# Latent class analysis

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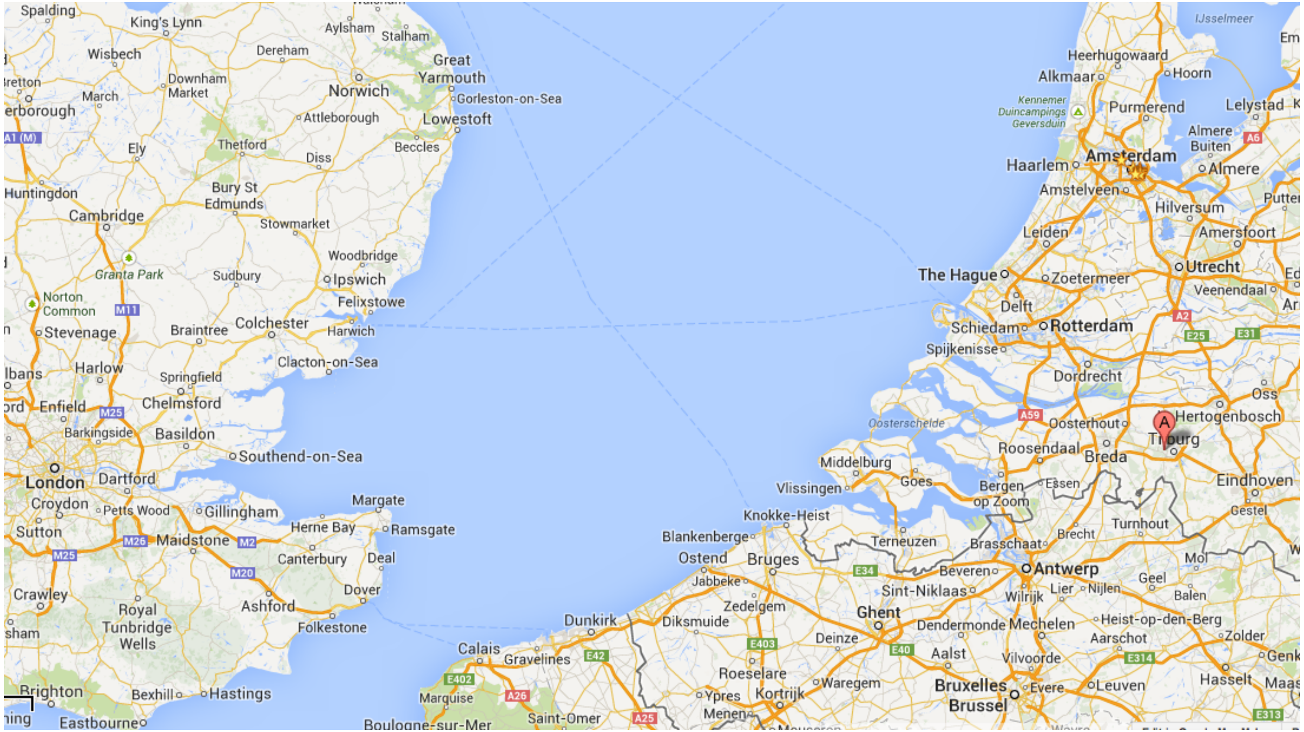
*(with material from Margot Sijssens-Bennink & Jeroen Vermunt)*

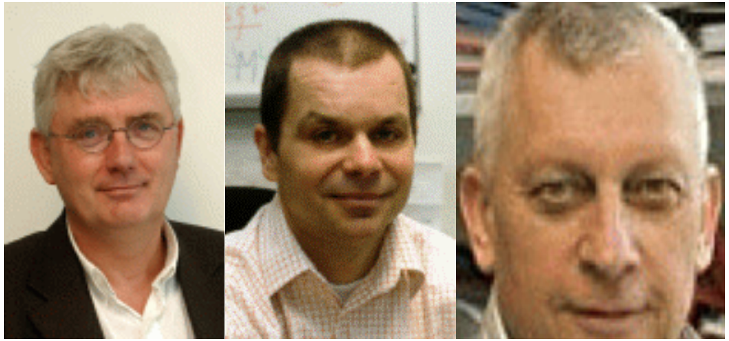
# About Tilburg University Methodology & Statistics



# About Tilburg University Methodology & Statistics

“Home of the latent variable”

Major contributions to latent class analysis:



Jacques Hagenaars *(emeritus)*

Jeroen Vermunt

Marcel Croon *(emeritus)*

lem

# More latent class modeling in Tilburg

Guy
Moors
(extreme
respnse)

Klaas
Sijtsma
(Mokken;
IRT)

Wicher
Bergsma
(marginal
models)
*(@LSE)*

Daniel
Oberski
(local fit of
LCM)

**Recent PhD’s**

Zsuzsa
Bakk
(3step LCM)

Dereje
Gudicha
(power
analysis in
LCM)

Margot
Sijssens-
Bennink
*(micro-
macro LCM)*

Daniel van
der Palm
(divisive
LCM)

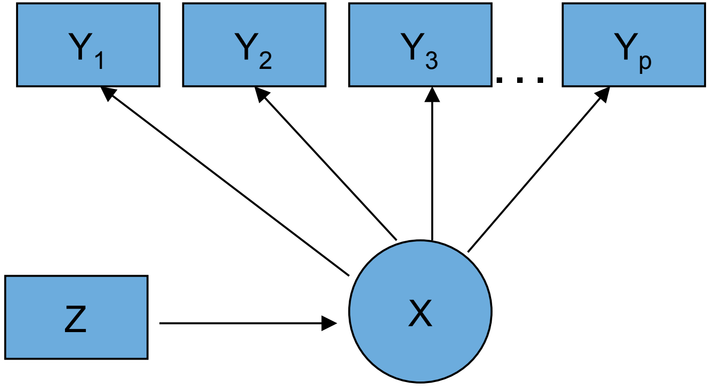
# What is a latent class model?

Statistical model in which parameters of interest differ across unobserved subgroups ("latent classes"; "mixtures")

Four main application types:

- Clustering (model based / probabilistic)
- Scaling (discretized IRT/factor analysis)
- Random-effects modelling (mixture regression / NP multilevel)
- Density estimation

# The Latent Class Model




- Observed Continuous or Categorical Items
- Categorical Latent Class Variable (X)
- Continuous or Categorical Covariates (Z)

Adapted from: Nylund (2003) Latent class anlalysis in Mplus. URL: http://www.ats.ucla.edu/stat/mplus/seminars/lca/default.htm

# Four main applications of LCM

- Clustering (model based / probabilistic)
- Scaling (discretized IRT/factor analysis)
- Random-effects modelling (mixture regression / nonparametric multilevel)
- Density estimation

# Why would survey researchers need latent class models?

## For substantive analysis:

- Creating typologies of respondents, e.g.:
  - McCutcheon 1989: tolerance,
  - Rudnev 2015: human values
  - Savage et al. 2013: “A new model of Social Class”
  - …
- Nonparametric multilevel model (Vermunt 2013)
- Longitudinal data analysis
  - Growth mixture models
  - Latent transition (“Hidden Markov”) models

# Why would survey researchers need latent class models?

## For survey methodology:

- As a method to evaluate questionnaires, e.g.
  - Biemer 2011: Latent Class Analysis of Survey Error
  - Oberski 2015: latent class MTMM
- Modeling extreme response style (and other styles), e.g.
  - Morren, Gelissen & Vermunt 2012: extreme response
- Measurement equivalence for comparing groups/countries
  - Kankaraš & Moors 2014: Equivalence of Solidarity Attitudes
- Identifying groups of respondents to target differently
  - Lugtig 2014: groups of people who drop out panel survey
- Flexible imputation method for multivariate categorical data
  - Van der Palm, Van der Ark & Vermunt

