1. Suppose that two variables x and z are statistically independent. Show that the mean and variance of their sum satisfies

$$\mathbb{E}[x+z] = \mathbb{E}[x] + \mathbb{E}[z]$$
$$\mathrm{var}[x+z] = \mathrm{var}[x] + \mathrm{var}[z].$$

(x와 z가 독립일 때 위 두식을 증명하세요.)

2. By setting the derivative of the log-likelihood function

$$\ln p(x|\mu,\sigma^2) = -\frac{1}{2\sigma^2}\sum_{n=1}^{N}(x_n-\mu)^2 - \frac{N}{2}\ln\sigma^2 - \frac{N}{2}\ln(2\pi) \quad \text{(EQ: 2-1)}$$

with respect to μ and σ^2 equal to zero, verify the following results.

$$\mu_{ML} = \frac{1}{N}\sum_{n=1}^{N} x_n \qquad \text{(EQ: 2-2)}$$

$$\sigma_{ML}^2 = \frac{1}{N}\sum_{n=1}^{N}(x_n-\mu_{ML})^2 \qquad \text{(EQ: 2-3)}$$

(EQ: 2-1이 주어졌을 때, 이를 μ 와 σ^2로 미분한 결과가 0으로 설정함으로써, EQ: 2-2와 EQ: 2-3을 보이세요.)

3. Evaluate the Kullback-Leibler divergence

$$KL(p||q) = -\int p(x)\ln\left\{\frac{q(x)}{p(x)}\right\}dx$$

between two Gaussians $p(x) = \mathcal{N}(x|\mu,\sigma^2)$ and $q(x) = \mathcal{N}(x|m,s^2)$.

Tip: Use following properties.

$$\mathbb{E}[x^2] = \int x^2\mathcal{N}(x|\mu,\sigma^2)\,dx = \mu^2+\sigma^2$$

$$\mathbb{E}[x] = \int x\mathcal{N}(x|\mu,\sigma^2)\,dx = \mu$$

$$\int x\mathcal{N}(x|\mu,\sigma^2)\,dx = 1$$

(두 가우시안 분포에 대한 KL-divergence를 구하시오.)

4. Show that the entropy $H[x]$ of a Bernoulli distributed random binary variable x is given by

$$H[x] = -\mu \ln \mu - (1-\mu)\ln(1-\mu).$$

(Bernoulli 분포에 대한 entropy가 위 식처럼 나온다는 것을 보이세요.)