Python History for problem 1 with Golden Section Method

```
[Command: python -u C:\Users\lxyxi\OneDrive\ME441_project\ME441_Engineering_Optimization_2017Fall\Homework4\GoldenSection.py]
cG= 0.61803398875
Step#:  0 a              b              c              d              fa             fb             fc             fd
Step#:  1 0.816311896062 1.71352549156 1.15901699437 1.37082039325 10.3383178603 8.28267744039 8.22290286525 7.73763198885
Step#:  2 1.15901699437 1.71352549156 1.37082039325 1.50172209269 8.22290286525 8.28267744039 7.73763198885 7.75174285191
Step#:  3 1.15901699437 1.50172209269 1.28991869381 1.37082039325 8.22290286525 7.75174285191 7.84885778028 7.73763198885
Step#:  4 1.28991869381 1.50172209269 1.37082039325 1.42082039325 7.84885778028 7.75174285191 7.73763198885 7.71470626413
Step#:  5 1.37082039325 1.50172209269 1.42082039325 1.45172209269 7.73763198885 7.75174285191 7.71470626413 7.71803738703
Step#:  6 1.37082039325 1.45172209269 1.40172209269 1.42082039325 7.73763198885 7.71803738703 7.71933192215 7.71470626413
Step#:  7 1.40172209269 1.45172209269 1.42082039325 1.43262379212 7.71933192215 7.71803738703 7.71470626413 7.71440066584
Step#:  8 1.42082039325 1.45172209269 1.43262379212 1.43991869381 7.71470626413 7.71803738703 7.71440066584 7.71518703728
Step#:  9 1.42082039325 1.43991869381 1.42811529494 1.43262379212 7.71470626413 7.71518703728 7.71428717069 7.71440066584
Step#:  10 1.42082039325 1.43262379212 1.42532889044 1.42811529494 7.71470626413 7.71440066584 7.71435931266 7.71428717069
Step#:  11 1.42532889044 1.43262379212 1.42811529494 1.42983738762 7.71435931266 7.71440066584 7.71428717069 7.71429693285
Step#:  12 1.42532889044 1.42983738762 1.42705098312 1.42811529494 7.71435931266 7.71429693285 7.71430189657 7.71428717069
Step#:  13 1.42705098312 1.42983738762 1.42811529494 1.42877307581 7.71430189657 7.71429693285 7.71428717069 7.71428599892
Step#:  14 1.42811529494 1.42983738762 1.42877307581 1.42917960675 7.71428717069 7.71429693285 7.71428599892 7.71428830345
Step#:  15 1.42811529494 1.42917960675 1.42852182588 1.42877307581 7.71428717069 7.71428830345 7.71428573151 7.71428599892
Step#:  16 1.42811529494 1.42877307581 1.42836654487 1.42852182588 7.71428717069 7.71428599892 7.71428600813 7.71428573151
Optimal Solution:    [ 7.71428583    1.42844419]
[Finished in 1.585s]
```

Python Code for problem 1 with Golden Section Method

