Week 4: Stream (2)

Instructor: Daejin Choi (djchoi@inu.ac.kr)

INCHEON
NATIONAL
UNIVERSITY

* Most slides are used from www.mmds.org with small updates

Revisit: Operations on Data Streams

- Stream Processing: choose **a subset of input streams**
  - **Sampling data from a stream**
    - Construct a random sample
  - **Queries over sliding windows**
    - Number of items of type ***x*** in the last ***k*** elements of the stream

Week 3

  - **Filtering a data stream**
    - Select elements with property ***x*** from the stream
  - **Counting distinct elements**
    - Number of distinct elements in the last ***k*** elements of the stream
  - **Estimating moments**
    - Estimate avg./std. dev. of last ***k*** elements
  - …

Week 4

Filtering a Data Stream: Bloom Filter

Filtering Data Streams

- **Each element of data stream is a tuple**
- Given a list of keys **S**
- **Determine which tuples of stream are in *S***
  - NOTE: it's different from user-based sampling
- **Obvious solution: Hash table**
  - But suppose we **do not have enough memory** to store all of ***S*** in a hash table
    - E.g., we might be processing millions of filters on the same stream

Example Application: Email Spam Filtering

- We know 1 billion "benign" email addresses

- If an email comes from one of these, it is **NOT** spam

  → All benign emails should be determined as benign address

  → Classifying spam as benign is acceptable

Bloom Filter Can Be a Solution

- Bloom Filter consists of
  - An array of bits
  - A number of hash functions

- Two processes
  - Setup process
    - Initially, all bits in the given array set **0**s
    - For each element x in S, set the bits $h(x)$ to 1, for each hash function h
  - Lookup process
    - For input y, check if the bits $h(y)$ set to 1, for each hash function h
    - Accept if all bits are set to 1, otherwise reject

Starting from Bloom Filter with 1 Hash

- **Given a set of keys *S* that we want to filter**
- Create a **bit array *B*** of ***n*** bits, initially all ***0*s**
- Choose a **hash function *h*** with range **[*0*,*n*)**
- Hash each member of ***s*** ∈ ***S*** to one of ***n*** buckets, and set that bit to **1**, i.e., ***B[h(s)]=1***
- Hash each element ***a*** of the stream and output only those that hash to bit that was set to **1**
  - **Output *a* if B[h(a)] == 1**

lookup

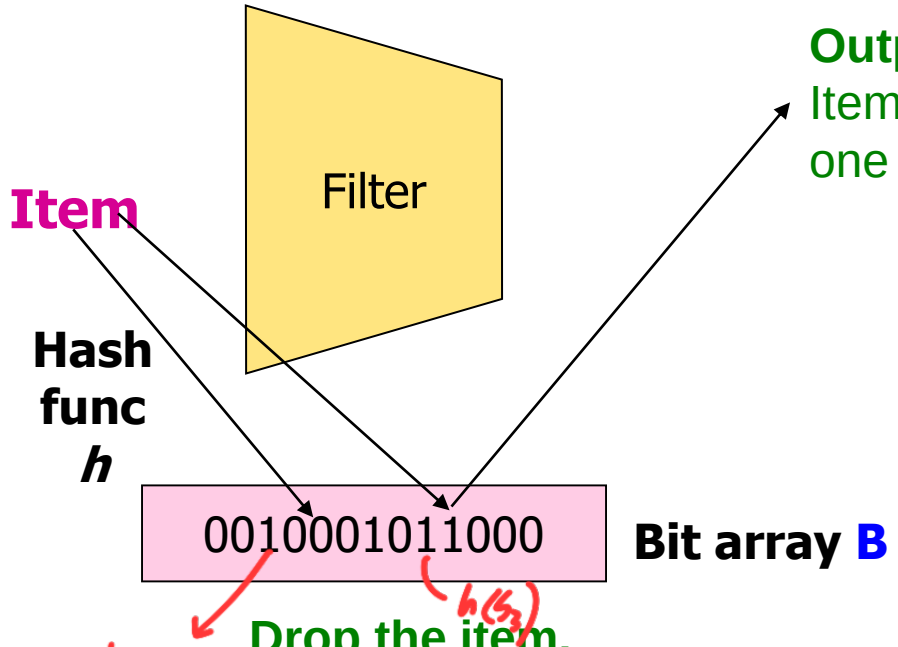
setup

B = [0 0 1 0 0 0]

h(s)=2

2

Bloom Filter Processing



Output the item since it may be in *S*.
Item hashes to a bucket that at least one of the items in **S** hashed to.

