Digital Image Processing

CHAP. 02

Digital Image Fundamentals

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Preview

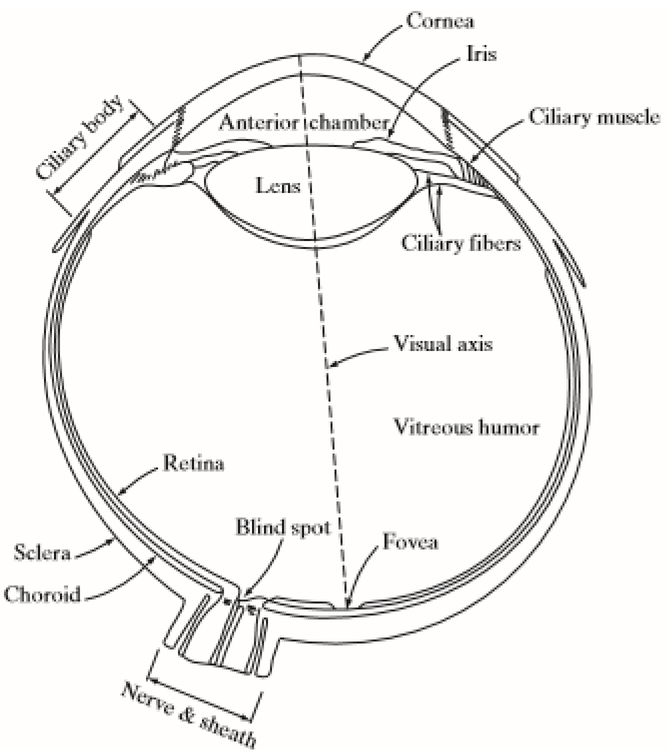
- We'll talk about
  - Several concepts related to digital images
  - Some of the notations
  - Summary if the mechanics of the HVS
  - Imaging sensors
  - Sampling and Quantization
  - Some basic relationships between pixels
  - Conditions for linear operations

Structure of the Human eye (1)

- Cross section of the human eye
  - Shape
    - Nearly a sphere 구
    - Diameter: about 20 mm

세포막

  - Three membranes enclosing the eye
    - Cornea (각막) and sclera (공막)
      - 안구를 보호
    - Choroid (맥락막)
      - 영양을 공급하고 과도한 빛을 감소
    - Retina (망막)
      - 상이 맺히며, light receptor가 분포

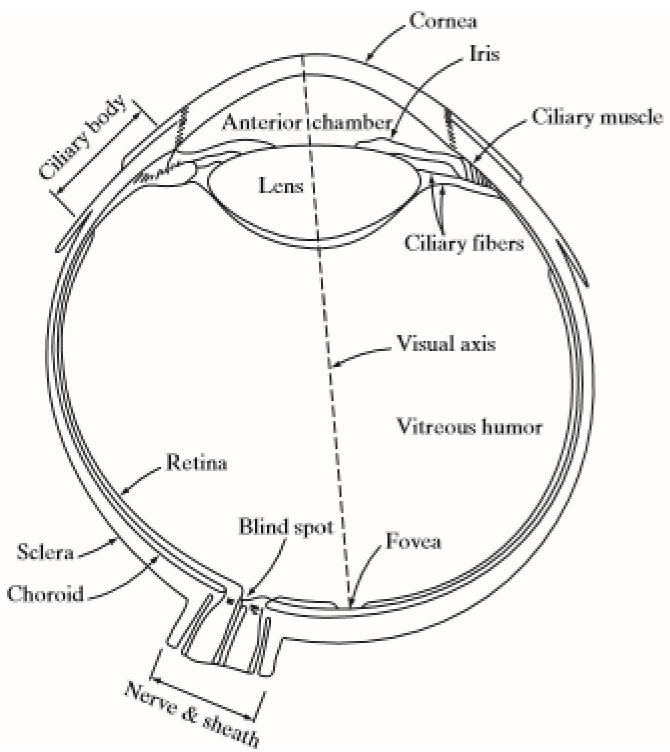


Structure of the Human eye (2)

- Outer cover
  - Cornea(각막): tough, transparent tissue that covers the anterior part
    투명
  - Sclera(공막): opaque membrane that encloses the optical globe
    불투명
- Choroid (맥락막)
  - 안구에 영양을 공급하는 혈관을 포함
  - 과도한 빛을 감소시키는 색소를 포함
  - Two parts at its anterior extreme

    1. Ciliary body (섬모체, 모양체)
       - 렌즈의 두께를 조절하여 초점을 맞춤

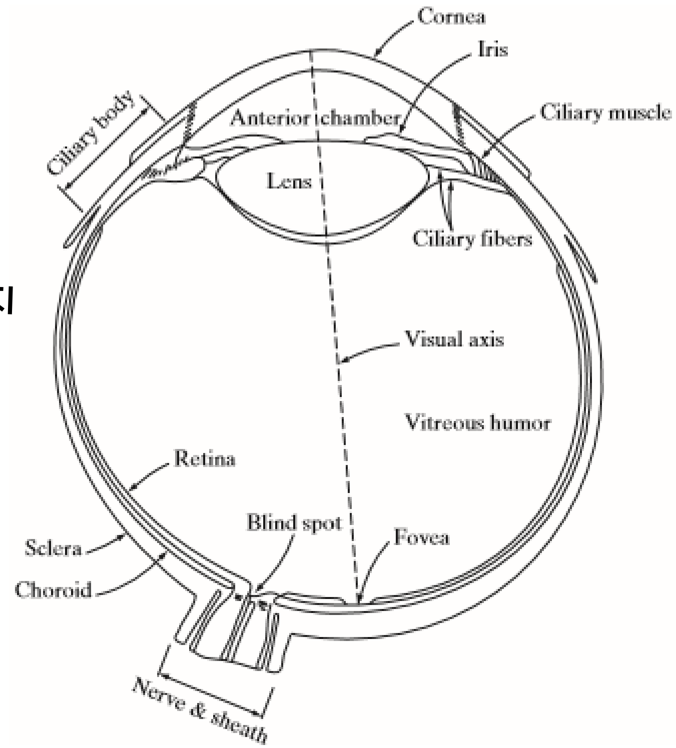
    2. Iris (홍채)
       - 얇은 막으로 막 중앙에 동공(pupil)
       - 동공을 축소, 확대하여 빛의 양을 조절



Structure of the Human eye (3)

- Lens (수정체)
  - Concentric layers of fibrous cell로 구성, 가시대역의 8%를 흡수
  - 60~70%의 수분, 6% 의 지방
- Retina (망막)

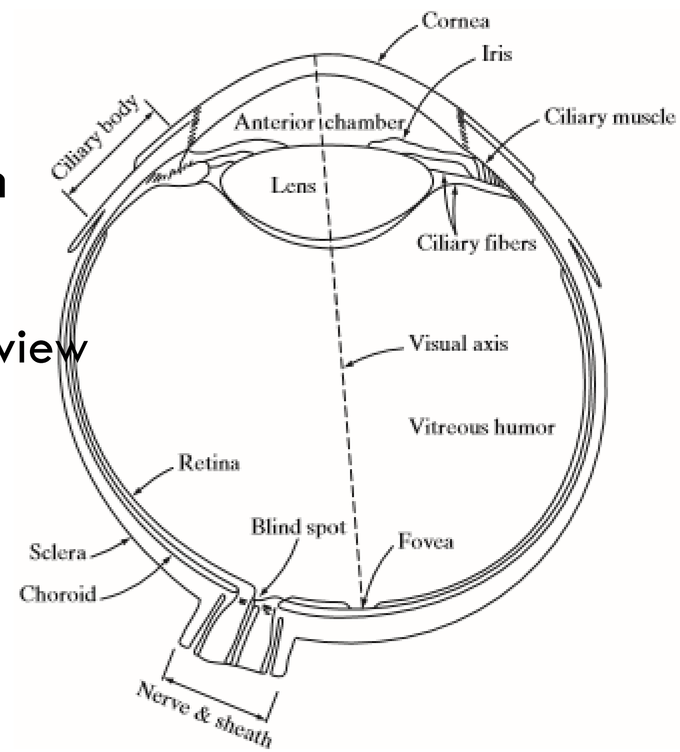
  수광체
  - 상이 맺히는 곳, light receptor가 분포
  - 신경세포와 연결되어 뇌에서 사물을 인지



Structure of the Human eye (4)

- Retina (망막)
  - Two kinds of light receptor

1. Cones (원추세포, 추상체) → 밝을때 동작
   - Highly sensitive to color
   - 6~7 백만개 있으며 fovea 근처에 집중
   - Photopic vision or bright-light vision
2. Rods (간상세포, 간상체) → 어두울 때 동작
   - Gives a overall picture of the field of view
   - 75~150 백만개가 망막 전역에 분포
   - Scotopic vision of dim-light vision



Structure of the Human eye (5)

- Blind spot (맹점)
  - 신경다발의 통로로 상이 맺히지 않는 지점
- Fovea (세포 밀집지역)
  - 지름 1.5mm 정도의 원형구역으로 150,000 elements/mm^{2}의 밀도
  - 337,000개의 cone이 집중적으로 분포

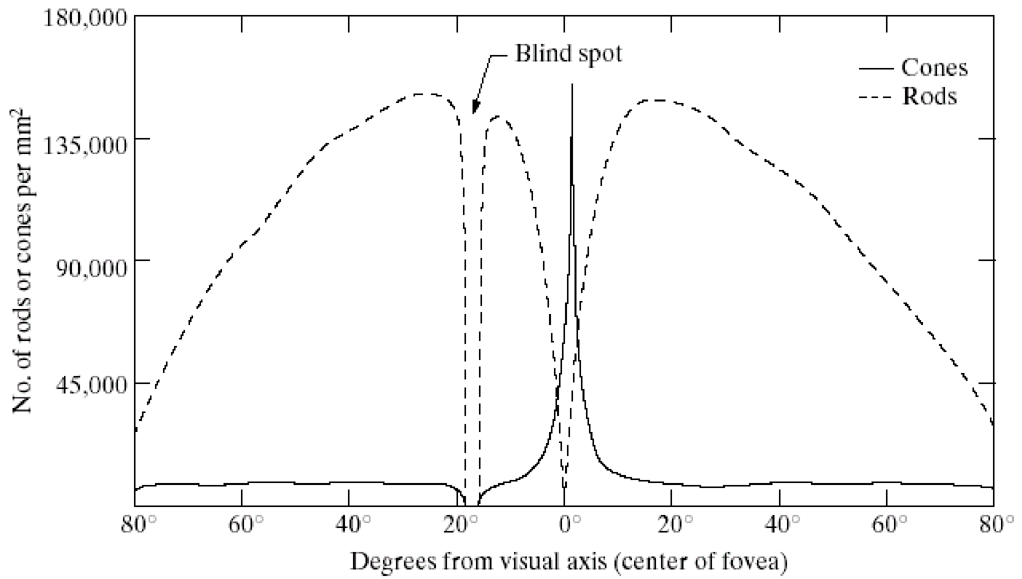
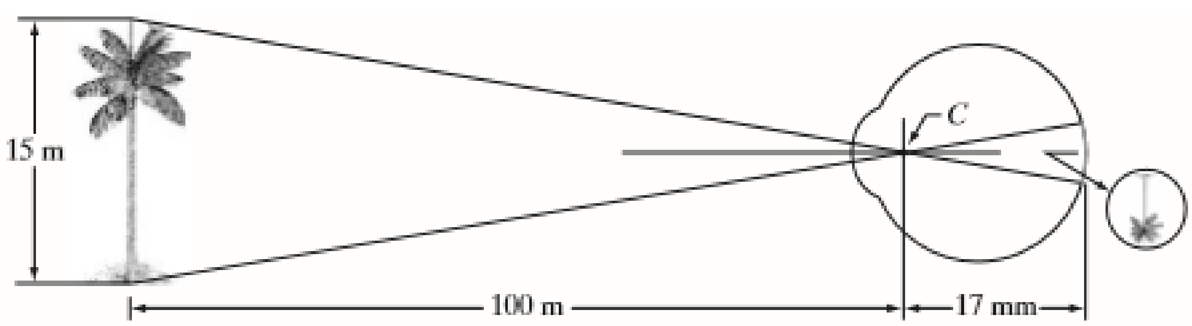


FIGURE 2.2 Distribution of rods and cones in the retina.

Image formation in the eye

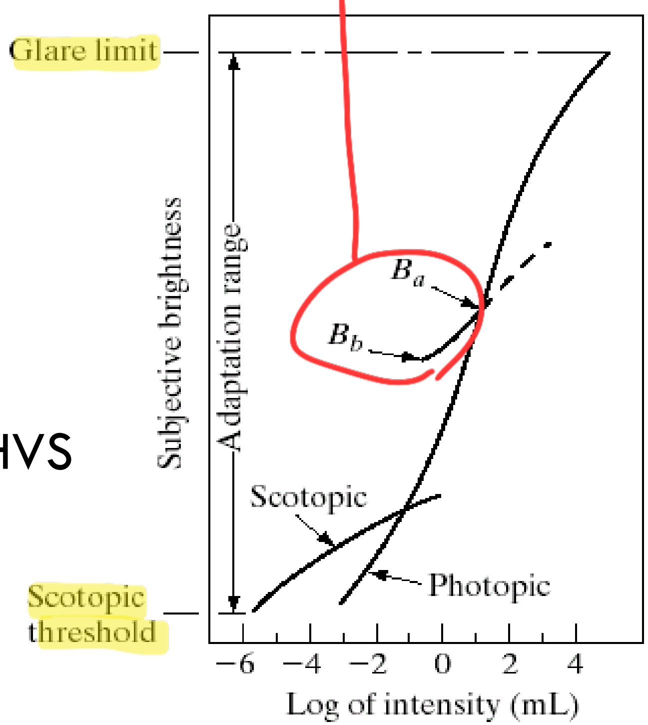
- Lens of eye
  - More flexible than optical lens
    - Ciliary fiber의 tension을 조절하여 lens의 두께를 조절
  - Focal length: 14 mm ~ 17 mm
  - 망막에는 역상이 맺힘



