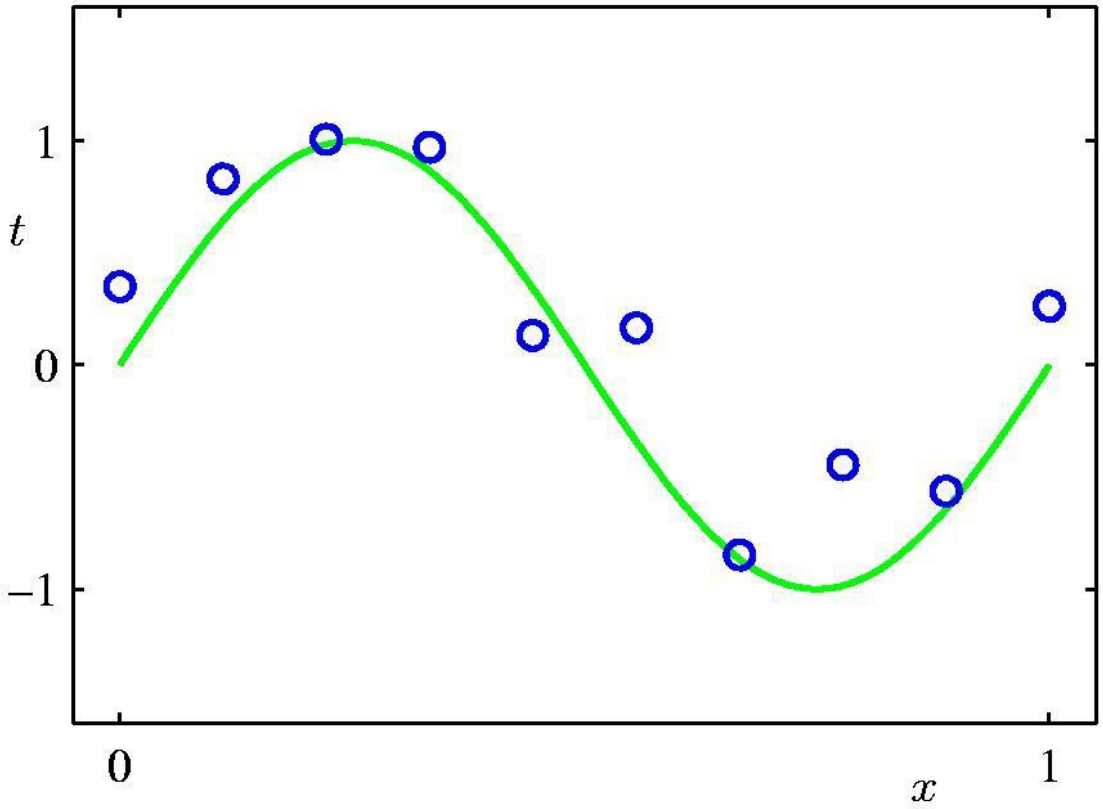
Artificial Intelligence (EI06024001)

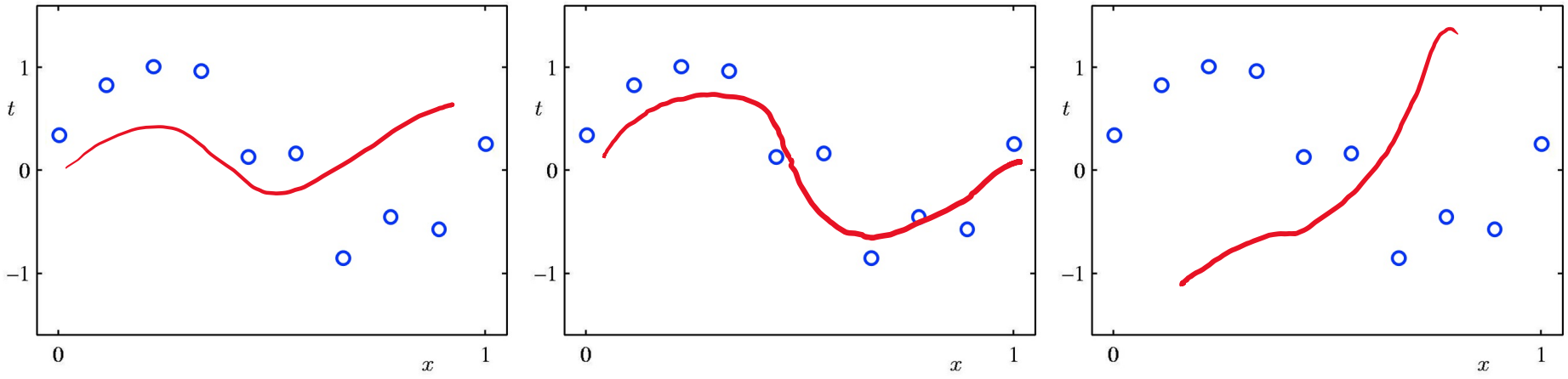
Polynomial Curve Fitting

Polynomial Curve Fitting Problem



$$y(x, \mathbf{w}) = w_0 + w_1 x + w_2 x^2 + \cdots + w_M x^M = \sum_{j=0}^{M} w_j x^j$$