Drop the item.
It hashes to a bucket set to **0** so it is surely not in ***S***.

핵심적 특징

FP는 있고 (= 긍정이라 예측, 사실은 부정)

(부정이라 예측, 실제로 긍정)
FN은 없다

Creates false positives but no false negatives

- If the item is in ***S*** we surely output it, if not we may still output it

Example

- Use n = **11** bits for our filter.
- Stream elements = integers.
- Use two hash functions:
  - $h_1(x)$ =
    - Take odd-numbered bits from the right in the binary re presentation of x.
    - Treat it as an integer i.
    - Result is i modulo 11.
  - $h_2(x)$ = same, but take **even**-numbered bits.

Example – Continued

Stream element	h_1	h_2	Filter contents
			00000000000
25 = 11001	5	2	00100100000
159 = 10011111	7	0	10100101000
585 = 1001001001	9	7	10100101010

Note: bit 7 was already 1.

Example: Lookup

- Suppose we have the same Bloom filter as be fore, and we have set the filter to 10100101010.
- Lookup element y = 118 = 1110110 (binary).
- $h_1(y)$ = 14 modulo 11 = 3.
- $h_2(y)$ = 5 modulo 11 = 5.
- Bit 5 is 1, but bit 3 is 0, so we are sure y is n ot in the set.

Return to Our Original Question

- **|S| = 1 billion (benign) email addresses**
  |B| = 1GB = 8 billion bits

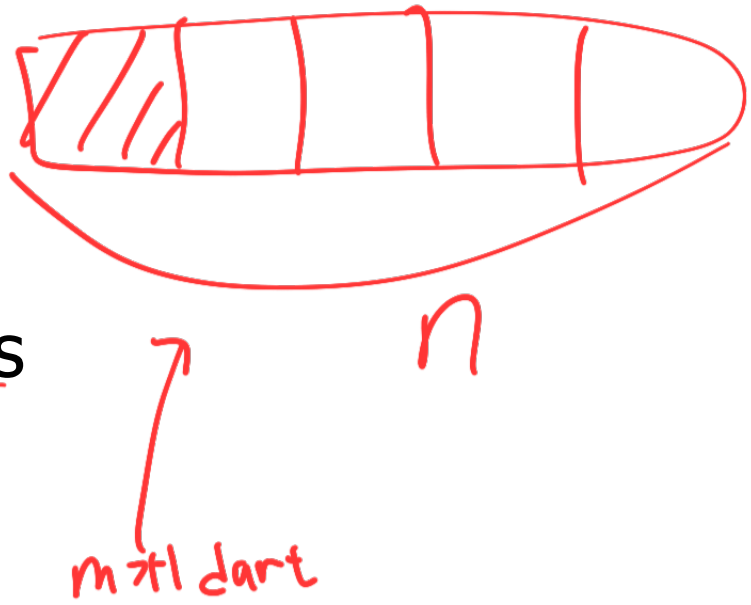
- If the email address is in ***S***, then it surely hashes to a bucket that has the big set to **1**,
  so it always gets through (***no false negatives***)

- However, the UNKNOWN email address can be determined a s benign (***false positives***) → We need to estimate

Analysis: Throwing Darts (1)