Discrimination of intensity levels

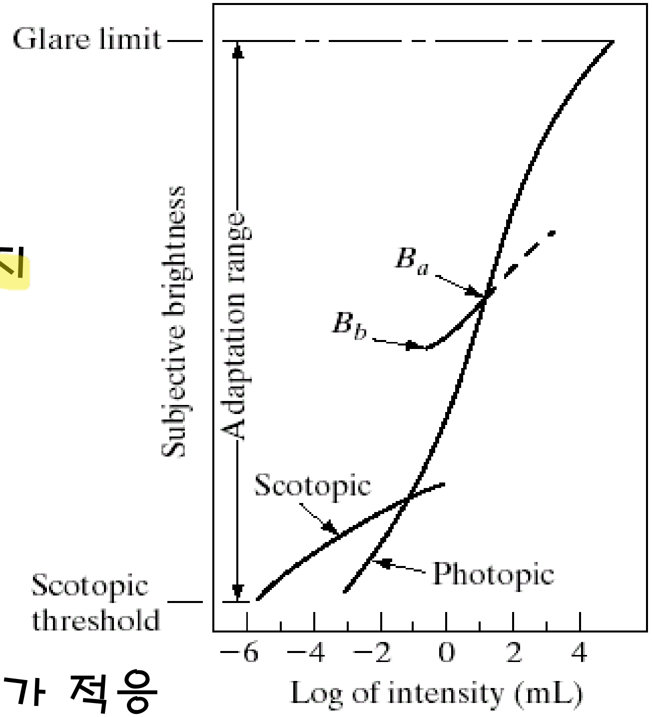
- Range
  - Order of 10^{10}
  - From scotopic threshold to glare limit
- Subjective brightness
  - Intensity perceived by HVS
  - Logarithmic function of the light intensity
    → ex) 빛 2배 ⇑ → 사람 1.4배 ⇑
- Long solid curve
  - Represents the range of adaptation of HVS
- Gradual transition
  - From scotopic to photopic vision

이부분 만 한번에 인식



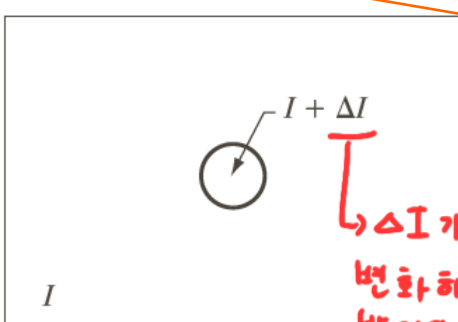
Brightness adaptation

- HVS는 전 영역을 동시에 구별할 수 없다
  - 상대적으로 좁은 영역에서 intensity level을 동시에 구별
- Brightness adaptation
  - B_b: brightness adaptation level
  - B_b를 변화시켜가며 적응하는 과정
  - Range: B_b-B_a, B_b이하는 검은색으로 인지
- Three kinds of adaptation
  - Dark adaptation (암적응)
  - Light adaptation (명적응)
  - Chromatic adaptation (색적응)
    - white balance를 맞추기 위하여 cones가 적응

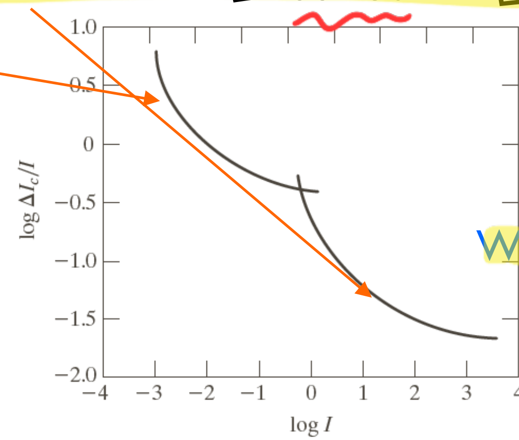


Brightness discrimination

- 사람이 구별할 수 있는 intensity의 변화를 측정
- Web ratio: $\frac{\Delta Ic}{I}$
  - I_c : increment of illumination discriminable 50% of the time
- Discrimination
  - Poor at low illumination
  - Improves significantly as background illumination increases
  - 낮은 조도에서는 rods가 높은 조도에서는 cones가 활성화



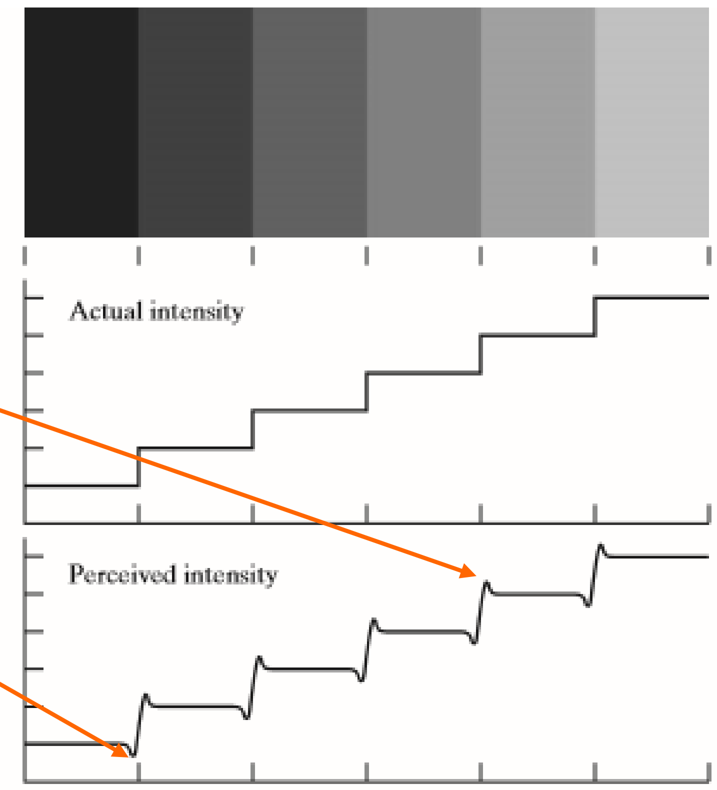
ㄴ ΔI가 얼마나 변화해야지 밝기가 바뀐걸 느꼈나?



Web ratio in log scale

Mach bands

- An example
  - Showing that perceived brightness is not a simple function of intensity

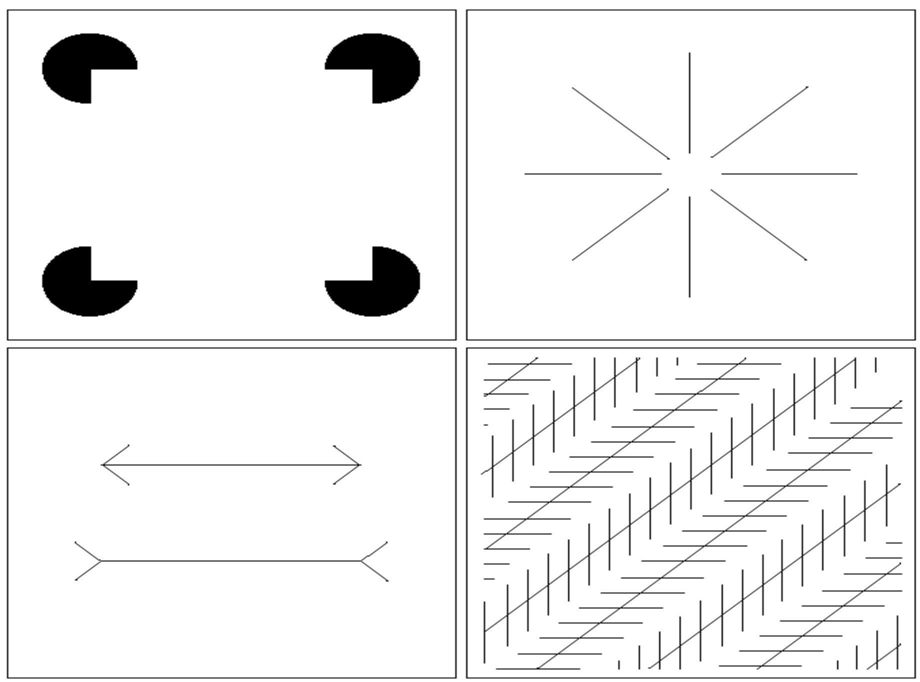


- HVS
  - Tends to undershoot or overshoot around the boundary of regions of different intensities

사람은 색의 경계선 부분에서 강조

Optical illusions

- Optical illusions (착시) ~ 과학적으로 증명은 안됨
  - are a characteristic of the HVS that is not fully understood

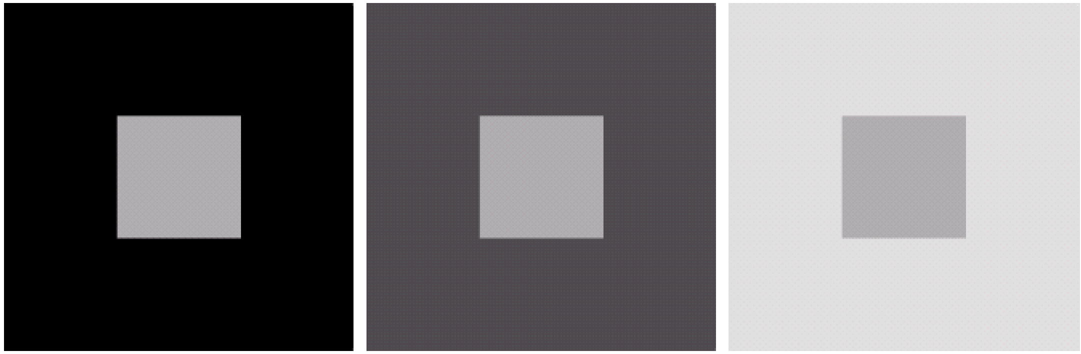


More on optical illusions

- Optical illusions

Simultaneous contrast

- Another example
  - Showing that perceived brightness does not depend simply on its intensity
  - 내부 사각형의 intensity는 모두 같지만, 배경이 밝아지면 내부 사각형이 어두워져 보인다.

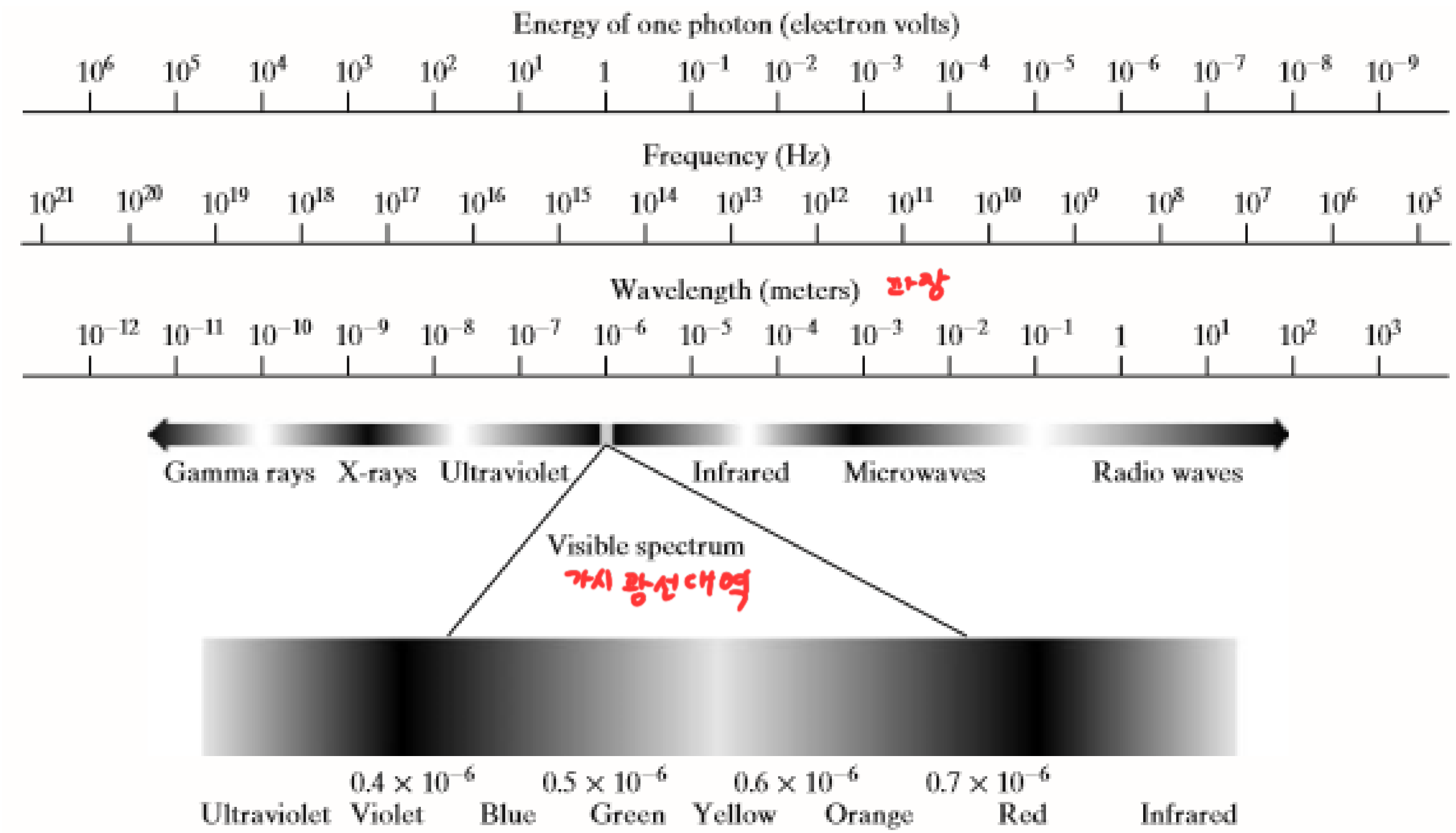


a b c

FIGURE 2.8 Examples of simultaneous contrast. All the inner squares have the same intensity, but they appear progressively darker as the background becomes lighter.

Light and the EM spectrum

- The EM spectrum



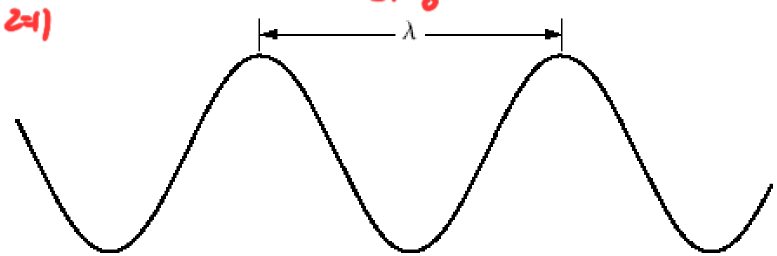
EM waves

- EM waves
  - Can be visualized as propagating waves with wavelength λ
  - Or, can be thought of as a stream of massless particles each traveling in a wavelike pattern and moving at the second of light
- Wavelength
  - $\lambda = \dfrac{c}{\nu}$, ν: frequency, c: speed of light, 2.998×10^8 m/s
- Energy
  - $E = h\nu$, h: Planck's constant, proportional to frequency

frequency에 비례

FIGURE 2.11 Graphical representation of one wavelength.