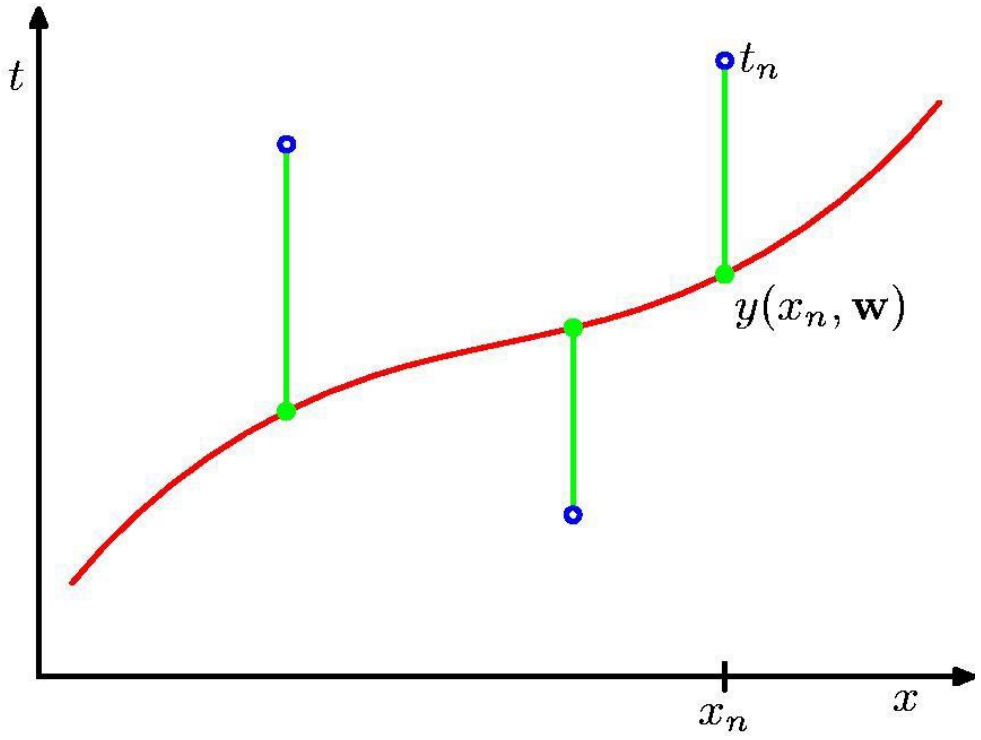
Polynomial Curve Fitting Problem



Q: How to determine which model is good?

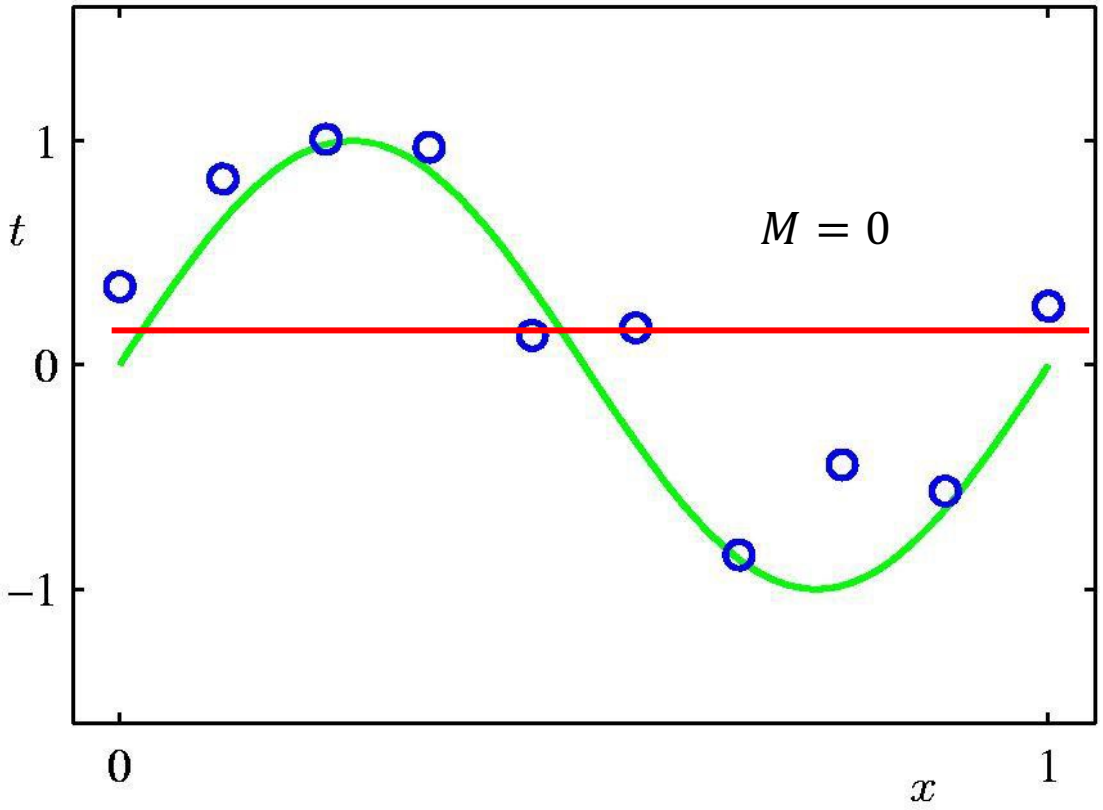
A: Use loss-function!

