Math 222: Analytic geometry and Calculus 3
Final exam: Summer 2011
July 29, 2011

Name..........................
Instructor...................

To receive credit you **must** show your work

(20 pts) Problem 1. The points P(0,1,1), Q(2,2,0) and R(1,3,2) determine a triangle in 3-space.

a) Find the angle at the vertex P in degree

b) find the area of the triangle

c) find the equation of the plane which contains the triangle

d) find symmetric equations of the line which passes through the point R and is orthogonal (perpendicular) to the plane containing the triangle

(20 pts) Problem 2. Let $P(3,4,5)$, $Q(2,5,3)$ and

$$f(x,y,z) = z - \sqrt{x^2+y^2}.$$

a) Find the directional derivative of f at P in the direction of Q

b) find the maximum rate of change of f at the point P and the direction in which it occurs

c) find the tangent plane to the surface $f(x,y,z) = 0$ at the point P

d) find the linear approximation to the function f at the point P and use it to approximate $f(2.9, 4, 5.1)$.

(20 pts) Problem 3.

Find the extreme values of the function

$$f(x, y) = x^2 + 2y^2$$

on the disk

$$x^2 + y^2 \leq 1.$$

(20 pts) Problem 4.

Use a **double integral** to find the volume of the solid which is bounded by the surfaces $z = xy^3 + \frac{2}{1+y}$, $z = xy^3$, $x = 1 - y^2$, and the planes $y = x - 1$, $y = 1$.

(20 pts) Problem 5.

Use **<u>cylindrical coordinates</u>** to set up an integral (but DO NOT solve, i.e., DO NOT compute the integrals) for computing:

a) the **<u>mass</u>** of the solid which is bounded by the surfaces $z = \sqrt{2(x^2 + y^2)}$ and $z = 4 - x^2 - y^2$ with density function $\delta(x, y, x) = x + 2y$.

<u>Use spherical coordinates</u> to set up a integral (but DO NOT solve, i.e., DO NOT compute the integrals) for computing:

b) the **<u>volume</u>** of the solid which is **<u>in the first octant</u>**, inside the spheres $x^2 + y^2 + z^2 = 2$ and above the plane $z = 1$.

(20 pts) Problem 6. Let

$$\mathbf{F} = \langle yze^{xyz} + \pi\cos(\pi x), xze^{xyz} + 3y^2, xye^{xyz} \rangle$$

(a) Find a function f such that $\nabla f = \mathbf{F}$

(b) use part (a) to evaluate $\int_C \mathbf{F} \bullet d\mathbf{r}$ along the curve C which is the line segment from (1,-1,0) to (2,1,1).

(20 pts) Problem 7. Find the area of the surface S which is the part of the plane $2x + 2y + z = 2$ that lies in the first octant.

(20 pts) Problem 8. Compute the surface integral $\iint_S \langle x, y, 1\rangle \bullet d\mathbf{S}$ where S is the part of the paraboloid $z = x^2 + y^2$ that lies inside the sphere $x^2 + y^2 + z^2 = 20$.

(20 pts) Problem 9. Use Stoke's theorem to calculate $\int_C \mathbf{F} \bullet d\mathbf{r}$ where

$$\mathbf{F}(x, y, z) = \langle 2y, 4x, yz \rangle$$

where C is the curve of intersection of the plane $x+z = 2$ and the cylinder $x^2+y^2 = 1$.

(20 pts) Problem 10. Use Divergence Theorem to calculate $\iint_S \mathbf{F} \bullet d\mathbf{S}$ where

$$\mathbf{F}(x, y, z) = \langle x^3 + \sin(yz), y^3, z^3 + \sin(xy) \rangle$$

and S is the surface of the solid which lies between the spheres $x^2 + y^2 + z^2 = 1$, $x^2 + y^2 + z^2 = 4$ and above the cone $z = \sqrt{x^2 + y^2}$.