파장

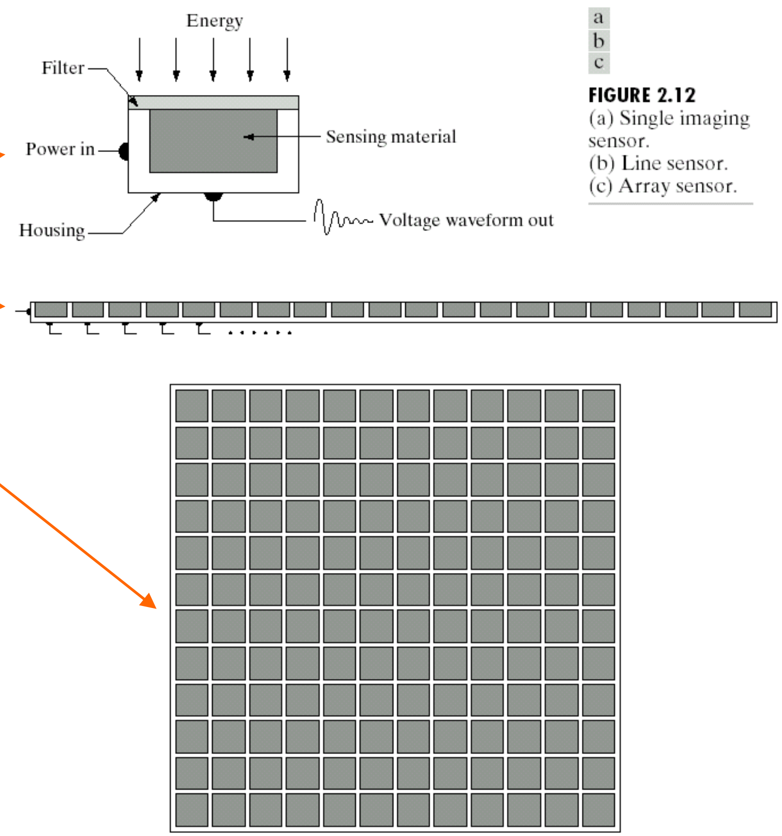


Light

- Light
  - A particular type of EM radiation that can be seen and sensed by the human eye
  - Range: from 0.43 µm (violet) to 0.79 µm (red)
  - Colors: violet, blue, green, yellow, orange, red 6 band = rainbow
  - Quantity to qualify color
    - Radiance: total amount of energy flowed from the light source measured in watts (W) light source로부터 방출되는 에너지
    - Luminance: a measure of the amount of perceived energy measured in lumens (lm) 사람이 인지되는 양
    - Brightness: a subjective descriptor of light perception that is practically impossible to measure 주관적인 빛의 밝기

Imaging sensing (1)

- Imaging sensor
  - Transform illumination energy into digital images
- Types of imaging sensors
  - Single sensor
  - Line sensor
  - Array sensor, or area sensor

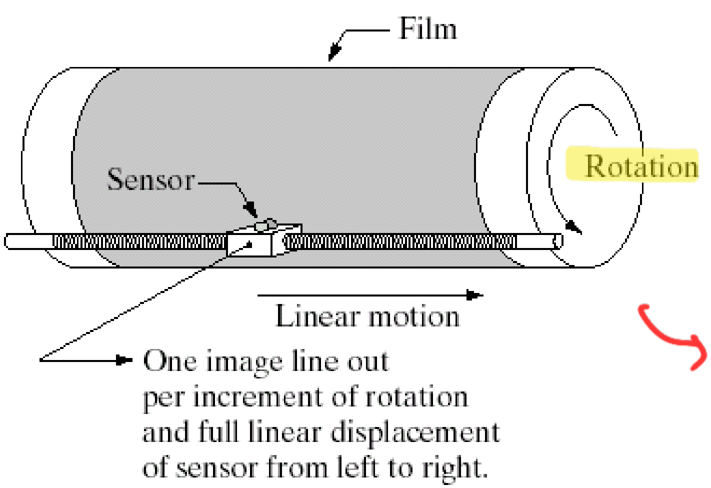


a
b
c

FIGURE 2.12
(a) Single imaging sensor.
(b) Line sensor.
(c) Array sensor.

Imaging sensing (2)

- Image acquisition using a single sensor
  - Inexpensive way to get high-resolution images



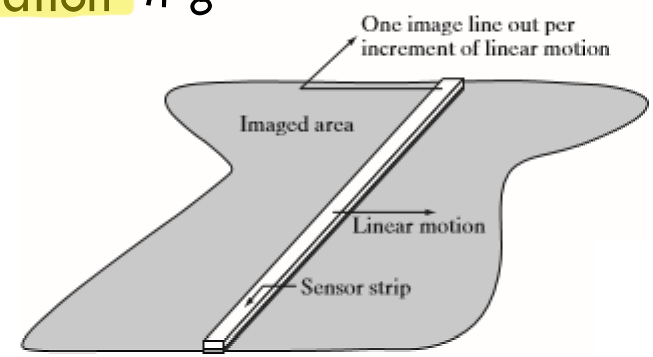
too long time.

FIGURE 2.13 Combining a single sensor with motion to generate a 2-D image.

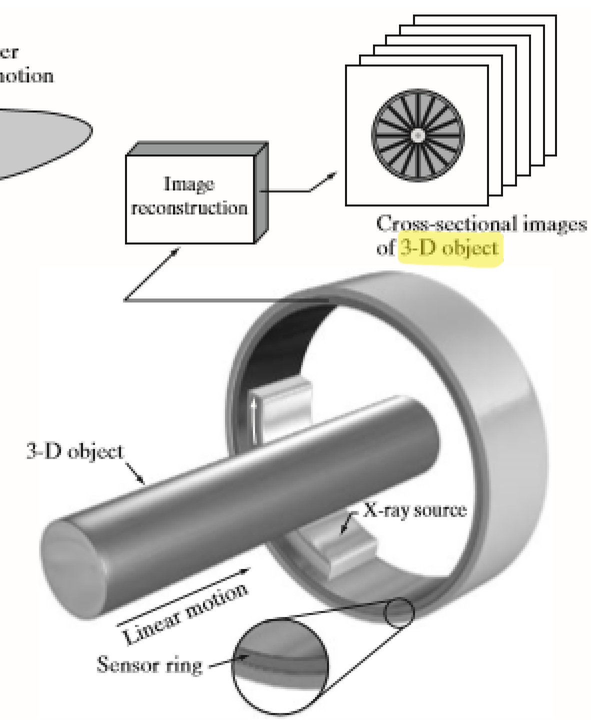
Imaging sensing (3)

- Image acquisition using sensor strips(line sensor)
  - Medical and industrial computerized axial tomography
    - To obtain cross-sectional (slice) images of 3-D objects
    - But, require extensive processing to reconstruct a 3-D shape
  - Structured illumination 가능

구조적 조명

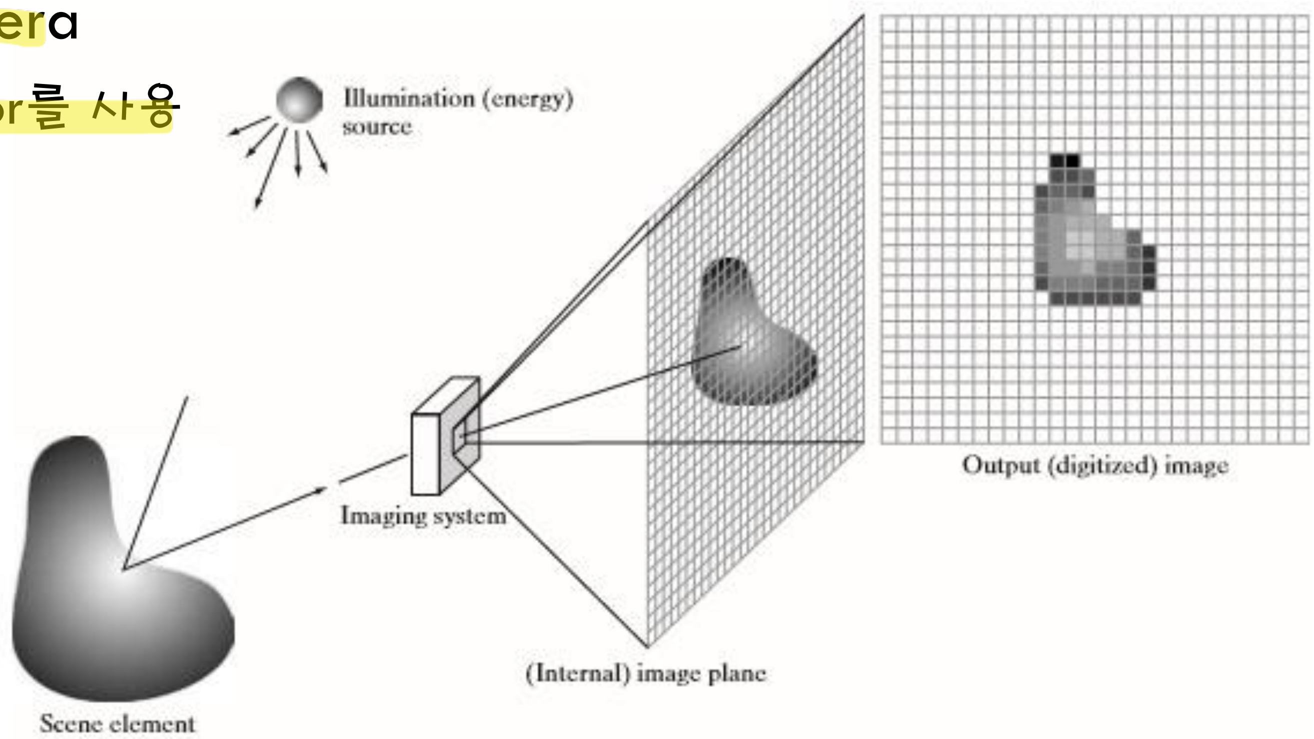


CT →



Imaging sensing (4)

- Image acquisition using 2-D array if sensors
  - Typical arrangement in numerous electromagnetic and some ultrasonic sensing device such as a digital camera
- Digital camera
  - CCD sensor를 사용

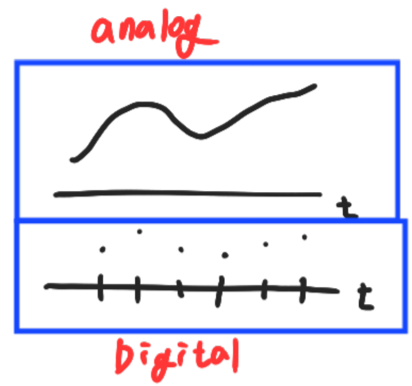


Simple image formation model

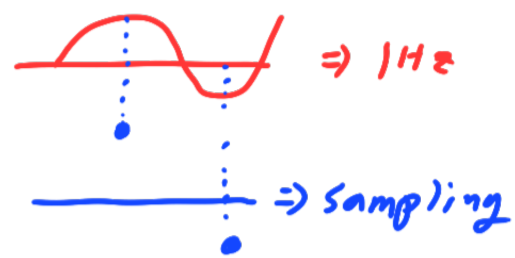
- Image
  - Denoted by 2-D function of the form f(x,y)
  - f(x,y): value of amplitude of function at spatial coordinates (x,y)
    - $0 < f(x,y) < \infty$ digital $\Rightarrow 0 \le f(x,y) < 256$
      어두움 밝음
- $f(x,y) \approx i(x,y) \times r(x,y)$
  - characterized by two components, illumination and reflectance
  - i(x,y): illumination,
    - $0 < i(x,y) < \infty$, E.g. 90,000 lm/m^2 for clear day
  - r(x,y): reflectance,
    - $0 < r(x,y) < \infty$, E.g. 0.01 for black velvet, 0.65 for stainless steel
      - 0: total absorption, 1: total reflectance

Image digitization (1)

- Digitization (10진수를 2진수로)
  - Conversion of the analog signal into digital form
    - Analog: continuous both coordinate and amplitude
    - Digital: discrete both coordinate and amplitude
  - involves sampling and quantization



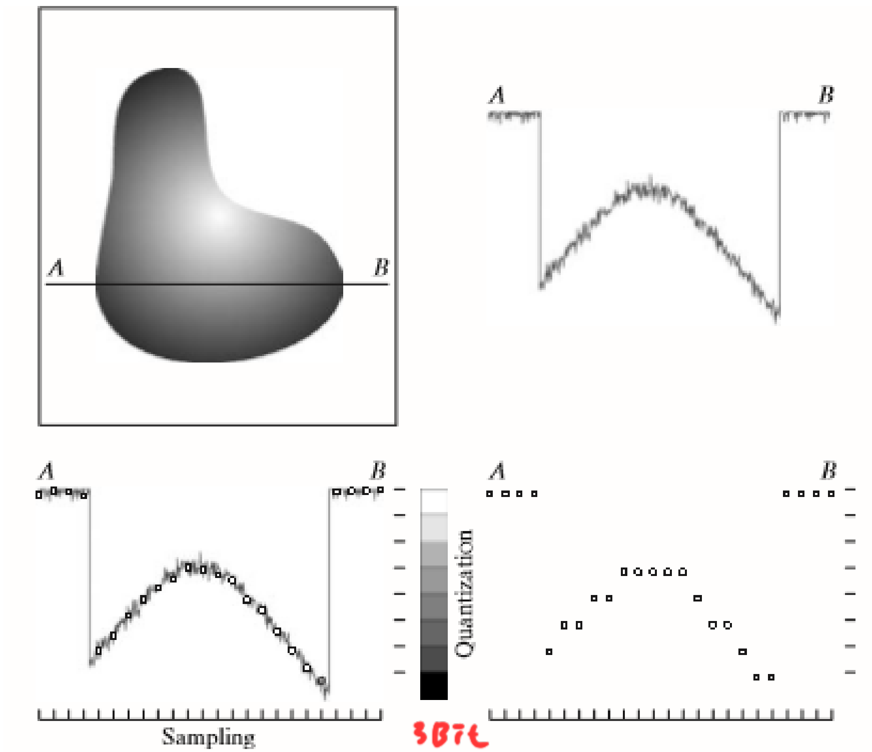
- Sampling (표본화)
- Digitization of coordinate
  - Nyquist rate: signal의 최대 주파수의 2배
  - Sampling grid: rectangular, hexagonal, etc
    ↳ 일반적

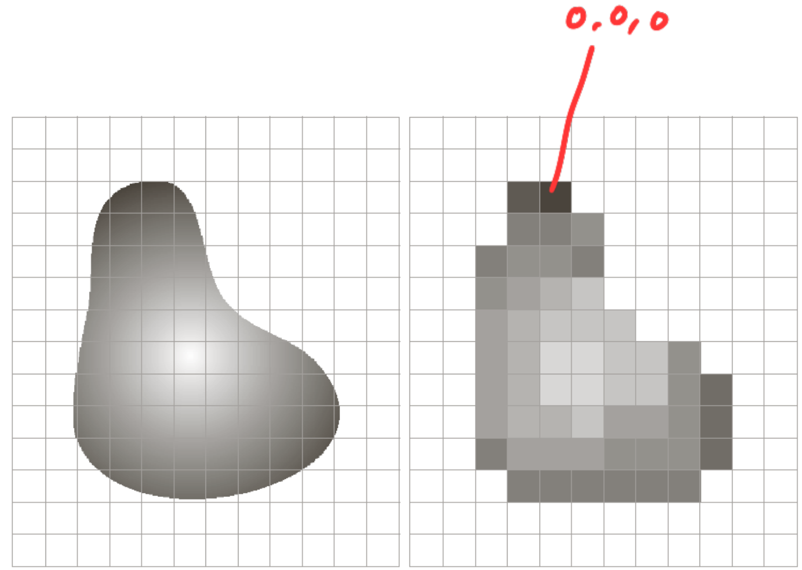


- Quantization (양자화) → 대표 보통 정수로
  - Digitization of amplitude
  - Monochrome의 경우 gray level이 된다