# Latent class analysis at ESRA!

latent class | Search for session or paper

| Search for paper author or session convenor

## Paper(s)

- Apathy is the Enemy. A study of UK environmental concern and its complicated relationship with pro-environmental behaviour. (Rebecca Rhead)
- Aspects of Validity: Scenario-Technique, Self-Report & Social Desirability (Lena Verneuer)
- Developing a diagnostic tool for detecting response styles, and a demonstration of its use in comparative research of single item measurements (Eva Van vlimmeren)
- Elimination and Selection by aspects decision rules in discrete choice experiments (Seda Erdem)
- Measurement equivalence in cross-cultural surveys: multigroup latent class analysis and MIMIC-models in prejudice research (Ekaterina Lytkina)
- Policy-Culture Gaps and the Role of Gender Norms (Daniela Grunow)
- Testing the Invariance of the Value Typology of Europeans Across Time Points (Maksim Rudnev)
- Testing the Theory of Social Integration (Ashley Amaya)
- Validating Schwartz value theory with confirmatory latent class analysis (Marko Sõmer)

# Software

**Commercial**

- Latent GOLD
- Mplus
- gllamm in Stata
- PROC LCA in SAS

**Free (as in beer)**

- ℓem

**Open source**

- R package poLCA
- R package flexmix
- (with some programming) OpenMx, stan

- Specialized models: HiddenMarkov, depmixS4,

## A small example

(showing the basic ideas and interpretation)

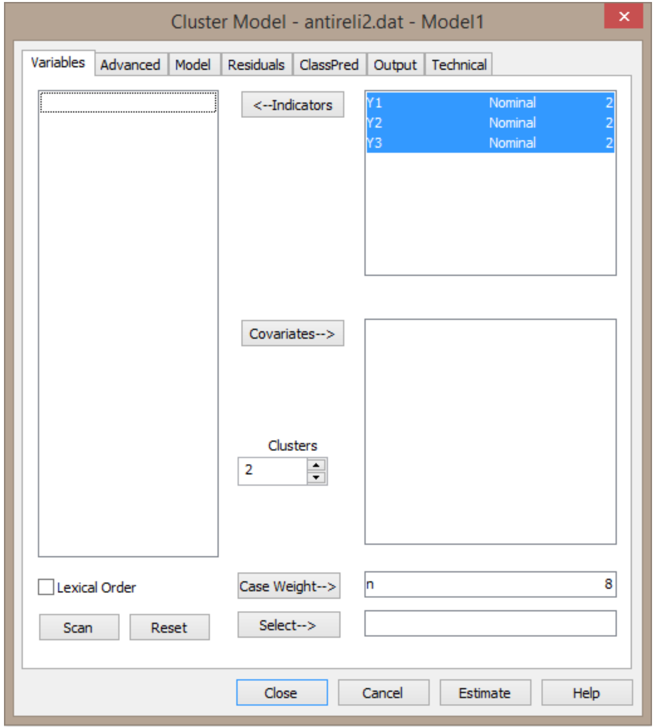
# Small example: data from GSS 1987

Y1: “allow anti-religionists to speak” (1 = allowed, 2 = not allowed),
Y2: “allow anti-religionists to teach” (1 = allowed, 2 = not allowed),
Y3: “remove anti-religious books from the library” (1 = do not remove, 2 = remove).

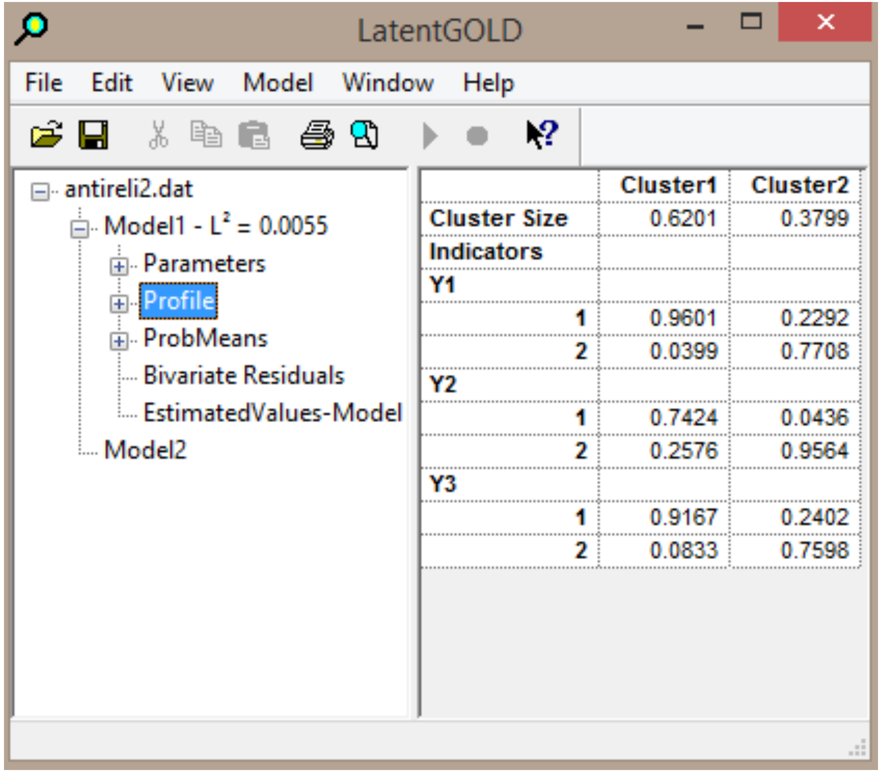
| | Y1 | Y2 | Y3 | Observed frequency (n) | Observed proportion (n/N) |
|---|---|---|---|---|---|
| | 1 | 1 | 1 | 696 | 0.406 |
| | 1 | 1 | 2 | 68 | 0.040 |
| | 1 | 2 | 1 | 275 | 0.161 |
| | 1 | 2 | 2 | 130 | 0.076 |
| | 2 | 1 | 1 | 34 | 0.020 |
| | 2 | 1 | 2 | 19 | 0.011 |
| | 2 | 2 | 1 | 125 | 0.073 |
| | 2 | 2 | 2 | 366 | 0.214 |

N = 1713

# 2-class model in Latent GOLD

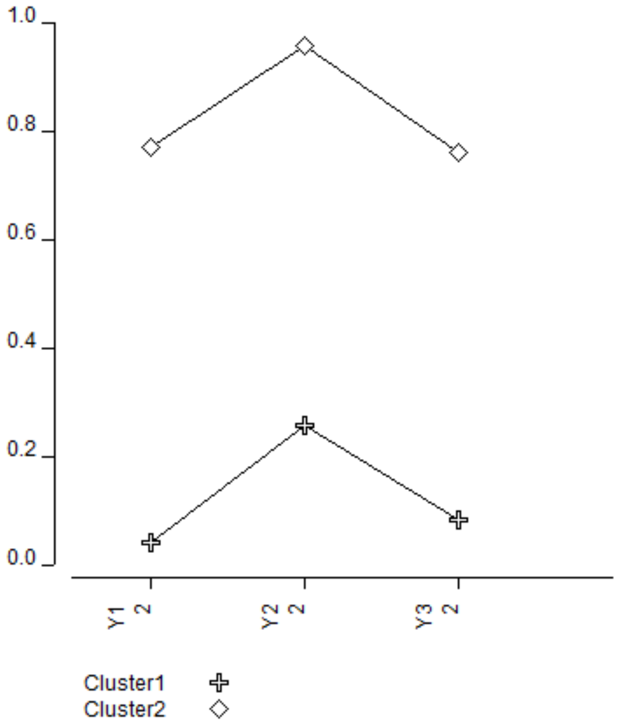


# Profile for 2-class model




| | | Cluster1 | Cluster2 |
|---|---|---|---|
| Cluster Size | | 0.6201 | 0.3799 |
| Indicators | | | |
| Y1 | | | |
| | 1 | 0.9601 | 0.2292 |
| | 2 | 0.0399 | 0.7708 |
| Y2 | | | |
| | 1 | 0.7424 | 0.0436 |
| | 2 | 0.2576 | 0.9564 |
| Y3 | | | |
| | 1 | 0.9167 | 0.2402 |
| | 2 | 0.0833 | 0.7598 |