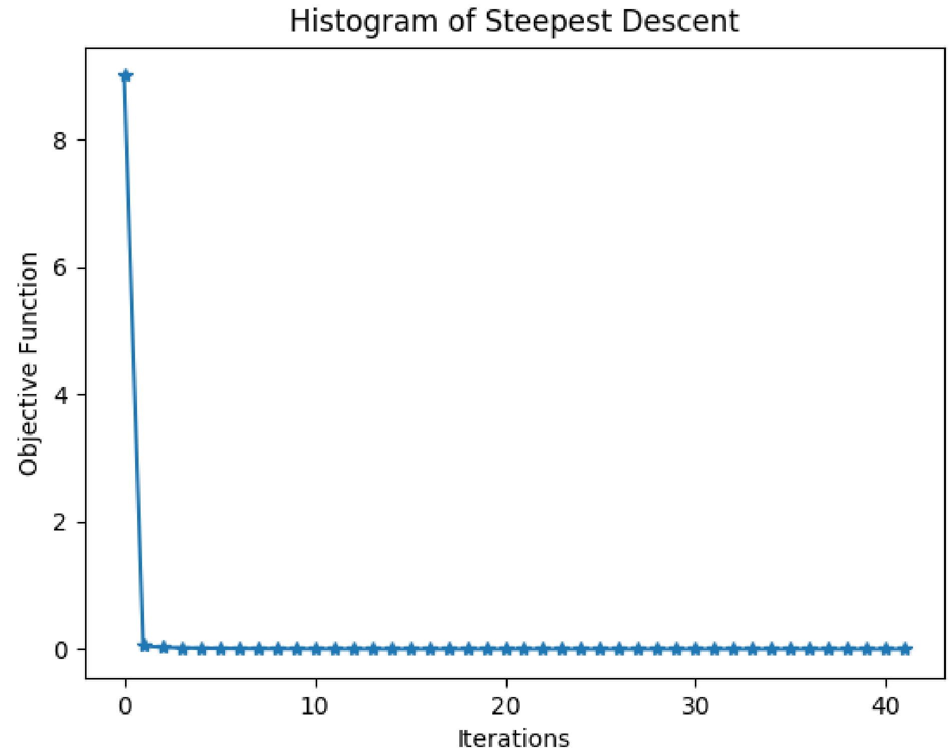
```
'''
GoldenSectionSearchAlgorithm
Date:     10/13/17
Class:    ME441_EngineeringOptimization
Author: Xiaoyi Liu
'''
import numpy as np

cG=(-1+np.sqrt(5))/2
print("cG=",cG)
delta=0.05

def fLinear(x):
    return    7*x*x-20*x+22

def InitialBracketing(fLinear):
    xl=0
    xm=xl+delta
    xu=xm+(1+cG)*delta
    while((fLinear(xl)-fLinear(xm))
            *(fLinear(xm)-fLinear(xu))>0):
        xl=xm
        xm=xu
        xu=xm+(1+cG)*(xm-xl)
    return(xl,xu)

def GoldenSection(fLinear,a,b):
#    (a,b)=(InitialBracketing(fLinear))
    c=a+(1-cG)*(b-a)
    d=a+cG*(b-a)
    fa=fLinear(a)
    fb=fLinear(b)
    fc=fLinear(c)
    fd=fLinear(d)
    i=0
    print('Step#: ', i, 'a                    b                    c                    d                    fa
fb                    fc                    fd\n')

    while((np.abs(a-b))>0.001):
        if(fc<fd):
            b=d
            fb=fd
            d=c
            fd=fc
            c=a+(1-cG)*(b-a)
            fc=fLinear(c)
        else:
            a=c
            fa=fc
            c=d
            fc=fd
            d=a+cG*(b-a)
            fd=fLinear(d)
        i+=1
        print('Step#: ', i, a,    b, c, d, fa, fb,fc,fd,'\n')
#    print(a,b)
    return np.array([fLinear((a+b)/2),(a+b)/2])

(a,b)=(InitialBracketing(fLinear))
print('Optimal Solution: ', GoldenSection(fLinear,a,b))
```

Python History for problem 2 with Steepest Descent Method

```
[Command: python -u C:\Users\lxyxi\OneDrive\ME441_project\ME441_Engineering_Optimization_2017Fall\Homework4\SteepestDesent.py]
Step # =  0
df=  [4 8 6] x=  [1, 1, 1] alpha=  0.154430856687 f0=  9 norm=  10.7703296143
Step # =  1
df=  [ 0.29365944 -0.03040455 -0.17723427] x=  [ 0.38227657 -0.23544685  0.07341486] alpha=  0.41238353925 f0=  0.0532030752104 norm=
0.344343271325
Step # =  2
df=  [ 0.07653547 -0.07627486  0.14019638] x=  [ 0.26117625 -0.22290852  0.14650335] alpha=  0.660003665687 f0=  0.0287654015479 norm=
0.177004399043
Step # =  3
df=  [ 0.07619146 -0.16099575 -0.12924075] x=  [ 0.21066256 -0.17256683  0.05397323] alpha=  0.223935688187 f0=  0.0184288369988 norm=
0.220063493694
Step # =  4
df=  [ 0.11417287  0.00697429  0.0586311 ] x=  [ 0.19360058 -0.13651414  0.08291485] alpha=  0.368362879187 f0=  0.0130066171457 norm=
0.128536733694
Step # =  5
