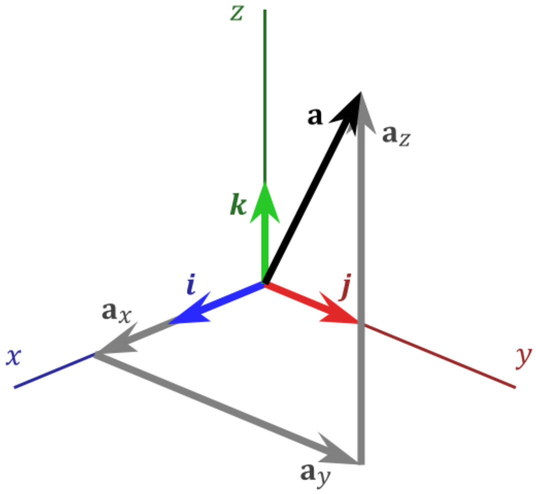
# Math Reference Sheet

## Vectors




Subtraction: $\boldsymbol{A - B = (A_x - B_x, A_y - B_y, A_z - B_z)}$

Addition: $\boldsymbol{A + B = (A_x + B_x, A_y + B_y, A_z + B_z)}$

Scalar Multiplication: $\boldsymbol{aB = (aB_x, aB_y, aB_z)}$

Magnitude: $\|A\| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

*Dot Product:* $\boldsymbol{A \cdot B = A_x B_x + A_y B_y + A_z B_z}$

*Cross Product:* $\boldsymbol{A \times B = (A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x)}$

*Dot Product in relation to cos α:* $\boldsymbol{A \cdot B = \|A\| \, \|B\| cos\ \alpha}$

Cross Product in relation to sin $\alpha$: $\boldsymbol{A \times B = \|A\| \, \|B\| sin\ \alpha}$

Projection of A onto B: $\boldsymbol{proj_B\ A = \frac{A \cdot B}{\|B\|^2}}$**Q**

## A useful property:

## If $A \cdot B = 0$ then A and B are perpendicular

$$[x' y' z' 1] = [x\ y\ z\ 1]\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{bmatrix}$$

**Figure 1 - Translate**

# Matrices

$$M_{n\times m} = A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} & \dots & A_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \vdots & Anm \end{bmatrix}$$

$${M_{ij}}^T = M_{ji}$$

$$aM = Ma = \begin{bmatrix} aM_{11} & aM_{12} \\ aM_{21} & aM_{22} \end{bmatrix}$$

$$M + N = \begin{bmatrix} M_{11} + N_{11} & M_{12} + N_{12} \\ M_{21} + N_{21} & M_{22} + N_{22} \end{bmatrix}$$

$$M - N = \begin{bmatrix} M_{11} - N_{11} & M_{12} - N_{12} \\ M_{21} - N_{21} & M_{22} - N_{22} \end{bmatrix}$$

$$I_n = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

$$[x' y' z' 1] = [x\ y\ z\ 1]\begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**Figure 2 - Scale**

$$[x' y' z' 1] = [x\ y\ z\ 1]\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

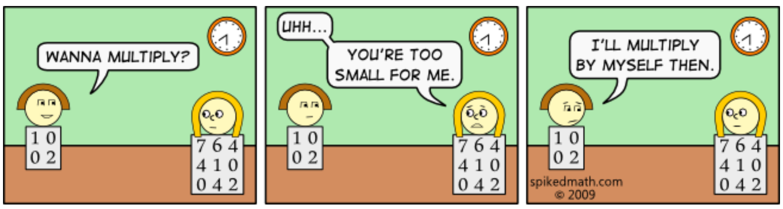
**Figure 3-Rotate around X-axis**

$$[x' y' z' 1] = [x\ y\ z\ 1]\begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**Figure 4-Rotate around Y-axis**

$$[x' y' z' 1] = [x\ y\ z\ 1]\begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**Figure 5 - Rotate around Z-axis**




***Matrix Multiplication:***

$$A_{n\times m} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} & \dots & A_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \vdots & Anm \end{bmatrix} \quad B_{m\times p} = \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1p} \\ B_{21} & B_{22} & \dots & B_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ B_{m1} & B_{m2} & \vdots & B_{np} \end{bmatrix}$$

$$C = \begin{bmatrix} (AB)_{11} & (AB)_{12} & \cdots & (AB)_{1m} \\ (AB)_{21} & (AB)_{22} & \dots & (AB)_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ (AB)_{n1} & (AB)_{n2} & \vdots & A_{np} \end{bmatrix}$$ au where $AB_{ij} = \sum_{k=1}^{m} A_{ij} B_{kj}$