# Profile plot for 2-class model



# Estimating the 2-class model in `R`

```
antireli  <- read.csv("antireli_data.csv")

library(poLCA)

M2 <- poLCA(cbind(Y1, Y2, Y3)~1, data=antireli, nclass=2)
```

# Profile for 2-class model

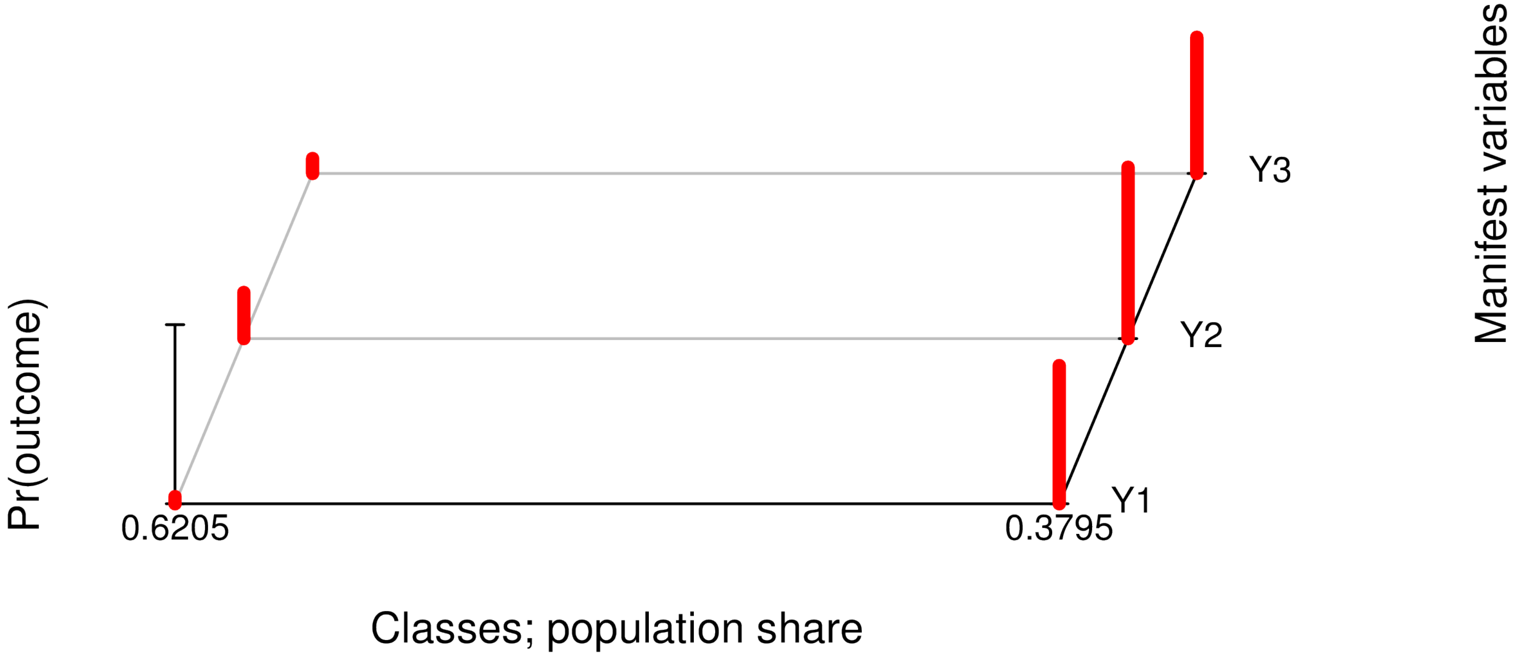
```
$Y1
           Pr(1)  Pr(2)
class 1:  0.9601 0.0399
class 2:  0.2284 0.7716

$Y2
           Pr(1)  Pr(2)
class 1:  0.7424 0.2576
class 2:  0.0429 0.9571

$Y3
           Pr(1)  Pr(2)
class 1:  0.9166 0.0834
class 2:  0.2395 0.7605

Estimated class population shares
 0.6205 0.3795
```

```
> plot(M2)
```



Manifest variables
Y3
Y2
Y1
Pr(outcome)
0.6205
0.3795
Classes; population share

# Model equation
# for 2-class LC model for 3 indicators

Model for

$$P(y_1, y_2, y_3)$$

the probability of a particular response pattern.

For example, how likely is someone to hold the opinion
“allow speak, allow teach, but remove books from library:

P(Y1=1, Y2=1, Y3=2) = ?

# Two key model assumptions

($X$ is the latent class variable)

**1. (MIXTURE ASSUMPTION)**

Joint distribution mixture of 2 class-specific distributions:

$$P(y_1, y_2, y_3) = P(X = 1)P(y_1, y_2, y_3 \mid X = 1) + P(X = 2)P(y_1, y_2, y_3 \mid X = 2)$$

**2. (LOCAL INDEPENDENCE ASSUMPTION)**

Within class $X$=$x$, responses are independent:

$$P(y_1, y_2, y_3 \mid X = 1) = P(y_1 \mid X = 1)P(y_2 \mid X = 1)P(y_3 \mid X = 1)$$

$$P(y_1, y_2, y_3 \mid X = 2) = P(y_1 \mid X = 2)P(y_2 \mid X = 2)P(y_3 \mid X = 2)$$

# Example: model-implied proprtion

| | X=1 | X=2 |
|---|---|---|
| P(X) | 0.620 | 0.380 |
| P(Y1=1\|X) | 0.960 | 0.229 |
| P(Y2=1\|X) | 0.742 | 0.044 |
| P(Y3=1\|X) | 0.917 | 0.240 |

**P(Y1=1, Y2=1, Y3=2) =**

*(Mixture assumption)*

P(Y1=1, Y2=1, Y3=2 | X=1) P(X=1) +

P(Y1=1, Y2=1, Y3=2 | X=2) P(X=2)

# Example: model-implied proprtion

| | X=1 | X=2 |
|---|---|---|
| P(X) | 0.620 | 0.380 |
| P(Y1=1\|X) | 0.960 | 0.229 |
| P(Y2=1\|X) | 0.742 | 0.044 |
| P(Y3=1\|X) | 0.917 | 0.240 |

**P(Y1=1, Y2=1, Y3=2) =**

*(Mixture assumption)*

P(Y1=1, Y2=1, Y3=2 | X=1) 0.620 +

P(Y1=1, Y2=1, Y3=2 | X=2) 0.380 =

*(Local independence assumption)*

P(Y1=1|X=1) P(Y2=1|X=1) P(Y2=2|X=1) 0.620 +

P(Y1=1|X=2) P(Y2=1|X=2) P(Y2=2|X=2) 0.380

# Example: model-implied proprtion

| | X=1 | X=2 |
|---|---|---|
| P(X) | 0.620 | 0.380 |
| P(Y1=1\|X) | 0.960 | 0.229 |
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**P(Y1=1, Y2=1, Y3=2) =**

*(Mixture assumption)*

P(Y1=1, Y2=1, Y3=2 | X=1) 0.620 +

P(Y1=1, Y2=1, Y3=2 | X=2) 0.380 =

*(Local independence assumption)*

(0.960) (0.742) (1-0.917) (0.620) +

(0.229) (0.044) (1-0.240) (0.380) ≈

**≈ 0.0396**

# Small example: data from GSS 1987

Y1: "allow anti-religionists to speak" (1 = allowed, 2 = not allowed),
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| | 2 | 1 | 2 | 19 | 0.011 |
| | 2 | 2 | 1 | 125 | 0.073 |
| | 2 | 2 | 2 | 366 | 0.214 |