df=  [ 0.02492064 -0.13061113 -0.03289712] x=  [ 0.15154353 -0.13908321  0.06131732] alpha=  0.240498834187 f0=  0.00996260641973 norm=
0.13697636773
Step # =  6
df=  [ 0.07575752 -0.00112716  0.0615734 ] x=  [ 0.14555014 -0.10767138  0.06922904] alpha=  0.366234255562 f0=  0.00770527430314 norm=
0.0976307126875
Step # =  7
df=  [ 0.02109313 -0.10006652 -0.02780214] x=  [ 0.11780514 -0.10725858  0.04667875] alpha=  0.240498834187 f0=  0.00596005119489 norm=
0.105977294555
Step # =  8
df=  [ 0.05907915 -0.00057598  0.04707515] x=  [ 0.11273227 -0.0831927   0.05336514] alpha=  0.365827724624 f0=  0.00461010752707 norm=
0.0755430139848
Step # =  9
df=  [ 0.01627498 -0.07740151 -0.02138901] x=  [ 0.09111948 -0.08298199  0.03614374] alpha=  0.240498834187 f0=  0.00356641026694 norm=
0.0819350911507
Step # =  10
df=  [ 0.0456767  -0.00048178  0.03641706] x=  [ 0.08720537 -0.06436702  0.04128777] alpha=  0.366234255562 f0=  0.00275892193413 norm=
0.0584191388164
Step # =  11
df=  [ 0.01257285 -0.0599071  -0.01657875] x=  [ 0.07047699 -0.06419057  0.0279506 ] alpha=  0.240498834187 f0=  0.00213409066513 norm=
0.0634176014177
Step # =  12
df=  [ 0.03534051 -0.00034992  0.02818511] x=  [ 0.06745324 -0.04978298  0.03193777] alpha=  0.365827724624 f0=  0.00165071065569 norm=
0.0452048053245
Step # =  13
```

```
df=   [ 0.00973945  -0.04631674  -0.01280245]  x=   [ 0.0545247    -0.04965497    0.02162687]  alpha=   0.240498834187  f0=   0.00127701024582  norm=
0.0490306063674
Step # =    14
df=   [ 0.02733304  -0.00028696    0.02179169]  x=   [ 0.05218237  -0.03851585    0.02470585]  alpha=   0.366234255562  f0=   0.000987868926663  norm=
0.0349579112552
Step # =    15
df=   [ 0.00752264  -0.03584891  -0.00992158]  x=   [ 0.04217208  -0.03841076    0.01672498]  alpha=   0.24024758425  f0=   0.000764145319843  norm=
0.0379495955081
Step # =    16
df=   [ 0.02113327  -0.00024578    0.01683819]  x=   [ 0.04036478  -0.02979814    0.01910862]  alpha=   0.366892036437  f0=   0.000591059044944  norm=
0.0270222173097
Step # =    17
df=   [ 0.00580636  -0.02774794  -0.00769265]  x=   [ 0.03261115  -0.02970797    0.01293082]  alpha=   0.24024758425  f0=   0.000457107307567  norm=
0.0293741169655
Step # =    18
df=   [ 0.01634918  -0.00017608    0.01303266]  x=   [ 0.03121619  -0.0230416     0.01477896]  alpha=   0.366234255562  f0=   0.000353512771629  norm=
0.0209087785596
Step # =    19