Sum-of-Squares Error Function

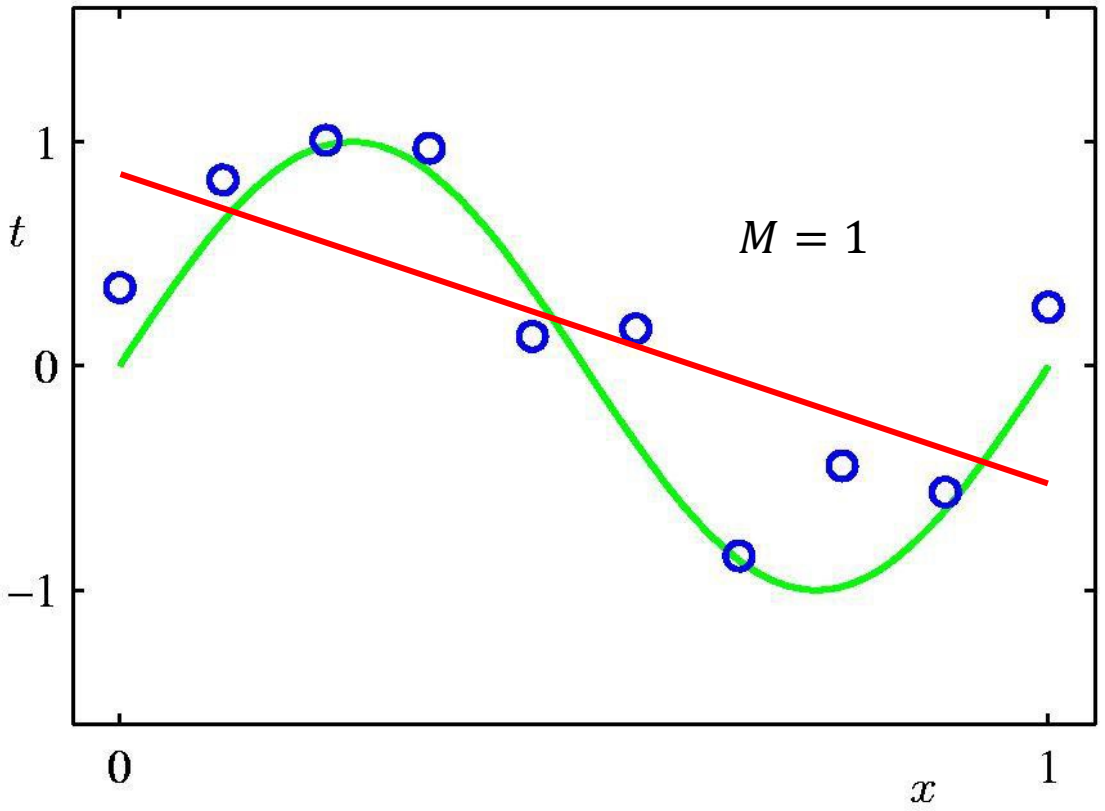


$$E(\mathrm{w}) = \frac{1}{2}\sum_{n=0}^{N}\{y(x_n, w) - t_n\}^2$$

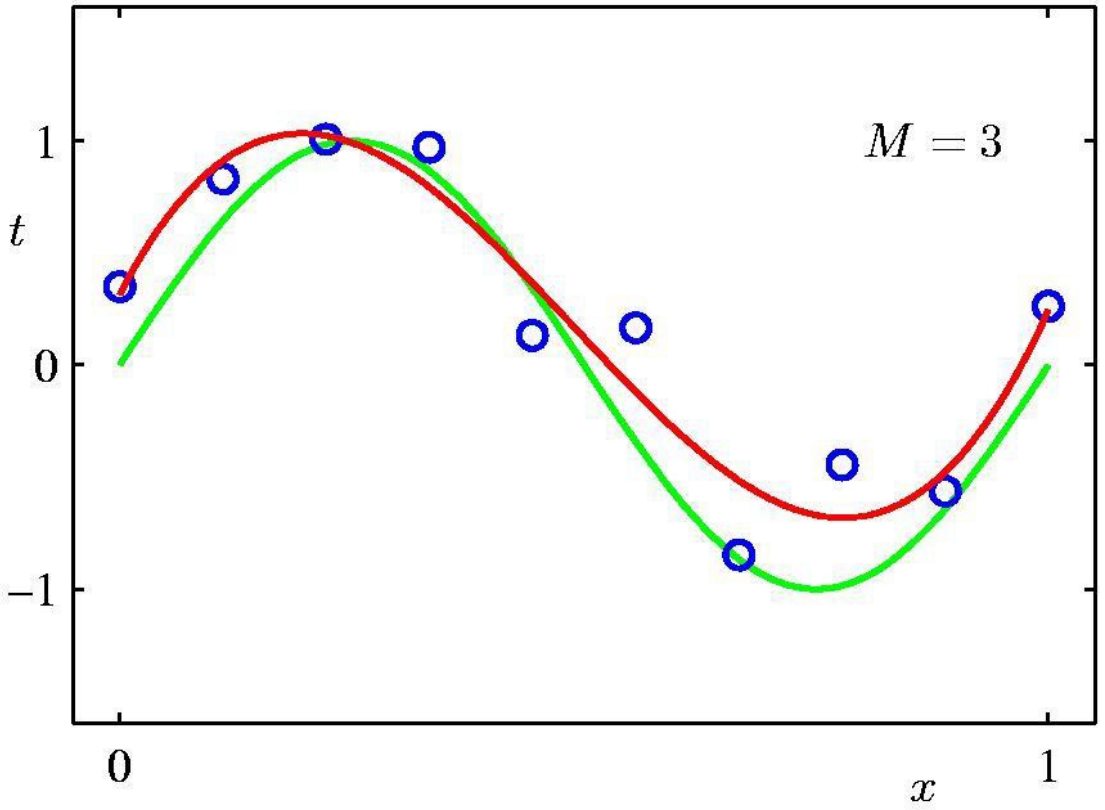
