Topology Qualifying Exam

Spring 2002

(Yetter and Muenzenberger)

Choose and work any 6 of the following 10 problems. Start each problem on a new sheet of paper. Include your name, the date and the question number on each sheet. **Do not turn in more than six (6) problems.** The word space below always means a topological space.

1. Use Zorn's Lemma to prove that for each set X and a (binary) relation R on X, there is a maximal $A \subset X$ such that $A \times A \subset X$.

2. Prove that if A and B are disjoint compact subsets of a Hausdorff space X, then there are disjoint open sets U and V such that $A \subset U$ and $B \subset V$.

3. Prove or disprove that the continuous image of a compact space is compact.

4. Let $f : X \to Y$ be a continuous surjective map from a compact space X to a Hausdorff space Y. Prove that f is a quotient map.

5. Prove that if Y is compact, then the projection $\pi_X : X \times Y \to X$ is closed (i.e. for each closed subset A of $X \times Y$, the image $\pi_X(A)$ is closed in X).

6. Prove that a compact metric space has a countable basis.

7. Prove that no continuous function $f : S^1 \to \mathbb{R}$ is one-to-one. (Here S^1 is the unit circle $\{(x, y) \in \mathbb{R}^2 | x^2 + y^2 = 1\}$, and both spaces have the usual topologies.)

8. Prove that a quotient of a locally connected space is locally connected.

9. Let X be a connected normal T_2 space having more than one point. Prove that X is uncountable.

10. Describe the fundamental groups of the following spaces, where each has the usual topology
    (a) the circle, S^1
    (b) the Moebius strip, M
    (c) the figure-eight (union of two circles in the plane intersecting in a single point)
    (d) the torus $S^1 \times S^1$
    (e) the real projective plane, $\mathbb{RP}^2$