Topology Qualifying Exam
Miller & Maginnis, Fall 2002

1. Given subsets A, V and W of a topological space X such that A is closed, V is open, W is open and $A \cap W \subset V$. Prove that $(A \cap W) \cup (V \backslash A)$ is open.

2. Let X be a topological space such that every sequence of points of X has a convergent subsequence. Show that every countable open cover of X has a finite subcover.

3. Prove that a countable product of separable spaces is separable (recall that a separable space is one which has a countable dense subset).

4. Suppose that X is a regular topological space and that $A \subset X$ is a closed subset. Show that the quotient space X/A is Hausdorff.

5. Show that every locally compact Hausdorff space is regular.

6. Let $f : S^2 \to S^2$ by $f(x, y, z) = (-x, -y, -z)$. Define an equivalence relation on S^2 by vRw iff $v = w$ or $w = f(v)$ for $v, w \in S^2$. Let $X = S^2/R$ with the quotient topology and $p : S^2 \to X$ be the quotient mapping.

a) Show that p is a covering map.

b) Use covering space theory to determine the fundamental group of X.

7. Let X be a compact, Hausdorff, locally connected space, Y be a Hausdorff space and $f : X \to Y$ a continuous onto mapping. Prove that Y is locally connected.