Test 3 Version B
Math 222 Fall 2022
November 10, 2022

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Time of Recitation: ______________

Initials of Recitation Instructor: ______________

You may not use any type of calculator whatsoever. (Cell phones off and away!) You are not allowed to have any other notes, and the test is closed book. Use the backs of pages for scrapwork, and if you write anything on the back of a page which you want to be graded, then you should indicate that fact (on the front). Do not unstaple or remove pages from the exam. **Except for matters of English, the proctors will not answer any questions.**

By taking this exam you are agreeing to abide by KSU's Academic Integrity Policy.

Simple or standard simplifications should be made. You must **show your work** for every problem, and in order to get credit or partial credit, your work must make sense!

GOOD LUCK!!!

Problem	Possible	Score	Problem	Possible	Score
			4	16	
1	15		5	14	
2	16		6	14	
3	16		7	9	
Total	47			53	

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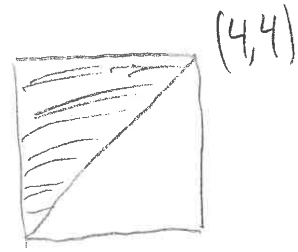
1. The region E is given by



$$\{(x, y)\ :\ 0 \le x \le y \le 4\}.$$

First compute

$$I = \iint_E (2+3y)\, dA\,.$$

Next find the average value of the function $f(x, y) = 2+3y$ on the region E.

$$I = \int_{y=0}^{4} \int_{x=0}^{y} (2+3y)\, dx\, dy = \int_{y=0}^{4} (2y+3y^2)\, dy$$

$$= y^2 + y^3 \Big|_0^4 = 16 + 64 = 80$$

$$\text{Avg Val.} = \frac{80}{\text{Area}(\Delta)} = \frac{80}{8} = 10$$

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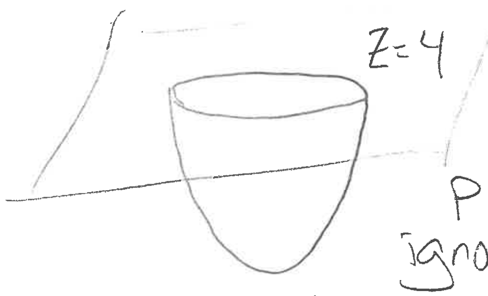
2. Let R be the region given by the inequalities:

$$x \leq 0,\ y \geq 0,\ x^2 + y^2 \leq z \leq 4\,.$$

Find

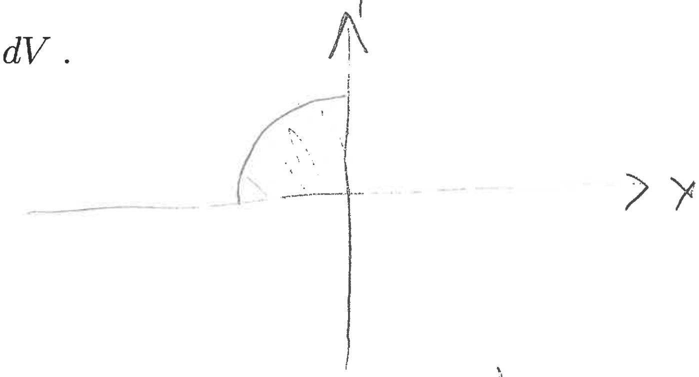
$$I = \iiint_R x\, dV\,.$$



$x^2+y^2 \leq z \leq 4$

Pic ignoring $x \leq 0,\ y \geq 0$

Pic from above



Cylindrical

$$I = \int_{\theta=\frac{\pi}{2}}^{\pi} \int_{r=0}^{2} \int_{z=r^2}^{4} r\cos\theta \, dz\, r\, dr\, d\theta$$

$$= \int_{\theta=\frac{\pi}{2}}^{\pi} \cos\theta\, d\theta \int_{r=0}^{2} (4-r^2)\, r^2\, dr$$

