Exam 3 Prep

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Contents

1 Material outline:

- Previous material:
  - All derivative rules
- Applied optimization
  - Closed interval method
  - 1st derivative test
  - 2nd derivative test
- L'Hôpital's rule
  - Remember: the limit must be of type $\frac{0}{0}$ or type $\frac{\infty}{\infty}$ for L'Hôpital's to apply.
- Evaluating Integrals
  - Visually/geometrically (area under a curve)
  - Antiderivatives
  - U-substitution

  - Symmetry? (even/odd)
  - Initial value problem

- Using integrals
  - "Net change theorem"
  - Calculating displacement / distance given a velocity function and endpoints in time.

- Approximating integrals
  - Left-Endpoint / Right-Endpoint / Midpoint

- Fundamental Theorem of Calculus (FTC)
  - Part 2:
$$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a)$$
    where $F'(x) = f(x)$.
  - Part 1:
    * Usual version:
$$\frac{\mathrm{d}}{\mathrm{d}x}\int_a^x g(t)\,\mathrm{d}t = g(x)$$
    * In general:
$$\frac{\mathrm{d}}{\mathrm{d}x}\int_a^{f(x)} g(t)\,\mathrm{d}t = g(f(x)) \cdot f'(x)$$

2 Past exam problems

The following problems all have written solutions in my exam archive at http://www.math.ksu.edu/~winstonc/exams/calc1.

Note, I have rephrased some of the problems here, to ease typing. Unless I've made a mistake, the exam listed should be where the problem was originally found.

2.1 Evaluating limits (L'Hôpital)

1. (2018 Fall Exam 3) $\displaystyle\lim_{\theta\to 0}\frac{\sin(\theta^2)}{\theta^2}$
2. (2018 Fall Exam 3) $\displaystyle\lim_{x\to\infty}\frac{x\ln x}{x^2+3x}$
3. (2017 Fall Exam 3) $\displaystyle\lim_{t\to 0}\frac{1+t-\cos t}{t^2+\sin(2t)}$
4. (2017 Fall Exam 3) $\displaystyle\lim_{x\to\infty} x^2 2^{-x}$

5. (2017 Fall Exam 3) $\lim_{x \to 0^+} x^{2x}$

6. (2017 Spring Exam 3) $\lim_{x \to \infty} \frac{e^x + 5x}{x + 3}$

7. (2017 Spring Exam 3) $\lim_{\theta \to 0} \frac{\sin(\theta^2)}{3\theta^2}$

8. (2016 Fall Exam 3) $\lim_{t \to 0} \frac{4t - \sin(2t)}{5t^2 + 3t}$

9. (2016 Fall Exam 3) $\lim_{x \to \infty} x \sin\left(\frac{2}{x}\right)$

10. (2016 Fall Exam 3) $\lim_{x \to \infty} (5x)^{1/x}$

2.2 Evaluating integrals

Some may require u-sub. Some may require geometry / symmetry.

1. (2018 Fall Exam 3) $\int (\sqrt{x} + 6\sec^2(x) - 5)\,\mathrm{d}x$

2. (2018 Fall Exam 3) $\int_0^4 (e^x - 3)\,\mathrm{d}x$

3. (2018 Fall Exam 3) $\int_0^{\pi/2} 2\sin^3(\theta)\cos(\theta)\,\mathrm{d}\theta$

4. (2018 Fall Exam 3) $\int x\sqrt{5+x}\,\mathrm{d}x$

5. (2017 Fall Exam 3) $\int \frac{\sqrt{x} - \sqrt{2}x^5}{x}\,\mathrm{d}x$

6. (2017 Fall Exam 3) $\int 3x\sin\left(5x^2\right)\mathrm{d}x$

7. (2017 Fall Exam 3) $\int x\sqrt{x+2}\,\mathrm{d}x$

8. (2017 Fall Exam 3) $\int_{-2}^{2} \sin^3(5x) + \sqrt{4 - x^2}\,\mathrm{d}x$

9. (2016 Fall Exam 3) $\int \frac{x^2 - 7x}{x^3}\,\mathrm{d}x$

10. (2016 Fall Exam 3) $\int \sqrt{\tan x}\sec^2 x\,\mathrm{d}x$

11. (2016 Fall Exam 3) $\displaystyle\int \frac{(\ln x)^3}{x}\,\mathrm{d}x$

12. (2016 Fall Exam 3) $\displaystyle\int_{-2}^{2} \left(\frac{\sin x}{1+x^2} + \cos\left(\frac{\pi}{4}x\right)\right)\mathrm{d}x$

