Name Solutions Rec. Instr.

Signature Rec. Time

Math 220

Exam 2

October 19, 2017

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam. Unless instructed otherwise, **show your work.**

Problem	Points	Points Possible	Problem	Points	Points Possible
1		14	6		8
2		8	7		18
3		9	8		5
4		8	9		10
5		6	10		14

Total =

1. Differentiate the following functions. You do not need to simplify your answers.

A. (7 points) $w(x) = \dfrac{\tan(e^x)}{1+x^3}$

$$w'(x) = \frac{(1+x^3)\sec^2(e^x)\cdot e^x - \tan(e^x)\cdot 3x^2}{(1+x^3)^2}$$

B. (7 points) $h(x) = \sin^{-1}(\ln(x))$.

$$h'(x) = \frac{1}{\sqrt{1-(\ln x)^2}} \cdot \frac{1}{x}$$

2. (8 points) Use logarithmic differentiation to find $\frac{dy}{dx}$ if $y = x^{5x}(2+3x^2)^4$.

$$\ln y = \ln(x^{5x}(2+3x^2)^4) = \ln(x^{5x}) + \ln((2+3x^2)^4)$$

$$\frac{d}{dx}\left[\ln y = 5x\ln x + 4\ln(2+3x^2)\right]$$

$$\frac{1}{y}\frac{dy}{dx} = 5x\cdot\frac{1}{x} + (\ln x)\cdot 5 + \frac{4}{2+3x^2}\cdot 6x$$

$$\frac{dy}{dx} = x^{5x}(2+3x^2)^4\left[5 + 5\ln x + \frac{24x}{2+3x^2}\right]$$

3. (9 points) Use implicit differentiation to find an equation for the tangent line to the curve $x^2y + y^2 = x^3 - 3$ at the point $(2, 1)$.

$$\frac{d}{dx} x^2 y + \frac{d}{dx} y^2 = \frac{d}{dx}(x^3-3)$$

$$x^2 y' + y \cdot 2x + 2y \cdot y' = 3x^2$$

$$y'(x^2+2y) = 3x^2 - 2xy$$

$$y' = \frac{3x^2-2xy}{x^2+2y}$$

$$y'\Big|_{(2,1)} = \frac{3\cdot 2^2 - 2\cdot 2}{2^2+2} = \frac{12-4}{6} = \frac{4}{3}$$

Pt slope:

$$y - y_0 = m(x-x_0)$$

$$\boxed{y - 1 = \frac{4}{3}(x-2)}$$

or $y = \frac{4}{3}x - \frac{5}{3}$

4. A. (5 points) Find the linear approximation of $f(x) = \cos(x)$ near $x = \frac{\pi}{4}$

$L(x) = f(a) + f'(a)(x-a)$, $\quad f\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$L(x) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$ $\quad f'(x) = -\sin(x)$, $f'\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

B. (3 points) Use your answer from part **A** to estimate $\cos(\frac{\pi}{4} + \frac{1}{10})$.

Insert $x = \frac{\pi}{4} + \frac{1}{10}$ into $L(x)$.

$$\cos\left(\frac{\pi}{4}+\frac{1}{10}\right) \approx L\left(\frac{\pi}{4}+\frac{1}{10}\right) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(\cancel{\frac{\pi}{4}} + \frac{1}{10} - \cancel{\frac{\pi}{4}}\right)$$

$$= \boxed{\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{20}} \quad \text{or} \quad \frac{9}{20}\sqrt{2}$$

5. (6 points) The volume of a cone of height 9 ft is given by $V = 3\pi r^2$, where r is the radius. Estimate the change in volume using the differentials dV and dr, if $r = 5$ ft and increased by $\frac{1}{10}$ ft.

$$\frac{dV}{dr} = \frac{d}{dr} 3\pi r^2 = 3\pi \cdot 2r = 6\pi r$$

$$\Rightarrow dV = 6\pi r \, dr$$

When $r = 5$, $dr = \frac{1}{10}$, $\quad dV = 6\pi \cdot 5 \cdot \frac{1}{10} = 3\pi$

Change in $V \approx dV = 3\pi \ (ft)^3$

6. (8 points) Determine the absolute minimum and absolute maximum value of the function $f(x) = x^4 - 2x^2$ over the interval $[0, 2]$.