$$= \int_{r=0}^{2} (r^4 - 4r^2)\, dr = \left(\frac{r^5}{5} - \frac{4r^3}{3}\right)\Big|_0^2$$

$$= 2^5\left(\frac{3-5}{15}\right) = \frac{-2^6}{15} = \frac{-64}{15}$$

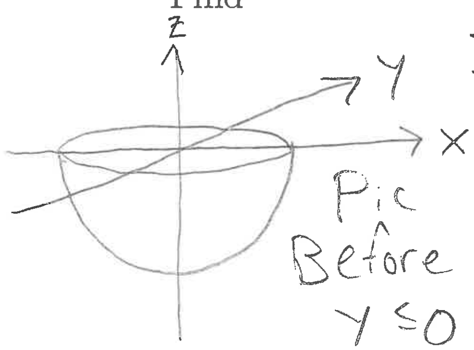
B

3. Let R be the region given by the inequalities:

$$y \le 0, z \le 0,\ x^2 + y^2 + z^2 \le 16\,.$$

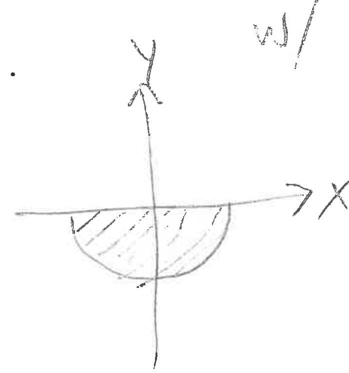
Find

$$I = \iiint_R y\, dV\,.$$



Pic Before taking $y \le 0$

From above w/ $y \le 0$



Spherical!

$$= \int_{\phi=\frac{\pi}{2}}^{\pi} \int_{\theta=-\pi}^{0} \int_{\rho=0}^{4} \rho \sin\theta \sin\phi\ \rho^2 \sin\phi\ d\rho\, d\theta\, d\phi$$

$$= \int_0^4 \rho^3\, d\rho \int_{\theta=-\pi}^{0} \sin\theta\, d\theta \int_{\phi=\frac{\pi}{2}}^{\pi} \frac{1-\cos 2\phi}{2}\, d\phi$$

$$= \frac{4^4}{4} \cdot (-2) \cdot \frac{\pi}{4} = -2\pi \cdot 4^2 = -32\pi$$

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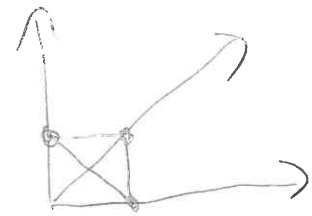

4. Let T be the tetrahedron with vertices

$$P_1 = (1,0,0),\ P_2 = (0,2,0),$$

$$P_3 = (0,0,4),\ P_4 = (0,0,0)\ .$$

$4x + 2y + z = 4$

$4x + 2y + z \leq 4$

Consider the integral:

$$\iiint_T (y+1)\, dV\ .$$

(a) Find an equation for each face, and write down the four inequalities which determine this tetrahedron. Hint: Three of the inequalities are easily found by drawing a sketch or by guessing. Hint 2: One of the three "easy" faces has the equation $x = 0$, and the corresponding inequality is $x \geq 0$. $y=0,\ y \geq 0$ $z=0,\ z \geq 0$

(b) Set up the integral as an iterated integral in each order where it could be computed without "chopping" the integral, and where the y integration is done last. (So y is the "outer" bound of integration.) **Do not bother to compute the integral.**

$$I = \int_{y=0}^{2} (y+1) \int_{x=0}^{\frac{2-y}{2}} \int_{z=0}^{4-4x-2y} 1\, dz\, dx\, dy$$

$$= \int_{y=0}^{2} (y+1) \int_{z=0}^{4-2y} \int_{x=0}^{\frac{4-2y-z}{4}} 1\, dx\, dz\, dy$$

