Partial Differential Equations
Qualifying Examination
September 12, 1988

1. A total charge q is distributed uniformly in the ball of radius a centered at the origin. Find the potential generated by this charge distribution in R^3.
2. Let
$$\begin{cases} (\Delta + k^2)u = 0 \quad \text{in} \quad |x| \geq a, \quad x \in R^3, \quad x = (x_1, x_2, x_3) \\ u \Big|_{|x|=a} = 0 \\ u = e^{ikx_3} + v \\ \text{where} \quad r\left(\frac{\partial v}{\partial r} - ikv\right) \to 0 \quad \text{as} \quad r = |x| \to \infty. \end{cases}$$
Find u in the region $|x| \geq a$.
3. Find the temperature of an infinite circular cylinder if its initial temperature $u\Big|_{t=0} = 1 - \rho^2$, $\rho = \sqrt{x_1^2 + x_2^2}$, $0 \leq \rho \leq 1$ and $u(x,t)\Big|_{\rho=1} = 0$, $x \in R^3$. How does $u(x,t)$ behave for large t?
4. Assume $\Delta u = 0$ in $\Omega := \{x : x \in R^3, |x| \geq 1\}$. Is it possible that $\frac{1}{|x|^{3/4}} < u < \frac{1}{|x|^{1/2}}$?
5. Assume
$$\begin{cases} \Delta u - u = 0 & \text{in} \quad R^3 \\ |x|e^{|x|}|u| \to 0 & \text{as} \quad |x| \to \infty. \end{cases}$$
Does it imply that $u \equiv 0$?
6.
$$\begin{cases} u_{tt} = \Delta u & \text{in} \quad R^2 \\ u\Big|_{t=0} = \phi(x), & u_t\Big|_{t=0} = 0. \end{cases}$$
$\phi(x)$ is a smooth function with support in the region $|x| \leq 1$. Calculate $u(x,t)$ at $t = 5$ on the circle $|x| = 10$.
7. Find all solutions of the equation $u_t = u_{xx}$ which are of the form $u = t^{-1/2} f\left(\frac{x}{2} t^{-1/2}\right)$, where $f(\xi) \in C^2$.

Please append this page to your solutions. Less than 6 problems – fail.