Geometry Manifolds Qualifying Exam
Spring 1995

Part A. Do all parts.

1. A Riemannian metric is a cross section of what bundle on a manifold?

2. Is the two handled torus (connected sum of two $S^1 \times S^1$'s) parallelizable?

3. Give an example of a compact manifold which is not orientable.

4. If we are given a 3-form on the unit ball in R^3, when will Stoke's theorem allow us to rewrite its integral as an integral on S^2?

5. Describe the universal covering space of (a) S^2, (b) $S^2 \times S^1$.

6. Consider the vector field on $R^3 (y^2 + z^2 + 1)\partial/\partial x$. Describe the family of its flows.

7. What is the dimension of the fiber of the bundle of 7-forms on a 9-manifold?

8. (a) Give an example of a lie group which is contractible as topological space and has dimension seven.

(b) Give an example of a lie group which is not contractable as a topological space. Is its lie algebra contractible?

Part B. Calculate – Do 3 of the following 5.

1. Let an atlas for the 2-sphere be given by choosing stereographic projection from two antipodal points. Pick a geodesic joining the two as 0-ray and write polar coordinates on each patch.

(a) Find the transition function $(r, \theta) \to (r', \theta')$.

(b) Write the round metric of radius 1 in each patch.

(c) Find the $\left\{\theta^{\theta}_{r}\right\}$ component of the Levi-Civita connection in one patch.

2. Write generators and relations for π_1 of the once punctured torus.

3. Give a set of generators for the lie algebra $su(2)$, and compute the bracket of each pair.

4. Use Stokes theorem to compute $\int_{S^2} z dx \wedge dy - y dx \wedge dz$ on the unit sphere in R^3.

5. Find the scalar curvature of the surface $z = x^2 - y^2$ at the point $(0, 0, 0)$.