NAME ______________________ Rec. Instructor: ______________

Signature ______________________ Rec. Time ______________

CALCULUS II - EXAM 1
July 1, 2019

<u>Show all work</u> for full credit. No books, notes or calculators are permitted. The point value of each problem is given in the left-hand margin. You have 75 minutes.

Problem	Points	Possible	Problem	Points	Possible
1		10	7		10
2		10	8		10
3		10	9		10
4		10			
5		10			
6		10	Total Score		90

You are free to use the following formulas on any of the problems.

$$\sin(ax)\sin(bx) = \tfrac{1}{2}\cos((a-b)x) - \tfrac{1}{2}\cos((a+b)x), \qquad \cos(ax)\cos(bx) = \tfrac{1}{2}\cos((a-b)x) + \tfrac{1}{2}\cos((a+b)x),$$

$$\sin^2(x) = \tfrac{1}{2}(1-\cos(2x)), \qquad \cos^2(x) = \tfrac{1}{2}(1+\cos(2x)),$$

$$\int \tan x\ dx = -\ln|\cos x| + C, \qquad \int \sec x\ dx = \ln|\sec x + \tan x| + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)+C, \qquad \int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C, \qquad \int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)+C$$

$$\int \sin^n x\ dx = -\frac{\sin^{n-1}x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2}x\ dx,$$

$$\int \tan^n x\ dx = \frac{\tan^{n-1}x}{n-1} - \int \tan^{n-2}x\ dx, \qquad \int \sec^n x\ dx = \frac{\sec^{n-2}x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}x\ dx$$

$$\int \tan x \; dx = -\ln|\cos x| + C, \qquad \int \sec x \; dx = \ln|\sec x + \tan x| + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C,$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C, \qquad \int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$$

Work = $\int$ Force $\cdot\, dx$; Units of work: ft-lbs, newton-meters = joules;

Hooke's Law for springs: $F = kx$, where x is the distance stretched from rest position.

$M_x = \frac{1}{2}\int_a^b f(x)^2 - g(x)^2 \; dx$, $\quad M_y = \int_a^b x(f(x) - g(x)) \; dx$.

(10) **1.** $\displaystyle\int \frac{2x^2+x+10}{x-4}\,dx$

$$\begin{array}{r} 2x + 9 \\ x-4\,\overline{)\,2x^2+x+10} \\ -\,(2x^2-8x) \\ \hline 9x+10 \\ -(9x-36) \\ \hline 46 \end{array}$$

$$= \int 2x+9+\frac{46}{x-4}\,dx$$

$$= \boxed{x^2+9x+46\ln|x-4|+C}$$

(10) **2.** A 50m cable with density 5N/m is attached to a 3000N wrecking ball. Find the work to raise the wrecking ball from the ground up to the top.

x = amt of cable pulled up

$F(x) = 5(50-x) + 3000$ ← force of cable + ball

$$W = \int_0^{50} F(x)\,dx$$

3000 + 250

$$= \int_0^{50} 3250 - 5x\,dx$$

$$= 3250x - \frac{5}{2}x^2 \Big]_0^{50}$$

$$\begin{array}{r} 325 \\ \times\ 5 \\ \hline 1625 \end{array}$$

$$\frac{5\cdot 50^2}{2} = 5\cdot 25\cdot 50 = 25\cdot 250 = 6250$$

$$= 162500 - 6250$$

$$= \boxed{156250 \ \text{N}\cdot\text{m}}$$

$$\begin{array}{r} 162500 \\ -\ 6250 \\ \hline 156250 \end{array}$$

(10) **3.** $\int e^x \cos x \, dx$

	D	I
+	e^x	$\cos x$
−	e^x	$\sin x$
+	e^x	$-\cos x$

$$= e^x \sin x + e^x \cos x - \int e^x \cos x \, dx$$

$$2I = e^x(\sin x + \cos x)$$

$$\Rightarrow I = \boxed{\frac{1}{2} e^x(\sin x + \cos x) + C}$$

(10) **4.** $\int \sin^6(x) \cos^3(x) \, dx$

$\sin^2 + \cos^2 = 1$

$u = \sin x$

$du = \cos x \, dx$

$$= \int \sin^6 x \, (1 - \sin^2 x) \cos x \, dx$$

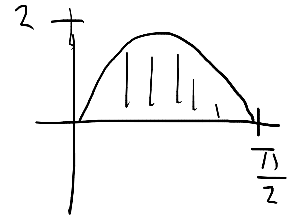
$$= \int u^6 (1 - u^2) \, du$$

$$= \int u^6 - u^8 \, du$$

$$= \frac{u^7}{7} - \frac{u^9}{9}$$

$$= \boxed{\frac{\sin^7 x}{7} - \frac{\sin^9 x}{9} + C}$$

(10) **5.** Determine the centroid $(\overline{x}, \overline{y})$ of the region from $x = 0$ to $x = \frac{\pi}{2}$ bounded by $y = 2\sin(2x)$, $y = 0$, and $x = 0$.