- **More accurate analysis for the number of false positives**

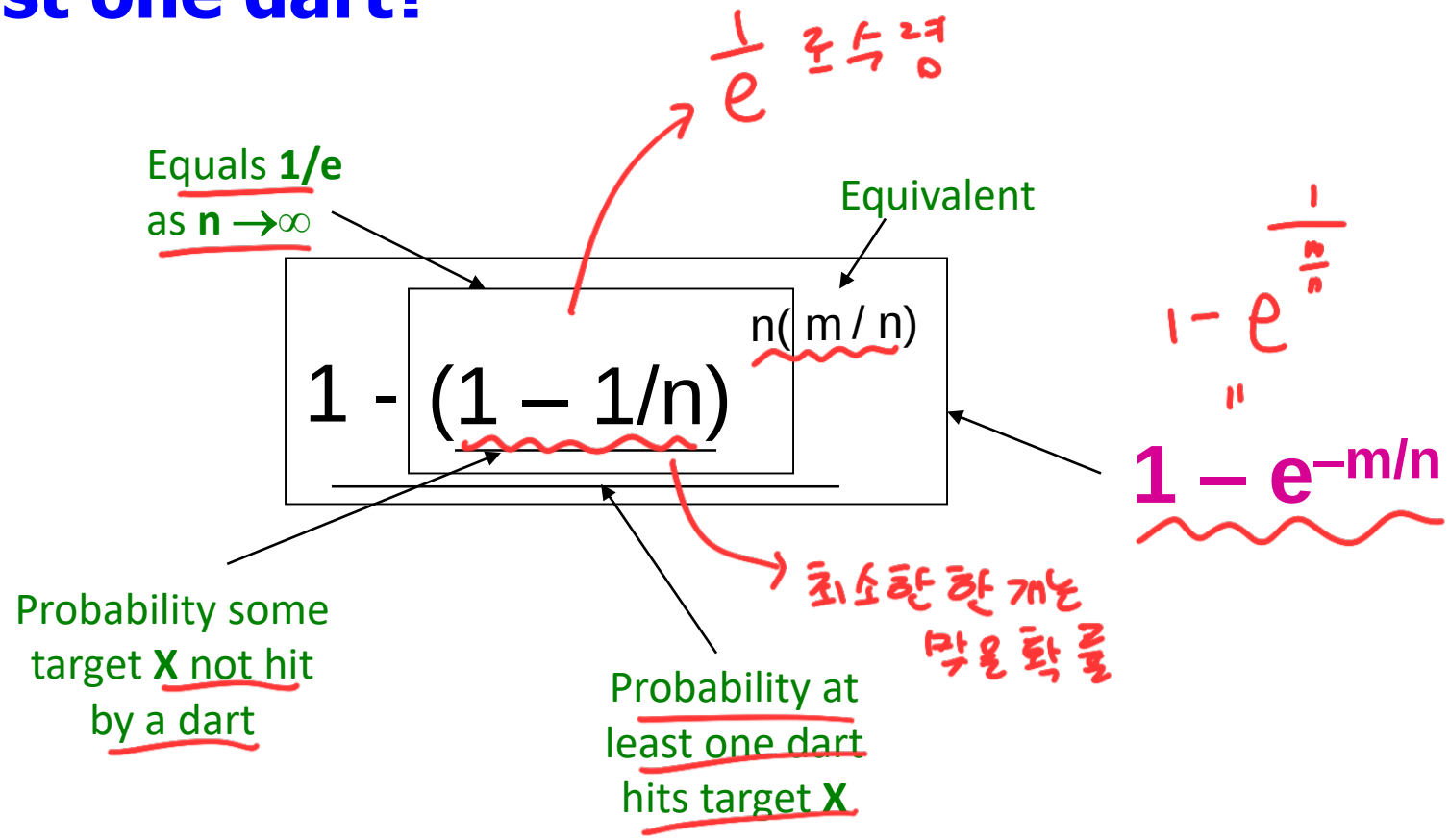
- **Consider:** If we throw ***m*** darts into ***n*** equally likely targets, **what is the probability that a target gets at least one dart?**



- **In our case:**
  - **Targets** = bits/buckets
  - **Darts** = input items

Analysis: Throwing Darts (2)

- We have ***m*** darts, ***n*** targets
- **What is the probability that a target gets at least one dart?**



Analysis: Throwing Darts (3)

- **Fraction of 1s (over input) in the array B**
  ≈ probability of false positive = $1 - e^{-m/n}$

- **Example: 10^9** darts, **$8 \cdot 10^9$** targets
  - Fraction of **1s** in **B = $1 - e^{-1/8} = 0.1175$**
    - Compare with our earlier estimate: **1/8 = 0.125**

Bloom Filter with More Hash Functions

- Consider: $|S| = m$, $|B| = n$
- Use k independent hash functions $h_1, \ldots, h_k$
- **Initialization:**
  - Set **B** to all **0s**

    (**note:** we have a single array B!)
  - Hash each element $s \in S$ using each hash function h_i, set $B[h_i(s)] = 1$ (for each $i = 1, \ldots, k$)
- **Run-time:** look up
  - When a stream element with key x arrives
    - If $B[h_i(x)] = 1$ for all $i = 1, \ldots, k$ then declare that x is in S
      - That is, x hashes to a bucket set to **1** for every hash function $h_i(x)$
    - Otherwise discard the element x

