Shimon Arm Path Planning

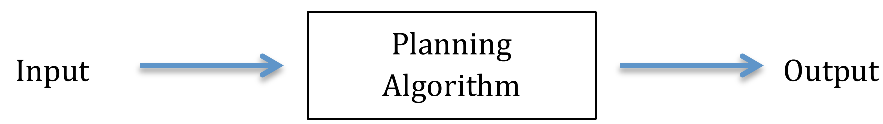
Representation:

Arm: a1, a2, a3, a4 (from lowest pitch to highest)

And let a21 be the 1st stick on arm 2.

Position: let p_i^k be the position of arm i at time k (here time is discrete value corresponding to the midi note time signature).

System architecture:



Input: MIDI file, note sequence to play with time signature. The node sequence should be mapped to position values before sending to the planning algorithm.

State: the position of four arms at the current time step, from 1 to 28 (four octave range).

Output: the position of four arms at each time signature

Goal:

Minimize the global distance traveled of four arms totally, represented as

$$\min \sum_{k=1}^{n} \sum_{i=1}^{4} (p_i^k - p_i^{k-1})^2$$

To achieve global minimum, we can first explore ways to achieve local minimum for each time step, which is to calculate the minimum distance traveled to play one particular note at that time. This may involve one arm to go to that position and play that node, or two arms, one moves away to make space for the second arm to go there and hit that note, we can set rules to make four arms not be too close to each other at any time, so that we can eliminate the possibility of moving two arms to make space for the third one.

On the other hand, the branching factor does not need to be very large. Since we have the position information of four arms at the previous time step, we can narrow down the search range to the two arms that are nearest to the target node to play at the current time step. Even though this may not result in a global optimal solution, it will largely shorten the search time, makes it more efficient. So if the performance is good enough, I think a non-optimal solution to be our first step doesn't sound too bad.

Constraints:

There are three cases of mechanical constraints.

1. Each arm has a position range that it can reach.

$$\begin{cases} 1 \leq p_1^k \leq 21 \\ 3 \leq p_2^k \leq 23 \\ 6 \leq p_2^k \leq 26 \\ 8 \leq p_2^k \leq 28 \end{cases}, \text{for } \forall\ k \in [1, n]$$

2. When arm 2 and arm 3 are together, at least 2 and at most 4 notes will be missing.

$$p_2^k - p_3^k \geq 4, \qquad \text{for } \forall\ k \in [1, n]$$

3. When arm 1 and arm 2 are together, or arm 3 and arm 4 are together, 2 notes will be missing.

$$p_1^k - p_2^k \geq 2, \ \ p_3^k - p_4^k \geq 2, \qquad \text{for } \forall\ k \in [1, n]$$