# Quaternions

$$q = w + xi + yj + zk$$

**Where**

$$i^2 = j^2 = k^2 = ijk = 1$$

$$q = cos\left(\frac{\theta}{2}\right) + i\left(x * sin\left(\frac{\theta}{2}\right)\right) + i\left(y * sin\left(\frac{\theta}{2}\right)\right) + i\left(z * sin\left(\frac{\theta}{2}\right)\right)$$

**Magnitude(Norm):** $\|q\| = \sqrt{q_w^2 + q_x^2 + q_y^2 + q_z^2}$

**Dot Product:** $a_1 \cdot b_2 = a_w b_w + a_x b_x + a_y b_y + a_z b_z$

**Multiplication of Quaternions:**

**If q = a * b where a and b are quaternions, then:**

$$q = (a_w b_w - a_x b_x - a_y b_y - a_z b_z)$$
$$+ i(a_w b_x + a_x b_w + a_y b_z + a_z b_y)$$
$$+ j(a_w b_y - a_x b_z - a_y b_w - a_z b_w)$$
$$+ k(a_w b_z + a_x b_y + a_y b + x + a_z b_w)$$

$$if\ q = w + xi + yj + zk\ then$$

$$q^t = w - xi - yj - zk$$

**Rotating a vector by a quaternion:**

$$v' = q \times v \times q^t$$

**Equations for Matrix -> Quaternion and Quaternion -> Matrix can be found in the “extra resources” section**

# Common Terminology and Extra Functions

**Spaces:**

**World space** – Transformations are in relation to the entire scene/world.

**Object/Model space** – Transformations are in relation to the object itself. This space is generally used to describe the transformations of vertices or other nodes within a model.

**Screen/Camera space –** Transformations are in relation to where they appear on the screen once all camera transformations have taken place.

**Interpolation and Curves**

Linear interpolation between A and B - $\boldsymbol{x = A + (B - A)t \; where \; t \; is \; 0 - 1}$

Bezier curve blending function: $\boldsymbol{B}(t) = (1-t)^3\boldsymbol{P_0} + 3t(1-t)^2\boldsymbol{P_1} + 3t^2(1-t)\boldsymbol{P_2} + t^3\boldsymbol{P_3}$

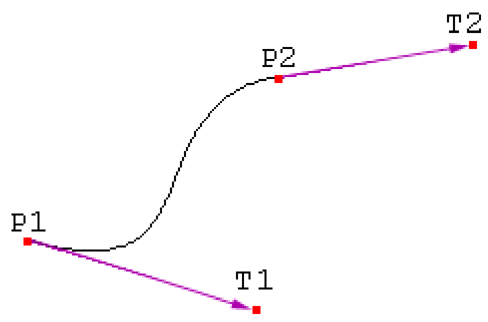
Hermite curve control functions, assuming points $P_1$as a start point, $P_2$ as an end point, and $T_1$ and $T_2$ as the tangent lines:

$\boldsymbol{h_1(t) = 2t^3 - 3t^2 + 1}$

$\boldsymbol{h_2(t) = \; -2t^3 + 3t^2}$

$\boldsymbol{h_3(t) = t^3 - -2t^2 + t}$

$\boldsymbol{h_4(t) = t^3 - t^2}$




$\boldsymbol{P(t) = h_1 * P_1 + h_2 * P_2 + h_3 * \; T_1 + h_4 * T_2}$

# Resources:

## Vector / Matrix:

Mathematics for 3D Game Programming and Computer Graphics – Chapter 1 – 2.

Mathematics and Physics for Programmers – Chapter 5

https://www.khanacademy.org/math/algebra

## Quaternions

Mathematics for 3D Game Programming and Computer Graphics – Chapter 3

http://www.euclideanspace.com/maths/algebra/realNormedAlgebra/quaternions/index.htm

## Curves / Other

Mathematics for 3D Game Programming and Computer Graphics – Chapter 15

http://cubic.org/docs/hermite.htm

http://cubic.org/docs/bezier.htm