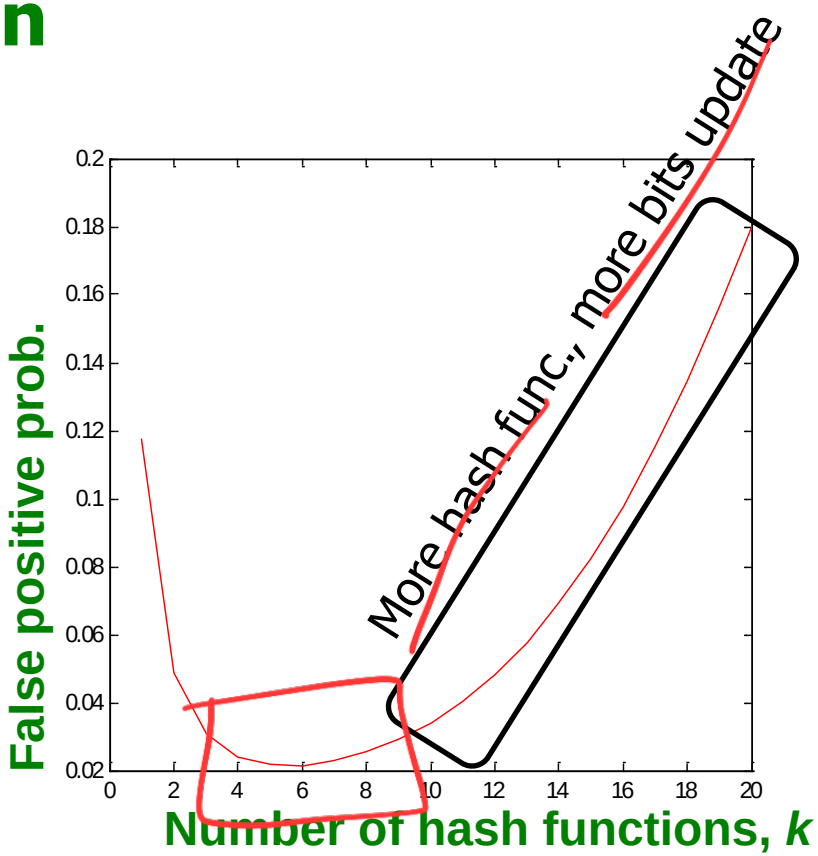
Bloom Filter – Analysis

- **What fraction of the bit vector B are 1s?**
  - Throwing $k \cdot m$ darts at n targets
  - So fraction of **1**s is $(1 - e^{-km/n})$
- But we have k independent hash functions and we only let the element x through **if all** k hash element x to a bucket of value **1**
- So, **false positive probability** $= (1 - e^{-km/n})^k$

$\therefore$ FP ⇓ $\because$ 검증 과정이 늘어남. $\left(1 - e^{-\frac{km}{n}}\right)^k$

Bloom Filter – Analysis (2)

- ***m* = 1 billion, *n* = 8 billion**
  - **k = 1**: $(1 - e^{-1/8}) = 0.1175$
  - **k = 2**: $(1 - e^{-1/4})^2 = 0.0493$



- **What happens as we keep increasing *k*?**

- "Optimal" value of ***k***: ***n/m* ln(2)**
  - **In our case:** Optimal **k = 8 ln(2) = 5.54 ≈ 6**
    - **Error at k = 6**: $(1 - e^{-1/6})^2 = 0.0235$

Bloom Filter: Wrap-up

- **Bloom filters guarantee no false negatives, and use limited memory**
  - Great for pre-processing before more expensive checks

- **Suitable for hardware implementation**
  - Hash function computations can be parallelized

Counting Distinct Elements

Flajolet-Martin Approach

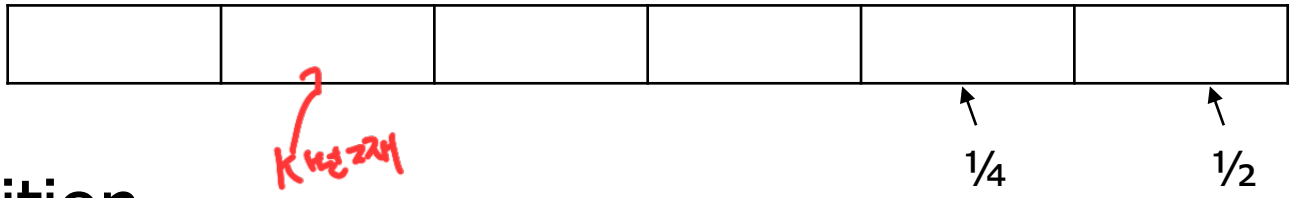
- Pick a hash function ***h*** that maps each of the ***N*** elements to at least $\log_2 N$ bits