df=   [ 0.0045029    -0.02143941  -0.00593039]  x=   [ 0.02522856  -0.02297711    0.01000596]  alpha=   0.240498834187  f0=   0.000273438949242  norm=
0.0226956787072
Step # =    20
df=   [ 0.01264932  -0.00012818    0.01008692]  x=   [ 0.02414561  -0.01782096    0.01143221]  alpha=   0.365827724624  f0=   0.00021151278573  norm=
0.0161792366698
Step # =    21
df=   [ 0.00348816  -0.01657571  -0.0045796  ]  x=   [ 0.01951814  -0.01777407    0.00774213]  alpha=   0.240498834187  f0=   0.000163622094971  norm=
0.0175469124556
Step # =    22
df=   [ 0.00978324  -0.00010497    0.00779883]  x=   [ 0.01867925  -0.01378763    0.00884352]  alpha=   0.366234255562  f0=   0.000126579979174  norm=
0.0125117766354
Step # =    23
df=   [ 0.00269421  -0.01282951  -0.00354908]  x=   [ 0.01509629  -0.01374918    0.00598732]  alpha=   0.240498834187  f0=   9.79091914365e-05  norm=
0.0135812739114
Step # =    24
df=   [      7.56926604e-03      -7.63933320e-05         6.03608184e-03]   x=     [  0.01444833   -0.0106637        0.00684087]   alpha=      0.365827724624   f0=
7.57349845423e-05 norm=    0.00968162735738
Step # =    25
df=   [ 0.00208706  -0.00991903  -0.00274069]  x=   [ 0.01167929  -0.01063575    0.0046327  ]  alpha=   0.240498834187  f0=   5.85875081001e-05  norm=
0.0105002110546
Step # =    26
df=   [      5.85422332e-03      -6.25778862e-05         4.66687281e-03]   x=     [  0.01117735   -0.00825024      0.00529184]   alpha=      0.366234255562   f0=
4.53236556762e-05 norm=    0.007487025341
Step # =    27
df=   [ 0.00161203  -0.00767728  -0.00212397]  x=   [ 0.00903333  -0.00822732    0.00358267]  alpha=   0.240498834187  f0=   3.50579528233e-05  norm=
0.00812714190446
```

```
Step # =    28

df=    [    4.52939721e-03    -4.55302756e-05    3.61203166e-03]  x=    [  0.00864564  -0.00638094    0.00409348]  alpha=    0.365827724624  f0=

2.711792513e-05 norm=    0.00579346914879

Step # =    29

df=   [ 0.00124875  -0.00593563  -0.00164018]  x=   [ 0.00698866  -0.00636429    0.0027721  ]  alpha=    0.240498834187  f0=    2.09781943648e-05  norm=

0.00628340927321

Step # =    30

df=    [    3.50312728e-03    -3.73047339e-05    2.79268779e-03]  x=    [  0.00668834  -0.00493678    0.00316656]  alpha=    0.366234255562  f0=

1.62287415327e-05 norm=    0.00448022293045

Step # =    31

df=   [ 0.00096452  -0.00459414  -0.0012711  ]  x=   [ 0.00540537  -0.00492311    0.00214378]  alpha=    0.240498834187  f0=    1.25530610562e-05  norm=