O^{th} Order Polynomial



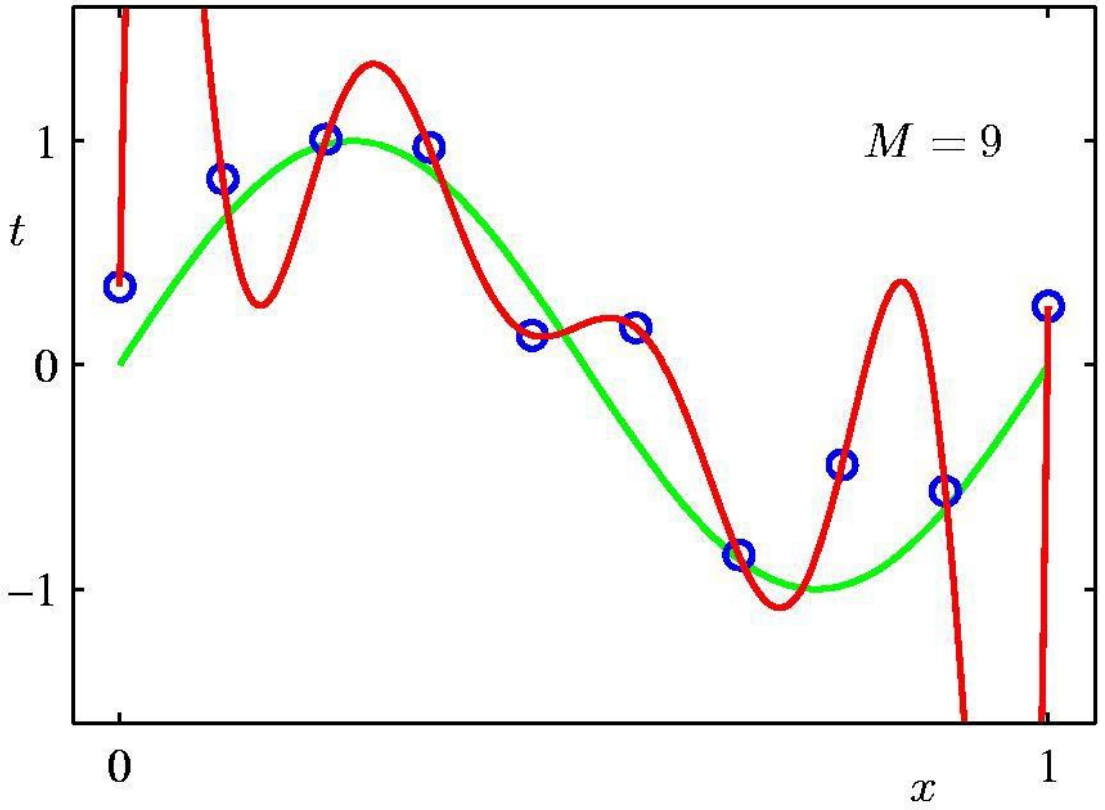
1th Order Polynomial



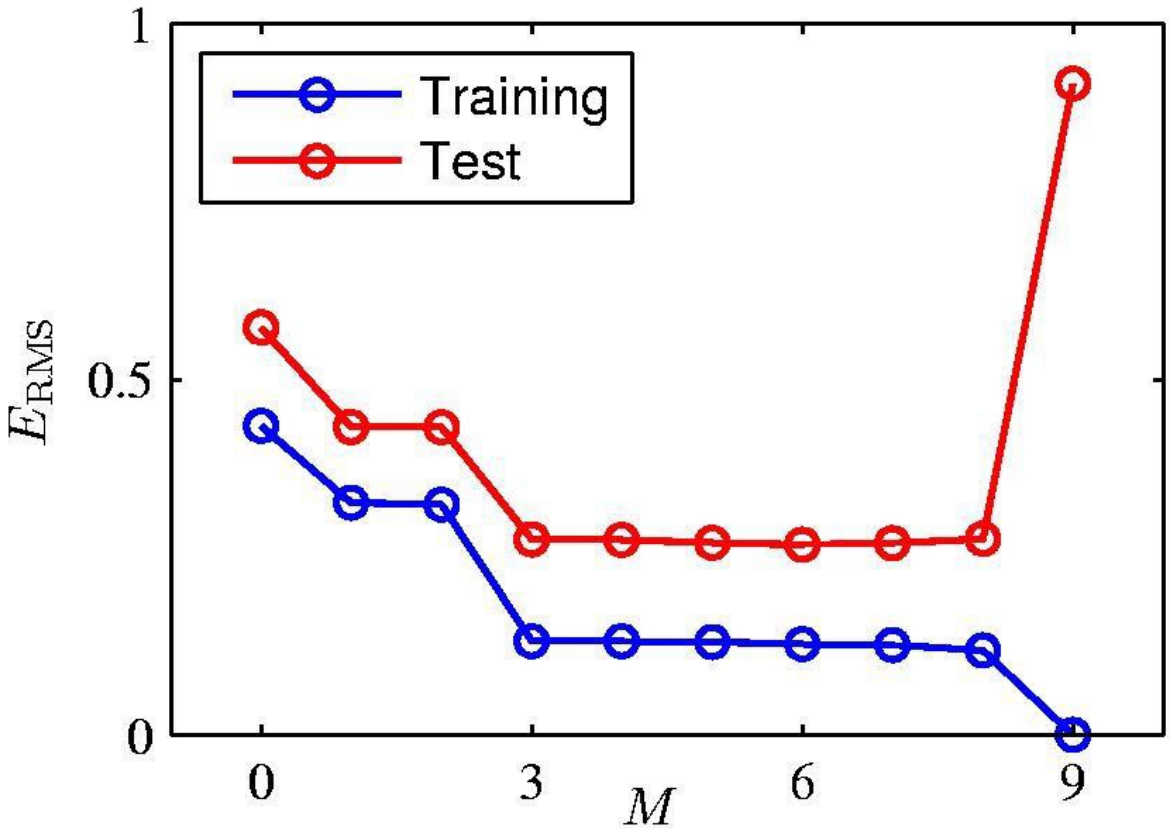
3rd Order Polynomial