- For each stream element ***a***, let ***r(a)*** be the number of trailing **0s** in ***h(a)***
  - **r(a)** = position of first 1 counting from the right
    - E.g., say ***h(a) = 12***, then ***12*** is ***1100*** in binary, so ***r(a) = 2***
- Record ***R* = the maximum *r(a)* seen**
  - **R** $= \max_a$ **r(a)**, over all the items ***a*** seen so far

- **Estimated number of distinct elements =** 2^R

Why It Works: Intuition

- Hash each item x to a bit, using exponential distribution
  - ½ map to bit 0, ¼ map to bit 1, …



- Intuition
  - The 0th bit is accessed with prob. ½
  - The 1st bit is accessed with prob. ¼
  - …
  - The kth bit is accessed with prob. $O(1/2^k)$
- Thus, if the kth bit is set, then we know that an event with prob. $O(1/2^k)$ happened

  → We inserted distinct items $O(2^k)$ times

Why It Works: More formally

- **Goal: showing that probability of finding a tail of *r* zeros:**
  - **Goes to 1 if $m \gg 2^r$**
  - **Goes to 0 if $m \ll 2^r$**

  where m is the number of distinct elements seen so far in the stream

$2^R \approx m$ ← 목적

→ **The goal also means that 2^R will almost always be around *m!***

Why It Works: More formally

- **The probability that a given *h*(*a*) ends in at least *r* zeros is 2^{-r}**
  - **h(a)** hashes elements uniformly at random
  - Probability that a random number ends in at least ***r*** zeros is $\mathbf{2^{-r}}$

hash 값 ⇒ 동일한 확률로 나타날 것임

$\ldots 100\ldots00 \rightarrow \frac{1}{2^r}$

- Then, the probability of **NOT** seeing a tail of length ***r*** among ***m*** elements:

$$\left(1 - 2^{-r}\right)^m$$

m개 만큼 element 들어왔을때 0이 r개보다 적은 숫자로 들어왔을 확률

Prob. all end in fewer than ***r*** zeros.

Prob. that given **h(a)** ends in fewer than ***r*** zeros

Why It Works: More formally

- **Note:** $(1-2^{-r})^m = (1-2^{-r})^{2^r (m2^{-r})} \approx e^{-m2^{-r}}$
- **Prob. of NOT finding a tail of length *r* is:**
  - If ***m* << 2^r^**, then prob. tends to **1**
    - $(1-2^{-r})^m \approx e^{-m2^{-r}} = 1$ as $\mathbf{m/2^r \to 0}$
    - So, the probability of finding a tail of length ***r*** tends to **0**
  - If ***m* >> 2^r^**, then prob. tends to **0**
    - $(1-2^{-r})^m \approx e^{-m2^{-r}} = 0$ as $\mathbf{m/2^r \to \infty}$
    - So, the probability of finding a tail of length ***r*** tends to **1**

- **Thus, 2^R will almost always be around *m*!**

Computing Moments

Generalization: Moments

- **Suppose a stream has elements chosen from a set *A* of *N* values**
- **Let m_i be the number of times value *i* occurs in the stream**
- **The k^{th} *moment* (적률) is**

$$\sum_{i \in A} (m_i)^k$$

Special Cases

$$\sum_{i \in A} (m_i)^k$$

k=0

- **0th moment =** number of distinct elements
  - The problem just considered

k=1

- **1st moment =** count of the numbers of elements = length of the stream
  - Easy to compute

k=2

- **2nd moment =** ***surprise number S*** **=** a measure of how uneven the distribution is

고르면 ↓ 아고르면 ↑

↳ 얼마나 불균형한지

skew 첨도

Example: Surprise Number

- **Stream of length 100**
- **11 distinct values**

- Item counts: **10, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9**
  Surprise $S = 910$ $= 10^2 + 9^2 \times 10 = 910$

- Item counts: **90, 1, 1, 1, 1, 1, 1, 1 ,1, 1, 1**
  Surprise $S = 8{,}110$

$= 90^2 + 1^2 \times 10$

Problem Definition

- **Q**: Given a stream, how can we estimate k-th moments efficiently, with small memory space?

