$\{(x_i, y_i)\}_{i=1}^{n}$

$f(x) = x\beta$ where $f(x_i) \approx y_i$

replace w/ $\phi(x)$

$f(x) = \phi(x)\tilde{\beta} = \langle \tilde{\beta}, \phi(x) \rangle$

$\tilde{\beta} \in \mathbb{R}^m$, $\phi(x) \in \mathbb{R}^m$

$$= \langle W, Z(x) \rangle$$

$$\uparrow \quad \uparrow$$

$$\mathbb{R}^{n_1 \times n_2 \times n_3 \cdots \times n_d}$$

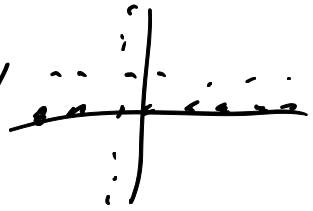
For Each x, $Z(x)$ is pure state

product state

$$Z(x) = z_1(x) \otimes z_2 \otimes \cdots \otimes z_d = \underbrace{[\,]}_{\text{rank-1}}$$

low-rank

$X \in \mathbb{R}^2$



Ex

$$K(x,y) = \langle z(x), z(y) \rangle = \left\langle \bigotimes_{i=1}^{d} z_i(x), \bigotimes_{i=1}^{d} z_i(y) \right\rangle$$

$$= \prod_{j=1}^{d} \langle z_j(x), z_j(y) \rangle$$

$z_j(x) \in \mathbb{R}^{n_j}$

particular to grid of RFF + fast that

$$z_j(x) = \left[e^{i w_{kj} x_j} \right]_{k=1}^{n_j}$$

$$z(x) = \left[e^{i \langle w, x \rangle} \right] = \prod_{k=1}^{d} e^{i w_k x_k}$$

Extension $f(x) = \langle \mathcal{W}, z(x) \rangle$ $\leftarrow (n_1 \cdots n_d)^2$

$\mathcal{W}$: rank r

$\mathcal{W} = \sum_{k=1}^{r} \mathcal{W}_k$ $\leftarrow$ rank 1

$$\langle \mathcal{W}, z(x) \rangle = \sum_{k=1}^{r} \langle \mathcal{W}_k, z(x) \rangle$$

$r d n_i$

$$= \sum_{k=1}^{r} \prod_{j=1}^{d} \langle w_{k_j}, z_j(x) \rangle$$

(r, d, n_j)

1) $x \sim \mathrm{Unif}([0,1]^2)$

2) Def $f(x) = \sin(x_1 x_2)$ or $= (x_1 + x_2)^2$

$\longrightarrow$ $y_i = f(x_i) + \varepsilon_i$ ε noise

3) Sample $w_{k_1} \sim \mathcal{N}(0,1)$ $k_1 = 1, \dots m$

$w_{k_2} \sim \mathcal{N}(0,1)$ $k_2 = 1, \dots m$

$w_{k_1, \dots k_d}$

4) $z(x_j) = \begin{bmatrix} e^{i w_{k_1 j} x_1} & e^{i w_{k_2 j} x_2} \\ 1 & 1 \end{bmatrix} \in \mathbb{R}^{m \times 2}$

efficient

$$\text{inff} = \left[- \; e^{i w_{k_1 j} x_1} \; e^{i w_{k_2 \ell} x_2} \; - \right]_{j,\ell} \in \mathbb{R}^{m \times m}$$

5) $y_i = \langle v, z(x_i) \rangle$

6) What is $\text{rank}(v)$?

MSE $(f(x_i) - \langle w, z(x_i) \rangle)$?

7) Sanity check: choose f_{sep}, is v rank-1 ?