9th Order Polynomial



Over-fitting



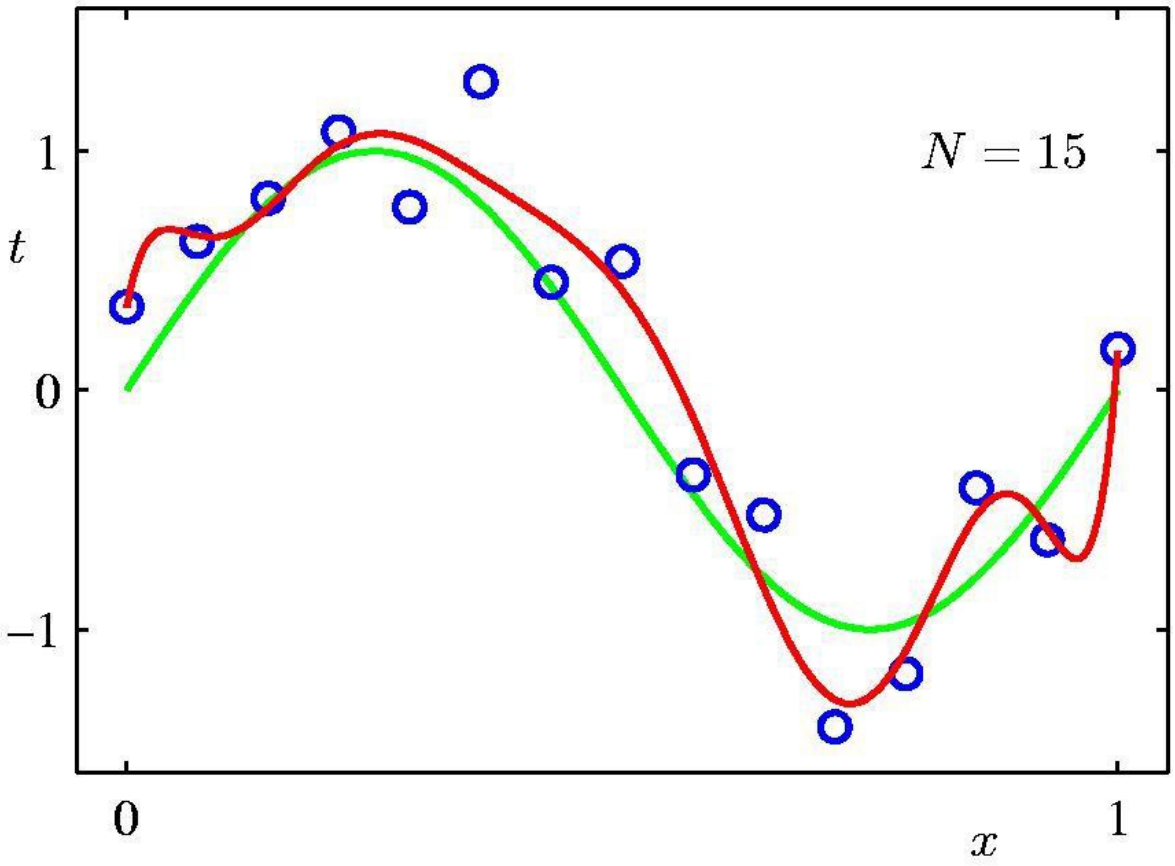
Root-Mean-Square (RMS) Error: $E_{\mathrm{RMS}} = \sqrt{2E(\mathrm{w}^*)/N}$