N = 1713

**Implied is 0.0396, observed is 0.040.**

# More general model equation

Mixture of C classes

$$P(\mathbf{y}) = \sum_{x=1}^{C} P(X = x) P(\mathbf{y} \mid X = x)$$

Local independence of K variables

$$P(\mathbf{y} \mid X = x) = \prod_{k=1}^{K} P(y_k \mid X = x)$$

Both together gives the likelihood of the observed data:

$$P(\mathbf{y}) = \sum_{x=1}^{C} P(X = x) \prod_{k=1}^{K} P(y_k \mid X = x)$$

# “Categorical data” notation

- In some literature an alternative notation is used

- Instead of Y1, Y2, Y3, variables are named A, B, C
- We define a model for the joint probability

$$P(A = i, B = j, C = k) := \pi_{ijk}^{ABC}$$

$$\pi_{ijk}^{ABC} = \sum_{t=1}^{T} \pi_t^X \pi_{ijk\,t}^{ABC|X} \qquad \text{with} \qquad \pi_{ijk\,t}^{ABC|X} = \pi_{i\,t}^{A|X} \pi_{j\,t}^{B|X} \pi_{k\,t}^{C|X}$$

# Loglinear parameterization

$$\pi_{ijk\ t}^{ABC|X} = \pi_{i\,t}^{A|X}\pi_{j\,t}^{B|X}\pi_{k\,t}^{C|X}$$

$$\ln(\pi_{ijk\ t}^{ABC|X}) = \ln(\pi_{i\,t}^{A|X}) + \ln(\pi_{j\,t}^{B|X}) + \ln(\pi_{k\,t}^{C|X})$$

$$:= \lambda_{i\,t}^{A|X} + \lambda_{j\,t}^{B|X} + \lambda_{k\,t}^{C|X}$$

# The parameterization actually used in most LCM software

$$P(y_k \mid X = x) = \frac{\exp(\beta^k_{0y_k} + \beta^k_{1y_k x})}{\sum_{m=1}^{M_k} \exp(\beta^k_{0m} + \beta^k_{1mx})}$$

$\beta^k_{0y_k}$ Is a logistic intercept parameter

$\beta^k_{1y_k x}$ Is a logistic slope parameter (loading)

So just a series of **logistic regressions**, with X as independent and Y dep’t!
Similar to CFA/EFA (but logistic instead of linear regression)

# A more realistic example

(showing how to evaluate the model fit)

# One form of political activism



61.31%

38.69%

# Another form of political activism

Relate to covariate?


ESTAMOS DESPIERTOS ¡ QUE HORA ES?
YA ES HORA DE QUE SE VAYAN !

There are different ways of trying to improve things in [country] or help prevent[9] things from going wrong. During the last 12 months, have you done any of the following?
Have you…**READ OUT…**

| | | Yes | No | (Don't know) |
|---|---|---|---|---|
| **B13** | …contacted a politician, government or local government official? | 1 | 2 | 8 |
| **B14** | …worked in a political party or action group? | 1 | 2 | 8 |
| **B15** | …worked in another organisation or association? | 1 | 2 | 8 |
| **B16** | …worn or displayed a campaign badge/sticker? | 1 | 2 | 8 |
| **B17** | …signed a petition? | 1 | 2 | 8 |
| **B18** | …taken part in a lawful public demonstration? | 1 | 2 | 8 |
| **B19** | …boycotted certain products? | 1 | 2 | 8 |

# Data from the European Social Survey round 4 Greece

| contplt | wrkprty | wrkorg | badge | sgnptit | pbldmn | bctprd | clsprty |
|---|---|---|---|---|---|---|---|
| 2 | 2 | 2 | 2 | 2 | 2 | 1 | 2 |
| 2 | 2 | 2 | 2 | 2 | 2 | 1 | 1 |
| 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 |
| 2 | 2 | 2 | 2 | 2 | 2 | 1 | 2 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

```
library(foreign)
ess4gr <- read.spss("ESS4-GR.sav", to.data.frame = TRUE,
      use.value.labels = FALSE)

K <- 4      # Change to 1,2,3,4,..
MK <- poLCA(cbind(contplt, wrkprty, wrkorg,
                  badge, sgnptit, pbldmn, bctprd)~1,
                  ess4gr, nclass=K)
```

# Evaluating model fit

In the previous small example you calculated the model-implied (expected) probability for response patterns and compared it with the observed probability of the response pattern:

observed - expected

The small example had $2^3 – 1= 7$ unique patterns and 7 unique parameters, so df = 0 and the model fit perfectly.

observed – expected = 0 <=> df = 0

# Evaluating model fit

Current model (with 1 class, 2 classes, …)

Has $2^7 - 1 = 128 - 1 = 127$ unique response patterns

But much fewer parameters

So the model can be **tested**.

Different models can be compared with each other.

# Evaluating model fit

- Global fit
- Local fit
- Substantive criteria

# Global fit

# Goodness-of-fit chi-squared statistics

- H0: model with C classes; H1: saturated model
- $L^2 = \sum 2\, n \ln (n / (P(y)*N))$
- $X^2 = \sum (n - P(y)*N)^2 / (P(y)*N)$
- df = number of patterns -1 - Npar
- Sparseness: bootstrap *p*-values

# Information criteria

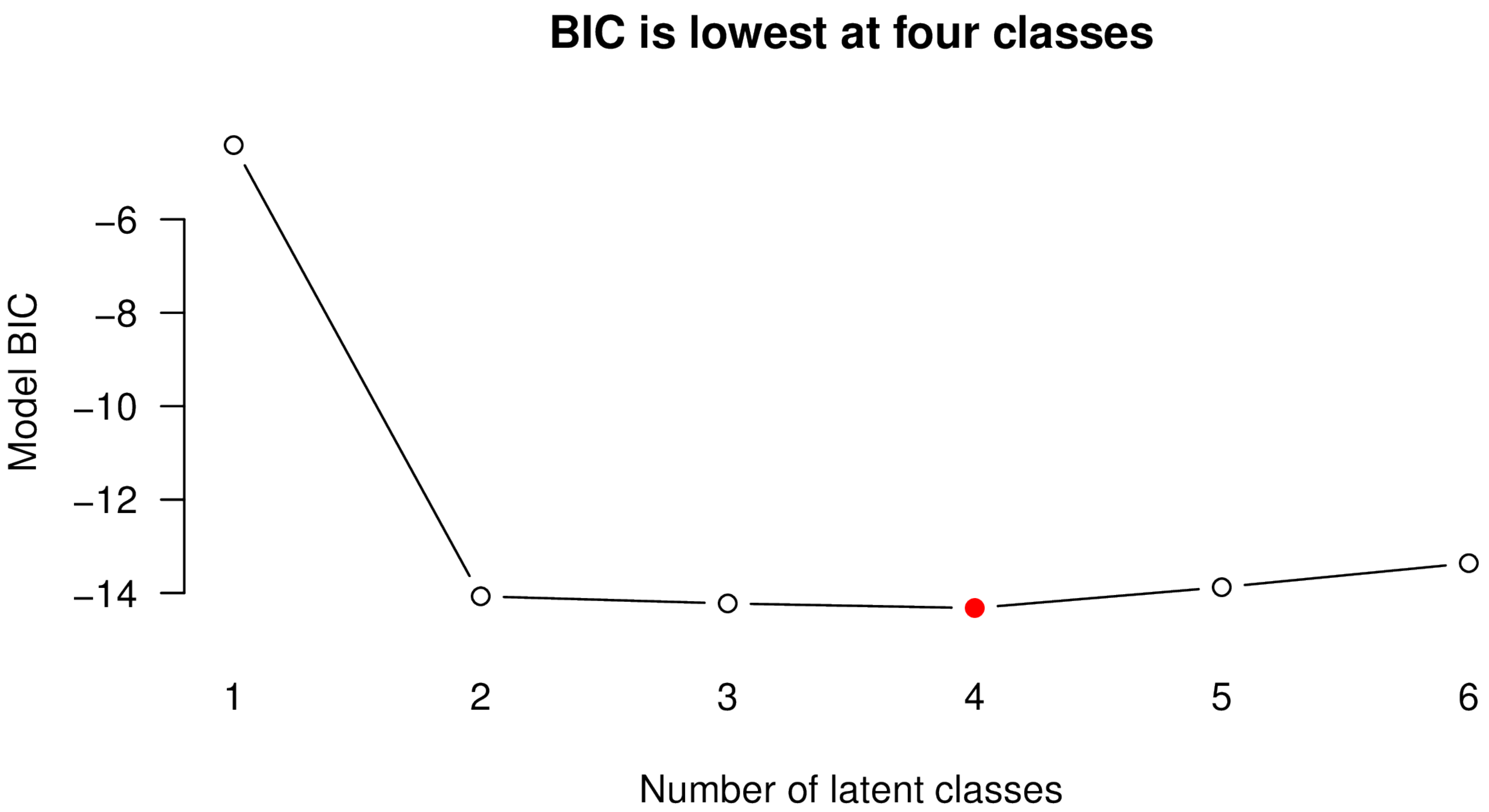
- for model comparison
- parsimony versus fit