- **A**: AMS method

AMS Method

[Alon, Matias, and Szegedy]

- **AMS method works for all moments**
- **Gives an unbiased estimate** → 기대값으로 추정
- **We will just concentrate on the 2nd moment *S***
- **We pick and keep track of many variables *X*:**
  - For each variable ***X*** we store ***X.el*** and ***X.val***
    - ***X.el*** corresponds to the item ***i***
    - ***X.val*** corresponds to the **count** of item ***i***
  - Note this requires a count in main memory, so number of ***X***s is limited
- **Our goal is to compute** $S = \sum_i m_i^2$

estimate

One Random Variable (X)

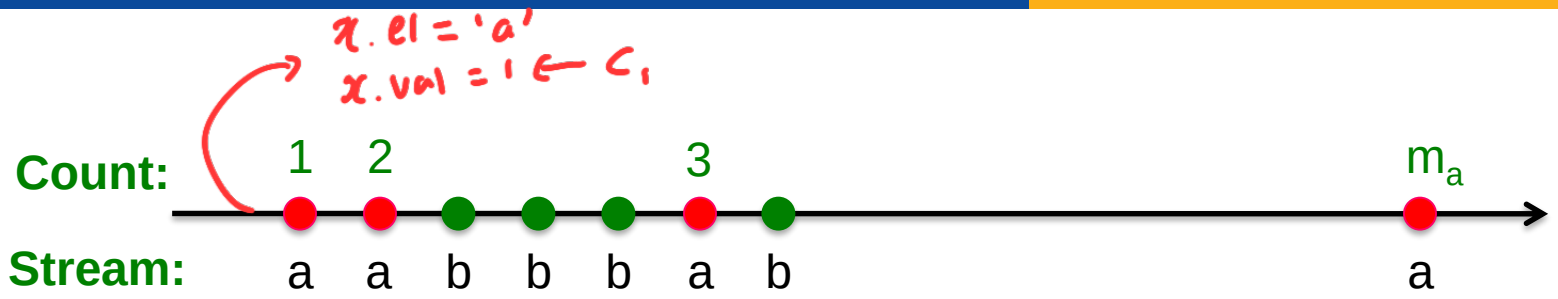
- **How to set *X.val* and *X.el*?**
  - Assume stream has length ***n*** (we relax this later)
  - Pick some random time ***t*** (***t<n***) to start, so that any time is equally likely
  - Let at time ***t*** the stream have item ***i***. ***We set X.el = i***
  - Then we maintain count ***c*** (***X.val = c***) of the number of ***i*** in the stream starting from the chosen time ***t***

- **Then the estimate of the 2nd moment ($\sum_i m_i^2$) is:**

$$S = f(X) = n\,(2 \cdot c - 1)$$

  - Note, we will keep track of multiple **X**s, (**X_1, X_2,… X_k**) and our final estimate will be $S = 1/k \sum_j^k f(X_j)$

Expectation Analysis



- **2nd moment is $S = \sum_i m_i^2$**
- c_t … number of times item at time t appears from time **t** onwards ($c_1=1$, $c_2=2$, $c_3=1$)
- $E[f(X)] = \frac{1}{n}\sum_{t=1}^{n} n(2c_t - 1)$

$$= \frac{1}{n}\sum_i n\,(1 + 3 + 5 + \cdots + 2m_i - 1)$$

m_i … total count of item i in the stream (we are assuming stream has length **n**)

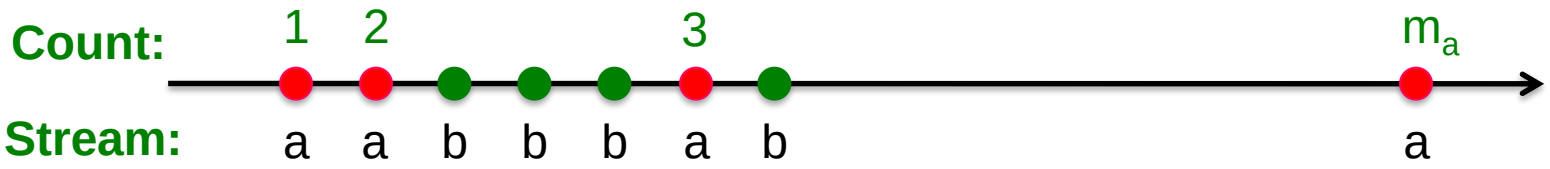
Group times by the value seen

Time t when the last i is seen ($c_t=1$)

Time t when the penultimate i is seen ($c_t=2$)

Time t when the first i is seen ($c_t=m_i$)

Expectation Analysis



- $E[f(X)] = \frac{1}{n}\sum_i n\,(1 + 3 + 5 + \cdots + 2m_i - 1)$
  - Little side calculation: $(1 + 3 + 5 + \cdots + 2m_i - 1) = \sum_{i=1}^{m_i}(2i - 1) = 2\frac{m_i(m_i+1)}{2} - m_i = (m_i\ )^2$
- **Then** $\boldsymbol{E[f(X)] = \frac{1}{n}\sum_i\ n\,(m_i)^2}$