$$M = \rho \int_0^{\frac{\pi}{2}} 2\sin(2x)\,dx$$

$$= -\rho\Big[\cos(2x)\Big]_0^{\frac{\pi}{2}}$$

$$= -\rho(\cos(\pi) - \cos(0))$$

$$= -\rho(-1-1) = 2\rho$$

$$M_x = \frac{\rho}{2}\int_0^{\frac{\pi}{2}} (2\sin(2x))^2 - 0^2\,dx$$

$$= \frac{\rho}{2}\int_0^{\frac{\pi}{2}} 4\sin^2(2x)\,dx = \frac{\rho}{\cancel{2}}\int_0^{\frac{\pi}{2}} \cancel{2}(1-\cos(4x))\,dx$$

$$= \rho\Big[x - \tfrac{1}{4}\cancelto{0}{\sin(4x)}\Big]_0^{\frac{\pi}{2}}$$

$$= \rho\frac{\pi}{2}$$

$$M_y = \rho\int_0^{\frac{\pi}{2}} x(2\sin(2x) - 0)$$

	D	I
+	$2x$	$\sin(2x)$
−	2	$-\frac{1}{2}\cos(2x)$
+	0	$-\frac{1}{4}\sin(2x)$

$$= \rho\Big[-x\cos(2x) + \tfrac{1}{2}\sin 2x\Big]_0^{\frac{\pi}{2}}$$

$$= \rho\Big[-\tfrac{\pi}{2}\cos\pi + \tfrac{1}{2}\cancelto{0}{\sin\pi} - 0\Big]$$

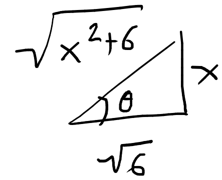
$$= \rho\,\frac{\pi}{2}$$

$$\boxed{\overline{x} = \frac{M_y}{M} = \frac{\pi}{4} \qquad \overline{y} = \frac{M_x}{M} = \frac{\pi}{4}}$$

(10) **6.** $\displaystyle\int \frac{dx}{\sqrt{6+x^2}}$

$x = \sqrt{6}\tan\theta$

$dx = \sqrt{6}\sec^2\theta\, d\theta$



$$= \int \frac{\cancel{\sqrt{6}}\sec^2\theta\, d\theta}{\sqrt{\cancel{6}(1+\tan^2\theta)}}$$

$$= \int \sec\theta\, d\theta = \ln\left|\sec\theta + \tan\theta\right|$$

$$= \boxed{\ln\left|\frac{\sqrt{x^2+6}}{\sqrt{6}} + \frac{x}{\sqrt{6}}\right| + C}$$

(10) **7.** $\frac{d}{d\theta}e^{\cosh(1+\theta^3)}$

$\frac{d}{dx}\cosh x = \sinh x$

$$= \boxed{e^{\cosh(1+\theta^3)} \cdot \sinh(1+\theta^3) \cdot 3\theta^2}$$

(10) **8.** $\displaystyle\int \frac{x^2 - 3x + 4}{x^3 - x}\,dx$

$$\frac{x^2-3x+4}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

$$A = \frac{4}{(-1)(1)} = -4$$

$$B = \frac{1-3+4}{1(2)} = \frac{2}{2} = 1$$

$$C = \frac{(-1)^2+3+4}{-1(-2)} = \frac{8}{2} = 4$$

$$= \int \frac{-4}{x} + \frac{1}{x-1} + \frac{4}{x+1}\,dx$$

$$= \boxed{-4\ln|x| + \ln|x-1| + 4\ln|x+1| + C}$$

(10) **9.** Evaluate the integral using proper limit notation.

$$\int_{-2}^{3} \frac{1}{x^3}\,dx$$

$$= \lim_{b\to 0^-} \int_{-2}^{b} x^{-3}\,dx + \lim_{b\to 0^+} \int_{b}^{3} x^{-3}\,dx$$

$$= \lim_{b\to 0^-} \frac{x^{-2}}{-2}\Bigg]_{-2}^{b} + \lim_{b\to 0^+} \frac{x^{-2}}{-2}\Bigg]_{b}^{3}$$

$$= \underbrace{\lim_{b\to 0^-} \left(-\frac{1}{2b^2} + \frac{1}{8}\right)}_{= -\infty} + \lim_{b\to 0^+} \left(-\frac{1}{18} + \frac{1}{2b^2}\right)$$

Integral diverges