**Common criteria**

- BIC(LL) = -2LL + ln(N) * Npar
- AIC(LL) = -2LL + 2 * Npar
- AIC3(LL) = -2LL + 3 * Npar
- BIC(L2) = L2 - ln(N) * df
- AIC(L2) = L2 - 2 * df
- AIC3(L2) = L2 - 3 * df

# Model fit comparisons

| | $L^2$ | BIC($L^2$) | AIC($L^2$) | df | p-value |
|---|---|---|---|---|---|
| 1-Cluster | 1323.0 | -441.0 | 861.0 | 120 | 0.000 |
| 2-Cluster | 295.8 | -1407.1 | -150.2 | 112 | 0.001 |
| 3-Cluster | 219.5 | -1422.3 | -210.5 | 104 | 0.400 |
| 4-Cluster | 148.6 | -1432.2 | -265.4 | 96 | 1.000 |
| 5-Cluster | 132.0 | -1387.6 | -266.0 | 88 | 1.000 |
| 6-Cluster | 122.4 | -1336.1 | -259.6 | 80 | 1.000 |


BIC is lowest at four classes
Model BIC
−6
−8
−10
−12
−14
1
2
3
4
5
6
Number of latent classes

# Local fit

# Local fit: bivariate residuals (BVR)

Pearson “chi-squared” comparing observed and estimated frequencies in 2-way tables.

Expected frequency in two-way table:

$$N \cdot P(y_k, y_{k'}) = N \cdot \sum_{x=1}^{C} P(X = x)\, P(y_k \mid X = x)\, P(y_{k'} \mid X = x)$$

Observed:

Just make the bivariate cross-table from the data!

# Example calculating a BVR

Observed

| | No | Yes |
|---|---|---|
| No | 3250 | 280 |
| Yes | 123 | 216 |

Expected

| | No | Yes |
|---|---|---|
| No | 3217 | 313 |
| Yes | 156 | 183 |

Bivariate residuals

| | No | Yes |
|---|---|---|
| No | 32.6 | -32.6 |
| Yes | -32.6 | 32.6 |

$$\mathrm{BVR}_{1,3} \quad = \quad r_{11}{}^2 \sum_{k,l} \hat{\mu}_{kl}^{-1} = (32.6)^2 \sum_{k,l} \hat{\mu}_{kl}^{-1} \approx 1063(0.0154) \approx 16.3$$

# 1-class model BVR's

| | contplt | wrkprty | wrkorg | badge | sgnptit | pbldmn | bctprd |
|---|---|---|---|---|---|---|---|
| contplt | . | | | | | | |
| wrkprty | 342.806 | . | | | | | |
| wrkorg | 133.128 | 312.592 | . | | | | |
| badge | 203.135 | 539.458 | 396.951 | . | | | |
| sgnptit | 82.030 | 152.415 | 372.817 | 166.761 | . | | |
| pbldmn | 77.461 | 260.367 | 155.346 | 219.380 | 272.216 | . | |
| bctprd | 37.227 | 56.281 | 78.268 | 65.936 | 224.035 | 120.367 | . |

# 2-class model BVR's

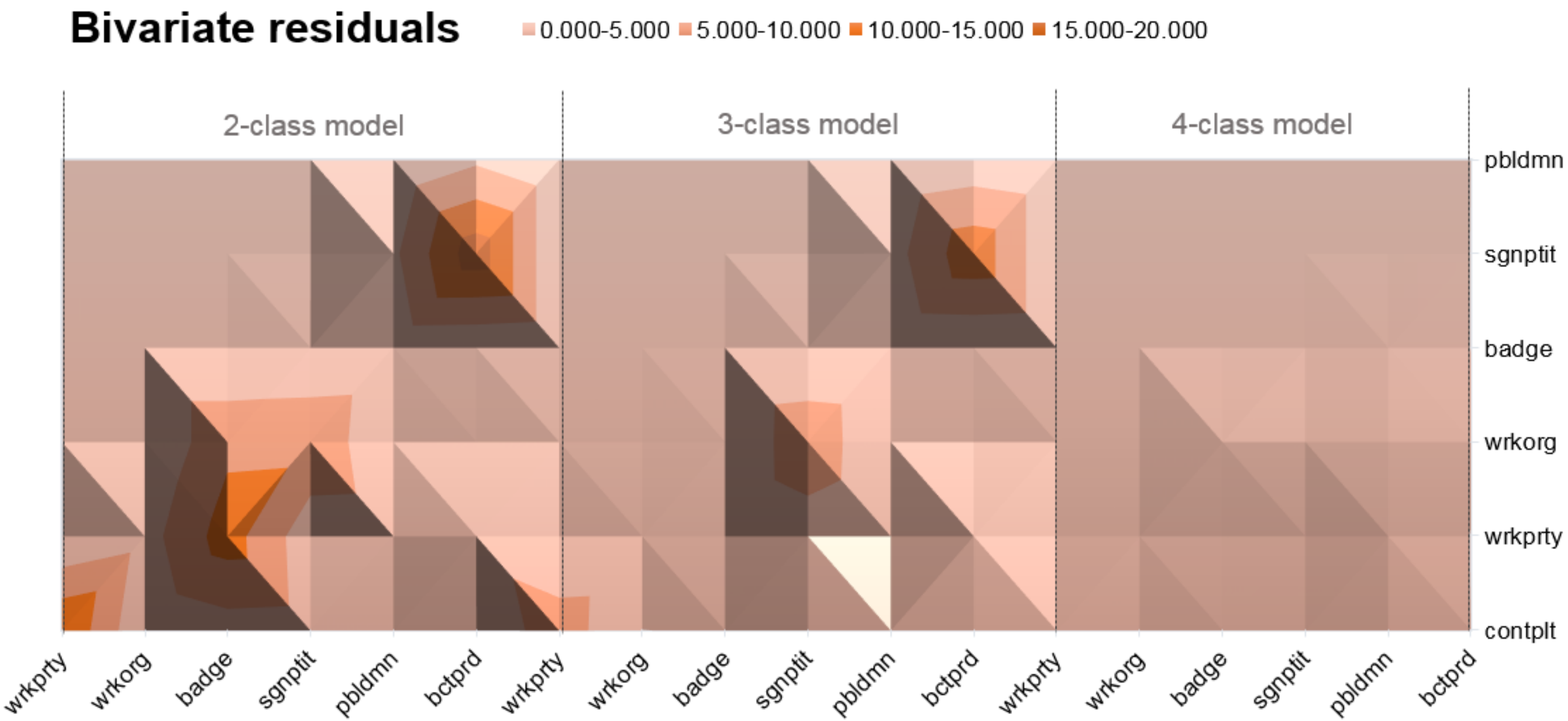
| | contplt | wrkprty | wrkorg | badge | sgnptit | pbldmn | bctprd |
|---|---|---|---|---|---|---|---|
| contplt | . | | | | | | |
| wrkprty | 15.147 | . | | | | | |
| wrkorg | 0.329 | 2.891 | . | | | | |
| badge | 2.788 | 12.386 | 8.852 | . | | | |
| sgnptit | 2.402 | 1.889 | 9.110 | 0.461 | . | | |
| pbldmn | 1.064 | 1.608 | 0.108 | 0.945 | 3.957 | . | |
| bctprd | 1.122 | 2.847 | 0.059 | 0.717 | 18.025 | 4.117 | . |

