Topology Qualifying Exam
Spring 1993

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1. Prove that the interval [0,1] with its usual topology is compact.

2. Prove or disprove that a closed subspace of a normal space must be normal.

3. Prove or disprove that if X and Y are any sets, $f : X \to Y$ is any function, $A \subseteq X$, and $B \subseteq Y$, then $f[A \cap f^{-1}[B]] = f[A] \cap B$.

Use the following definition for problems 4 and 5.

For any topological property P a space is X called **locally** P if and only if every point x of X has a base of neighborhoods each of which has property P (as a subspace).

4. Prove that every quotient of a locally pathwise connected space is locally pathwise connected.

5. (a) Prove that every locally T_1 space is T_1.

(b) Prove that a locally T_2 space need not be T_2.

6. Prove that a space X is Hausdorff if and only if the diagonal $\Delta \subseteq X \times X$ is closed.

7. Let X be a regular space and assume that $\mathcal{U}$ is a countable open covering of X with the property that the closure of each of its members is paracompact. Prove that X is paracompact.

8. State the Axiom of Choice and prove that it is equivalent to Statement S below.

Statement S: The product of any set (including the empty set) of nonempty sets is nonempty.

9. Prove that if $A \times B$ is a compact subset of the space $X \times Y$ that is contained in an open set U of $X \times Y$, then there is a "rectangular" open set of the form $V \times W$ such that $A \times B \subseteq V \times W \subset U$.

10. Prove that if A is a retract of a Hausdorff space X, then A is closed in X.

11. (a) Prove that in a first-countable Hausdorff space X, every one point set is a countable intersection of open subsets of X.

(b) Give an example of a space in which every one point set is a countable intersection of open sets, but which is not first-countable.

12. Let X be a completely regular space. Prove that X is connected if and only if its Stone-Čech compactifiction βX is connected.