Instructions: Wait to open the exam until instructed to do so. Then answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. You will have 1 hour and 50 minutes to complete this exam.

Question	Points	Score
1	15	
2	10	
3	20	
4	15	
5	10	
6	20	
7	20	
8	15	
9	15	
Total:	140	

Name: ____

Recitation Instructor: ________________________________

Recitation Time: __________________________________

1. Let $\mathbf{u} = \langle 1, 1, 0 \rangle$, $\mathbf{v} = \langle 1, 0, 1 \rangle$ and $\mathbf{w} = \langle 0, 1, 1 \rangle$.

   (a) (5 points) Compute the volume of the parallelepiped spanned by $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$.

   (b) (5 points) Compute the angle θ between $\mathbf{v}$ and $\mathbf{w}$.

   (c) (5 points) Give the equation for the plane parallel to $\mathbf{u}$ and $\mathbf{v}$ and passing through the origin.

2. Consider the curve $\mathcal{C}$ given by the parametrization

$$\mathbf{r}(t) = \langle \sin(t), \cos(t), e^t \rangle \qquad \text{for } 0 \leq t \leq \pi$$

(a) (5 points) Find the speed of $\mathbf{r}(t)$ as a function of t.

(b) (5 points) Compute the scalar line integral

$$\int_{\mathcal{C}} 3x^2z^2 + 3y^2z^2 \, ds.$$

3. Calculate the following quantities if they exist. Otherwise, explain why they do not exist. Justify either response.

(a) (5 points)

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+y^2}$$

(b) (5 points) For

$$f(x,y,z) = \cos(z^2 - y^2) + y\sin(x)$$

compute

$$f_{xy}(x,y,z)$$

(c) (5 points) For $f(x, y, z) = x + y^2 + z^3$ find the change in $f(x, y, z)$ as one moves in the direction of the unit vector $\mathbf{u} = \frac{\sqrt{3}}{3}\langle 1, 1, 1\rangle$ starting at $(2, 0, -1)$.

(d) (5 points) Find the equation for the tangent plane to the surface

$$z = xy$$

at the point $(1, 2, 2)$.

4. Let

$$f(x,y) = x^3 - 12x + y^2$$

and $\mathcal{D}$ be the square $[-3,3] \times [-3,3]$.

(a) (5 points) Find the critical points of $f(x,y)$ in the interior of $\mathcal{D}$.

(b) (5 points) Describe the local behavior of $f(x,y)$ at the critical points found in part (a).

(c) (5 points) Find the maximum value of f on $\mathcal{D}$.

5. (10 points) Let $\mathcal{W} = [0,1] \times [-1,0] \times [0,2]$. Evaluate the triple integral

$$\iiint_{\mathcal{W}} (2x + z)e^y \; \mathrm{d}V$$

6. Evaluate the following integrals.

   (a) (10 points) Let $\mathcal{D}$ be the region $x^2 + y^2 \leq 4$, $0 \leq y$, $x \leq 0$. Evaluate

$$\iint_{\mathcal{D}} 3x \; \mathrm{d}A.$$

(b) (10 points) Let $\mathcal{D}$ be region between the lines $y = -x$, $y = -1$ and $x = -1$. Compute the integral

$$\iint_{\mathcal{D}} 2y \ \mathrm{d}A.$$

7. Let

$$\mathbf{F} = \langle 2x + yz, xz, xy \rangle .$$

(a) (5 points) If $\mathbf{F}$ is a conservative vector field, find a potential. Otherwise, explain why it is not conservative.

(b) (5 points) Let $\mathcal{C}$ be the oriented curve with parametrization

$$\mathbf{r}(t) = \left\langle \sin^6(\pi t) + t + 1, e^t + e^{-t}, e^{t^2-1} - 1 \right\rangle$$

for $-1 \le t \le 1$. Compute

$$\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}.$$

(c) (5 points) Is there a vector potential for $\mathbf{F}$ (a vector field $\mathbf{A}$ that satisfies $\mathbf{F} = \text{curl}(\mathbf{A})$)? Explain your response.

(d) (5 points) Let $\mathcal{S}$ be the sphere $x^2 + y^2 + z^2 = 9$ oriented outwardly. Compute the surface integral

$$\iint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S}.$$

State any theorems used in the computation.

8. Let $\mathcal{D}$ be the lower half disc

$$\mathcal{D} = \{(x, y) : x^2 + y^2 \leq 1, y \leq 0\}.$$

The boundary of $\mathcal{D}$ consists of the line segment $\mathcal{C}_1$ along the x-axis oriented from $(1, 0)$ to $(-1, 0)$ and the semi-circle

$$\mathcal{C}_2 = \{(x, y) : y = -\sqrt{1 - x^2}, -1 \leq x \leq 1\}$$

oriented counter-clockwise. Let $\mathbf{F}$ be the vector field

$$\mathbf{F} = \left\langle -yx^2, xy^2 \right\rangle.$$

(a) (5 points) Using polar coordinates, calculate the double integral

$$\iint_{\mathcal{D}} x^2 + y^2 \, \mathrm{d}A$$

(b) (5 points) Compute the line integral

$$\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$$

(c) (5 points) Using only Green's Theorem and the computations in parts (a) and (b), compute the vector line integral

$$\int_{C_2} \mathbf{F} \cdot \mathrm{d}\mathbf{r}$$

9. Let $\mathcal{S}$ be the cylinder
$$\{(x, y, z) : x^2 + y^2 = 1, 0 \leq z \leq 3\}.$$
oriented outward and $\mathbf{F} = \langle zy, -zx, 0\rangle$.

