Topology Qualifying Exam
Spring 1994

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1. Prove or disprove: The continuous image of a locally connected space is locally connected.

2. Prove or disprove: A space X is Hausdorff if and only if the diagonal $\Delta : X \hookrightarrow X \times X$ is closed.

3. (a) State the Axiom of Choice.

(b) Give another statement equivalent to the Axiom of Choice.

4. Prove or disprove: A metric space is compact if and only if it is complete and totally bounded.

5. Give an example of a space which is connected but not path connected.

6. Let $f : X \to Y$ be a continuous surjective map from a compact space X to a Hausdorff space Y. Prove that f is a quotient map.

7. Prove or disprove: Every compact Hausdorff space is a Baire space.

8. Prove or disprove: S_Ω, the minimal uncountable well-ordered set equipped with the order topology is not Lindelöf.

9. Prove or disprove: The product of a family of connected spaces is connected.

10. Let $C(X, Y)$ be the set of continuous functions fro X to Y, given the compact-open topology. Let $e : C(X < Y) \times X \to Y$ be the evaluation map $e(f, x) = f(x)$. Prove that if X is locally compact Hausdorff, then e is continuous.

11. (a) State Urysohn's Lemma

(b) Use Urysohn's Lemma to prove that for every finite open cover $\mathcal{U} = \{U_i\}_{i=1}^n$ of a normal space X, there is a partition of unity on X subordinated to $\mathcal{U}$.

12 (a) Prove that [0,1] in the usual topology is connected.

(b) Use your result in (a) to prove that every path-connected space is connected.

13. Let X be a completely regular space. Prove that X is connected if and only if its Stone-Čech compactification $\beta(X)$ is connected.