**Differential Equations Qualifying Exam**
**Spring, 2003**
**NAME:**

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**1.** Solve the problem:

$$\begin{cases} u\,\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 1, & -\infty < x < +\infty,\ y < 2, \\ u(x,x) = \frac{x}{2}\,, & \text{for } -\infty < x < 2. \end{cases}$$

**2.** Show that $(4\pi|x|)^{-1}e^{-c|x|}$ is a fundamental solution of $-\Delta + c^2$ in $\mathbb{R}^3$.

**3.** Describe the distributional derivative, $f'$, of the function $f(x) = \ln|x|$ on $\mathbb{R}$, i.e., find (and justify) a formula for $\langle f', \eta\rangle$ for any test function $\eta \in \mathcal{D}(\mathbb{R})$. Is $f'$ a tempered distribution?

**4.** Let $\Omega$ be a bounded domain in $\mathbb{R}^3$ with smooth boundary $\partial\Omega$. Let $u(x,t)$ be a sufficiently smooth solution of the problem

$$\begin{cases} u_t - \Delta u + u^3 = 0, & x \in \Omega,\ t > 0, \\ u\big|_{\partial\Omega} = 0. \end{cases}$$

Show that $\int_\Omega |u(x,t)|^2\,dx \to 0$ as $t \to +\infty$.

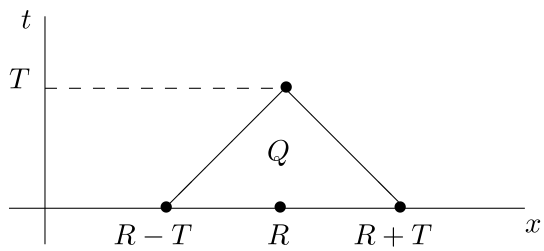
**5.** Find the 1-periodic in $x$ solution $u(x,t)$ of the problem

$$u_{tt} + 4\pi\,u_t - u_{xx} = 0, \qquad u(x,0) = 0,\ \ u_t(x,0) = \cos(2\pi x)\,.$$

**6.** Consider the problem

$$\begin{cases} u_{tt} - u_{xx} - u_t + u = h(x,t), \\ u(0,x) = 0, \ \ u_t(0,x) = 0. \end{cases}$$

Show that if $h(x,t) = 0$ inside the right triangle $Q$,




then $u(x,t) = 0$ in $Q$.