13. (2016 Spring Exam 3) $\displaystyle\int (\sec^2 x + 4)\,\mathrm{d}x$

14. (2016 Spring Exam 3) $\displaystyle\int (\sqrt{x} + 5e^x)\,\mathrm{d}x$

15. (2015 Fall Exam 3) $\displaystyle\int (x^2+4)^2\,\mathrm{d}x$

16. (2015 Fall Exam 3) $\displaystyle\int_1^2 2^t\,\mathrm{d}t$

17. (2015 Fall Exam 3) $\displaystyle\int \tan^3 x \sec^2 x\,\mathrm{d}x$

18. (2015 Fall Exam 3) Suppose that $\displaystyle\int_0^6 f(x)\,\mathrm{d}x = 9$, $\displaystyle\int_4^6 f(x)\,\mathrm{d}x = 5$, and $\displaystyle\int_0^4 g(x)\,\mathrm{d}x = 8$. Compute $\displaystyle\int_0^4 (5f(x) - 3g(x))\,\mathrm{d}x$.

19. (2015 Spring Exam 3) Find the most general antiderivative of $\sec^2 x + 3x^4 + 2$.

20. (2014 Fall Exam 3) $\displaystyle\int \left(\cos x + 4x + \frac{1}{x}\right)\mathrm{d}x$

21. (2014 Fall Exam 3) $\displaystyle\int \left(3e^x + 4\sin x + 7\sec^2 x\right)\mathrm{d}x$

22. (2014 Spring Exam 3) $\displaystyle\int (7 + 2x + 3e^x)\,\mathrm{d}x$

23. (2014 Spring Exam 3) $\displaystyle\int (\sec^2\theta + \cos\theta)\,\mathrm{d}\theta$

2.2.1 with initial conditions

1. (2018 Fall Exam 3) Find $f(x)$ if $f''(x) = 6x$, $f'(0) = 1$, and $f(0) = 2$.

2. (2017 Fall Exam 3) Solve the initial value problem for $f(t)$: $f'(t) = 2e^{-2t}$, $f(0) = 1$.

3. (2016 Fall Exam 3) Solve the initial value problem for $f(t)$: $f'(t) = 4e^{3t}$, $f(0) = 5$.

4. (2016 Spring Exam 3) Find the function $v(x)$ satisfying $v''(x) = 2$, $v'(0) = -3$, and $v(0) = 5$.

5. (2015 Fall Exam 3) A ball thrown vertically from the roof of a building 150 feet tall hits the ground 3 seconds later. Was the ball thrown upward or downward? With what speed was it thrown? (Recall that acceleration due to gravity is $a = -32\text{ft}/\text{sec}^2$.) [1]

6. (2015 Spring Exam 3) Find the function $g(x)$ satisfying $g'(x) = \sin x + 1$ and $g(0) = 3$.

7. (2014 Fall Exam 3) Find $v(x)$ if $v''(x) = 6x + 2$, $v'(0) = 1$, and $v(0) = 2$.

8. (2014 Spring Exam 3) Find the function $k(x)$ provided that $k'(x) = 2x^3 + 3x + 2$ and $k(0) = 2$.

2.2.2 for distance / displacement

1. (2018 Fall Exam 3) Suppose a particle has position $s(t)$ feet at time t seconds and a velocity function $s'(t) = 3\cos(t)$ ft/s. Find the displacement from time $t = 0$ seconds to time $t = \pi/2$ seconds.

2. (2016 Fall Exam 3) An object moves along a straight line with velocity $v(t) = 4 - t^2$ m/sec. Find the displacemenet of the object over the time interval $[0, 3]$ seconds. Find the total distance the object travels over the same time interval.

3. (2015 Fall Exam 3) An object moves along the x-axis with velocity $v = 12t^3 - 12t^2$ cm/sec. Find the total distance traveled for the interval $-1 \leq t \leq 2$ seconds.

2.3 Approximating integrals

1. (2017 Fall Exam 3) Estimate the area below the curve $y = \sqrt{x}$ over the interval $[1, 4]$ using L_3. Sketch a graph of the curve and illustrate the rectangles used on the graph.

2. (2016 Fall Exam 3) Estimate the area below the curve $y = \sqrt{x} + 1$ over the interval $[0, 6]$ using R_3. Sketch a graph of the curve and illustrate the rectangles used on the graph.

3. (2015 Fall Exam 3) Approximate the area under the curve $y = 12x - 4x^2$ between $x = 0$ and $x = 2$ using four rectangles and the right endpoint method.

4. (2015 Spring Exam 3) Estimate the area between $y = x^2$ and the x-axis over the interval $[0, 4]$. Use $n = 2$ rectangles, taking the sampling points to be the midpoints (in other words, compute M_2). Sketch the rectangles on the graph.

[1] This is a neat problem.