$$f'(x) = 4x^3 - 4x = 4x(x^2 - 1) = 4x(x-1)(x+1)$$

Critical points: $x = 0, 1, \cancel{-1}$ ← Note -1 is not in $[0, 2]$

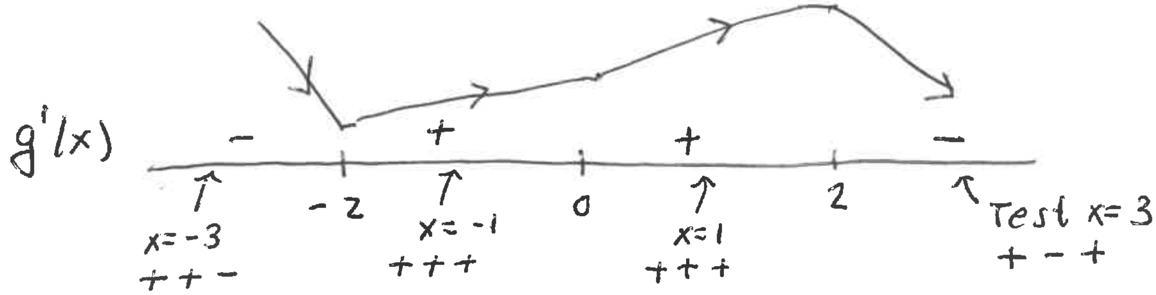
	x	$x^4 - 2x^2$
end pts.	0	0
	2	$16 - 8 = 8$ → Absolute Max $= 8$, (at $x = 2$)
critical pts	1	$1 - 2 = -1$ → Absolute Min $= -1$, (at $x = 1$)

7. Let $g(x) = 20x^3 - 3x^5$.

A. (4 points) Find the critical points of $g(x)$ and draw a number line showing where $g'(x)$ is positive and negative.

$$g'(x) = 60x^2 - 15x^4 = 15x^2(4 - x^2) = 15x^2(2-x)(2+x)$$

critical pts: $x = 0, 2, -2$



B. (2 points) Give the open interval(s) where $g(x)$ is increasing: $(-2, 2)$ or $(-2, 0), (0, 2)$ Both OK

C. (3 points) Classify each critical point as a local minimum, local maximum or neither.

$x = -2$ is local min

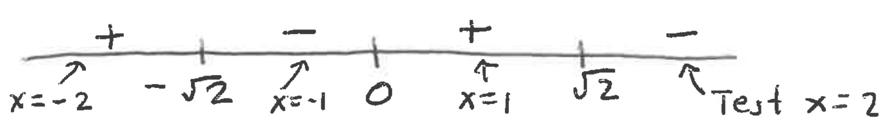
$x = 0$ is neither

$x = 2$ is local max

D. (4 points) Draw a number line showing where $g''(x)$ is positive and negative.

$$g''(x) = 120x - 60x^3 = 60x(2 - x^2)$$

$g''(x) = 0$ when $x = 0$ and $x^2 = 2$, $x = \pm\sqrt{2}$



E. (2 points) Give the open interval(s) where $g(x)$ is concave up: $(-\infty, -\sqrt{2}), (0, \sqrt{2})$

F. (3 points) Give the x-coordinates of all inflection points of $g(x)$:

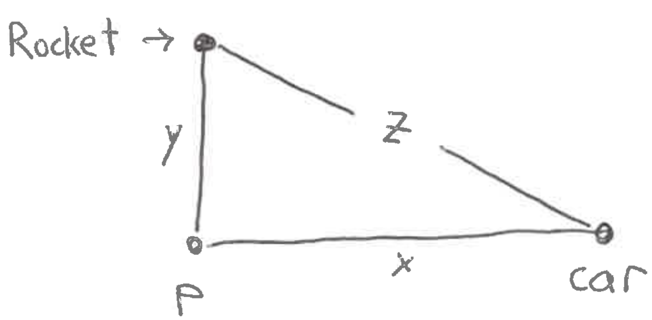
$x = -\sqrt{2}, 0, \sqrt{2}$

8. (5 points) Evaluate $\displaystyle\lim_{x\to-\infty} \frac{7-3x^5}{2x^5-15x^3}$. Show your work.

$$= \lim_{x\to-\infty} \frac{(7-3x^5)\,\frac{1}{x^5}}{(2x^5-15x^3)\,\frac{1}{x^5}}$$

$$= \lim_{x\to-\infty} \frac{\frac{7}{x^5}-3}{2-\frac{15}{x^2}} = \frac{0-3}{2-0} = \boxed{-\frac{3}{2}}$$

9. (10 points) A rocket is launched vertically upward from a point P. At the same time, a car is driving on a straight line away from the the point P. Use related rates to determine the rate that the distance between the rocket and the car is increasing at the instant when the rocket is 3 miles up and travelling 500 miles per hour, and the car is 4 miles away from P and travelling 30 miles per hour. (Assume the ground is flat.)



