Goemetry of Manifolds Qualifying Exam
Fall 1994

Part A. Short answers. Answer all of the following.

1. What is the fundamental group of

(a) S^3

(b) $S^1 \times S^1$

(c) The Euclidean plane with two pts. deleted.

2. A differential 1-form is a section of what bundle?

3. What is the topology of the underlying manifold of the lie group $SU(2)$? What is the topology of the lie algebra $\mathcal{SU}(2)$?

4. State Stokes' theorem.

5. (a) What kind of differential form can we integrate on a 4-manifold?

(b) What kind of differential form can we integrate on a surface in a 4-manifold?

6. (a) Give an example of a compact surface whose tangent bundle is trivial.

(b) Give an example of a compact surface whose tangent bundle is not trivial.

7. Suppose we have a non-zero vector field on R^n all of whose covariant derivatives with respect to the standard metric and Levi-Civita connection vanish. Describe the family of its integral curves.

8. If M is a 4-dimensional manifold, what is the dimension of the fibers of its bundle of differential 2-forms.

9. Consider the complex of differential forms used to define the de Rham cohomology of a 5-manifold. How many of these spaces are non-vanishing.

Part B. Do any two of the following problems.

1. (a) Write the metric tensor for the Euclidean plane in polar coordinates.

(b) Compute the $\theta^\theta r$ component of the Levi-Civita connection for the Euclidean plane in polar coordinates.

2. Give an example of a topological space every point of which has a neighborhood homeomorphic to R^2 which is not a manifold.

3. Prove the Jacobi identity holds for $\mathcal{SU}(3)$.

4. Compute the DeRham cohomology of the Torus T^2.

5. Use Stokes' theorem to compute the area of the unit ball in R^2.