5. (2014 Fall Exam 3) Estimate $\displaystyle\int_0^6 (x^2+1)\,\mathrm{d}x$ by using $n=3$ subintervals, taking the sampling points to be midpoints (in other words, compute M_3). Sketch the rectangles on a graph.

2.4 FTC Part 1

1. (2018 Fall Exam 3) $\displaystyle\frac{\mathrm{d}}{\mathrm{d}x}\int_3^x e^{2t}\sin\left(t^3\right)\mathrm{d}t$

2. (2017 Fall Exam 3) $\displaystyle\frac{\mathrm{d}}{\mathrm{d}x}\int_2^x \frac{\cos\left(t^2\right)}{2+t}\,\mathrm{d}t$

3. (2017 Fall Exam 3) $\displaystyle\frac{\mathrm{d}}{\mathrm{d}x}\int_{x^3}^5 \frac{\cos\left(t^2\right)}{2+t}\,\mathrm{d}t$

4. (2016 Fall Exam 3) $\displaystyle\frac{\mathrm{d}}{\mathrm{d}x}\int_2^x \frac{\sin t}{1+t}\,\mathrm{d}t$

5. (2016 Fall Exam 3) $\displaystyle\frac{\mathrm{d}}{\mathrm{d}x}\int_2^{x^3} \frac{\sin t}{1+t}\,\mathrm{d}t$

6. (2015 Fall Exam 3) Define $F(x) = \displaystyle\int_1^x \frac{\sin\left(\frac{\pi t}{6}\right)}{t^2}\,\mathrm{d}t$. Find an equation of the tangent line to $y=F(x)$ at $x=1$. [2]

2.5 Applied Optimization

In these problems, one must justify why the min/max is an *absolute* min/max.
Include units!

1. (2018 Fall Exam 3) A rectangular open-topped aquarium is to have a square base and volume $8\,\mathrm{m}^3$. The material for the base costs \$2 per m^2, and the material for the sides costs \$1 per m^2. What dimensions minimize the cost of the aquarium?

2. (2017 Fall Exam 3) A box with square base and open top is formed from two materials. The base costs \$4 per square foot, while the four sides cost \$1 per square foot. If the total cost for the base and four sides is fixed to ber \$120, find the dimensions that maximize the volume of the box.

3. (2017 Spring Exam 3) Let $p(x) = 100 - 2x$ be the price in dollars per cake a bakery can charge if it sells x cakes. What cake price will maximize revenue?

4. (2017 Spring Exam 3) A rectangular open-topped aquarium is to have a square base and volume $5\,\mathrm{m}^3$. The material for the base costs \$10 per m^2, and the material for the sides costs \$1 per m^2. What dimensions minimize the cost of the aquarium?

[2] interesting twist

5. (2016 Fall Exam 3) A rectangular fence consists of three sides costing \$2 per meter and one side costing \$1 per meter. If the area of the rectangle is 12 square meters, find the dimensions that minimize the cost of the fence.

6. (2016 Spring Exam 3) A farmer has 24 feet of fencing and wants to fence off a rectangular area that borders a straight river. The farmer needs no fencing along the river. What dimensions will maximize the fenced-in area?

7. (2016 Spring Exam 3) A rectangular open-topped aquarium is to have a square base and volume $8\,\mathrm{m}^3$. The material for the base costs \$2 per m^2, and the material for the sides costs \$1 per m^2. What dimensions minimize the cost of the aquarium?

8. (2015 Fall Exam 3) A landscaper is designing a fence along the four sides of a rectangular garden, which is to have an area of 5000 square feet. The fencing for three sides costs \$10 per foot, but the fencing along the front side of the garden will cost \$30 per foot. Find the length and width of the garden in order to minimize the total cost.

9. (2015 Spring Exam 3) If a bakery charges x dollars per cake, it makes a total profit of $P(x) = -x^2 + 100x - 30$. If the bakery wants to maximize profit, what should it charge per cake?

10. (2015 Spring Exam 3) Find the dimensions of the box with square base that has volume 8 and minimal surface area.

11. (2014 Fall Exam 3) A homeowner with 16 feet of fencing wants to enclose a rectangular area against the side of her house. What dimensions will maximize the fenced-in area? (Note that 3 sides of the rectangle will be formed from fencing, and the house will serve as the fourth side of the rectangle.)

12. (2014 Fall Exam 3) A rectangular open-topped box is to have a square base and volume $12\,\mathrm{ft}^3$. If material for the base costs \$3 per ft^2 and material for the sides costs \$1 per ft^2, what dimensions minimize the cost of the box?

13. (2014 Spring Exam 3) What is the smallest perimeter possible for a rectangle of area $4\,\mathrm{ft}^2$?