# 3-class model BVR's

| | contplt | wrkprty | wrkorg | badge | sgnptit | pbldmn | bctprd |
|---|---|---|---|---|---|---|---|
| contplt | . | | | | | | |
| wrkprty | 7.685 | . | | | | | |
| wrkorg | 0.048 | 0.370 | . | | | | |
| badge | 0.282 | 0.054 | 0.273 | . | | | |
| sgnptit | 2.389 | 2.495 | 8.326 | 0.711 | . | | |
| pbldmn | 2.691 | 0.002 | 0.404 | 0.086 | 2.842 | . | |
| bctprd | 2.157 | 2.955 | 0.022 | 0.417 | 13.531 | 1.588 | . |

# 4-class model BVR's

| | contplt | wrkprty | wrkorg | badge | sgnptit | pbldmn | bctprd |
|---|---|---|---|---|---|---|---|
| contplt | . | | | | | | |
| wrkprty | 0.659 | . | | | | | |
| wrkorg | 0.083 | 0.015 | . | | | | |
| badge | 0.375 | 0.001 | 1.028 | . | | | |
| sgnptit | 0.328 | 0.107 | 0.753 | 0.019 | . | | |
| pbldmn | 0.674 | 0.939 | 0.955 | 0.195 | 0.004 | . | |
| bctprd | 0.077 | 0.011 | 0.830 | 0.043 | 0.040 | 0.068 | . |



Bivariate residuals
0.000-5.000
5.000-10.000
10.000-15.000
15.000-20.000
2-class model
3-class model
4-class model
pbldmn
sgnptit
badge
wrkorg
wrkprty
contplt
wrkprty
wrkorg
badge
sgnptit
pbldmn
bctprd
wrkprty
wrkorg
badge
sgnptit
pbldmn
bctprd
wrkprty
wrkorg
badge
sgnptit
pbldmn
bctprd

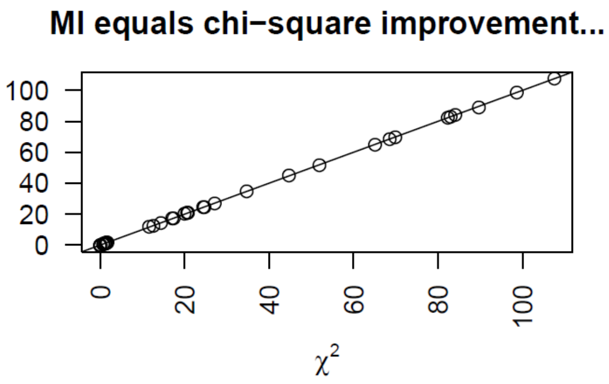
# Local fit: beyond BVR

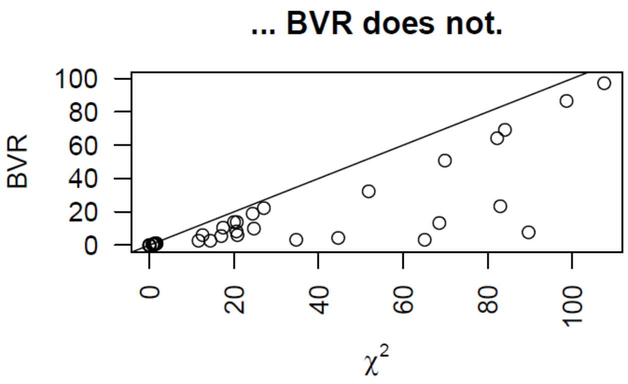
The bivariate residual (BVR) is not actually chi-square distributed!

(Oberski, Van Kollenburg & Vermunt 2013)

Solutions:

- Bootstrap p-values of BVR (LG5)
- "Modification indices" (score test) (LG5)