- **So,** $\mathbf{E[f(X)] = \sum_i (m_i)^2 = S}$
- **We have the second moment (in expectation)!**

Higher-Order Moments

- **For estimating k^{th} moment we essentially use the same algorithm but change the estimate:**
  - For **k=2** we used $n\,(2 \cdot c - 1)$
  - For **k=3** we use: $n\,(3 \cdot c^2 - 3c + 1)$ (where **c=X.val**)
- **Why?**
  - **For k=2:** Remember we had $(1 + 3 + 5 + \cdots + 2m_i - 1)$ and we showed terms ***2c-1*** (for **c=1,…,m**) sum to m^2
    - $\sum_{c=1}^{m} 2c - 1 = \sum_{c=1}^{m} c^2 - \sum_{c=1}^{m} (c-1)^2 = m^2$
    - **So:** $2c - 1 = c^2 - (c-1)^2$
  - **For k=3:** $c^3 - (c-1)^3 = 3c^2 - 3c + 1$
- **Generally:** Estimate $= n\,(c^k - (c-1)^k)$

Combining Samples

- **In practice:**
  - Compute $f(X) = n(2c - 1)$ for as many variables X as you can fit in memory
  - Average them in groups
  - Take median of averages

- **Problem: Streams never end**
  - We assumed there was a number n, the number of positions in the stream
  - But real streams go on forever, so n is a variable – the number of inputs seen so far

Streams Never End: Fixups

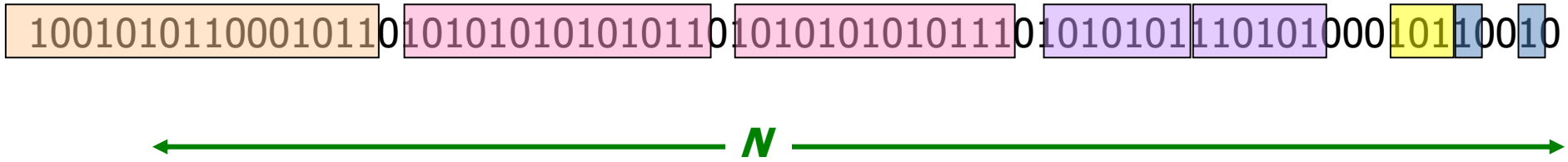
- The variables ***X*** have ***n*** as a factor – keep ***n*** separately; just hold the count in ***X***
- Suppose we can only store ***k*** counts. We must throw some ***X***s out as time goes on:
  - **Objective:** Each starting time ***t*** is selected with probability ***k/n***
  - **Solution: (fixed-size sampling!)**
    - Choose the first ***k*** times for ***k*** variables
    - When the n^{th} element arrives (***n*** > ***k***), choose it with probability ***k/n***
    - If you choose it, throw one of the previously stored variables **X** out, with equal probability

Counting Itemsets

Counting Itemsets

- **New Problem:** Given a stream, which items appear more than *s* times in the window?

- **Possible solution:** Think of the stream of baskets as one binary stream per item
  - **1** = item present; **0** = not present
  - Use **DGIM** to estimate counts of **1**s for all items



Extensions

- **In principle, you could count frequent pairs or even larger sets the same way**
  - **One stream per itemset**
  - E.g., for a basket {i, j, k}, assume 7 independent streams: (i) (j) (k) (i, j) (i, k) (j, k) (i, j, k)

↳ $2^3 - 1$

↳ itemset 이 몇번 나타나는지

- **Drawbacks:**
  - **Number of itemsets is way too big**

Exponentially Decaying Windows

- **Exponentially decaying windows: A heuristic for selecting likely frequent item(sets)**
  - **What are "currently" most popular movies?**
    - Instead of computing the raw count in last ***N*** elements
    - Compute a **smooth aggregation** over the whole stream

element 당 점수를 매김

- If stream is $a_1, a_2, \ldots$ and we are taking the sum of the stream, take the answer at time t to be: $= \sum_{i=1}^{t} a_i (1-c)^{t-i}$ ⇒ 점점 줄어든다
  - **c** is a constant, presumably tiny, like 10^{-6} or 10^{-9}

먼저 들어온 item = weight ⇑

- **When new a_{t+1} arrives:**
  Multiply current sum by **(1-c)** and add a_{t+1}

