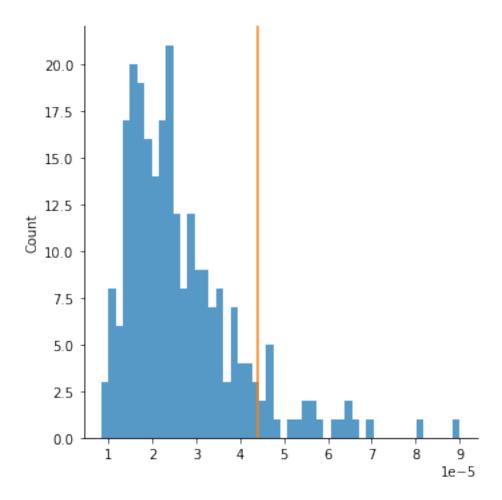
Image saved to images\_latentdim6\_8v3v8/same\_encoder\_s2\_8v8



100%| | 250/250 [00:34<00:00, 7.22it/s]

bandwidth: 8
p\_value: 0.092

Image saved to images\_latentdim6\_8v3v8/same\_encoder\_s3\_8v8



```
CPU times: user 5min 25s, sys: 14min 27s, total: 19min 52s Wall time: 4min 28s
```

## []: same\_encoder\_stats

```
[]: {-3: {'p': 0.1, 'time': 34.74849581718445},

-2: {'p': 0.124, 'time': 39.05711221694946},

-1: {'p': 0.744, 'time': 40.66038990020752},

0: {'p': 0.768, 'time': 37.54035305976868},

1: {'p': 0.76, 'time': 37.13159108161926},

2: {'p': 0.376, 'time': 38.414994955062866},

3: {'p': 0.092, 'time': 39.6552369594574}}
```

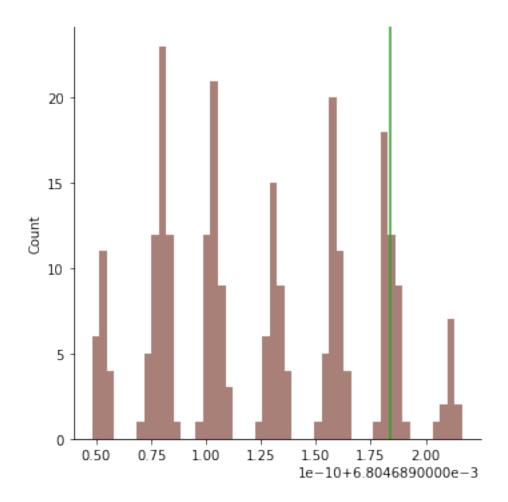
## 0.2.8 Statistics for Gaussian Kernel

```
[]: # gaussian mmd

same_gaussian_stats = {}
for val in [scale_sigma_squared, auto_sigma_squared, -3, -2, -1, 0, 1, 2, 3]:
```

```
start = time.time()
  observed_mmd_gaussian8v8, simulated_mmds_gaussian8v8 =__
→permutation_test_gaussian_mmd(
      x1_batch, x2_batch, n_perms=n_perms, sigma_squared=(2 ** val)
  )
  end = time.time()
  p = p_val(simulated_mmds_gaussian8v8, observed_mmd_gaussian8v8)
  same_gaussian_stats[val] = {'p': p, 'time': end - start}
  hist_color = 'tab:brown'
  line_color = 'tab:green'
  filepath = 'images_latentdim6_8v3v8/same_gaussian_s' + str(val) + '_8v8'
  print('bandwidth:', str(np.round(2 ** val, 3)))
  plot_permutation(
      simulated_mmds_gaussian8v8,
      observed_mmd_gaussian8v8,
      hist_color = hist_color,
      line_color = line_color,
      filepath = filepath
  plt.show()
```

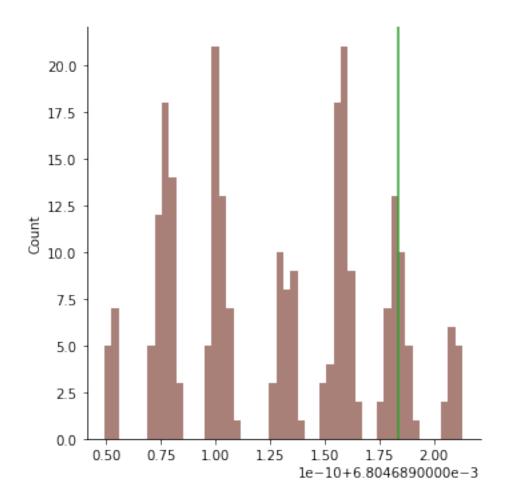
100%| | 250/250 [00:04<00:00, 56.00it/s]
bandwidth: 1.0
p\_value: 0.116
Image saved to images\_latentdim6\_8v3v8/same\_gaussian\_s2.8848260237648284e-05\_8v8



