TOPOLOGY

QUALIFYING EXAMINATION

SPRING 1997

(MUENZENBERGER - MARR)

Do as many of the following 10 problems as you can.

1. Prove that a space X is locally connected if each component of each open set is open.
2. (a) Prove that every compact Hausdorff space is normal.

   (b) Prove that the product space $[0,1]^{[0,1]}$ is normal, but not metrizable.
3. Prove that Zorn's Lemma implies the Axiom of Choice.
4. Prove that if V is a convex open subset of $\mathbb{R}^2$, then $\overline{V}^{\circ} = V$.
5. Prove that if A is a connected subset of a connected space X and if C is a component of $X - A$, then $X - C$ is connected.
6. Prove that the Cantor set $2^{\mathbb{N}}$ is *homogeneous*. That is, prove that given x and y in $2^{\mathbb{N}}$ there is a homeomorphism $h : 2^{\mathbb{N}} \to 2^{\mathbb{N}}$ such that $h(x) = y$.
7. Prove that for a metric space X the following are equivalent.

   (a) X is separable.

   (b) Every family of pairwise disjoint non-empty open subsets of X is countable.
8. Prove that the class of Baire spaces is invariant under continuous open surjections.
9. Prove that every contractible space is pathwise connected.
10. Show that every connected, locally compact, paracompact Hausdorff space is Lindelöf.