0.00486334611275

Step # =    32

df=    [    2.71036042e-03    -2.71355128e-05    2.16146386e-03]  x=    [  0.00517341  -0.00381823    0.00244948]  alpha=    0.365827724624  f0=

9.709936145e-06 norm=    0.00346680197598

Step # =    33

df=   [ 0.00074716  -0.00355192  -0.00098158]  x=   [ 0.00418188  -0.0038083     0.00165876]  alpha=    0.240498834187  f0=    7.51157804383e-06  norm=

0.00376004177229

Step # =    34

df=    [    2.09624746e-03    -2.22382549e-05    1.67116299e-03]  x=    [  0.00400219  -0.00295407    0.00189482]  alpha=    0.366234255562  f0=

5.81091811726e-06 norm=    0.00268095760831

Step # =    35

df=   [ 0.0005771    -0.00274917  -0.0007607  ]  x=   [ 0.00323447  -0.00294592    0.00128279]  alpha=    0.240498834187  f0=    4.49482441027e-06  norm=

0.00291026485615

Step # =    36

df=    [    1.62186120e-03    -1.61721834e-05    1.29343440e-03]  x=    [  0.00309568  -0.00228475    0.00146573]  alpha=    0.365827724624  f0=

3.47677263474e-06 norm=    0.00207452834545

Step # =    37

df=   [ 0.00044705  -0.0021255    -0.00058743]  x=   [ 0.00250236  -0.00227883    0.00099256]  alpha=    0.240498834187  f0=    2.68964067234e-06  norm=

0.00225003872044

Step # =    38

df=    [    1.25438018e-03    -1.32565684e-05    1.00003507e-03]  x=    [  0.00239484  -0.00176765    0.00113384]  alpha=    0.366234255562  f0=

2.08067700876e-06 norm=    0.00160428037293

Step # =    39

df=   [ 0.0003453    -0.00164512  -0.00045524]  x=   [ 0.00193545  -0.0017628     0.00076759]  alpha=    0.240498834187  f0=    1.60944381773e-06  norm=

0.00174152555783

Step # =    40

df=    [    9.70510692e-04    -9.63811484e-06    7.73999782e-04]  x=    [  0.0018524     -0.00136715    0.00087707]  alpha=    0.365827724624  f0=

1.24490499023e-06 norm=    0.00124139420006

Step # =    41

df=   [ 0.00026748  -0.00127192  -0.00035155]  x=   [ 0.00149736  -0.00136362    0.00059392]  alpha=    0.240498834187  f0=    9.63068867191e-07  norm=

0.00134644096035

[Finished in 2.247s]
```



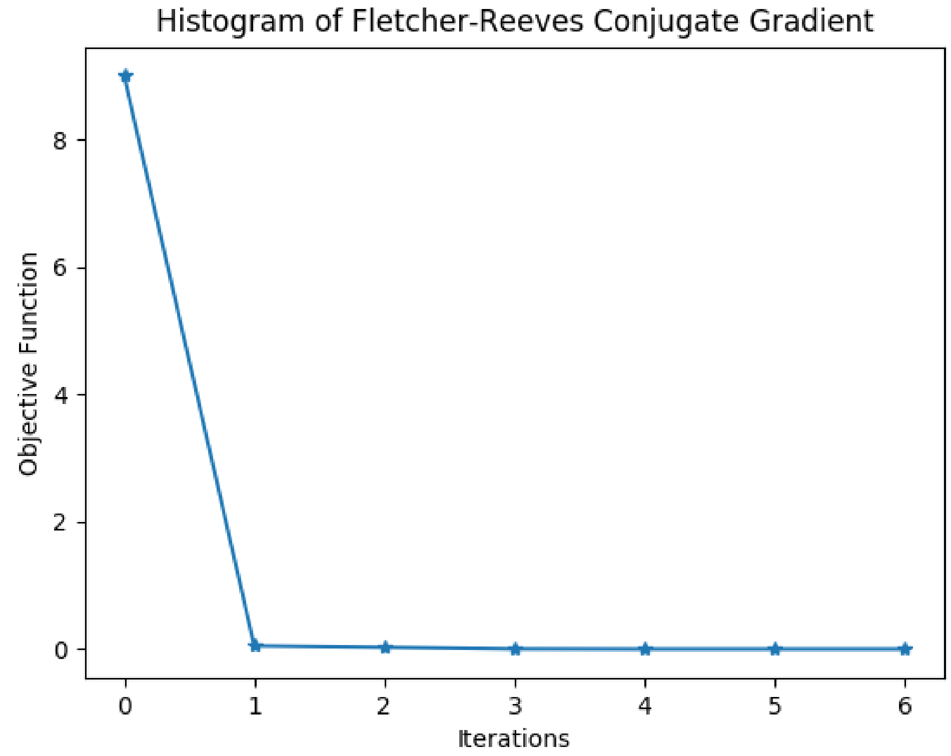
Python Code for problem 2 with Steepest Descent Method

```
'''
SteepestDescentAlgorithm
Date:     10/13/17
Class:    ME441_EngineeringOptimization
Author: Xiaoyi Liu
'''
import numpy as np
import matplotlib.pyplot as plt
from GoldenSection_For_Multiopt import GoldenSection

def fQuar(x):
    return x[0]**2+2*x[1]**2+2*x[2]**2 + 2*x[0]*x[1]+2*x[1]*x[2]