100%| | 250/250 [00:04<00:00, 54.35it/s]

bandwidth: 1.001
p\_value: 0.116

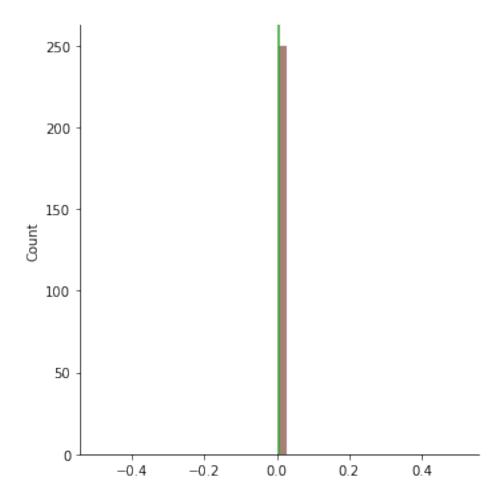
Image saved to images\_latentdim6\_8v3v8/same\_gaussian\_s0.0012755102040816326\_8v8



100%| | 250/250 [00:05<00:00, 49.20it/s]

bandwidth: 0.125
p\_value: 0.0

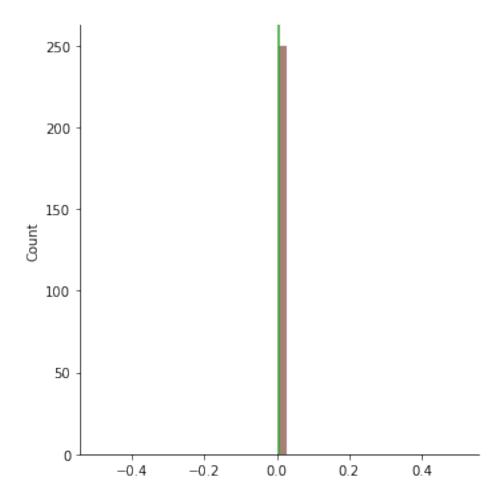
Image saved to images\_latentdim6\_8v3v8/same\_gaussian\_s-3\_8v8



100%| | 250/250 [00:03<00:00, 63.44it/s]

bandwidth: 0.25
p\_value: 0.0

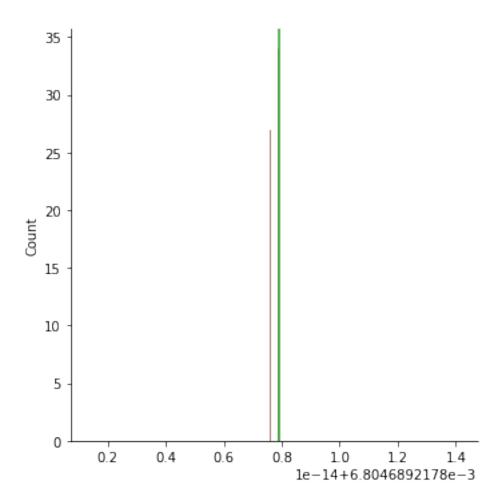
 ${\tt Image \ saved \ to \ images\_latentdim6\_8v3v8/same\_gaussian\_s-2\_8v8}$ 



100%| | 250/250 [00:03<00:00, 68.13it/s]

bandwidth: 0.5
p\_value: 0.188

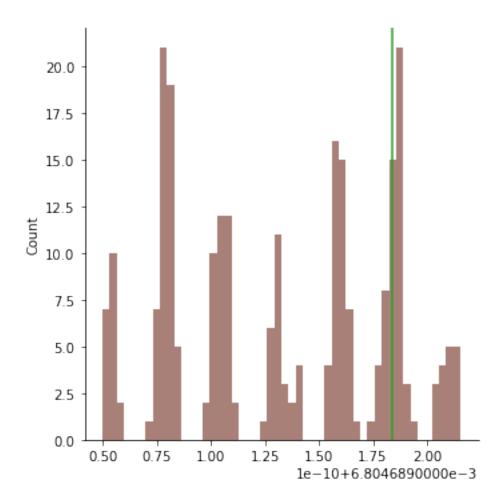
 ${\tt Image \ saved \ to \ images\_latentdim6\_8v3v8/same\_gaussian\_s-1\_8v8}$ 