Image digitization (2)

- Digitization process of one scan line





a b

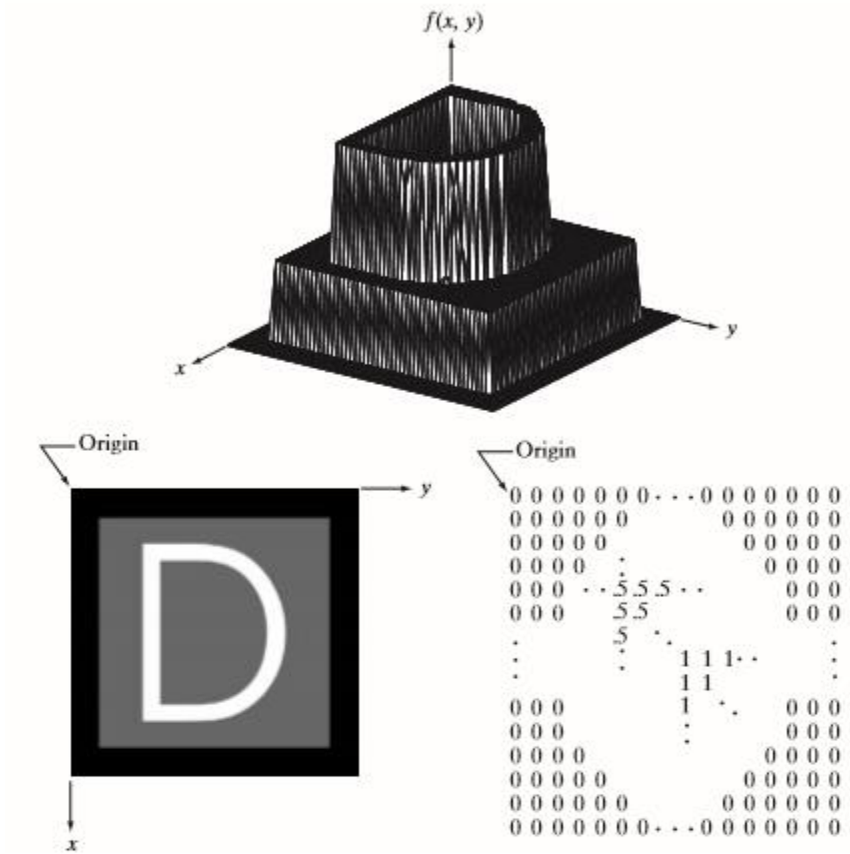
FIGURE 2.17 (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

Image digitization (3)

- What is a digital image ? (2차원 공간에 빛의 세기를 숫자로 표기)
  - Converts brightness (intensity) into numbers
  - Set of numbers at 2-D discrete coordinates
    - 2-D discrete coordinates: spatial domain
- What is a digital audio ? (파형을 숫자로)
  - Converts waveform (intensity) into numbers
  - Set of numbers at 1-D discrete coordinates
    - 1-D discrete coordinates: temporal domain

Representing digital images

- Digital image in computer
  - 2-D array of memory에 표현



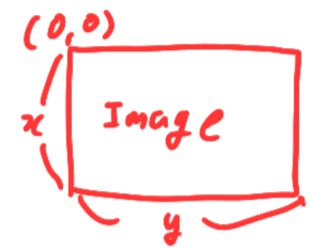
0: black
1: gray
5: white

Digital image in matrix notation

- Mathematical representation of digital image
  - Represents as a 2-D matrix of real number

$$
f(x,y) = \begin{array}{cc} & N \\ M & \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & \vdots & & \vdots \\ f(M-1,0) & f(M-1,1) & \cdots & f(M-1,N-1) \end{bmatrix} \end{array}
$$

M x N matrix



Or

$$
\mathbf{A} = \begin{bmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,N-1} \\ \vdots & \vdots & & \vdots \\ a_{M-1,0} & a_{M-1,1} & \cdots & a_{M-1,N-1} \end{bmatrix}
$$

pixel
pel, image element,
picture element

where $a_{i,j} = f(x=i, y=j) = f(i,j)$

Attributes of digital image

1. Spatial resolution (분해능, 해상도) => 분해할수 있는 가장 작은 detail

- Measure of the smallest discernible detail
- 영상의 detail을 표현할 수 있는 능력
  - E.g. Line pair per unit distance, dots per unit distance
    - E.g. Dots per inch (dpi)
      - E.g. 75 dpi for newspaper, 133 dpi for magazine, 2400 dpi for text

2. Intensity resolution (계조도)

- Gray level, or gray scale
- 명암의 등급을 표현
- k bit를 할당하면, L = 2^k level 표현, 일반적으로 k는 2의 배수 k=8이 적당

Number of bits

- Total bits
  - M x N x k bits, or N^2 x k bits if M=N

TABLE 2.1
Number of storage bits for various values of N and k.

N/k	1 (L = 2)	2 (L = 4)	3 (L = 8)	4 (L = 16)	5 (L = 32)	6 (L = 64)	7 (L = 128)	8 (L = 256)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

Images of various spatial resolutions (1)

- From 32x32 to 1024x1024

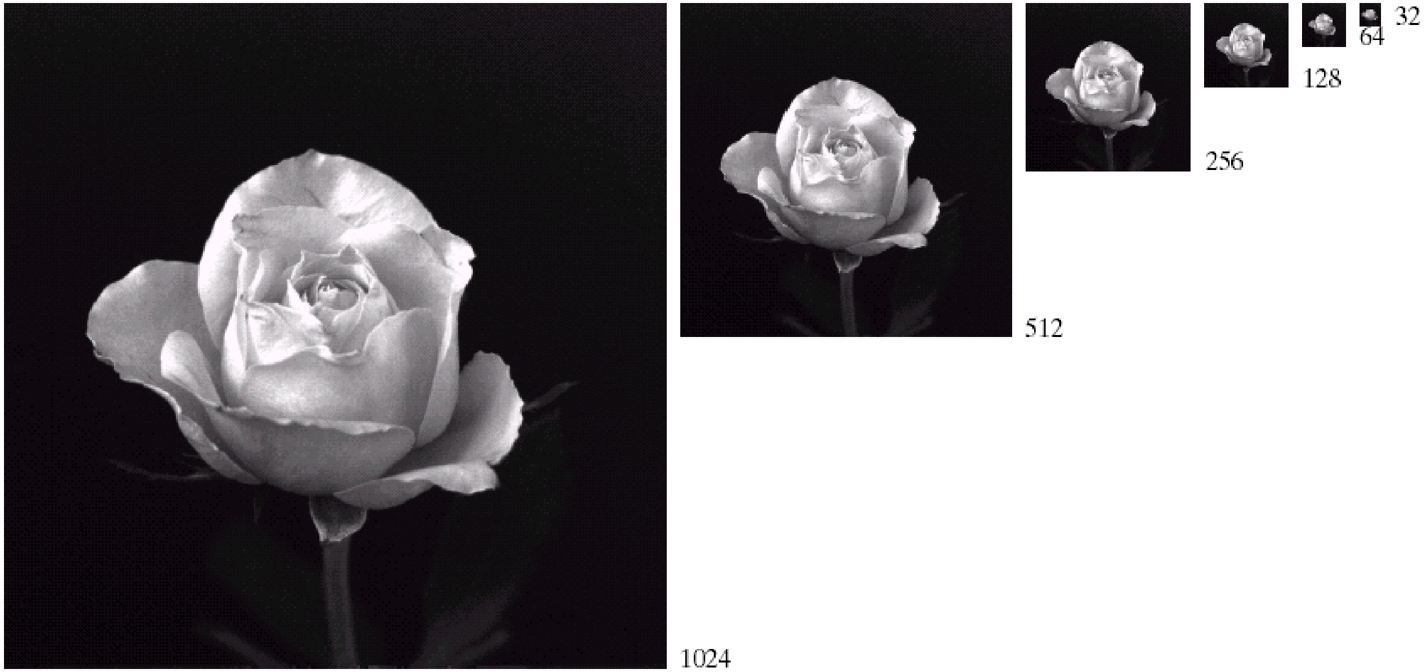
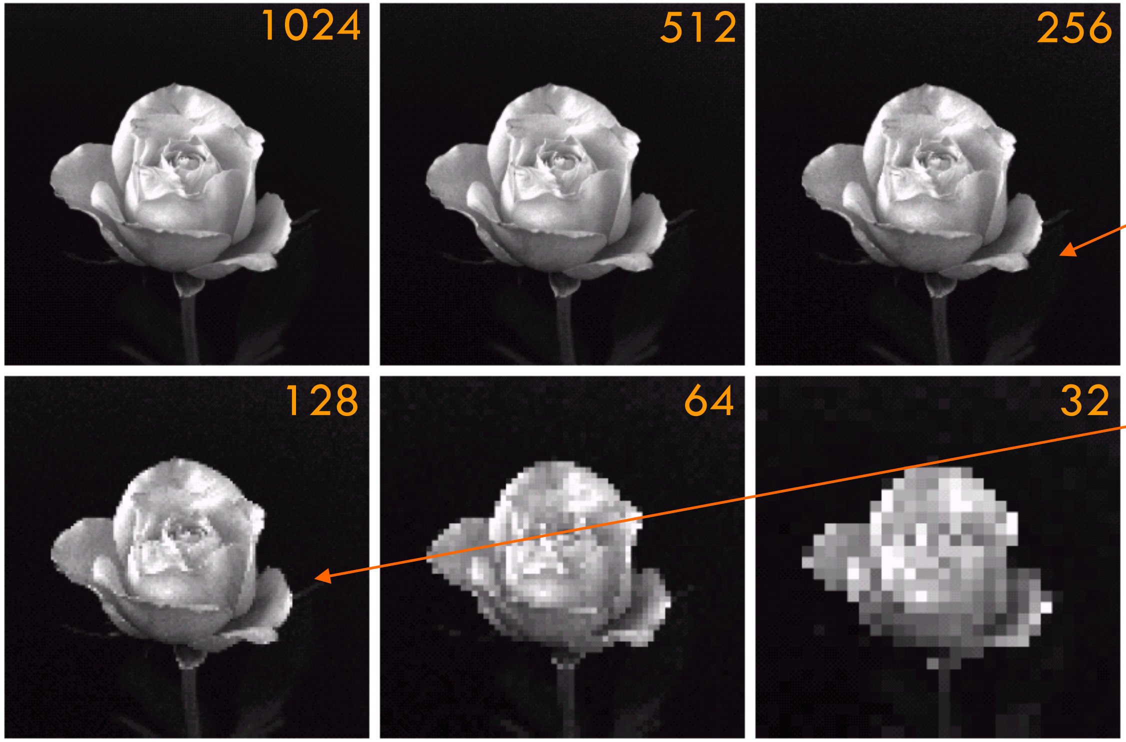


FIGURE 2.19 A 1024 × 1024, 8-bit image subsampled down to size 32 × 32 pixels. The number of allowable gray levels was kept at 256.

Images of various spatial resolutions (2)

- Zoomed images by pixel duplication for comparison



미세한
바둑판 패턴

visible
degradation

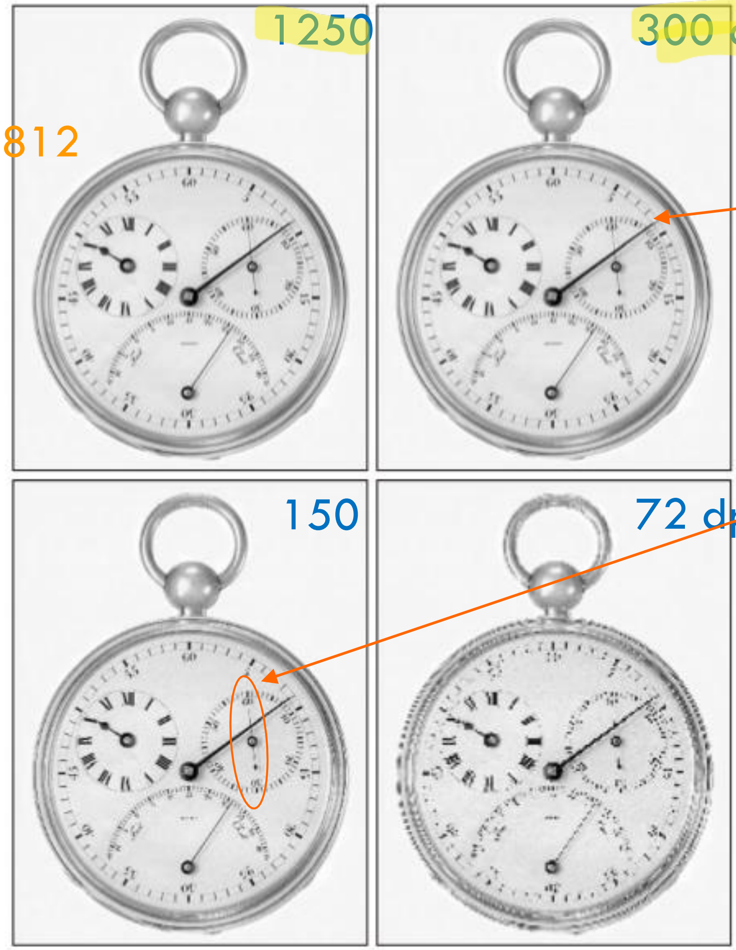
interpolation
→ Inter Linear

a b c
d e f

FIGURE 2.20 (a) 1024 × 1024, 8-bit image. (b) 512 × 512 image resampled into 1024 × 1024 pixels by row and column duplication. (c) through (f) 256 × 256, 128 × 128, 64 × 64, and 32 × 32 images resampled into 1024 × 1024 pixels.