Polynomial Coefficients

	$M = 0$	$M = 1$	$M = 3$	$M = 9$
$w_0^\star$	0.19	0.82	0.31	0.35
$w_1^\star$		-1.27	7.99	232.37
$w_2^\star$			-25.43	-5321.83
$w_3^\star$			17.37	48568.31
$w_4^\star$				-231639.30
$w_5^\star$				640042.26
$w_6^\star$				-1061800.52
$w_7^\star$				1042400.18
$w_8^\star$				-557682.99
$w_9^\star$				125201.43

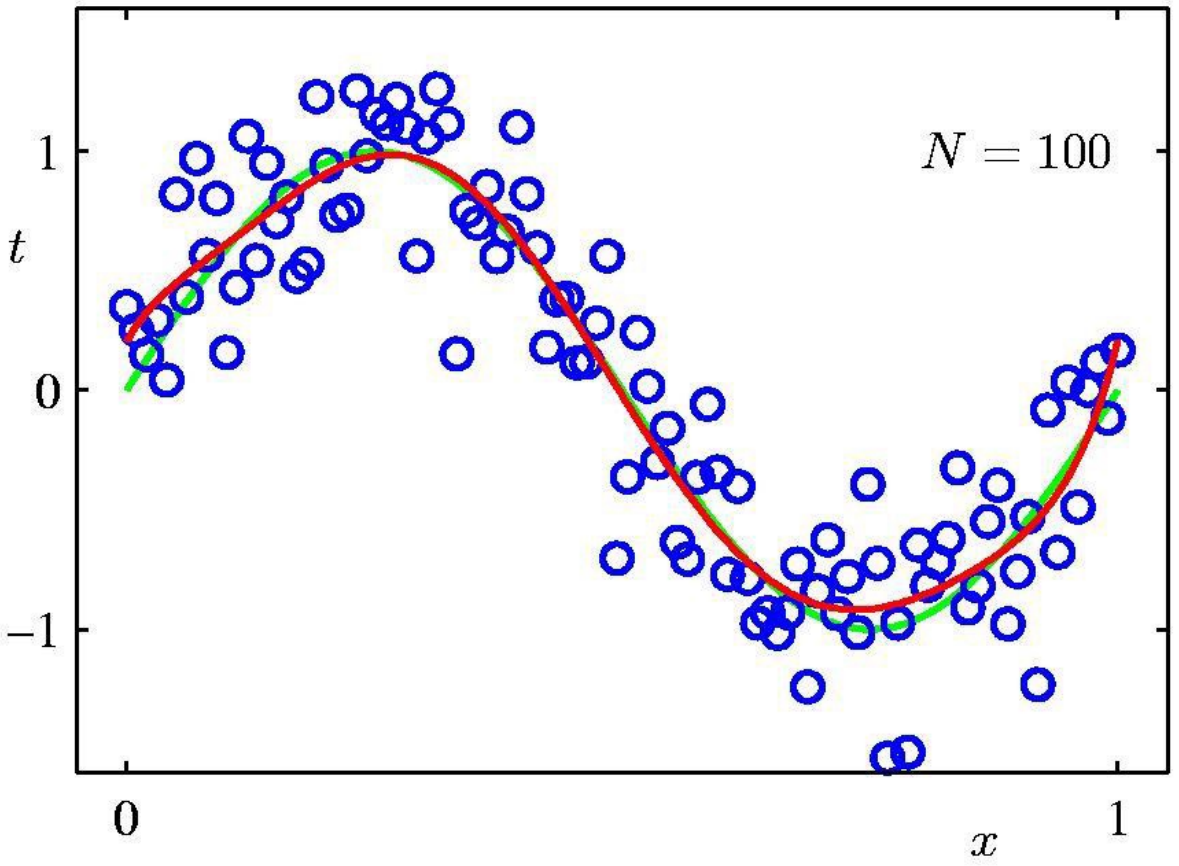
Data Set Size: $N = 15$

9th Order Polynomial



Data Set Size: $N = 100$

9th Order Polynomial



Model Complexity vs. Data Size

$y_1(x) = w_0 + w_1 x$

Model complexity ↑ Requires more train data!

