NAME Soln Rec. Instructor: ______

Signature ______ Rec. Time ______

CALCULUS II - EXAM 1
February 5, 2019

Show all work for full credit. No books, notes or calculators are permitted. The point value of each problem is given in the left-hand margin. You have 75 minutes.

Problem	Points	Possible	Problem	Points	Possible
1a		10	4a		10
1b		10	4b		10
2a		10	5		10
2b		10	6		10
3a		10			
3b		10	Total Score		100

You are free to use the following formulas on any of the problems.

$$\sin(ax)\sin(bx) = \tfrac{1}{2}\cos((a-b)x) - \tfrac{1}{2}\cos((a+b)x), \qquad \cos(ax)\cos(bx) = \tfrac{1}{2}\cos((a-b)x) + \tfrac{1}{2}\cos((a+b)x),$$

$$\sin^2(x) = \tfrac{1}{2}(1-\cos(2x)), \qquad \cos^2(x) = \tfrac{1}{2}(1+\cos(2x)),$$

$$\int \tan x\,dx = -\ln|\cos x| + C, \qquad \int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C, \qquad \int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C, \qquad \int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sin^n x\,dx = -\frac{\sin^{n-1}x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2}x\,dx,$$

$$\int \tan^n x\,dx = \frac{\tan^{n-1}x}{n-1} - \int \tan^{n-2}x\,dx, \qquad \int \sec^n x\,dx = \frac{\sec^{n-2}x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}x\,dx$$

1. Evaluate the following integrals.

(10) a) $\displaystyle\int \frac{e^x}{(1+e^x)^3}\,dx$

$u = 1+e^x$
$du = e^x\,dx$

$= \displaystyle\int \frac{1}{u^3}\,du$ $= u^{-3}$

$= \dfrac{u^{-2}}{-2}$

$= \boxed{-\dfrac{1}{2(1+e^x)^2} + C}$

(10) b) $\displaystyle\int x\sqrt{x-1}\,dx$

$u = x-1 \rightarrow u+1 = x$
$du = dx$

$= \displaystyle\int (u+1)\sqrt{u}\,du$

$= \displaystyle\int u^{\frac{3}{2}} + u^{\frac{1}{2}}\,du$

$= \dfrac{2}{5}u^{\frac{5}{2}} + \dfrac{2}{3}u^{\frac{3}{2}}$

$= \boxed{\dfrac{2}{5}(x-1)^{\frac{5}{2}} + \dfrac{2}{3}(x-1)^{\frac{3}{2}} + C}$

2. Evaluate the following integrals.

(10) a) $\int x^2 \ln(x)\, dx$

	D	I
+	$\ln x$	x^2
−	$\frac{1}{x}$	$\frac{x^3}{3}$

$= \frac{x^3}{3} \ln x - \int \frac{1}{3} x^2\, dx$

$= \boxed{\frac{x^3}{3} \ln x - \frac{x^3}{9} + C}$

(10) b) $\int \tan^{-1} x\, dx$, where $\tan^{-1} x = \arctan x$.

	D	I
+	$\tan^{-1} x$	1
−	$\frac{1}{1+x^2}$	x

$= x \tan^{-1} x - \int \frac{x}{1+x^2}$

$= x \tan^{-1} x - \frac{1}{2} \int \frac{2x}{1+x^2}$

$= \boxed{x \tan^{-1} x - \frac{1}{2} \ln(x^2+1) + C}$

3. Evaluate the following integrals.

(10) a) $\displaystyle\int_0^1 \frac{dx}{\sqrt{4-x^2}}$

$x = 2\sin\theta \rightarrow \frac{x}{2} = \sin\theta$

$dx = 2\cos\theta\, d\theta$

$$= \int \frac{\cancel{2\cos\theta}\, d\theta}{\sqrt{\cancel{4}(\cancel{1-\sin^2\theta})}}$$

$$= \int 1\, d\theta$$

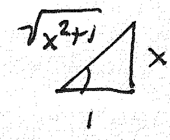
$$= \theta = \sin^{-1}\left(\frac{x}{2}\right)\Big]_0^1 = \sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(0)$$

$$= \frac{\pi}{6} - 0 = \boxed{\frac{\pi}{6}}$$

(10) b) $\displaystyle\int \frac{dx}{\sqrt{1+x^2}}$

$x = \tan\theta$

$dx = \sec^2\theta\, d\theta$



$$= \int \frac{\sec^2\theta\, d\theta}{\sec\theta}$$

$$= \int \sec\theta\, d\theta$$

$$= \ln|\sec\theta + \tan\theta|$$

$$= \boxed{\ln\left|\sqrt{x^2+1} + x\right| + C}$$

4. Evaluate the following integrals.

(10) a) $\int \sin^3(x)\cos^8(x)\,dx$

$\sin^2 + \cos^2 = 1$

$= \int \sin x\,(1-\cos^2 x)\cos^8 x\,dx$

$u = \cos x$

$du = -\sin x\,dx$

$= -\int (1-u^2)\,u^8\,du$

$= u^8 - u^{10}$

$= -\left(\frac{u^9}{9} - \frac{u^{11}}{11}\right)$

$= \boxed{-\frac{\cos^9(x)}{9} + \frac{\cos^{11}(x)}{11} + C}$

$\tan^2 x + 1 = \sec^2 x$

(10) b) $\int \tan^4(x)\,dx$

$\int \tan^2 x\,(\sec^2 x - 1)\,dx$

$= \int \tan^2 x \sec^2 x - \tan^2 x\,dx$

$= \frac{\tan^3 x}{3} - \int \sec^2 x - 1\,dx$

$= \boxed{\frac{\tan^3 x}{3} - \tan x + x + C}$

(10) **5.** An object moves along a straight line with velocity function $v(t) = te^{-t}$, in meters per second. Determine its change in position over the time interval $t = 0$ to $t = 4$ seconds.

	D	I
+	t	e^{-t}
−	1	$-e^{-t}$
+	0	e^{-t}

$$\int_0^4 te^{-t}\,dt =$$

$$= -te^{-t} - e^{-t}$$

$$= (-t-1)e^{-t}\Big]_0^4$$

$$= -5e^{-4} - (-1e^{0})$$

$$= \boxed{-5e^{-4} + 1 \quad \text{meters}}$$

(10) **6.** Find a function $f(s)$ such that $f'(s) = s\tan(s^2) - \sec^2(s)$.

$$\int s\tan(s^2) - \sec^2(s)\,ds$$

$u = s^2$

$du = 2s\,ds$

$$= \frac{1}{2}\int 2s\tan(s^2)\,ds - \int \sec^2(s)\,ds$$

$$= \frac{1}{2}\int \tan(u)\,du - \tan(s)$$

$$= -\frac{1}{2}\ln|\cos(u)| - \tan(s)$$

$$= \boxed{-\frac{1}{2}\ln|\cos(s^2)| - \tan(s) + C}$$