def dfQuar(x):
    return np.array([2*x[0]+2*x[1], 2*x[0]+4*x[1]+2*x[2], 2*x[1]+4*x[2]])

#SteepestDescentAlgorithm
x0=[1,1,1]
f0=fQuar(x0)
df0=dfQuar(x0)
epsilon=0.001
i=0
y=[]
```

```
while (np.linalg.norm(df0)>epsilon):
    y.append(f0)
    print('Step # = ',i)
    dr=-df0
    def localf(alpha):
        return fQuar(x0+alpha*dr)
    [ff,alpha0]=GoldenSection(localf,-1,1)
    print('df= ', df0, 'x= ', x0,'alpha= ',alpha0, 'f0= ',f0,'norm= ',np.linalg.norm(df0))
    x0=x0+alpha0*dr
    f0=fQuar(x0)
    df0=dfQuar(x0)
    i=i+1
plt.plot(y,'-*')
plt.xlabel('Iterations')
plt.ylabel('Objective Function')
plt.title('Histogram of Steepest Descent')
plt.show()
```

Python History for problem 2 with Conjugate Method

```
[Command: python -u C:\Users\lxyxi\OneDrive\ME441_project\ME441_Engineering_Optimization_2017Fall\Homework4\FRConjugate.py]

Step # =    0

df=   [-4 -8 -6] x=   [1, 1, 1] alpha=   0.154430856687 f0=   9 norm=   10.7703296143

c0=   [4 8 6] beta= 0.00102217490092 c1= [ 0.29365944 -0.03040455 -0.17723427]

[-0.29774814   0.02222715   0.17110122]

Step # =    1

df=   [-0.29774814   0.02222715   0.17110122] x=   [ 0.38227657 -0.23544685   0.07341486] alpha=   0.415576474687 f0=   0.0532030752104 norm=

10.7703296143

c0=   [ 0.29365944 -0.03040455 -0.17723427] beta= 0.250626550618 c1= [ 0.06465936 -0.09871919   0.12566246]

[-0.13928294   0.1042899   -0.08277995]

Step # =    2

df=   [-0.13928294   0.1042899   -0.08277995] x=   [ 0.25853945 -0.22620977   0.1445205 ] alpha=   1.73918327244 f0=   0.0286045204994 norm=

0.344127341699

c0=   [ 0.06465936 -0.09871919   0.12566246] beta= 1.08048567787 c1= [-0.05705927 -0.14561831 -0.08745704]

[-0.09343396   0.25830205 -0.00198551]

Step # =    3

df=   [-0.09343396   0.25830205 -0.00198551] x=   [ 0.01630088 -0.04483052   0.000551   ] alpha=   0.172871376375 f0=   0.00277491949107 norm=

0.192687940581

c0=   [-0.05705927 -0.14561831 -0.08745704] beta= 7.16122555095e-06 c1= [ -5.73195041e-05    3.23008828e-06    4.76072645e-04]

[   5.66504025e-05   -1.38032900e-06   -4.76086863e-04]

Step # =    4

df=   [    5.66504025e-05   -1.38032900e-06   -4.76086863e-04] x=   [ 0.00014883 -0.00017749   0.00020776] alpha=   0.251237921249 f0=

4.49027964009e-08 norm=   0.274688547566

c0=   [ -5.73195041e-05    3.23008828e-06    4.76072645e-04] beta= 0.193647045967 c1= [ -2.95476274e-05   -2.08913765e-04   -3.06523283e-06]