$y_9(x) = w_0 + w_1 x + w_2 x^2 + \cdots + w_9 x^9$

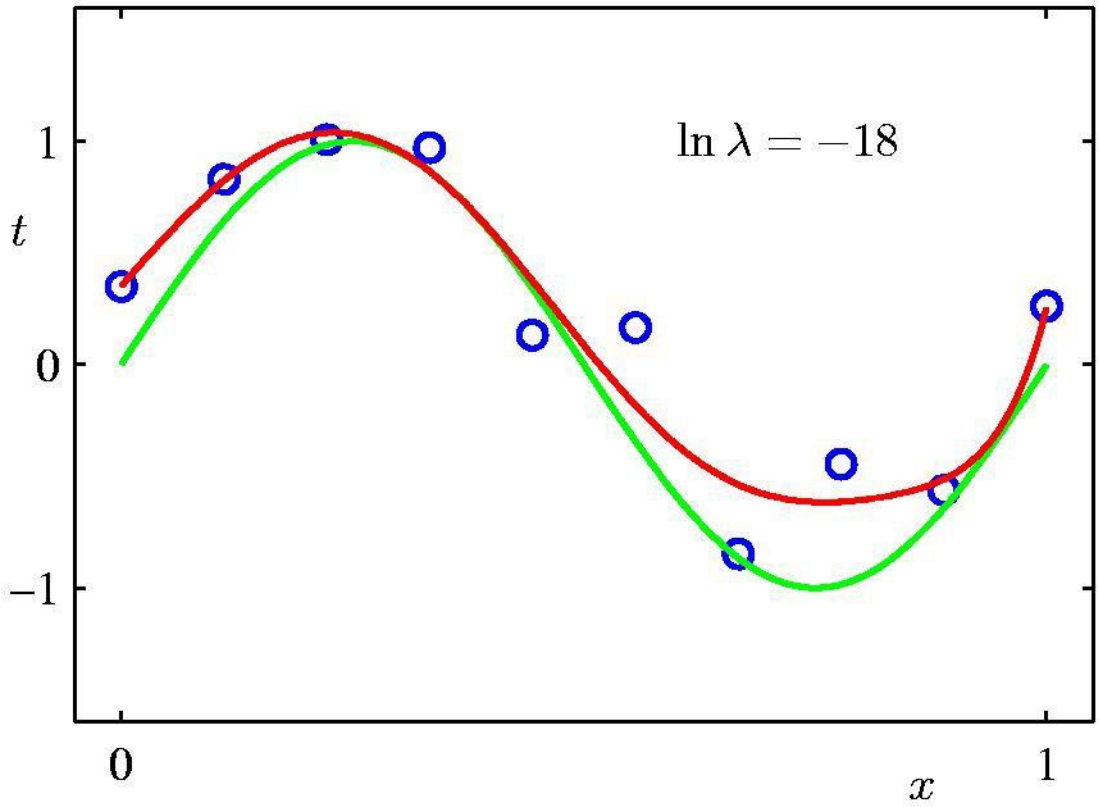
How to prevent overfitting when data size is fixed?

Regularization

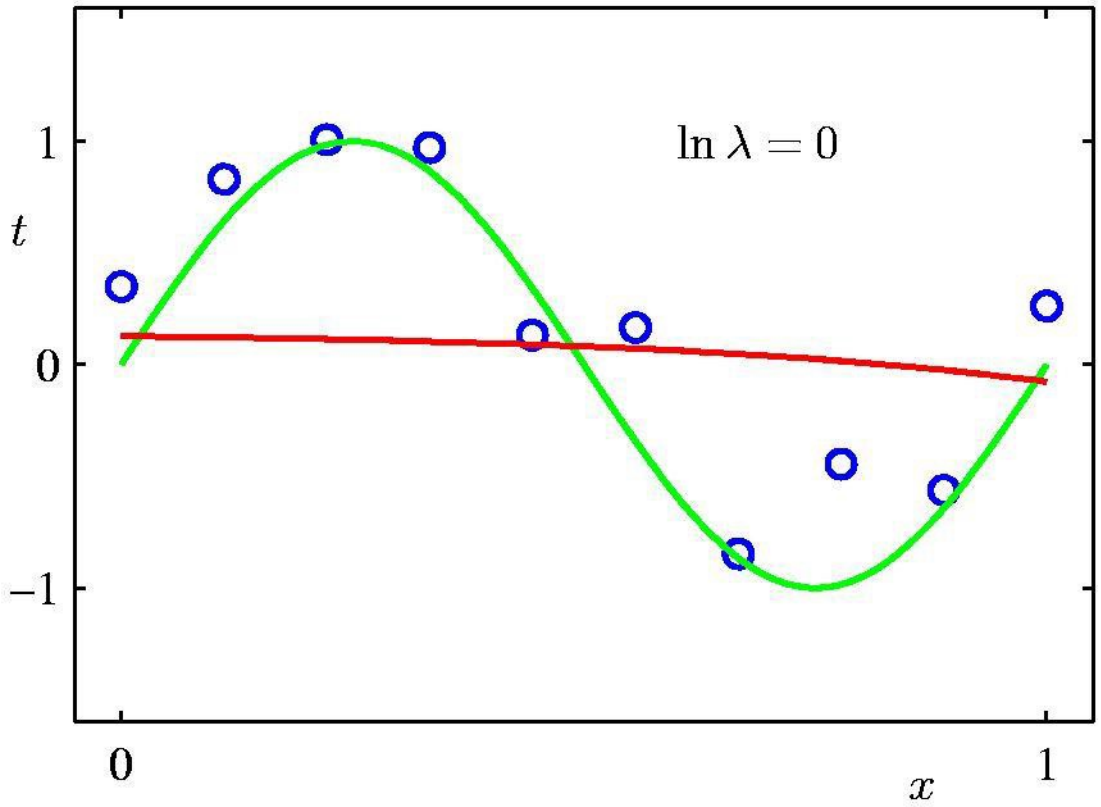
Penalize large coefficient values!