Images of various spatial resolutions (3)

- Effects of spatial resolution



3692x2812

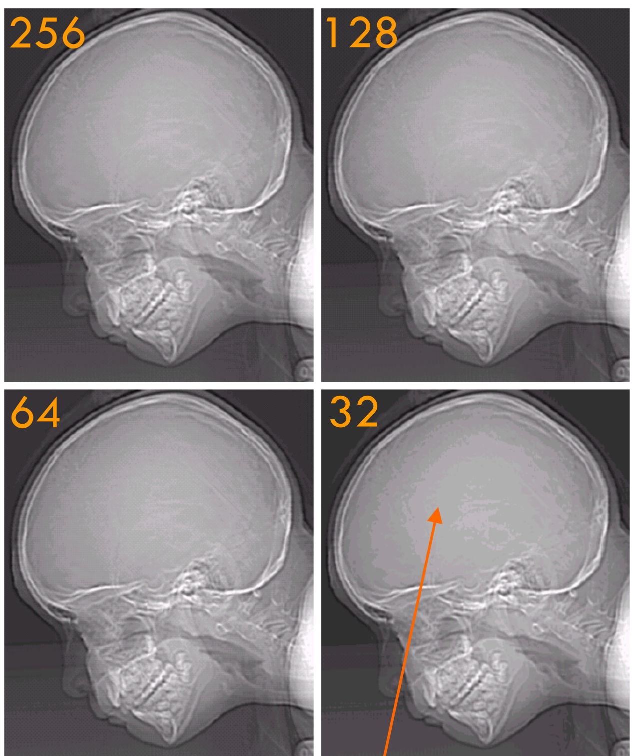
slight distortion, but acceptable
minimum resolution for book publishing

visible degradation

213x162

Images of various gray levels

- From 256 to 2 for fixed spatial resolution (452x374)

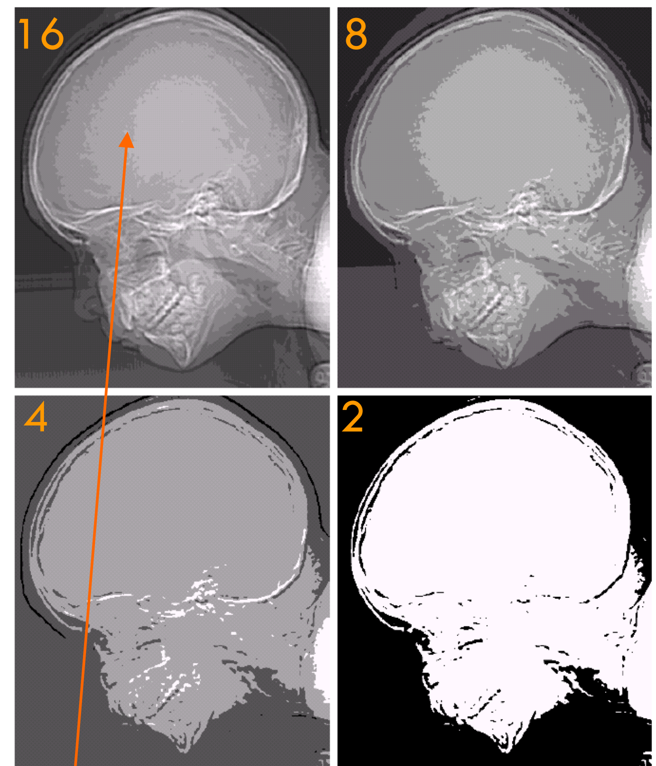


a b
c d

FIGURE 2.21 (a) 452 × 374, 256-level image. (b)–(d) Image displayed in 128, 64, and 32 gray levels, while keeping the spatial resolution constant.

e f
g h

FIGURE 2.21 *(Continued)* (e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)



Imperceptible very fine ridge-like structure in areas of constant intensity

visible false contouring appears

Standard images

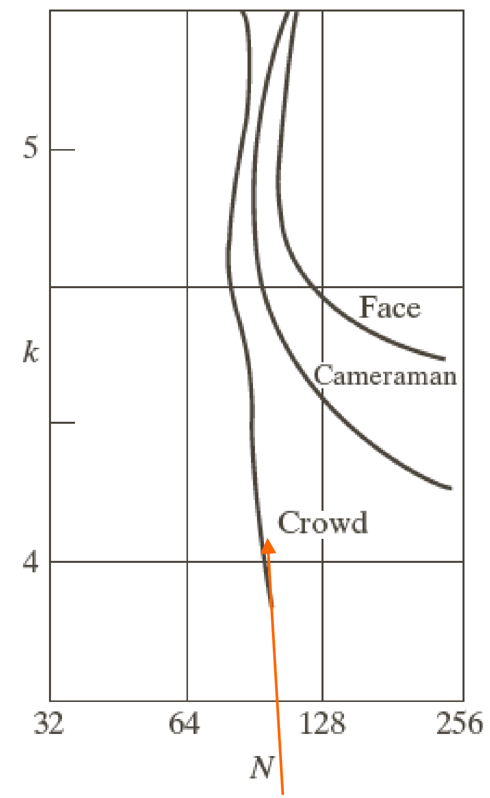
- Three types of images
  - Low level of detail : Lena
  - Medium level of detail: Cameraman
  - Large amount of detail: Crowd

a b c

FIGURE 2.22 (a) Image with a low level of detail. (b) Image with a medium level of detail. (c) Image with a relatively large amount of detail. (Image (b) courtesy of the Massachusetts Institute of Technology.)

Isopreference curves

- Curves of points of equal subjective quality
  - Complex image ⇒ 영향 거의 X
    - Less influence on N and k
  - Simple image ⇒ 영향 많이 받음
    - More influence on N and k
      - Small k for large N
      - Large k for small N



nearly vertical
for complex image

Anti-aliasing

- Aliasing
  - Undersampling에 의한 degradation 현상
    - 원인: function of unlimited duration을 finite duration(band-limited)으로 만들기 위한 window 함수가 high frequency 성분을 포함
    - 대책: reducing high-frequency component by blurring
- 주기함수의 경우 최대 주파수의 2배 이상으로 sampling하면 exact recovery가 가능

Moiré pattern

- 주기성의 break up에 의하여 발생하는 물결 모양의 무늬
- 입체영상을 rough하게 표현

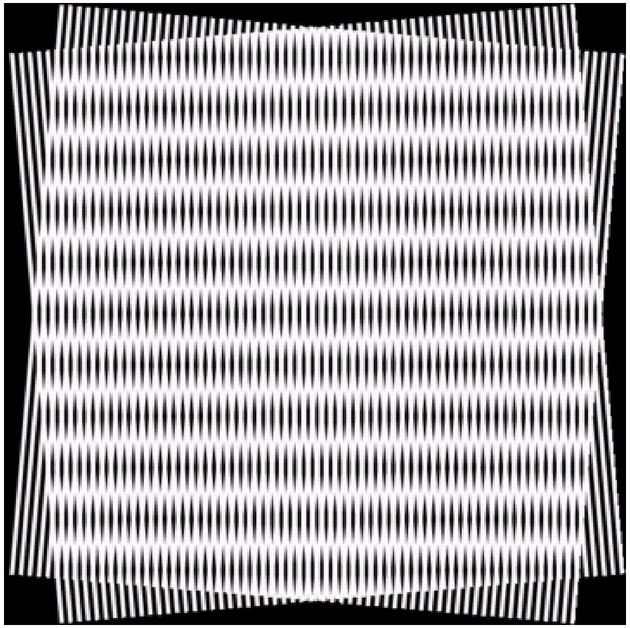


FIGURE 2.24 Illustration of the Moiré pattern effect.

Image interpolation (1)

- Interpolation = 보간법
  - The process to estimate values of unknown locations using data of known locations, basically resampling method
  - A basic tool for resizing task such as zooming, shrinking, rotating, and geometric correction
  - Regards as a kind of approximation

- Steps of resizing
  - Computation of new pixel locations
  - Assignment of gray levels to new locations: interpolation

Image interpolation (2)

- Methods of interpolation
  - Nearest neighbor interpolation
    - Assignthe intensity of its nearest neighbor
    - 가장가까운 위치의 값을 복사
  - Bilinear interpolation
    - $v(x,y) = ax + by + cxy + d$
      - Linear interpolation function
      - 4 NN for 4 unknown
  - Bicubic interpolation
    - $v(x,y) = \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij}x^i y^j$
      - Cubic interpolation function
      - 16 NN for 16 unknowns
  - Spline interpolation

Image interpolation (3)

- Effect of zooming from shrinking

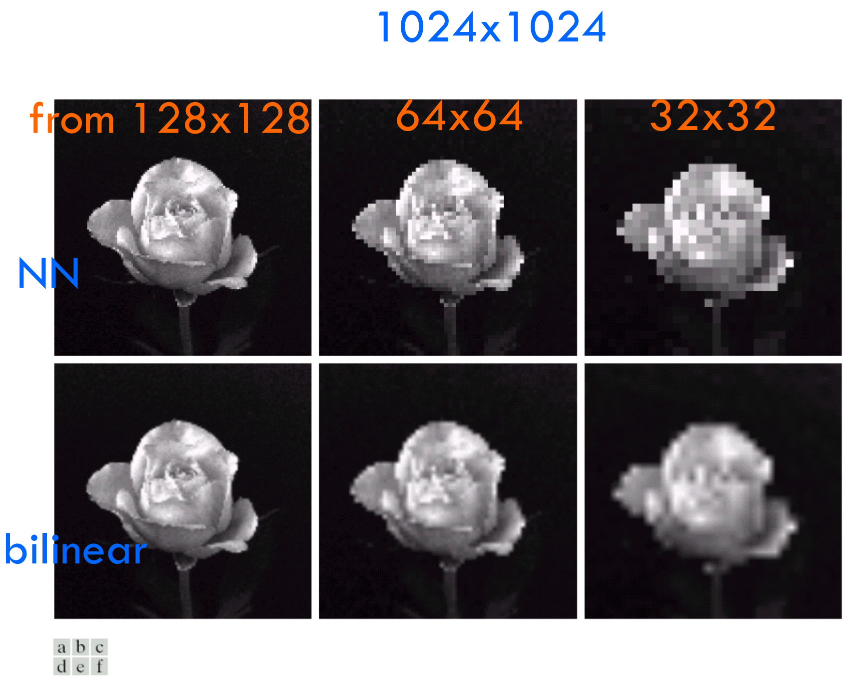
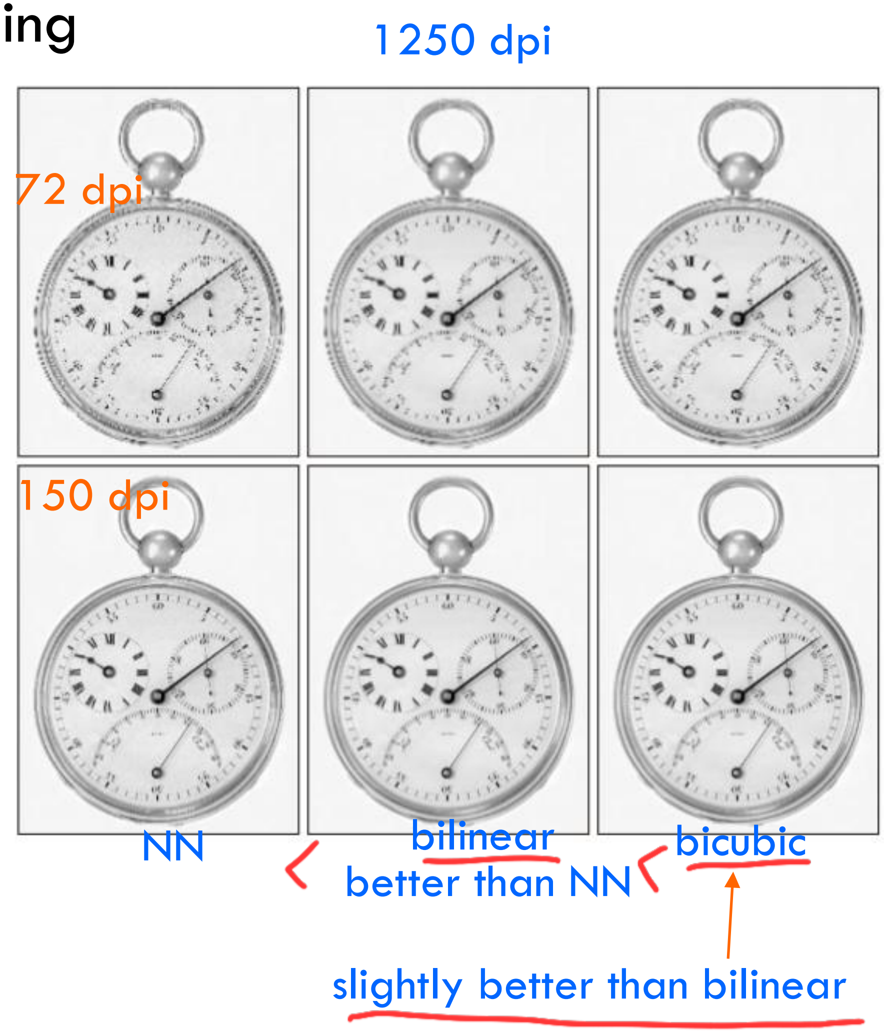


FIGURE 2.25 Top row: images zoomed from 128 × 128, 64 × 64, and 32 × 32 pixels to 1024 × 1024 pixels, using nearest neighbor gray-level interpolation. Bottom row: same sequence, but using bilinear interpolation.



Neighbor of a pixel

- 4 NBs of a pixel p at coordinate (x,y): $N_4(p)$
  - (x+1, y), (x-1,y), (x,y+1), (x, y-1)
- 8 NBs of a pixel p at coordinate (x,y): $N_8(p)$
  - 4 NB + (x+1,y+1), (x+1,y-1), (x-1,y+1), (x-1, y-1)

(x−1,y−1)	(x−1,y)	(x−1,y+1)
(x,y−1)	(x,y)	(x,y+1)
(x+1,y−1)	(x+1,y)	(x+1,y+1)

Adjacency (1) 인접성

- 4-adjacency: p and q are 4-adjacent if $q \in N_4(p)$
- 8-adjacency: p and q are 8-adjacent if $q \in N_8(p)$
- M-adjacency (mixed adjacency)
  - ① q is in $N_4(p)$, or
  - ② q is $N_D(p)$ and the set $N_4(p) \cap N_8(p) = \varnothing$

Ambiguity arises for 8-adjacency

거리가 1 아 2가 될수있음

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{matrix}\Bigg\} R_i \qquad \begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix} \qquad \begin{matrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{matrix}$$

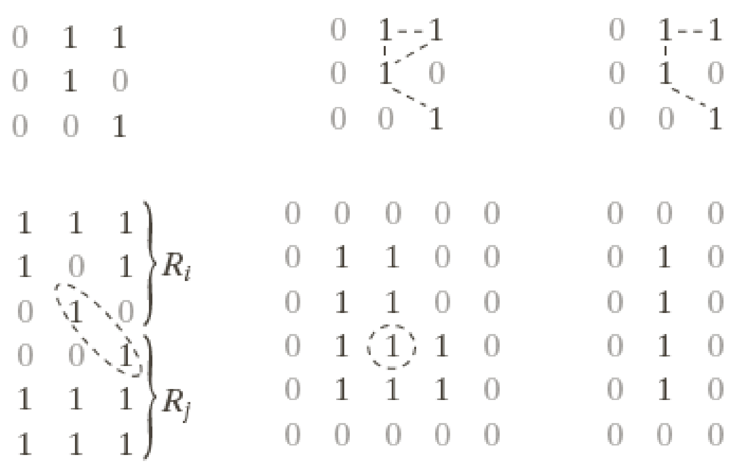
$$\begin{matrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}\Bigg\} R_j$$

Adjacency (2)