100%| | 250/250 [00:03<00:00, 64.51it/s]

bandwidth: 1
p\_value: 0.188

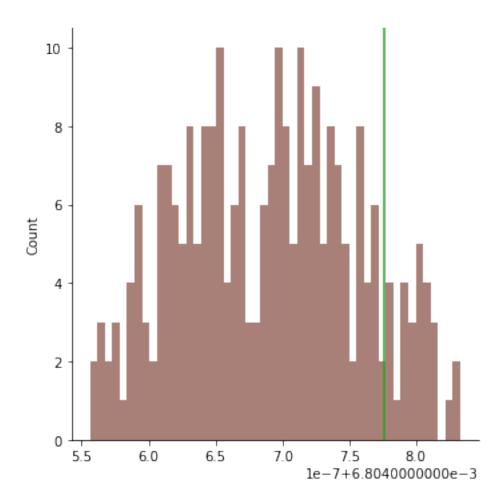
Image saved to images\_latentdim6\_8v3v8/same\_gaussian\_s0\_8v8



100%| | 250/250 [00:03<00:00, 80.19it/s]

bandwidth: 2
p\_value: 0.112

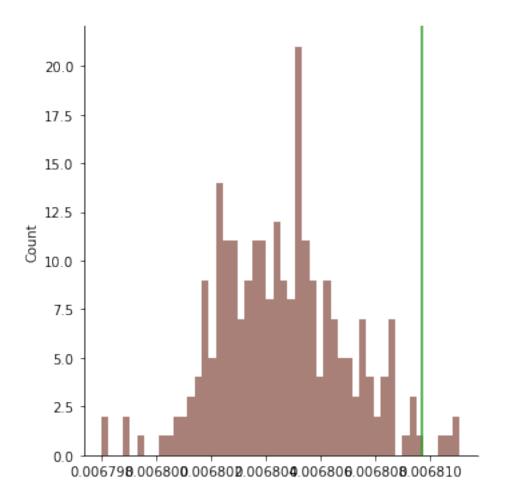
Image saved to images\_latentdim6\_8v3v8/same\_gaussian\_s1\_8v8



100%| | 250/250 [00:03<00:00, 64.11it/s]

bandwidth: 4
p\_value: 0.016

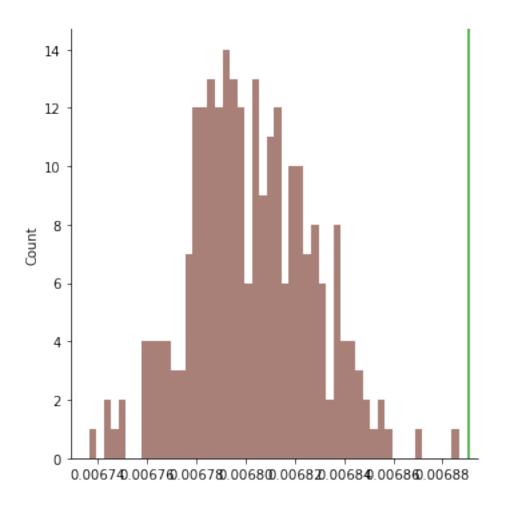
Image saved to images\_latentdim6\_8v3v8/same\_gaussian\_s2\_8v8



100%| | 250/250 [00:03<00:00, 67.34it/s]

bandwidth: 8
p\_value: 0.0

Image saved to images\_latentdim6\_8v3v8/same\_gaussian\_s3\_8v8



## []: same\_gaussian\_stats

```
[]: {2.8848260237648284e-05: {'p': 0.116, 'time': 4.476850986480713}, 0.0012755102040816326: {'p': 0.116, 'time': 4.629081726074219}, -3: {'p': 0.0, 'time': 5.093536138534546}, -2: {'p': 0.0, 'time': 3.9547619819641113}, -1: {'p': 0.188, 'time': 3.6981630325317383}, 0: {'p': 0.188, 'time': 3.8854548931121826}, 1: {'p': 0.112, 'time': 3.12951397895813}, 2: {'p': 0.016, 'time': 3.914917230606079}, 3: {'p': 0.0, 'time': 3.726551055908203}}
```