Given $\frac{dy}{dt} = 500\,\frac{mi}{hr}$ when $y=3$

$\frac{dx}{dt} = 30\,\frac{mi}{hr}$ when $x=4$

Find $\frac{dz}{dt}$ when $x=4, y=3$

$$\frac{d}{dt}\left[z^2 = x^2 + y^2 \right]$$

$$\cancel{2}z\,\frac{dz}{dt} = \cancel{2}x\,\frac{dx}{dt} + \cancel{2}y\,\frac{dy}{dt}$$

$$\frac{dz}{dt} = \frac{1}{z}\left(x\,\frac{dx}{dt} + y\,\frac{dy}{dt}\right)$$

when $x=4, y=3$

$z = \sqrt{3^2+4^2} = 5$

When $x=4, y=3$ we get

$$\frac{dz}{dt} = \frac{1}{5}\left(4\cdot 30 + 3\cdot 500\right) = 4\cdot 6 + 3\cdot 100 = 324\,\frac{mi}{hr}$$

10. Let $f(x) = \frac{2-x}{x^2-1}$. Given: $f'(x) = \frac{x^2-4x+1}{(x^2-1)^2}$.

A. (2 points) Give the equation of the horizontal asymptote for $f(x)$, if any. (No work needs to be shown.)

$y = 0$ (since denominator has larger degree than numerator)

B. (2 points) Give the equation(s) of all vertical asymptotes for $f(x)$, if any. (No work needs to be shown.)

Denom $= 0$

$x = 1$ and $x = -1$

C. (2 points) Give the x-intercepts, if any: $y = 0$ when $2 - x = 0$

$\boxed{x = 2}$

D. (2 points) Give the y-intercept, if any: When $x = 0$, $y = \frac{2-0}{0-1} = -2$

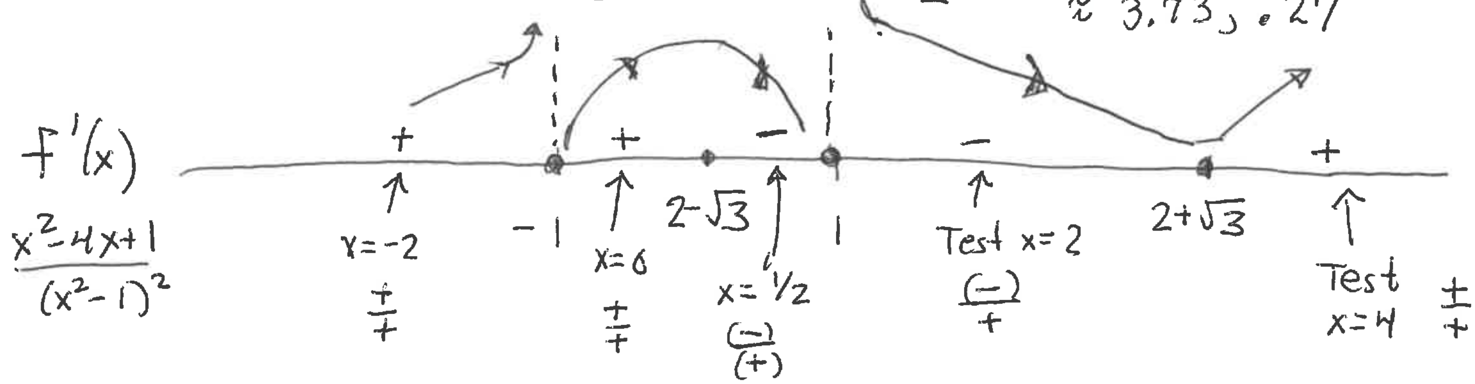
E. (6 points) Find all critical points of $f(x)$ (if any) and classify them as local minima, local maxima or neither. ($\sqrt{3} \approx 1.732$)

(Note: $x = \pm 1$ are not c.p.)

C.p. occur when $x^2 - 4x + 1 = 0$

$$\Rightarrow \quad x = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

$\approx 3.73, .27$



$\boxed{\begin{array}{l}\text{Local maximum at } x = 2 - \sqrt{3} \\ \text{Local minimum at } x = 2 + \sqrt{3}\end{array}}$