Qualifying Examination Syllabi

Algebra Qualifying Examination Syllabus

I. Groups
- A. Elementary concepts (subgroups, subgroup generated by a set, normal subgroups, homomorphisms, isomorphisms, automorphism groups, and constructions such as direct or semi-direct products)
- B. The Fundamental Isomorphism Theorems
- C. Group actions. The following topics are often discussed as corollaries to group actions:
  1. The "class equation" (in particular, p-groups have non-trivial centers)
  2. The Sylow Theorems
- D. Nilpotent and solvable groups

II. Rings and Modules
- A. Homomorphisms and ideals (submodules) of rings (modules)
- B. The Chinese Remainder Theorem
- C. Euclidean domains, principal ideal domains and unique factorization domains
- D. The structure theorem of finitely generated modules over a principal ideal domain as it applies to the determination of:
  1. The structure of finitely generated Abelian groups
  2. The various canonical forms (especially, rational and Jordan) of a linear transformation on a finite dimensional vector space
- E. Irreducible, indecomposable, and completely reducible modules
- F. Tensor product of modules

III. Field and Galois Theory
- A. Characteristic of a field and prime subfield
- B. Field extensions and degree of an extension
- C. Algebraic and transcendental elements, algebraic closure
- D. Simple extensions (especially algebraic ones)
- E. Normal extensions and separable extensions
- F. Galois extensions and the Fundamental Theorem of Galois Theory
- G. Computation of the Galois group of a polynomial

IV. Linear Algebra
- A. Elementary concepts (vector spaces, subspaces, linear transformations and their kernels, linear independence, bases and dimension, matrices and the Rank-Nullity Theorem)
  <>Dual spaces,
- B. Invariant subspaces and invariant subspace direct sum decompositions
- C. Canonical forms (Jordan and rational canonical forms, primarily over the complex, real and rational fields; see II.D.2)
- D. Determinants
- E. Tensor products and bilinear forms

Notes: The majority of the above topics can be found Hungerford's book: Algebra, Graduate Texts in Mathematics, 73. *Springer-Verlag, New York-Berlin,* 1980. Another book of reference is Groves' book: Algebra, Academic Press, 1983.

Analysis Qualifying Examination Syllabus

1. s-algebras.
2. Measurable functions and measure spaces
3. Lebesgue measure on the real line
4. Limit theorems: Fatou's lemma, Lebesgue's monotone convergence theorem, Lebesgue's dominated convergence theorem, Beppo Levi's theorem
5. Jensen's inequality, Hölder's and Minkowski's inequalities
6. Definition of L^p spaces, completeness of L^p spaces, density of continuous functions in L^p
7. Definition of Hilbert space, closed convex sets have elements of minimal norm, projections
8. Orthonormal sets, maximal orthonormal sets, trigonometric systems
9. Product spaces, Fubini's theorem
10. Complex differentiation, Cauchy-Riemann equations
11. Integration over paths
12. Liouville's theorem

13. Power series
14. Cauchy theorem
15. Local theory
16. Types of singularities, Laurent series
17. The residue theorem
18. Computation of real integrals using contour integration
19. Principle of the argument, Rouche's theorem
20. Maximum modulus theorem, Schwarz's lemma
21. Conformal mapping, including Möbius transformations

Geometry/Topology Qualifying Examination Syllabus

Notes: Students taking the geometry/topology qualifying exam will be expected to know basic definitions and facts from point set topology, algebraic toopology, and differential topology. The following is an outline of topics in these areas.

I. Point set topology
   - A. Sets and mappings, axiom of choice, well ordering theorm and Zorn's Lemma
   - B. Definition of a topology, bases and subbases, closed sets and the closure of a set, limit points of a set
   - C. Examples; subspace topology, product topology, quotient topology, order topology, topology generated by a metric
   - D. Continuous mappings and homeomorphisms
   - E. Topological properties; 1st and 2nd countability, connectedness, path connectedness, compactness, paracompactness, separations properties, local properties
   - F. Theorems; characterization of compactness for metrizable spaces, every metric space is paracompact, Tychonoff product theorem, Urysohn's lemma, metrization theorem for 2nt countable spaces

II. Algebraic Topology
   - A. Classification of compact surfaces (briefly introducing Euler numbers and orientations)
   - B. Fundamental group, the Seifert-Van Kampen Theorem, and covering spaces
   - C. Singular, simplicial, and cellular homology theories; simplicial sets and CW-complexes
   - D. Axioms for homology: homotopy invariance, exact sequences for a pair, excision, and the Mayer-Vietories sequence
   - E. Betti numbers and Euler number
   - F. Hurewicz Theorem (in dimension one, relating $H_1(X;\mathbf{Z})$ and $p_1(X)$)
   - G. Cohomology and cup products
   - H. Universal Coefficient Theorem and the Kunneth Theorem
   - I. Orientations of manifolds; the degree of a map

III. Differential topology
   - A. Basic definitions: smooth maps between Euclidean spaces, smooth manifolds, smooth manifolds with boundary, smooth maps between manifolds (with boundary), smooth partitions of unity
   - B. Tangency: tangent spaces (various definitions and equivalence), tangent bundles, derivatives and tangent mappings
   - C. Inverse Function Theorem and related topics: immersion, embedding, submersion
   - D. Sard's Theorem (without proof), Morse function, and manifolds defined as inverse images of regular values
   - E. Embedding Theorem for manifolds into Euclidean space (with proof in compact case, without proof of optimum dimension)
   - F. Orientations and orientability (via tangent bundle)
   - G. Cotangent bundle and differential 1-forms
   - H. Exterior algebra and exterior bundles
   - I. Differential forms
   - J. Vector fields and the Lie derivative
   - K. Interation on manifolds
   - L. DeRham cohomology and DeRham's Theorem

References

1. James Dugundji, Topology, 1965, Allyn and Bacon.
2. William S. Massey, Algebraic Topology; an Introduction, 1977, Springer.
3. Steen and Seebach, Counterexamples in Topology, 1978, Springer.
4. Marvin Greenberg and J. R. Harper, Algebraic Topology, 1982, Benjamin-Cummings.
5. Edwin H. Spanier, Algebraic Topology 1966, McGraw-Hill.
6. Michael Spivak, A Comprehensive Introduction fo Differential Geometry, Volume I, 1979, Publish or Perish.
7. John Milnor, Topology from the Differentiable Viewpoint, 1965, University of Virginia Press.
8. V. Guillemin and A. Pollack, Differential Topology, 1974, Prentice Hall.
9. James R. Munkres, Topology, Second Edition, 2000, Prentice Hall.