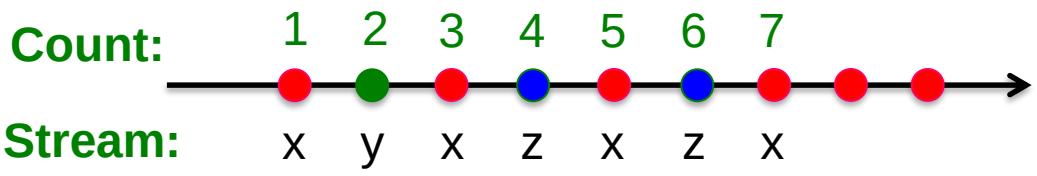
Example: Counting Items

- If each a_i is an "item" we can compute the **characteristic function** of each possible item x as an Exponentially Decaying Window
  - That is: $\sum_{i=1}^{t} \delta_i \cdot (1-c)^{t-i}$ → 0 or 1
    where $\delta_i = 1$ if $a_i = x$, and **0** otherwise
  - Imagine that for each item x we have a binary stream (**1** if x appears, **0** if x does not appear)
  - New item x arrives:
    - Multiply all counts by **(1-c)**
    - Add **+1** to count for element x

$(1-c)^{?}+1$ 새로운게 들어오면 weight 급격히 증가

- **Call this sum the "weight" of item x**

Example: Counting Items



$$\sum_{i=1}^{t} \delta_i \cdot (1-c)^{t-i}$$

Assume c = 0.2, Keep items with weights >= 1/2

- (T1) x: 1
- (T2) x: 0.8*1, y: 1
- (T3) x: 0.8*0.8 + 1, y: 0.8*1
- (T4) x: 0.8*1.64, y: 0.8*0.8, z = 1
- (T5) x: 1.312+1, y: 0.8*0.64=0.512, z: 0.8*1
- (T6) x: 0.8*2.312, y: 0.8*0.512, z:0.8*0.8
  - Remove y
- (T7) x: 0.8*1.8496+1, z: 0.8*0.64
- …

Example: Counting Items

- **What are "currently" most popular movies?**
- **Suppose we want to find movies of weight > ½** = threshold
  - **Important property:** Sum over all weights $\sum_t (1-c)^t$ is $1/[1-(1-c)] = \mathbf{1/c}$
- **Thus:**
  - There cannot be more than **2/c** movies with weight of **½**
- So, **2/c** is a limit on the number of movies being counted at any time

⇒ element의 개수는 $\frac{2}{c}$ 보다 적다

Extension to Itemsets

- **Count (some) itemsets**
  - **What are currently "hot" itemsets?**
  - **Problem:** Too many itemsets to keep counts of all of them in memory

item 들의 집합

- **When a basket B comes in:**
  - Multiply all counts by **(1-c)**
  - For uncounted items in **B**, create new count
  - Add **1** to count of any item in **B** and to any **itemset** contained in **B** that is already being counted
  - **Drop counts < ½**
  - Initiate new counts (next slide)

Initiation of New Counts

- Start a count for an itemset $S \subseteq B$ if every proper subset of S had a count prior to arrival of basket B
  - **Intuitively:** If all subsets of **S** are being counted this means they are "**frequent/hot**" and thus S has a potential to be "**hot**"
- **Example:**
  - Start counting S=**{i, j}** iff both **i** and **j** were counted prior to seeing B
  - Start counting S=**{i, j, k}** iff **{i, j}**, **{i, k}**, and **{j, k}** were all counted prior to seeing B

Summary

- **Filtering a data stream:** Bloom Filter
- **Counting distinct elements:** Flajolet-Martin
  - 일반화
  - $m << 2^r$
  - ↳ 범위 내에 존재한다.
- **Computing moments**: AMS Method
  - $n((^k - (c-1)^k)$
- **Counting itemsets**: Exponentially Decaying Windows
  - weight 사용

Intermediate Summary

- We have covered **basic operations** for **big data processing**
  - How to deal with **large (static) data**? MapReduce
  - How to deal with **stream data**?
    - Sampling, sliding window, …
    - Estimation + summarized results

- In practice,
  - Processing frameworks are well developed (& easy to use!)
  - The opportunities for you to develop operations from the beginning are rare
  - However, understanding the problems and concept of solutions will be helpful to design

- Now we are moving to **Data Analysis**

Thank you!

Instructor: Daejin Choi (djchoi@inu.ac.kr)