$$\tilde{E}(\mathrm{w}) = \frac{1}{2}\sum_{n=1}^{N}\{y(x_n, \mathrm{w}) - t_n\}^2 + \frac{\lambda}{2}\|\mathrm{w}\|^2$$

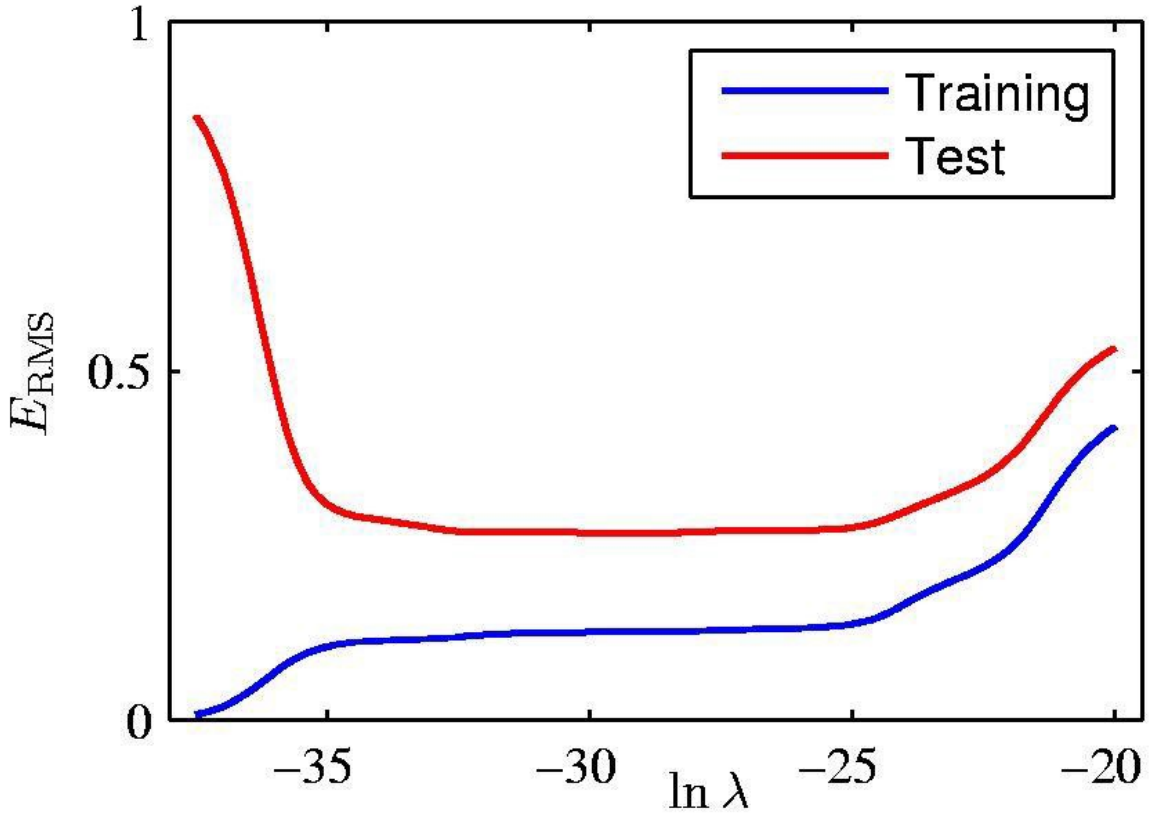
Regularization: $\ln\lambda = -18$



Regularization: $\ln\lambda = 0$



Regularization: E_{RMS} vs. $\ln\lambda$



Polynomial Coefficients

	$\ln \lambda = -\infty$	$\ln \lambda = -18$	$\ln \lambda = 0$
$w_0^\star$	0.35	0.35	0.13
$w_1^\star$	232.37	4.74	-0.05
$w_2^\star$	-5321.83	-0.77	-0.06
$w_3^\star$	48568.31	-31.97	-0.05
$w_4^\star$	-231639.30	-3.89	-0.03
$w_5^\star$	640042.26	55.28	-0.02
$w_6^\star$	-1061800.52	41.32	-0.01
$w_7^\star$	1042400.18	-45.95	-0.00
$w_8^\star$	-557682.99	-91.53	0.00
$w_9^\star$	125201.43	72.68	0.01

Next Lecture

Probability theory …