(a) (5 points) Compute curl($\mathbf{F}$).

(b) (5 points) Calculate
$$\iint_{\mathcal{S}} \text{curl}(\mathbf{F}) \cdot \, d\mathbf{S}$$

(c) (5 points) The boundary of $\mathcal{S}$ consists of a unit circle $\mathcal{C}_1$ on the x, y-plane oriented counter-clockwise and a unit circle $\mathcal{C}_2$ on the $z = 3$ plane oriented clockwise. Noting that the vector field is zero on the x, y-plane, one easily sees that

$$\int_{\mathcal{C}_1} \mathbf{F} \cdot \, \mathbf{dr} = 0$$

Using only this fact, Stokes' Theorem and your result from part (b), compute the vector line integral

$$\int_{\mathcal{C}_2} \mathbf{F} \cdot \, \mathbf{dr}$$

Coordinate systems

Polar	Cylindrical	Spherical
$x = r\cos(\theta)$ $y = r\sin(\theta)$	$x = r\cos(\theta)$ $y = r\sin(\theta)$ $z = z$	$x = \rho\cos(\theta)\sin(\phi)$ $y = \rho\sin(\theta)\sin(\phi)$ $z = \rho\cos(\phi)$
$r = \sqrt{x^2+y^2}$ $\tan(\theta) = \frac{y}{x}$	$r = \sqrt{x^2+y^2}$ $\tan(\theta) = \frac{y}{x}$ $z = z$	$\rho = \sqrt{x^2+y^2+z^2}$ $\tan(\theta) = \frac{y}{x}$ $\cot(\phi) = \frac{z}{\sqrt{x^2+y^2}}$
$\mathrm{d}x\,\mathrm{d}y = r\,\mathrm{d}r\,\mathrm{d}\theta$	$\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}z$	$\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \rho^2\sin\phi\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta$

Unit vectors

$$\mathbf{T} = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|} \qquad \mathbf{N} = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$$

Useful area and volume formulas

$$\text{Surface area of sphere of radius } R = 4\pi R^2$$
$$\text{Volume of sphere of radius } R = \frac{4}{3}\pi R^3$$

Derivative formulas

$$\text{Directional derivative}: D_{\mathbf{u}}f(x,y) = \nabla f(x,y)\cdot\mathbf{u},$$
$$\text{Discriminant}: D = f_{xx}(x_0,y_0)f_{yy}(x_0,y_0) - f_{xy}(x_0,y_0)^2$$

Trig identities

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta) \qquad \cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\sin^2(\theta) = \frac{1}{2}(1-\cos(2\theta)) \qquad \cos^2(\theta) = \frac{1}{2}(1+\cos(2\theta))$$

Change of variables

$$G : \mathcal{D}_0 \to \mathcal{D}$$

$$G(u,v) = (x(u,v), y(u,v))$$

$$\mathrm{Jac}(G) = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$$

$$\iint_{\mathcal{D}} f(x,y)\,\mathrm{d}x\,\mathrm{d}y = \iint_{\mathcal{D}_0} f(x(u,v), y(u,v))\ |\mathrm{Jac}(G)|\ \mathrm{d}u\,\mathrm{d}v$$

Line integrals

$$\mathbf{r}(t) \text{ for } a \le t \le b \text{ parametrizing } \mathcal{C}$$

$$\int_{\mathcal{C}} f(x,y,z)\,\mathrm{d}s = \int_a^b f(\mathbf{r}(t))\,\|\mathbf{r}'(t)\|\,\mathrm{d}t$$

$$\int_{\mathcal{C}} \mathbf{F}\cdot\mathrm{d}\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t))\cdot\mathbf{r}'(t)\,\mathrm{d}t$$

Surface integrals

$$G(u,v) = (x(u,v), y(u,v), z(u,v)) \text{ for } (u,v)\in\mathcal{D} \text{ parametrizing } \mathcal{S}$$

$$\mathbf{n}(u,v) = \mathbf{T}_u \times \mathbf{T}_v$$

$$\iint_{\mathcal{S}} f(x,y,z)\,\mathrm{d}S = \iint_{\mathcal{D}} f(G(u,v))\,\|\mathbf{n}(u,v)\|\,\mathrm{d}u\,\mathrm{d}v$$

$$\iint_{\mathcal{S}} \mathbf{F}\cdot\mathrm{d}\mathbf{S} = \iint_{\mathcal{D}} \mathbf{F}(G(u,v))\cdot\mathbf{n}(u,v)\,\mathrm{d}u\,\mathrm{d}v$$