

Exam 1 Prep

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1 Material outline:

- Prior knowledge
 - Evaluation of trig functions at special angles
 - Sketching graph of sin/cos with transformations
- Sided limits, limits, continuity
- Sided continuity?
- Evaluating limits
 - Direct substitution
 - Simplification
 - * Factorization
 - * Multiplying by the conjugate
 - * Combining fractions
 - Squeeze Theorem
 - Trig limits:

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad \lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$$

- Using the limit definition of derivative
- Average velocity and instantaneous velocity
- “Derivative is the slope of tangent line”
- The process of finding the tangent line
- Derivative properties:
 - Power rule
 - Product rule
 - Quotient rule
 - Chain rule
 - Trig derivatives
- Using IVT to find a root / to show a solution to an equation exists.

2 Past exam problems

2.1 Limit evaluation (including Squeeze)

1. $\lim_{\theta \rightarrow 0} \frac{\sin(\theta)}{2\theta}$
2. $\lim_{\theta \rightarrow \pi} \frac{\cos \theta}{\theta}$
3. $\lim_{\theta \rightarrow 1} \frac{1 - \sqrt{x}}{1 - x}$
4. $\lim_{\theta \rightarrow 0} \frac{\cos \theta}{\theta - 3}$
5. $\lim_{x \rightarrow 1} \left(\frac{1}{x - 1} - \frac{1}{x^2 - x} \right)$
6. $\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x - 3}$
7. $\lim_{x \rightarrow 0} x^2 \sin\left(\frac{5}{x}\right)$
8. $\lim_{\theta \rightarrow 0} \theta^2 \sin\left(\frac{1}{\theta}\right)$
9. $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 4}$

10. $\lim_{t \rightarrow 2} \frac{\sqrt{2+t} - 2}{t - 2}$
11. $\lim_{x \rightarrow \pi} \sin x$
12. $\lim_{x \rightarrow 3} \frac{x - 3}{x^2 - 5x + 6}$
13. $\lim_{t \rightarrow 3} \frac{\sqrt{t+6} - 3}{3 - t}$
14. $\lim_{x \rightarrow 2} (x - 2)^2 \cos\left(\frac{\pi}{x - 2}\right)$
15. $\lim_{x \rightarrow 5} \frac{x - 5}{x^2 - 25}$
16. $\lim_{x \rightarrow 4} \frac{2 - \sqrt{x}}{x - 4}$
17. $\lim_{\theta \rightarrow 0} \tan(\theta) + \theta + 1$
18. $\lim_{\theta \rightarrow 0} \frac{3 \sin \theta}{7\theta}$
19. $\lim_{x \rightarrow 5} \frac{x^2 - 4x - 5}{x - 5}$
20. $\lim_{t \rightarrow 9} \frac{3 - \sqrt{t}}{9 - t}$
21. Find $\lim_{x \rightarrow 1} h(x)$, where $4x - 1 \leq h(x) \leq 2x^2 + 1$ for all x .
22. $\lim_{\theta \rightarrow 0} \frac{\cos \theta}{\theta^2 + 3}$
23. $\lim_{w \rightarrow 3} \frac{w^2 - 2w - 3}{w - 3}$
24. $\lim_{t \rightarrow 1} \left(\frac{1}{t^2 - t} - \frac{1}{t - 1} \right)$
25. Find $\lim_{x \rightarrow 1} h(x)$, where $-2x^4 + 5 \leq h(x) \leq 11 - 8x$ for all x .
26. $\lim_{x \rightarrow 1} \frac{\sin(x - 1)}{x^2 - 1}$
27. $\lim_{x \rightarrow 3} \frac{x^2 + x - 12}{x^2 - x - 6}$

$$28. \lim_{\theta \rightarrow 0} \frac{\cos \theta - \cos^2 \theta}{\theta}$$

$$29. \lim_{x \rightarrow 0} 5x^2 \cos\left(\frac{2}{x}\right)$$

2.2 Continuity

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$$1. \text{ Is } q(x) = \begin{cases} 4 & x = 2 \\ \frac{x^2 - 4}{x - 2} & x \neq 2 \end{cases} \text{ continuous?}$$

2. Find a constant c that makes the following function continuous:

$$q(x) = \begin{cases} 3 & x > 2 \\ x + c & x \leq 2. \end{cases}$$

2.3 Using IVT

1. Show there is a root of $f(x) = x^3 + 2x - 1$ in the interval $(0, 1)$.
2. Show there is a root of $x^5 + x - 1 = 0$ in the interval $(0, 1)$.
3. Show there is a root of $3x^7 - 2^x = 0$ in the interval $(0, 1)$.
4. Show there is a root of $3x^7 + x - 2 = 0$ in the interval $(0, 1)$.
5. Let $f(x) = x^4 + 2x - 1$. Show there is a point c between -1 and 1 such that $f(c) = 0$.
6. Show that $2 \sin x = \cos 4x$ has a solution on the interval $[0, \frac{\pi}{4}]$
7. Show $2e^t = t^4$ has a solution on the interval $[-1, 0]$

2.4 Using limit definition of derivative (maybe make tangent line)

1. Let $v(x) = x^2$. Find $v'(3)$.
2. Let $f(x) = \sqrt{x}$. Find $f'(4)$.
3. Let $z(t) = 3t^2 + t$. Find $z'(1)$.
4. Let $g(x) = x^2 + 8$. Find $g'(3)$
5. Let $z(t) = \frac{7}{t}$. Find $z'(1)$.
6. Let $g(x) = x^2 + 3x$. Find $g'(2)$.
7. Let $f(x) = \sqrt{x + 3}$. Find $f'(1)$.

¹Shows up on older exams. May not be as relevant

2.5 Take derivatives

1. $\frac{d}{dx} \left(\frac{5}{x^3} + \sqrt{x} \right)$

2. $\frac{d}{dx} \left(5x^3 + 3\sqrt{x} + \frac{2}{x} \right)$

3. $\frac{d}{d\theta} (\sin \theta \cdot \cos \theta)$

4. $\frac{d}{d\theta} \tan(2\theta^7)$

5. $\frac{d}{dx} \frac{x^2 - 2}{x - 1}$

6. $\frac{d}{dx} \frac{x^5 + 4}{3x^2 + 9}$

7. $\frac{d}{dx} \cos(\sec(x))$

8. $\frac{d}{dx} \frac{3x^2 + 2}{x^8 + x^4}$

9. $k(x) = 5x^2 - 9\sqrt{x} + 3$

10. $v(\theta) = \cos(\tan(\theta))$

11. $p(t) = \frac{\sin(3t)}{2x^2 + 7}$

12. $\frac{d}{dx} \frac{\sqrt{x}}{\cos x}$

13. $h(x) = \frac{x^3}{(3x^2 + 4)^5}$

14. $\frac{d}{dx} \frac{\tan x}{x^8 + x^4}$

15. $\frac{d}{dx} \sin(\cos(\tan(x)))$

16. $v(x) = \frac{\cos x}{x^2 + 1}$

17. $h(x) = (\sin(\sqrt{x}))^5$

18. $p(x) = (\tan(x) + x)^3$

19. $f(x) = \frac{1}{x^2} + \sqrt[3]{x}$

20. $g(x) = (2x^{2/3} - 3x^2)(x^{-1/2} + 5x^{3/7})$