- Path
  - A sequence of distinct adjacent pixels from p(x,y) to q(s,t)
    - Sequence $(x,y), (x_1, y_1), \ldots, (x_n, y_n)$ form a path if $(x_n, y_n)=(s,t)$ and (x_i, y_i) and (x_{i+1}, y_{i+1}) are adjacent
      - The length of the path is n and the path is closed if $(x,y) = (s,t)$
- Connectivity
  - Connected
    - p and q are connected in S if there exist a path between p and q consisting entirely of pixels in a subset of pixels S
  - Connected component
    - Set of pixels that are connect to it in S
  - Connected set
    - The set S with only one connected component

Adjacency (3)

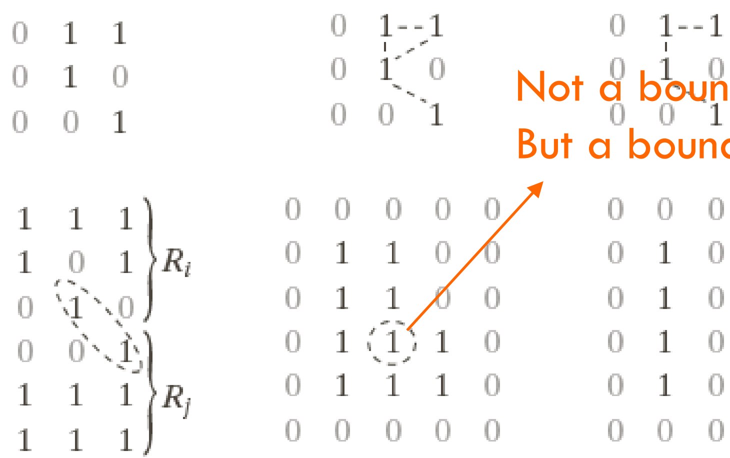
- Region
  - A connected set, usually 4-connectivity is used
  - Regions R_i and R_j are said to be adjacent if their union forms a connected set
  - Regions R_i and R_j are said to be disjoint if they are not adjacent



R_i and R_j are 8-adjacent

Adjacency (4)

- Region
  - R_u : union of all the disjoint regions, foreground
  - $(R_u)^c$: complement of R_u, background, usually 8-connectivity is used
- Boundary (border or contour)
  - Inner border: the set of points that are adjacent to points in $(R_u)^c$
  - Outer border: the corresponding border in the background

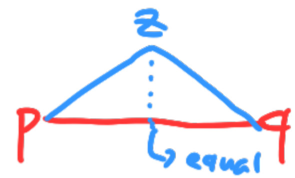


Not a boundary if 4-adjacency is used,
But a boundary if 8-adjacency between FG and BG

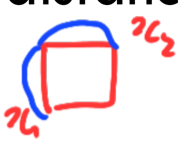
Inner boundary is region itself
Does not form a closed path

Distance measures (1)

- D: distance function or metric ⇒ 유사도
  1. $D(p,q) \geq 0$, equality holds iff $p=q$
  2. $D(p,q) = D(q,p)$
  3. $D(p,q) \leq D(p,z) + D(z,q)$, equality holds when z is on the line pq
  - where, $p(x_1, y_1)$, $q(x_2, y_2)$, $z(x_3, y_3)$



- D_e : Euclidean distance
  - $D_e = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
- D_4 : D_4 distance, or city-block distance, or Manhattan distance
  - $D_4 = |x_1 - x_2| + |y_1 - y_2|$



- D_8 : D_8 distance, or chessboard distance
  - $D_8 = \max(|x_1 - x_2|, |y_1 - y_2|)$

Distance measures (2)

Distance of D_4

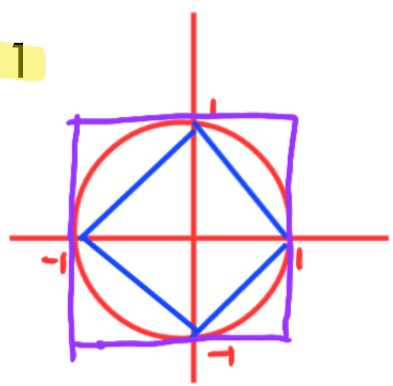
		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

4 NBs if $D_4 = 1$

Distance of D_8

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

8 NBs if $D_8 = 1$



— : D_4 Distance

— : D_8 Distance

Array vs. Matrix operations

- Consider following 2x2 images

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{and} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

- Array product
  - Elementwise operation

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}a_{22} \end{bmatrix}$$

- Matrix product
  - Inner product

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

$2\times 2 \cdot 2\times 2$ $\qquad\qquad$ 2×2

Linear operations (1)

- H: general operator

  $$g(x, y) = H[f(x, y)]$$

- Linear operator
  - Linear if

    $$\boxed{\begin{aligned} H\left[a_i f_i(x,y) + a_j f_j(x,y)\right] &= a_i H[f_i(x,y)] + a_j H\left[f_j(x,y)\right] \\ &= a_i g_i(x,y) + a_j g_j(x,y) \end{aligned}}$$

  - Properties of linear operator
    - Additivity
      - Output of the sum of two inputs is the same as the sum of the individual output
    - Homogeneity
      - Output of the input multiplied by a constant is the same as the output multiplied by a constant

Linear operations (2)

- Eg. Summing operation is linear ?

  linear: $H[a_i f_i(x,y) + a_j f_j(x,y)] = a_i H[f_i(x,y)] + a_j H[f_j(x,y)]$
  $= a_i g_i(x,y) + a_j g_j(x,y)$

  - Replace H with summing operator $\sum$

  $$\sum[a_i f_i(x,y) + a_j f_j(x,y)] = \sum a_i f_i(x,y) + \sum a_j f_j(x,y)$$

  $$= a_i \sum f_i(x,y) + a_j \sum f_j(x,y) = a_i g_i(x,y) + a_j g_j(x,y)$$

  - Thus, summing operation is linear

- Eg. Max operation is linear ?

  $f_1 = \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}$, $f_2 = \begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix}$ and $a_1 = 1$, $a_2 = -1$

  - Left hand: $max\left\{(1)\begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} + (-1)\begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix}\right\} = max\left\{\begin{bmatrix} -6 & -3 \\ -2 & -4 \end{bmatrix}\right\} = -2$
  - Right hand: $(1)\, max\left\{\begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}\right\} + (-1)\, max\left\{\begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix}\right\} = 3 + (-1)7 = -4$
  - Thus, max operation is not linear $L \neq R$

Arithmetic operations (1)

- Arithmetic operations
  - Array operation on pixel-by pixel basis
    - Addition: $s(x,y) = f(x,y) + g(x,y)$
    - Subtraction: $d(x,y) = f(x,y) - g(x,y)$
    - Multiplication: $p(x,y) = f(x,y) \times g(x,y)$
    - Division: $v(x,y) = f(x,y) \div g(x,y)$
- Image scaling
  - The range of gray scale after arithmetic operation may exceed 8 bits or shrink to few bits. Thus scaling is needed to span the dynamic range of full 8 bit scale
  - Scaled image: f_s

$f_s = K\,[f_m/\max(f_m)]$ where $f_m = f - \min(f)$ for range $[0, K]$

Arithmetic operations (2)

- Additive noise => noise의 위치가 random 할 때
  - $g(x,y) = f(x,y) + \eta(x,y)$

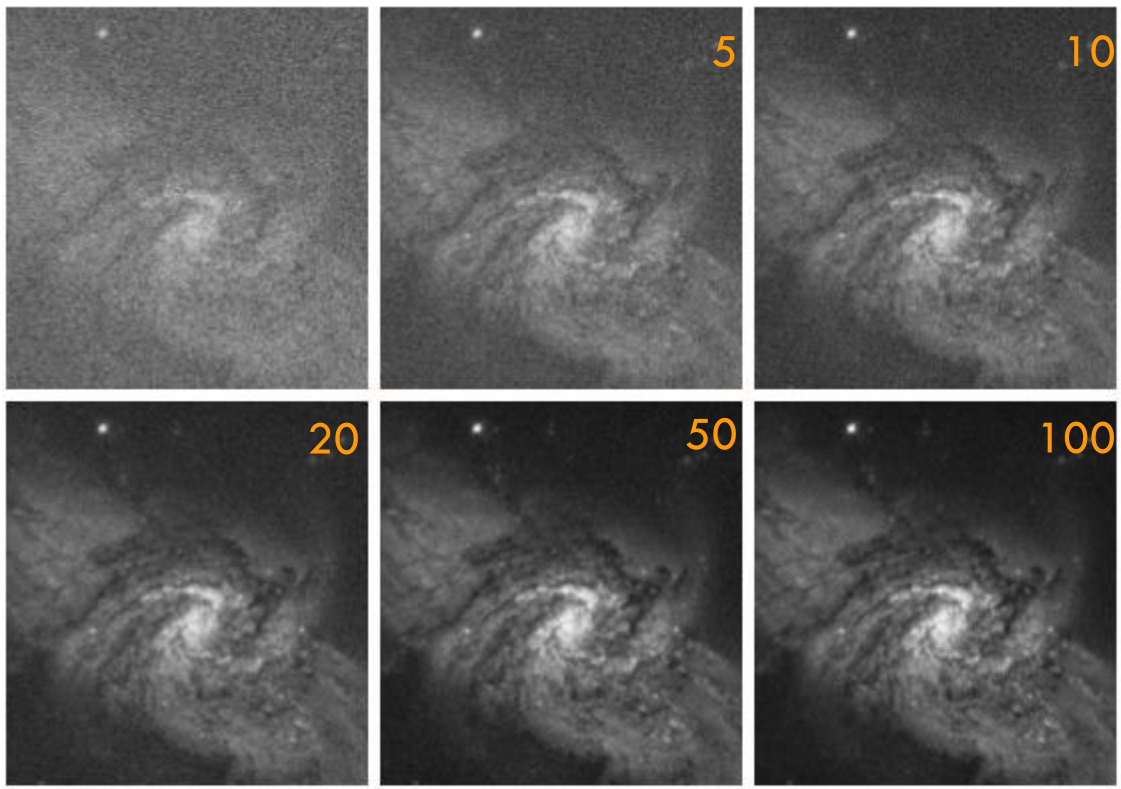
    where, $\eta(x,y)$: noise

    - Assumption: random noise meaning that noise is uncorrelated with pixel coordinates and has zero average
- Image averaging
  - Average of K different noisy images $\bar{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$
  - As k increases, the standard deviation decreases and the expected value approaches to noiseless image f(x,y)
    - Expected value: $E\{\bar{g}(x,y)\} = f(x,y)$
    - Variance: $\sigma^2_{\bar{g}(x,y)} = \frac{1}{K}\sigma^2_{\eta(x,y)}$ => output $\sigma^2 = \frac{1}{K}$ noise σ^2
    - Standard deviation: $\sigma_{\bar{g}(x,y)} = \frac{1}{\sqrt{K}}\sigma_{\eta(x,y)}$

Arithmetic operations (3)

noise < additive + / multiplicative ×

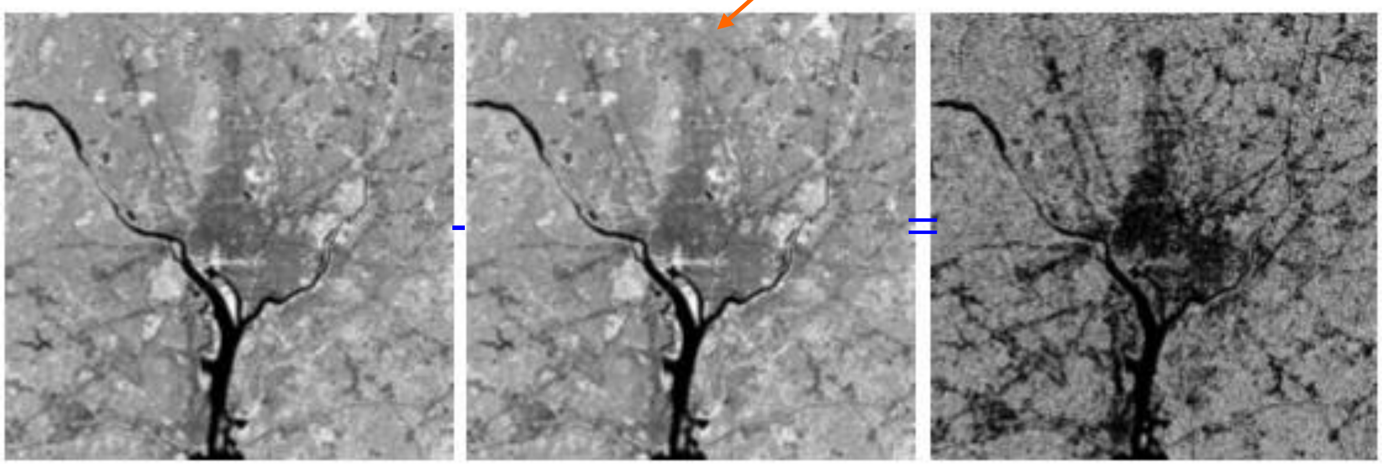
- Noise reduction by image averaging
  - Corrupted by additive Gaussian noise, zero mean, SD = 64
  - Noise reduction by observing the same scene over long periods of time



Arithmetic operations (4)

- Image subtraction: enhancement of differences
  - To show the difference of two images where black pixels indicate locations of no difference

set all LSBs to 0



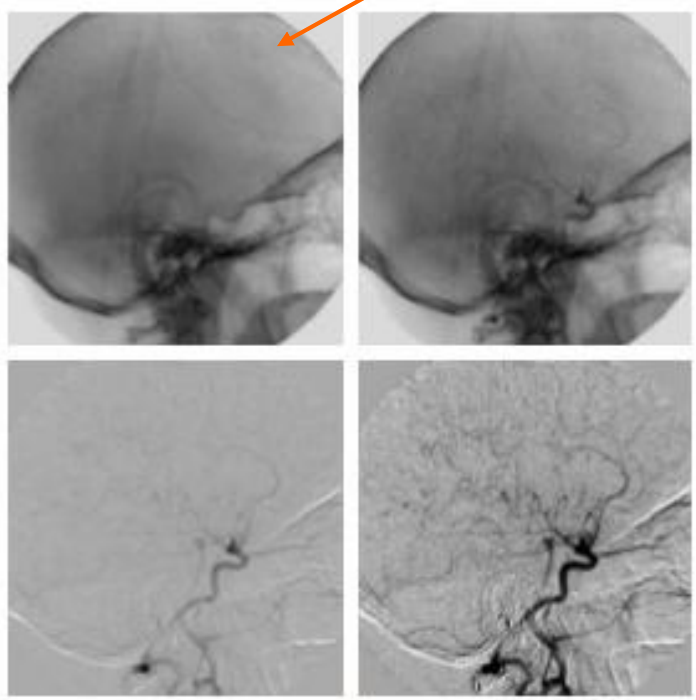
XXXXXXXX − XXXXXXX0 = 0000000X

Arithmetic operations (5)

