COMPLEX VARIABLES QUALIFYING EXAM
Spring 1997
(Burckel and Bennett)

1. **Prove that there is no** *continuous* **logarithm on the circle $\mathbb{T}$, that is, no continuous function $f : \mathbb{T} \to \mathbb{C}$ satisfies $e^{f(z)} = z$ for all $z \in \mathbb{T}$.**

2. *State* **the Open Mapping Theorem (for holomorphic functions) and use it to prove the Maximum Modulus Principle.**

3. **$f_n : \mathbb{D} = \{z \in \mathbb{C} : |z| < 1\} \to \mathbb{C}$ are holomorphic and $\lim_{n\to\infty} f_n = f$ uniformly on each compact subset of $\mathbb{D}$. Prove that f is holomorphic on $\mathbb{D}$. What important (even pre-eminent) result in real analysis demonstrates that no such conclusion holds for differentiable functions on $[-1, 1]$ which converge uniformly there?**

4. **There is a local and a global way to define** *meromorphic* **function. Give both definitions and prove the easier half of the equivalence.**

5. **What is the image of $D = \{z \in \mathbb{C} : |z| < 1,\ \mathbf{Re}(z) + \mathbf{Im}(z) > 1\}$ under the mapping $w = \dfrac{1-z}{1+z}$?**

6. **For what values of z does $\prod_{n=1}^{\infty} \sqrt[n]{z}$ converge?**

7. **Suppose λ is a positive real number, $f(z) := \sum_{n=0}^{\infty} a_n z^n$ is entire and $|f(z)| < \exp(|z|^\lambda)$ for all z. Show $|a_n| \le \left(\dfrac{e\lambda}{n}\right)^{\frac{n}{\lambda}}$ for all $n \ge 1$.**

8. **Given a domain $\Omega \subset \mathbb{C}$, show there is a function f which is holomorphic on Ω but which can't be analytically continued to any domain Ω' that properly contains Ω.**