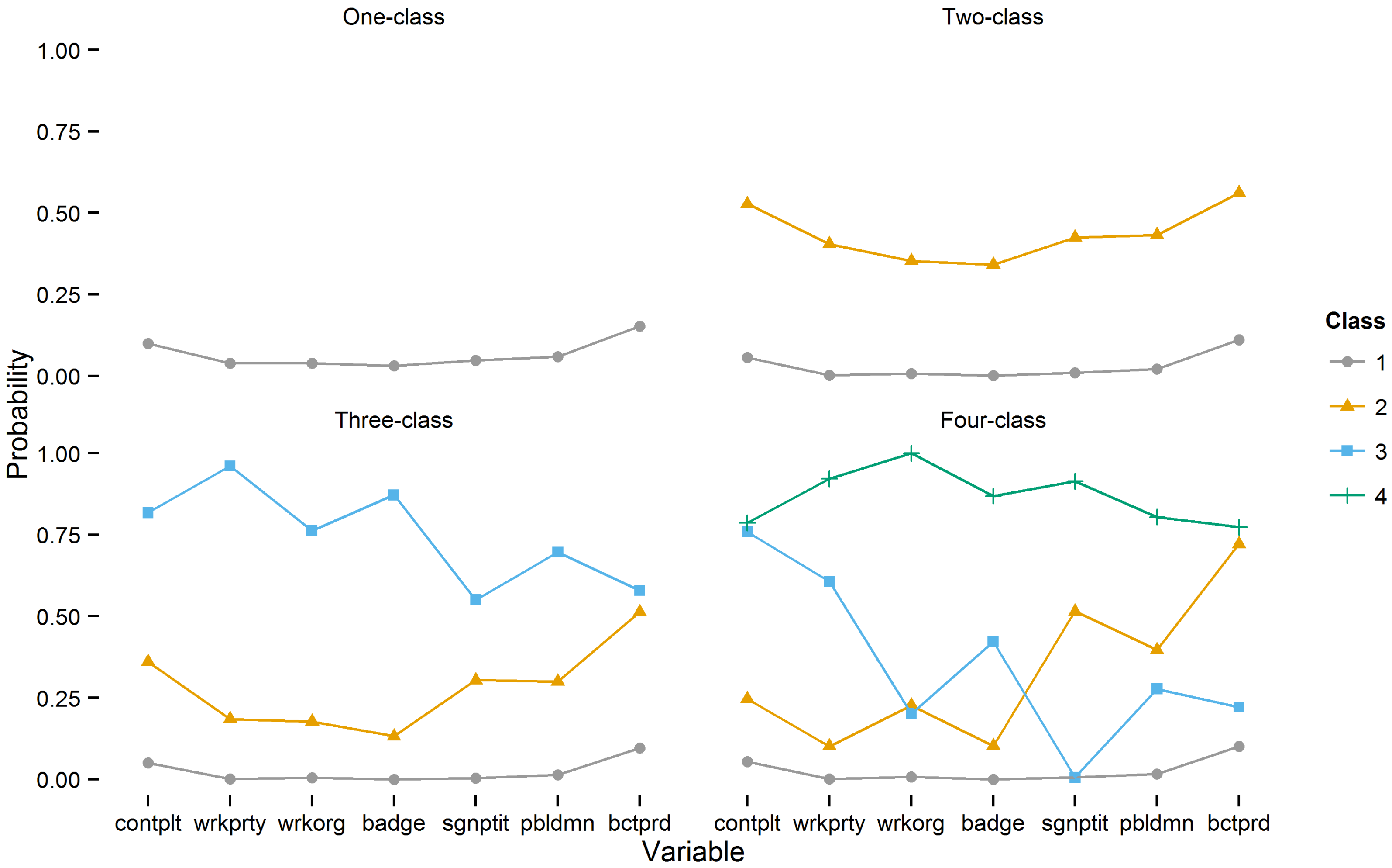



# Example of modification index (score test) for 2-class model

| Covariances / Associations | | | | | | | |
|---|---|---|---|---|---|---|---|
| term | | | coef | EPC(self) | Score | df | BVR |
| contplt | <-> | wrkprty | 0 | 1.7329 | 28.5055 | 1 | 15.147 |
| wrkorg | <-> | wrkprty | 0 | 0.6927 | **4.3534** | 1 | **2.891** |
| badge | <-> | wrkprty | 0 | 1.3727 | 16.7904 | 1 | 12.386 |
| sgnptit | <-> | bctprd | 0 | 1.8613 | 37.0492 | 1 | 18.025 |

**wrkorg <-> wrkparty is “not significant” according to BVR but is when looking at score test!**
(but not after adjusting for multiple testing)

# Interpreting the results and using substantive criteria


One-class
Two-class
Three-class
Four-class
Probability
1.00
0.75
0.50
0.25
0.00
contplt
wrkprty
wrkorg
badge
sgnptit
pbldmn
bctprd
Variable
Class
1
2
3
4

# EPC-interest for looking at change in substantive parameters

After fitting two-class model, how much would loglinear “loadings” of the items change if local dependence is accounted for?

| term | | | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 | Y7 |
|---|---|---|---|---|---|---|---|---|---|
| contplt | <-> | wrkprty | -0.44 | -0.66 | 0.05 | 1.94 | 0.05 | 0.02 | 0.00 |
| wrkorg | <-> | wrkprty | 0.00 | -0.19 | -0.19 | 0.63 | 0.02 | 0.01 | 0.00 |
| badge | <-> | wrkprty | 0.00 | -0.37 | 0.03 | -1.34 | 0.03 | 0.01 | 0.00 |
| sgnptit | <-> | bctprd | 0.01 | 0.18 | 0.05 | 1.85 | -0.58 | 0.02 | -0.48 |

*See Oberski (2013); Oberski & Vermunt (2013); Oberski, Moors & Vermunt (2015)*

# Model fit evaluation: summary

Different types of criteria to evaluate fit of a latent class model:

- **Global**
  BIC, AIC, L2, X2

- **Local**
  Bivariate residuals, modification indices (score tests), and expected parameter changes (EPC)

- **Substantive**
  Change in the solution when adding another class or parameters

# Model fit evaluation: summary

- Compare models with different number of classes using BIC, AIC, bootstrapped L2

- Evaluate overall fit using bootstrapped L2 and bivariate residuals

- Can be useful to look at the profile of the different solutions: if nothing much changes, or very small classes result, fit may not be as useful

# Classification

(Putting people into boxes, while admitting uncertainty)

# Classification

- After estimating a LC model, we may wish to classify individuals into latent classes

- The latent classification or **posterior** class membership probabilities $P(X = x \mid \mathbf{y})$ can be obtained from the LC model parameters using Bayes' rule:

$$P(X = x \mid \mathbf{y}) = \frac{P(X = x)P(\mathbf{y} \mid X = x)}{P(\mathbf{y})} = \frac{P(X = x)\prod_{k=1}^{K} P(y_k \mid X = x)}{\sum_{c=1}^{C} P(X = c)\prod_{k=1}^{K} P(y_k \mid X = c)}$$

# Small example: posterior classification

| Y1 | Y2 | Y3 | P(X=1 \| Y) | P(X=2 \| Y) | Most likely (but not sure!) |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 0.002 | 0.998 | 2 |
| 1 | 1 | 2 | 0.071 | 0.929 | 2 |
| 1 | 2 | 1 | 0.124 | 0.876 | 2 |
| 1 | 2 | 2 | 0.832 | 0.169 | 1 |
| 2 | 1 | 1 | 0.152 | 0.848 | 2 |
| 2 | 1 | 2 | 0.862 | 0.138 | 1 |
| 2 | 2 | 1 | 0.920 | 0.080 | 1 |
| 2 | 2 | 2 | 0.998 | 0.003 | 1 |

# Classification quality

## Classification Statistics

- classification table: true vs. assigned class
- overall proportion of classification errors

## Other reduction of "prediction" errors measures

- How much more do we know about latent class membership after seeing the responses?
- Comparison of $P(X=x)$ with $P(X=x \mid \mathbf{Y}=\mathbf{y})$
- R-squared-like reduction of prediction (of X) error

```
posteriors <- data.frame(M4$posterior, predclass=M4$predclass)

classification_table <-
    ddply(posteriors, .(predclass), function(x) colSums(x[,1:4])))

> round(classification_table, 1)
 predclass post.1 post.2 post.3 post.4
1         1 1824.0   34.9    0.0   11.1
2         2    7.5   87.4    1.1    3.0
3         3    0.0    1.0   19.8    0.2
4         4    4.0    8.6    1.4   60.1
```

# Classification table for 4-class

| | post.1 | post.2 | post.3 | post.4 |
|---|---|---|---|---|
| 1 | **0.99** | 0.26 | 0.00 | 0.15 |
| 2 | 0.00 | **0.66** | 0.05 | 0.04 |
| 3 | 0.00 | 0.01 | **0.89** | 0.00 |
| 4 | 0.00 | 0.07 | 0.06 | **0.81** |
| | 1 | 1 | 1 | **1** |

**Total classification errors:**

```
> 1 - sum(diag(classification_table)) / sum(classification_table)
[1] 0.0352
```

# Entropy $R^2$

```
entropy <- function(p) sum(-p * log(p))
error_prior <- entropy(M4$P) # Class proportions
error_post <- mean(apply(M4$posterior, 1, entropy))

R2_entropy  <- (error_prior - error_post) / error_prior
```

```
> R2_entropy
[1] 0.741
```

This means that we know a lot more about people's political participation class after they answer the questionnaire.