- Image subtraction: mask mode radiography
  - To track the propagation of blood vessels in the brain
  - $g(x,y) = f(x,y) - h(x,y)$, where $h(x,y)$: mask

prior to injection
of iodine medium

after injection



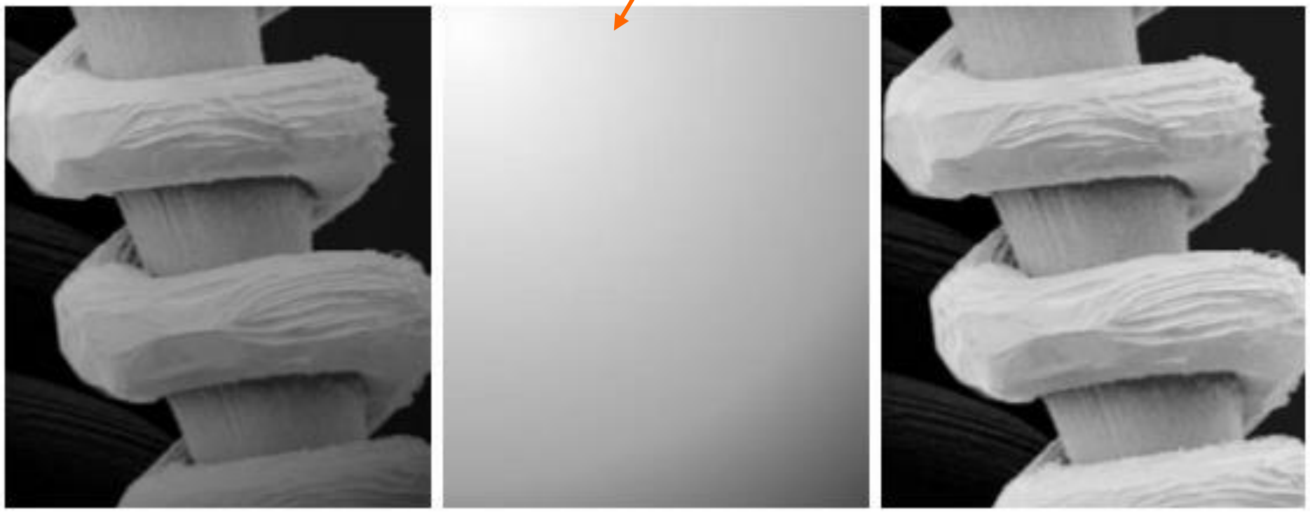
difference image

contrast enhanced
difference image

Arithmetic operations (6)

- Image multiplication (division): shading correction
  - To get a perfect image by shading correction when the shading pattern is known
  - $g(x,y) = f(x,y)h(x,y)$, where $h(x,y)$: shading pattern

    $f(x,y) = g(x,y)/h(x,y)$

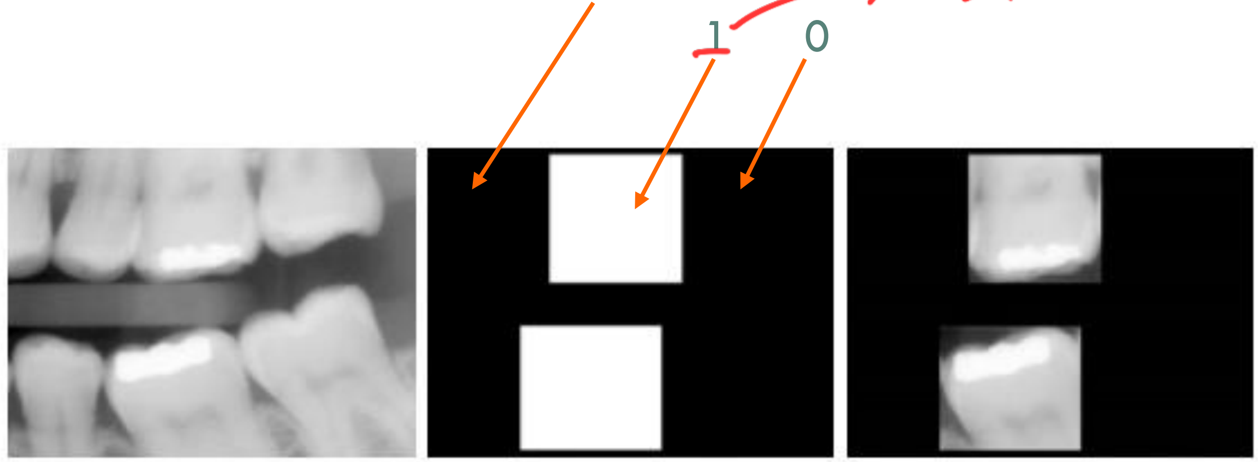


Arithmetic operations (7)

ROI 빼기

- Image multiplication (division): masking (region of interest)
  - To get a subimage of interest by multiplying by a mask consisting of 0 and 1 (region of interest)
  - g(x,y) = f(x,y)h(x,y), where h(x,y): mask

→ mask



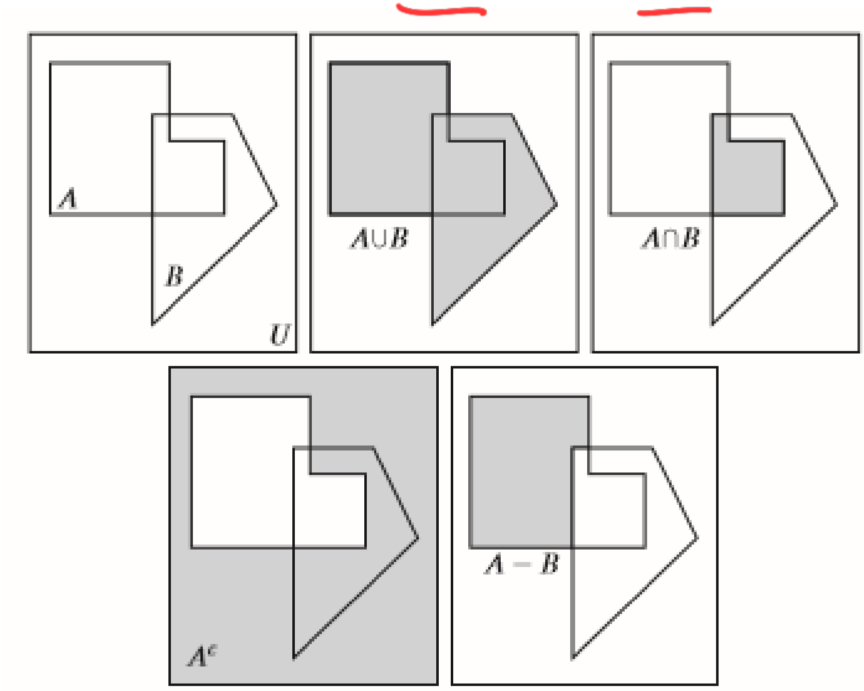
Set operations (1)

- Basic set operations
  - Definitions
  - A, B : sets in Z^2
  - Subset: $A \subseteq B$
    - If every element of A is also an element of B
  - Union: $A \cup B$
    - The set of all elements belonging to either A, B, or both
  - Intersection: $A \cap B$
    - The set of all element belonging to both A and B
  - Disjoint or mutually exclusive if $A \cap B = \varnothing$
  - Complement: $A^c = \{w \mid w \notin A\}$
  - Difference: $A\text{-}B = \{w \mid w \in A,\ w \notin B\}$

Set operations (2)

- Basic set operations
  - Graphical illustration

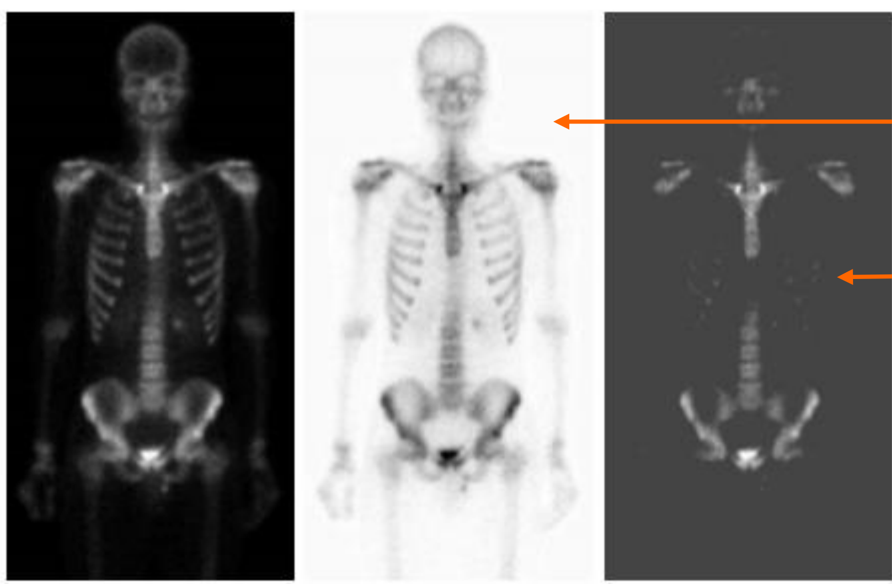
$A \cup B$ $A \cap B$



A^c $A-B$

Set operations (3)

- Set operations for gray-scale images
  - Elementwise array operation and different concept from binary
  - Union: $A \cup B = \{\max_z(a, b) \mid a \in A, b \in B\}$ ⇒ gray scaling 에서
  - Intersection: $A \cap B = \{\min_z(a, b) \mid a \in A, b \in B\}$
  - Complement: $A^c = \{(x,y,K-z) \mid (x,y,z) \in A\}$, z: intensity, K: max intensity



negative of A by complement

union of A and A∪B
a constant image (평균의 3배)
mid-gray

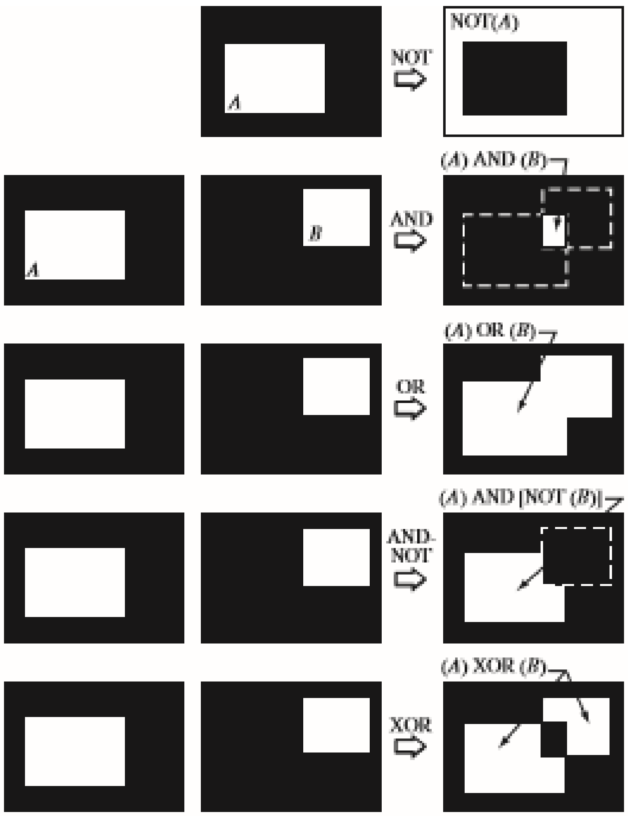
원 영상에서 mid-gray보다
밝은 화소만 남음

Logical operations

- Basic set operations
  - Binary image
    - 1: foreground (object), 0: background
  - AND: intersection $A \cap B$
  - OR: union $A \cup B$
  - NOT: complement A^c
  - XOR

similar to set difference

set of foreground pixels
belonging A or B, but not both



Fuzzy sets

- Fuzzy logic
  - Introduce a membership function instead of binary threshold
  - Provides a flexible tool to deal with imprecise concepts
  - Explore fuzzy logic in detail in section 3.8

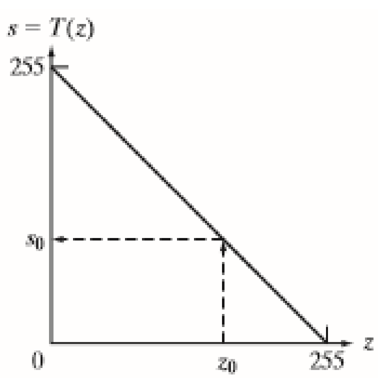
- Eg. Categorize a set of young people
  - Set the threshold as age of 20
  - What about a person of age of 20 years and 1 second ?
  - 50% young, 90% young과 같은 표현이 가능

Spatial operations (1)

- Spatial operations
  - Operations performed directly on the pixels of an image
  - Three types of spatial operations
    - Single-pixel operations
    - Neighborhood operations
    - Geometric spatial transformations
- Single-pixel operations
  - The simplest operation to alter the values of its individual pixels based on their intensity
  - Transformation function: T

    $s = T(z)$ $z_0 \rightarrow s_0$ $s = -x+255$

    - Eg. Intensity transformation to get a negative image

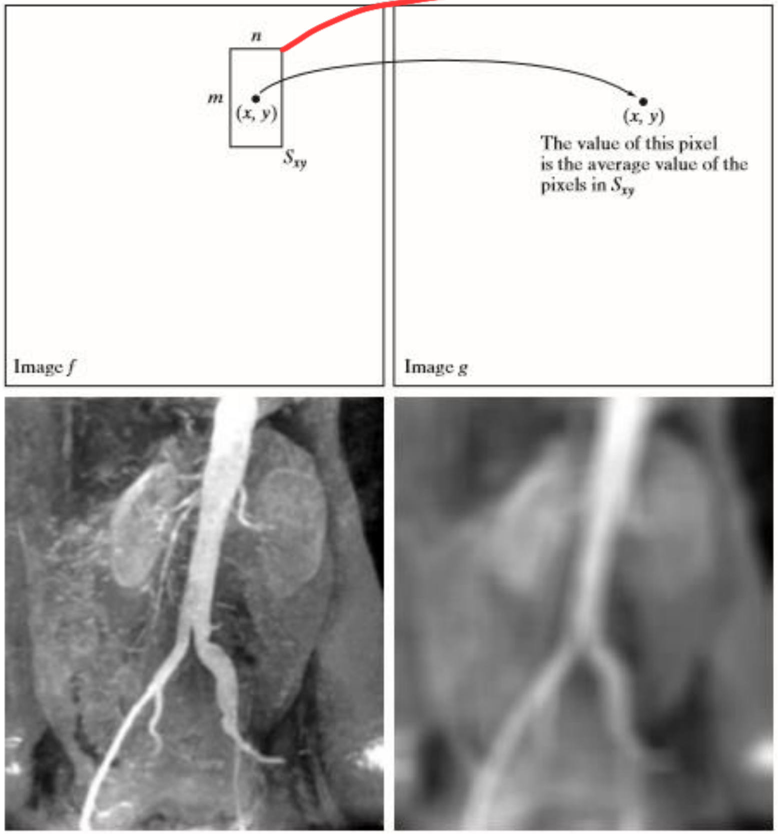


Spatial operations (2)