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5. Short Questions Part I

(a) Find the divergence and curl of the following vector field:

$$\vec{F}(x, y, z) := \langle e^{5x},\ x - 2y,\ x - 2z \rangle \ .$$

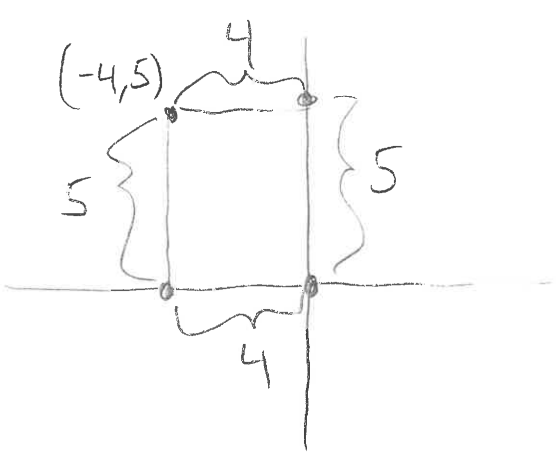
$$\nabla \cdot \vec{F} = 5e^{5x} - 2 - 2 = 5e^{5x} - 4$$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ e^{5x} & x-2y & x-2z \end{vmatrix} = \langle 0, -1, 1 \rangle$$

(b) Let C be the rectangle with vertices: $(0, 0)$, $(-4, 0)$, $(-4, 5)$, and $(0, 5)$. Assume that $f(x, y) \equiv 2$ on the x-axis, $f(x, y) \equiv 1$ on the y-axis (except at the origin), and $f(x, y) \equiv 3$ everywhere else. Find:

$$\int_C f(\vec{r})\, ds \ .$$

Clockwise from $\vec{O}$



$$= \underbrace{2 \cdot 4 + 3 \cdot 9 + 1 \cdot 5}$$

$$= 40$$

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6. Short Questions Part II

(a) Let C be the line segment from $(5,2)$ to $(1,4)$. Let $f(x,y) = \sin(x^2y^3)$. Express

$$\int_C f(\vec{r})\, ds$$

as a definite integral of one variable. (In other words, it should have the form $\int_a^b g(t)\,dt$.) Do **not** attempt to compute the integral under any circumstances!

$\vec{r}(t) = t(1,4) + (1-t)(5,2) = (5,2) + t\langle -4, 2\rangle$

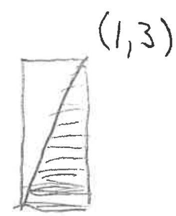
$\vec{r}\,'(t) = \langle -4, 2\rangle \qquad |\vec{r}\,'| = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

$$I = \int_{t=0}^{1} \sin\left((5-4t)^2 \cdot (2+2t)^3\right) \cdot 2\sqrt{5}\, dt$$

(b) Use Fubini's Theorem to evaluate the integral:

$y = 3x$

$$\int_{y=0}^{3} \int_{x=y/3}^{1} x e^{x^3}\, dx\, dy\ .$$



$$\int_{x=0}^{1} x e^{x^3} \int_{y=0}^{3x} 1\, dx\, dy$$

$$= \int_0^1 3x^2 e^{x^3}\, dx = e^1 - e^0$$

$$= e - 1$$

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7. For the following vector field, there is exactly one number M that makes the vector field conservative. First find that value of M, and then find the potential function for that vector field with that choice of M.

$$\vec{F}(x,y,z) = \langle 5yz + 3x^2, Mxz + 2y, 5xy + 4 \rangle.$$

$$\text{Curl } \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ 5yz+3x^2 & Mxz+2y & 5xy+4 \end{vmatrix} = \langle 5x - Mx, 5y - 5y, Mz - 5z \rangle$$

$M = 5 \qquad \vec{F} = \nabla f$

$$f = \int (5yz + 3x^2)\,dx = 5xyz + x^3 + \overline{C}(y,z)$$

$$= \int (5xz + 2y)\,dy = 5xyz + y^2 + \tilde{C}(x,z)$$

$$= \int (5xy + 4)\,dz = 5xyz + 4z + \hat{C}(x,y)$$

$$\boxed{f(x,y,z) = 5xyz + x^3 + y^2 + 4z + C}$$