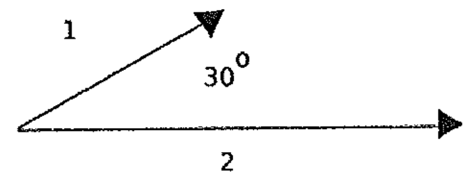
Short answer questions (6 points each):

Let the horizontal vector in the picture be denoted **u** and the diagonal vector be denoted **v**, with the magnitudes and angle between the vectors as indicated.



1. Find $|\mathbf{u} \times \mathbf{v}|$ $= |u||v| \sin\theta$

$= 2 \cdot 1 \cdot 1/2 = 1$

2. Regarding the page as a plane in 3-space, does $\mathbf{u} \times \mathbf{v}$ point up out of the page toward you, or down toward the desk or table?

up (RH rule

3. Find $\mathbf{u} \cdot \mathbf{v}$ $= |u||v| \cos\theta$

$= 2 \cdot 1 \cdot \sqrt{3}/2 = \sqrt{3}$

4. Suppose **u** is taken to be a displacement vector with magnitude in meters and **v** is taken to be a force with magnitude in Newtons. Write a brief word problem for which solving question 3 gives the computational work in solving the word problem, and tell the physical units that should be applied to the answer in 3 to make it a solution to the word problem.

A force of 1 N is applied to a box, and drags it 2 m along level ground find the work done.

units: N·m

Short answer questions, continued.

5. Find the equation of the line perpendicular to the plane $3x + 2y - z = 8$ and passing through the point $\langle 2, -1, 0 \rangle$.

$\langle 3, 2, -1 \rangle$ is a normal vector to the plane and thus suffices as a direction vector for any line $\perp$ to the plane

So $r(t) = \langle 2, -1, 0 \rangle + t\langle 3, 2, -1 \rangle$ is an equation for the desired line

6. Find $\langle 2, \frac{1}{2}, -1 \rangle \times \langle \frac{3}{2}, -2, 0 \rangle$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1/2 & -1 \\ 3/2 & -2 & 0 \end{vmatrix} = \left(\tfrac{1}{2}\cdot 0 - (-1)(-2)\right)\vec{i} - \left(2\cdot 0 - (-1)(3/2)\right)\vec{j} + \left(-2\cdot 2 - 1/2 \cdot 3/2\right)\vec{k}$$

$$= -2\vec{i} - 3/2\,\vec{j} - \frac{19}{4}\vec{k}$$

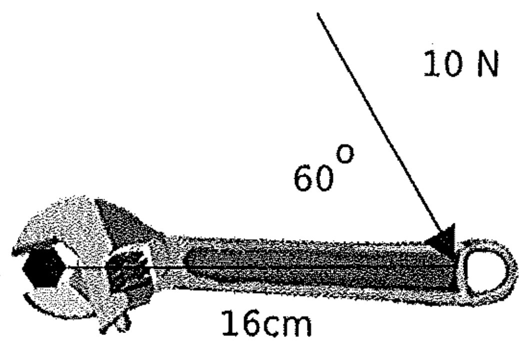
$$4 - 3/4 = \frac{16}{4} - \frac{3}{4} = \frac{13}{4}$$

Yet more short anwer questions:

7. What is the curvature of a circle of radius 7?

$\frac{1}{7}$ (curvature of a circle is the reciprocal of the radius)

8. What is the torque applied to the bolt when the force is applied to the wrench as illustrated?



$$|\vec{\tau}| = 10\,\mathrm{N} \cdot .16\,\mathrm{m} \cdot \sin 60^\circ$$

$$= 1.6\,\frac{\sqrt{3}}{2}\ \mathrm{N \cdot m}$$

$$= \frac{4\sqrt{3}}{5}\ \mathrm{N \cdot m}$$

Long questions, point values follow the question number in parentheses.

9. (20) Consider the vector-valued function $\mathbf{r}(t) = \cos(t)\mathbf{i} + 2\sin(t)\mathbf{j}$ as a trajectory (i.e. giving the position of a particle at time t, say in seconds after some chosen time $t = 0$ as a displacement from the origin in some units, say cm).

