Name _____ Rec. Instr. _____

Signature _____ Rec. Time _____

Math 220

Exam 3

April 6, 2023

No books, calculators, or notes are allowed. Please make sure that your cell phone is turned off. You will have 75 minutes to complete the exam. Unless instructed otherwise, **show your work** on each problem.

Problem	Points	Points Possible	Problem	Points	Points Possible
1		16	6		10
2		10	7		10
3		12	8		6
4		6	9		12
5		18	Total Score		100

1. The function $f(x)$ and its first and second derivatives are:

$$f(x) = \frac{x^2}{x^2 + 3} \qquad f'(x) = \frac{6x}{(x^2 + 3)^2} \qquad f''(x) = \frac{-18(x-1)(x+1)}{(x^2 + 3)^3}.$$

Find the information below about $f(x)$, and use it to sketch the graph of $f(x)$. When appropriate, write NONE. No work needs to be shown on this problem.

A. (1 point) Domain of $f(x)$: _____

B. (1 point) y-intercept: _____

C. (1 point) x-intercept(s): _____

D. (1 point) Horizontal asymptote(s): _____

E. (1 point) Interval(s) $f(x)$ is increasing: _____

F. (1 point) Interval(s) $f(x)$ is decreasing: _____
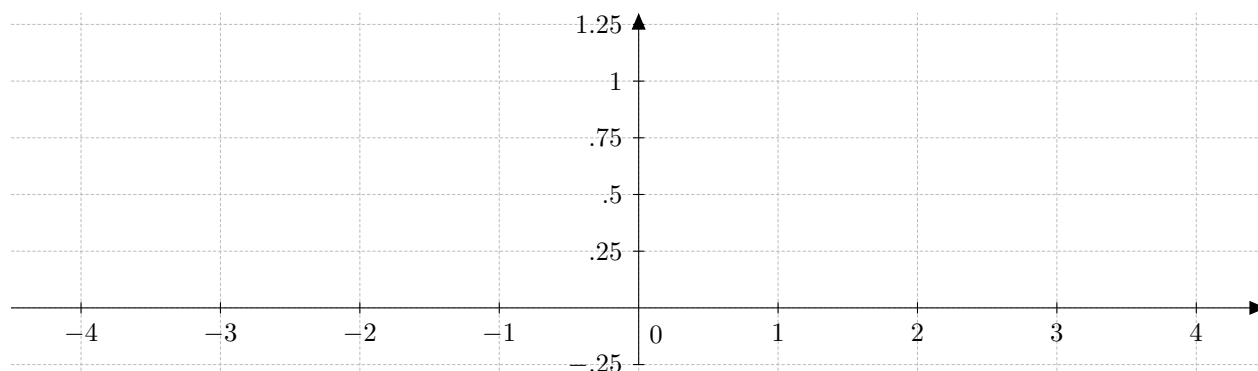
G. (1 point) Local maximum(s) (x, y): _____

H. (1 point) Local minimum(s) (x, y): _____

I. (1 point) Interval(s) $f(x)$ is concave up: _____
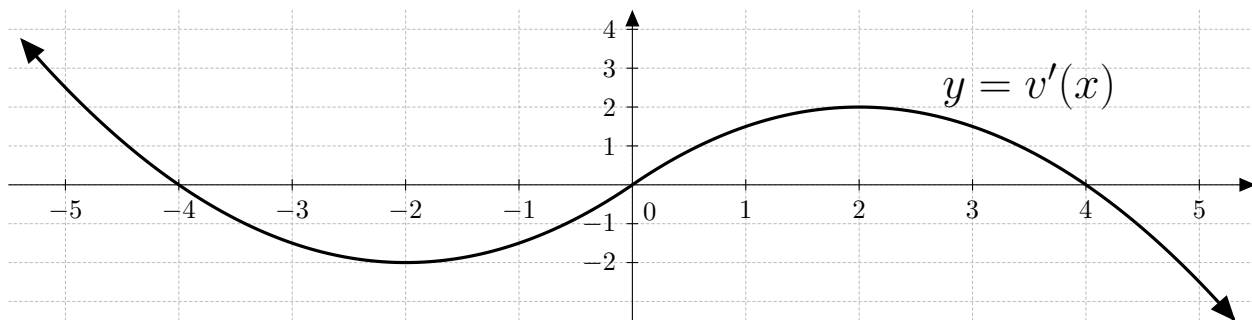
J. (1 point) Interval(s) $f(x)$ is concave down: _____

K. (1 point) Inflection point(s) (x, y): _____

L. (5 points) Sketch $y = f(x)$ on the graph below.

2. (10 points) Find the absolute maximum and absolute minimum of
$w(x) = x^3 - 3x + 2$ on $[0, 2]$.

3. (3 points each) $y = v'(x)$ is plotted above. Find:

A. Interval(s) where $v(x)$ is increasing: _____ decreasing: _____

B. x-coordinate(s) where $v(x)$ has a local max: _____ local min: _____

C. Interval(s) where $v(x)$ is concave up: _____ concave down: _____

D. x-coordinate(s) where $v(x)$ has an inflection point: _____

4. (3 points each) In each of the following blanks, fill in "**max**" or "**min**".

A. If $h'(e) = 0$ and $h''(e) = 8$, then $h(x)$ has a local _____ at $x = e$.

B. If $h'(-7) = 0$ and $h''(-7) = -5$, then $h(x)$ has a local _____ at $x = -7$.

5. (6 points each) Find the following limits. (Use limit notation correctly.)

A. $\lim\limits_{x \to \infty} \dfrac{3 + 5x}{e^x + 7}$

B. $\lim\limits_{\theta \to \pi} \dfrac{\sin(\theta)}{\theta - \pi}$

C. $\lim\limits_{x \to \infty} \dfrac{\sqrt{4x^2 + 3x}}{6x + 7}$

6. (10 points) A 5 meter ladder is leaning against a wall. If the top of the ladder slides down the wall at a rate of $\dfrac{1}{2}$ m/s, how fast will the bottom of the ladder be moving along the ground when the top of the ladder is 4 meters above the ground? (Include units with your answer.)

7. A. (7 points) Find the linearization of $g(x) = \ln(x)$ at $x = 1$.

B. (3 points) Use your answer from Part **A** to estimate $\ln(1.07)$.

8. (6 points) The surface area A of a sphere of radius r is given by $A = 4\pi r^2$. Find the differential dA.

9. (12 points) Suppose that you have 24 meters of fencing to make two adjacent rectangular kennels of length x meters and width y meters (see the diagram below). Find the values of x and y that maximize the enclosed area. (Justify why your answer corresponds to an absolute maximum, and include units in your answer.)