- Neighborhood operations
  - Operations performed on the neighborhoods of a pixel window
    - Eg. Local averaging

$$g(x,y) = \frac{1}{mn}\sum_{(r,c)\in S_{xy}} f(r,c)$$

where S_{xy}: set of NB of a pixel (x,y)



blurring

Spatial operations (3)

- Geometric spatial transformations
  - Modify the spatial relationship between pixels
  - AKA rubber-sheet transformations
  - Two steps of geometric transformation

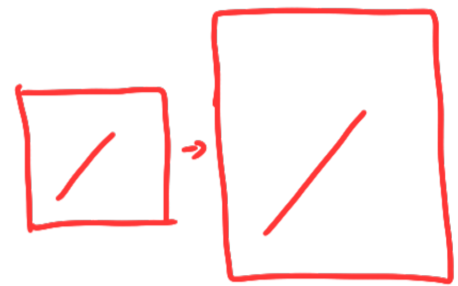
    1. a spatial coordinate transformation

       $(x,y) = T\{(v,w)\}$ ⇒ 대응되는 위치 찾기
       = 공간 좌표 변환

       - Eg. $(x,y) = T\{(v,w)\} = (v/2, w/2)$ ⇒ 축소
         - Shrinks the original image to half its size in both spatial directions

    2. intensity interpolation ⇝ 주변 픽셀을 이용해 좌표값 얻음

       - assigns intensity value to the transformed pixels



Spatial operations (4)

- Affine transformation
  - General spatial transformation especially in computer graphics

$$[x \quad y \quad 1] = [v \quad w \quad 1]\mathbf{T} = [v \quad w \quad 1]\begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{21} & t_{22} & 0 \\ t_{31} & t_{32} & 1 \end{bmatrix}$$

3차 원소로 만들기 위해 임의로 채워줌

    - Eg. Scaling, rotation, translation, sheer transformations

$$\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

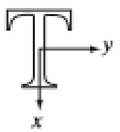
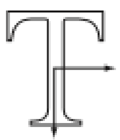
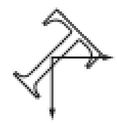
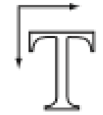
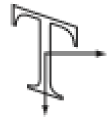
  - Coordinate mapping

intepolation 방법 달라짐

1. Forward mapping
    - For each pixel of input image, computes corresponding location of output
2. Inverse mapping
    - For each pixel location of output image, computes corresponding locations of input image and interpolate

Spatial operations (5)

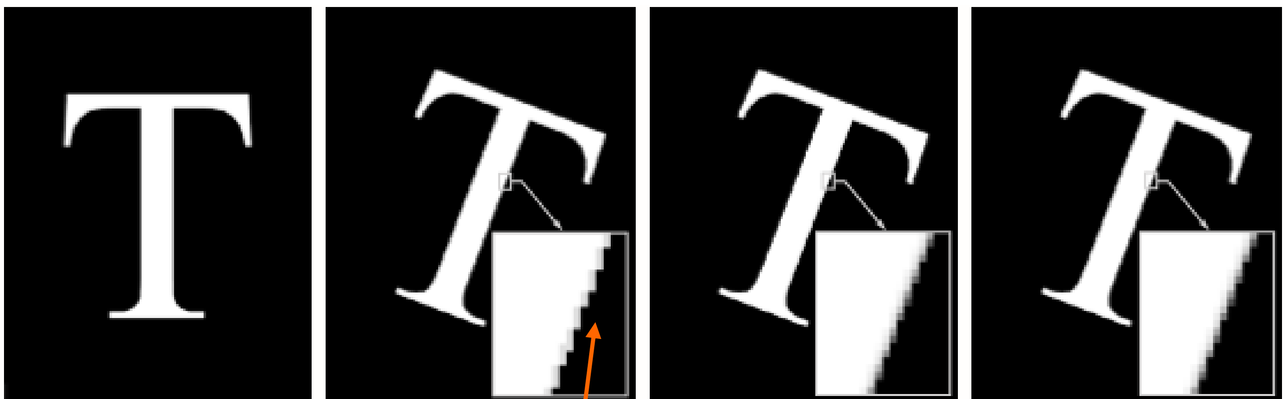
- Affine transformation

Transformation Name	Affine Matrix, T	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = w$	
Scaling c_x c_y	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = c_x v$ $y = c_y w$	
Rotation θ	$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v\cos\theta - w\sin\theta$ $y = v\cos\theta + w\sin\theta$	
Translation	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$	$x = v + t_x$ $y = w + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v + s_v w$ $y = w$	
Shear (horizontal)	$\begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = s_h v + w$	

Spatial operations (6)

- Eg. Effect of various interpolations

nearest neighbor　　bilinear　　bicubic



jagged edge

=

edge 선명

edge 부드럽

Image registration (1)

- To align (register) two or more images of the same scene
  - 두영상의 이미지 조합 (위치)
  - To combine images or perform quantitative analysis and comparison
  - To compensate the geometric distortions caused by differences in viewing angle, distance, and orientation; sensor resolution; shift in positions; and other factors
  - Need to estimate unknown specific transformation
    - Simple model based on a bilinear approximation

      $x = c_1 v + c_2 w + c_3 vw + c_4$, $y = c_5 v + c_6 w + c_7 vw + c_8$

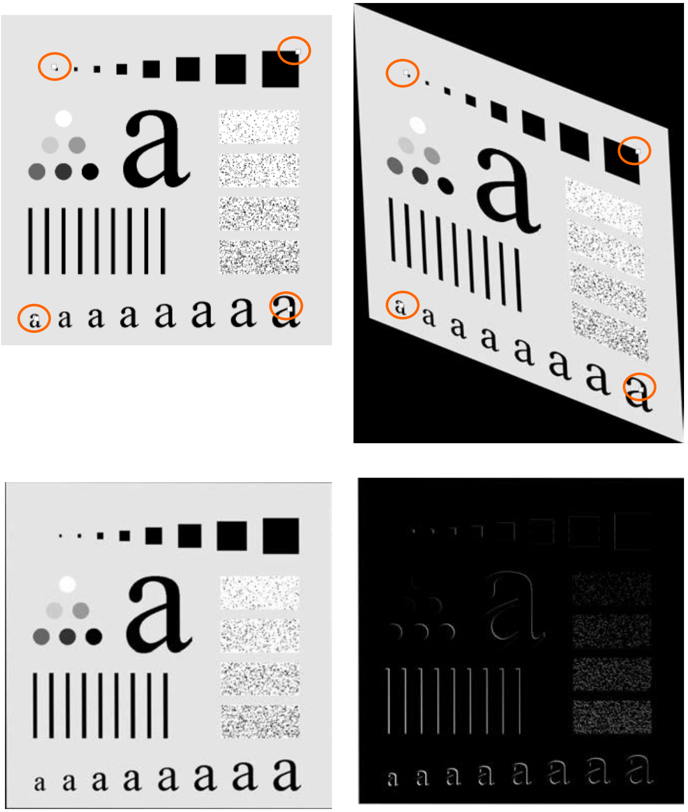
      - To solve equation, we needs four pairs of tie control points (control points) which are corresponding points whose locations are known (reseau marks) in the input and reference images

        대상의 위치 mask
    - The number of control points and sophistication of the model required to is dependent on the severity of the geometric distortion

Image registration (2)

- Eg. Registration of geometrically distorted image

reference



Input, distorted

tie points,
small white square

registered

difference
(registration error)

Vector and Matrix operations

- Vector and matrix representations
  - Eg. Multispectral image processing
    - Eg. Column vector to represent RGB color space
    - $\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$, z_1, z_2, z_3: intensity of red, green, and blue, respectively
  - Eg. Euclidean distance in n-dimensional space
    - $\mathbf{D}(\mathbf{z}, \boldsymbol{a}) = [(\mathbf{z} - \boldsymbol{a})^T(\mathbf{z} - \boldsymbol{a})]^{\frac{1}{2}} = [(z_1 - a_1)^2 + (z_2 - a_2)^2 + \cdots + (z_n - a_n)^2\,]^{\frac{1}{2}}$
    - $\|\mathbf{z} - \mathbf{a}\|$: vector norm, same as $\mathbf{D}(\mathbf{z}, \boldsymbol{a})$ L1 norm
  - Eg. Linear transformation in matrix notation
    - $\mathbf{w} = \mathbf{A}(\mathbf{z} - \boldsymbol{a})$, **A**: matrix of size mxn; **Z** and $\boldsymbol{a}$: nx1 column vector
  - Eg. Linear process
    - **g** =**Hf**+ **n**, **H:** linear process of size MNxMN; **f:** MNx1input image; **n:** MxN noise pattern

Image transforms (1)

- Processing in transform domain

- 2-D linear transform
  - Forward transform: $T(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) r(x,y,u,v)$
    - r(x,y,u,v): forward transformation kernel
    - Eg. Discrete Fourier transform: $r(x,y,u,v) = e^{-j2\pi(ux/M + vy/N)}$
  - Inverse transform: $f(x,y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} T(u,v) s(x,y,u,v)$
    - s(x,y,u,v): inverse transformation kernel
    - Eg. Discrete inverse Fourier transform: $s(x,y,u,v) = \frac{1}{MN} e^{j2\pi(ux/M + vy/N)}$
  - Kernel is separable if $r(x,y,u,v) = r_1(x,u) r_2(y,v)$
  - Kernel is symmetric if $r_1(x,u) = r_2(y,v)$

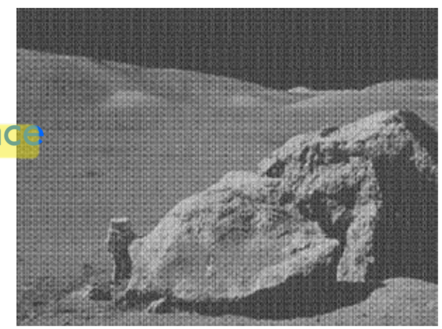
Normalize

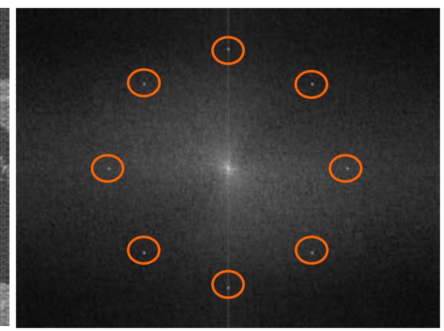
Image transforms (2)

- Eg. 2-D Fourier transform
  - Forward Fourier transform: $T(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux/M+vy/N)}$
  - Inverse Fourier transform: $f(x,y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} T(u,v)s(x,y,u,v)$

주기적인 노이즈 있을때

① Image corrupted by sinusoidal interference



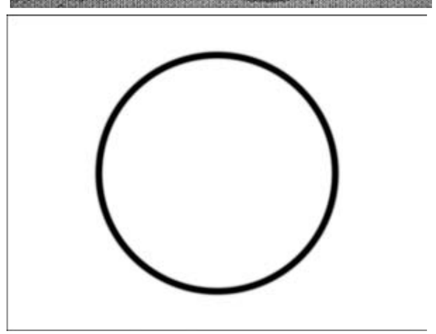


② noise 탐색

Magnitude of Fourier transform

○ Energy burst

③ Mask to remove the energy bursts



⇒ ④ inverse transform

Image transforms (3)

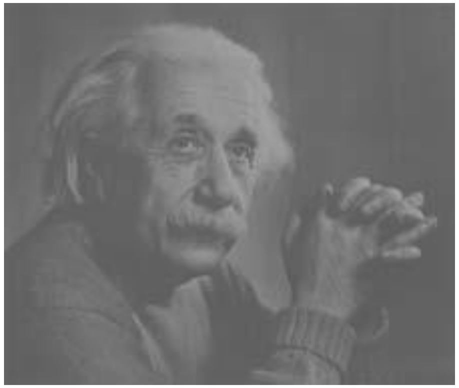
- 2-D linear transform
  - can be computed as cascade of 1-D transform if the kernel is separable and symmetric for a square image of size MxM
  - Matrix notation of transform
    - $\mathbf{T} = \mathbf{AFA}$
      - where F: MxM of f(x,y); A: MxM whose element $a_{ij} = r_1(i, j)$
      - 1-D transform of the result of 1-D transform($\mathbf{FA}$)
    - $\mathbf{F} = \mathbf{BAFAB} = \mathbf{BTB}$, if $\mathbf{B} = \mathbf{A}^{-1}$
    - Or, $\hat{\mathbf{F}} = \mathbf{BAFAB}$, if $\mathbf{B} \neq \mathbf{A}^{-1}$

Probabilistic methods

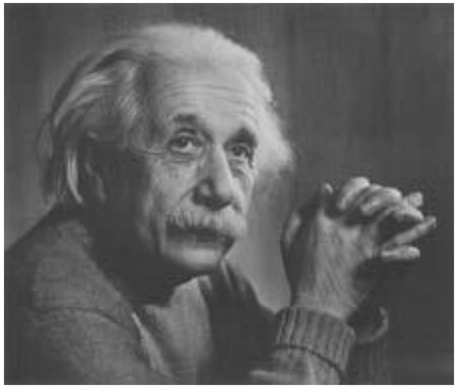
- Probabilistic moments
  - Probability density function: $p(z_k) = \frac{n_k}{MN}$
  - Nth moment of random variable z about mean
    - $\mu_n(z) = \sum_{k=0}^{L-1} (z_k - m)^n p(z_k)$
    - 0th moment: $\mu_0(z) = \sum_{k=0}^{L-1} p(z_k)$=1
    - 1st moment (mean): $m = \sum_{k=0}^{L-1} z_k p(z_k)$
    - 2nd moment (variance): $\sigma^2 = \sum_{k=0}^{L-1} (z_k - m)^2 p(z_k)$
      - Measure of the spread if the values of z about mean
      - Useful measure of image contrast
    - 3rd moment (skewness) ⇒ 첨도 $\mu >$
      - Positive: the intensities are biased to values higher than the mean
      - Negative: the intensities are biased to values lower than the mean $\mu <$
      - Zero: the intensities are distributed approximately equally on both sides of the mean

Probabilistic methods (2)

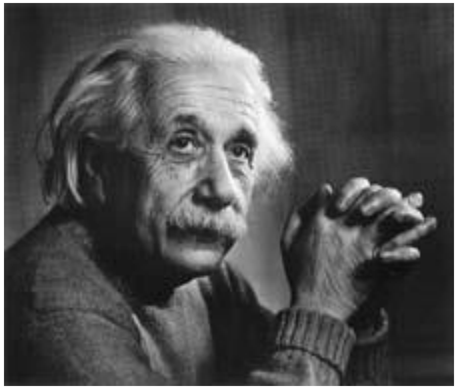
- Eg. Images of different variances



low contrast image
σ=14.3



medium contrast image
σ=31.6



high contrast image
σ=49.2