   (a) Find the velocity function.

   $$r'(t) = -\sin t\,\vec{i} + 2\cos t\,\vec{j}$$

   (b) Find the acceleration function.

   $$r''(t) = -\cos t\,\vec{i} - 2\sin t\,\vec{j}$$

   (c) Find the function giving the tangential component of the acceleration at time t

   $$\text{proj}_{\vec{r}'(t)} r''(t) = \frac{\vec{r}'(t)\cdot r''(t)}{|r'(t)|^2} r'(t)$$

   $$= \frac{\sin t\cos t - 4\sin t\cos t}{\sin^2 t + 4\cos^2 t} \langle -\sin t, 2\cos t\rangle$$

   $$= \frac{-3\sin t\cos t}{1+3\cos^2 t} \langle -\sin t, 2\cos t\rangle$$

   (d) Find the function giving the normal component of the acceleration at time t.

   $$\text{orth}_{\vec{r}'(t)} r''(t) = r''(t) - \text{proj}\, r''(t)$$

   $$= \left\langle -\cos t - \frac{3\sin^2 t\cos t}{1+3\cos^2 t}, -2\sin t + \frac{6\sin t\cos^2 t}{1+3\cos^2 t} \right\rangle$$

10. (16)

(a) Find an equation for the plane passing through the points $(1,0,1)$, $(1,2,3)$ and $(-1,2,3)$.

$\vec{p}$ $\vec{q}$ $\vec{r}$

Find two differences to get vectors ∥ to the plane:

$\vec{q}-\vec{p} = \langle 1-1,\ 2-0,\ 3-1\rangle = \langle 0, 2, 2\rangle$

$\vec{r}-\vec{p} = \langle -1-1,\ 2-0,\ 3-1\rangle = \langle -2, 2, 2\rangle$

take their cross product + find a normal vector

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & 2 \\ -2 & 2 & 2 \end{vmatrix} = 0\vec{i} - 4\vec{j} + 4\vec{k} = \langle 0, -4, 4\rangle$$

So $\langle 0, -4, 4\rangle \cdot (\langle x, y, z\rangle - \langle 1, 0, 1\rangle) = 0$ is an eq

$-4y + 4z - 4 = 0$ $\qquad -y + z = 1$

(b) Find the distance from the point $(4,0,-1)$ to the plane in part (a).

we need $\text{comp}_{\vec{n}}\, \vec{d}$ where $\vec{d}$ is the displacement from $(4,0,-1)$ to any point in the plane

let $\vec{d} = \langle 4,0,-1\rangle - \langle 1,0,1\rangle = \langle 3, 0, -2\rangle$

$$\text{comp}_{\langle 0,-4,4\rangle} \langle 3,0,-2\rangle = \left| \frac{0\cdot 3 + -4\cdot 0 + 4\cdot -2}{\sqrt{0+16+16}} \right|$$

$$= \left| \frac{-8}{4\sqrt{2}} \right| = \sqrt{2}$$

11. (16) Find

$$\int_0^3 3\mathbf{i} + te^{-2t}\mathbf{j} - t\mathbf{k}\, dt$$

$$\int_0^3 3\bar{i} + te^{-2t}\bar{j} - t\bar{k}\, dt$$

$$= 9\bar{i} + \left(\tfrac{1}{4} - \tfrac{7}{4}e^{-6}\right)\bar{j} - \frac{9}{2}\bar{k}$$

$$\int_0^3 3\, dt = 3t\Big|_0^3 = 9$$

$$\int_0^3 te^{-2t}\, dt$$

uk parts

$dv = e^{-2t}dt$

$u = t$

$du = dt$

$v = -\frac{1}{2}e^{-2t}$

$$= -\tfrac{1}{2}te^{-2t}\Big|_0^3 - -\frac{1}{2}\int_0^3 e^{-2t}\, dt$$

$$= -\tfrac{1}{2}te^{-2t} - \tfrac{1}{4}e^{-2t}\Big|_0^3$$

$$= -\tfrac{3}{2}e^{-6} - \tfrac{1}{4}e^{-6} - \left(0 - \tfrac{1}{4}\right)$$

$$= \tfrac{1}{4} - \tfrac{7}{4}e^{-6}$$

$$\int_0^3 -t\, dt = -\frac{t^2}{2}\Big|_0^3 = -\frac{9}{2}$$