Topology Qualifying Exam
Spring 2003 – Miller & Maginnis

NAME:__________________________

1. Let X be a topological space which is connected and locally path connected. Prove that X is path connected.

2. Let $\{X_a | a \in I\}$ be a collection of connected topological spaces. Prove that $\prod_{a \in I} X_a$ with the product topology is connected.

3. Prove that a compact Hausdorff space is metrizable if and only if it is second countable.

4. Let X be a well ordered set with the order topology. Assume that X has a maximal element. Prove that X is compact.

5. Prove that a metric space is sequentially compact if and only if it is limit point compact.

6. a) Let X be a topological space and x_0 an element of X. Define, in detail, what we mean by $\pi_1(X, x_0)$, the fundamental group of X relative to x_0.

b) Prove that the construction you gave in part a) defines a covariant functor from the category of pointed topological spaces to the category of groups.

c) If X is path connected, $x_0 \in X$ and $x_1 \in X$ show that $\pi_1(X, x_0)$ is isomorphic to $\pi_1(X, x_1)$.

d) Suppose that X and Y are path connected, $x_0 \in X, y_0 \in Y$ and $\pi_1(X, x_0)$ is not isomorphic to $\pi_1(Y, y_0)$. Show that X and Y cannot be homeomorphic.