Geometry Qualifying Exam
Fall 2000

Part A. Short answers. Work all of the following.

1. What is the fundamental group of

a) $S^2 \times S^2$

b) $T^*(S^3 \times S^1)$ the total space of the cotangent bundle of $S^3 \times S^1$.

c) R^3 with 2 parallel lines deleted.

2. A Riemannian metric on a manifold is a cross section of what bundle?

3. In R^3 with the standard euclidean flat metric, describe the flow and integral curves of a covariant-constant vector field. (Covariant constant means all covariant derivatives vanish).

4. Let Γ be the ellipsoid $x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$ in R^3 calculate $\int_\Gamma z dx \wedge dy - y dz \wedge dx$.

5. Find the scalar curvature of the surface $z = x^2 + y^2$ at $(0,0,0)$.

6. Can we integrate a 3-form on a surface in a 4-manifold? Why or why not?

7. **a)** What is the fiber dimension of the bundle of 5-forms on S^7?

b) What is the fiber dimension of the bundle of 7-forms on S^7?

8. Are all vector spaces (a) parallelizable? Why or why not? (b) Simply connected? Why or why not?

9. What is the scalar curvature of the euclidean plane in polar coordinates.

Part B. Choose 3 (and only 3) of the following.

1. Let G be a Lie group. Prove G is orientable.

2. Compute the Levi-Civita connection at a point on the standard unit 2-sphere in R^3 in latitude-longitude coordinates.

3. Compute the De-Rham cohomology of $S^1 \times S^2$.

4. Show an explicit isomorphism between the Lie algebras $so(3)$ and $su(2)$.

5. Let C be the 2-dimensional subbundle of the tangent bundle to R^4 determined by $V_1 = x_2 \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_3}$ and $V_2 = \frac{\partial}{\partial x_2} + \frac{\partial}{\partial x_3}$. Use the Frobenius theorem to determine if C is integrable.