[   4.05178105e-05    2.08646468e-04   -8.91275819e-05]

Step # =    5

df=   [   4.05178105e-05    2.08646468e-04   -8.91275819e-05] x=   [    1.63059707e-04   -1.77833521e-04    8.81504522e-05] alpha=   0.264105631937

f0=   1.60318206036e-08 norm=   0.000479447468228

c0=   [ -2.95476274e-05   -2.08913765e-04   -3.06523283e-06] beta= 0.242244313259 c1= [    1.02063751e-04   -1.41711643e-05    1.29877964e-05]

[ -9.22485419e-05    6.47145847e-05   -3.45784463e-05]

Step # =    6

df=   [ -9.22485419e-05    6.47145847e-05   -3.45784463e-05] x=   [    1.73760689e-04   -1.22728814e-04    6.46113559e-05] alpha=   1.88361046344

f0=   1.01565185216e-08 norm=   0.000230475090734

c0=   [    1.02063751e-04   -1.41711643e-05    1.29877964e-05] beta= 0.00332715195415 c1= [ -1.66274892e-06   -4.36817325e-06   -3.74735904e-06]

[   1.35582401e-06    4.58348851e-06    3.63231129e-06]

[Finished in 0.874s]
```



Python Code for problem 2 with Conjugate Method

```
'''
Fletcher-Reeves Conjugate Gradient Algorithm
Date:     10/13/17
Class:    ME441_EngineeringOptimization
Author: Xiaoyi Liu
'''
import numpy as np
import matplotlib.pyplot as plt
from GoldenSection_For_Multiopt import GoldenSection
def fQuar(x):
    return x[0]**2+2*x[1]**2+2*x[2]**2 + 2*x[0]*x[1]+2*x[1]*x[2]
def dfQuar(x):
    return np.array([2*x[0]+2*x[1], 2*x[0]+4*x[1]+2*x[2], 2*x[1]+4*x[2]])
#SteepestDescentAlgorithm
x0=[1,1,1]
f0=fQuar(x0)
c0=dfQuar(x0)
c1=c0
d0=-c0
d1=d0
epsilon=0.001
i=0
y=[]
```

```
while (np.linalg.norm(d1)>epsilon and i<10):
        y.append(f0)
        print('Step # = ',i)
        def localf(alpha):
                return fQuar(x0+alpha*d1)
        [ff,alpha0]=GoldenSection(localf,-1,1)
        print('df= ', d1, 'x= ', x0,'alpha= ',alpha0, 'f0= ',f0,'norm= ',np.linalg.norm(d0))
        x0=x0+alpha0*d1
        f0=fQuar(x0)
        c0=c1
        c1=dfQuar(x0)
        d0=d1
        d1=-c1+d0*((np.linalg.norm(c1)/np.linalg.norm(c0))**2)
        print('c0= ',c0,'beta=',(np.linalg.norm(c1)/np.linalg.norm(c0))**2,'c1=',c1)
        print(d1)
        i=i+1;
plt.plot(y,'-*')
plt.xlabel('Iterations')
plt.ylabel('Objective Function')
plt.title('Histogram of Fletcher-Reeves Conjugate Gradient')
plt.show()
```

Comparison of Steepest Descent and Conjugate methods:

	Steepest Descent	Conjugate method
Iteration numbers:	41	6
Optimal Solution:	9.631e-7	1.018e-8
Run Time:	2.247s	0.874s