Compared with if we only knew the overall proportions of people in each class

# Classify-analyze does not work!

- You might think that after classification it is easy to model people's latent class membership
- "Just take assigned class and run a multinomial logistic regression"
- Unfortunately, this **does not work** (biased estimates and wrong se's) *(Bolck, Croon & Hagenaars 2002)*
- (Many authors have fallen into this trap!)
- Solution is to **model class membership and LCM simulaneously**
- (Alternative is 3-step analysis, not discussed here)

## Predicting latent class membership

*(using covariates; concomitant variables)*

# Fitting a LCM in poLCA with gender as a covariate

```
M4 <- poLCA(
     cbind(contplt, wrkprty, wrkorg,
          badge, sgnptit, pbldmn, bctprd) ~ gndr,
          data=gr, nclass = 4, nrep=20)
```

This gives a **multinomial logistic regression** with X as dependent and gender as independent (“concomitant”; “covariate”)

# Predicting latent class membership from a covariate

$$P(X = x \mid Z = z) = \frac{\exp(\gamma_{0x} + \gamma_{zx})}{\sum_{c=1}^{C} \exp(\gamma_{0c} + \gamma_{zc})}$$

$\gamma_{0x}$ Is the logistic intercept for category x of the latent class variable X

$\gamma_{zx}$ Is the logistic slope predicting membership of class x for value z of the covariate Z

```
=========================================================
Fit for 4 latent classes:
=========================================================
2 / 1
            Coefficient  Std. error  t value  Pr(>|t|)
(Intercept)    -0.35987     0.37146   -0.969     0.335
gndrFemale     -0.34060     0.39823   -0.855     0.395
=========================================================
3 / 1
            Coefficient  Std. error  t value  Pr(>|t|)
(Intercept)     2.53665     0.21894   11.586     0.000
gndrFemale      0.21731     0.24789    0.877     0.383
=========================================================
4 / 1
            Coefficient  Std. error  t value  Pr(>|t|)
(Intercept)    -1.57293     0.39237   -4.009     0.000
gndrFemale     -0.42065     0.57341   -0.734     0.465
=========================================================
```

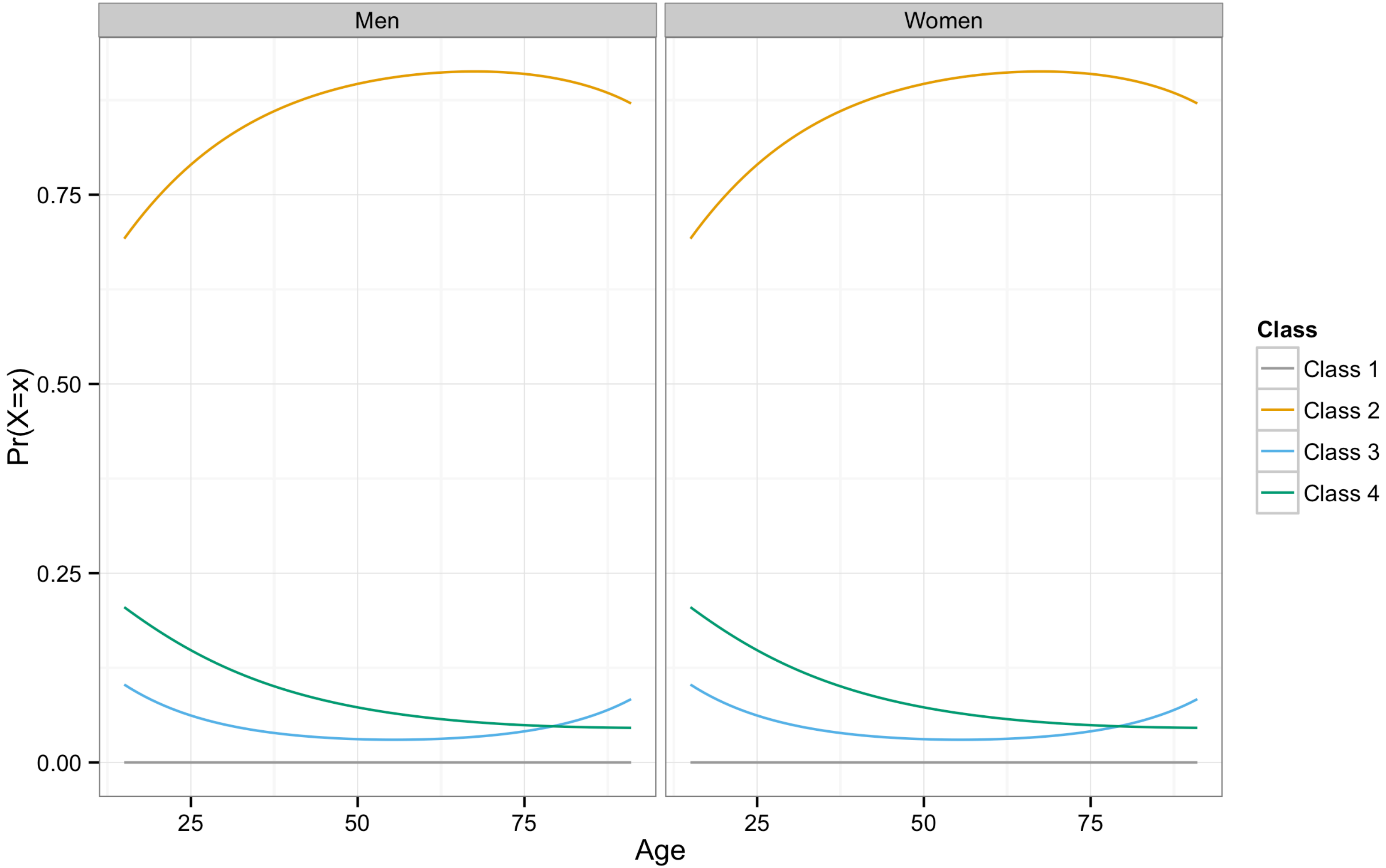
Class 1 Modern political participation
Class 2 Traditional political participation
Class 3 No political participation
Class 4 Every kind of political participation

Women more likely than men to be in classes 1 and 3
Less likely to be in classes 2 and 4

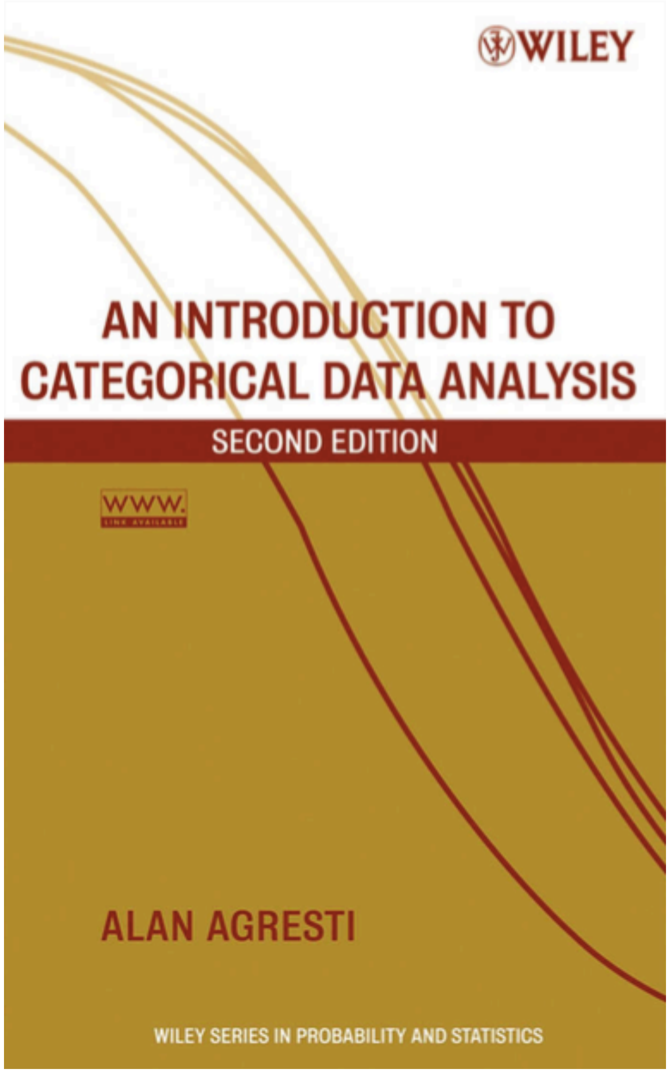
# Multinomial logistic regression refresher

For example:

- Logistic multinomial regression coefficient equals -0.3406
- Then log odds ratio of being in class 2 (compared with reference class 1) is -0.3406 smaller for women than for men
- So odds ratio is smaller by a factor exp(-0.3406) = 0.71
- So odds are 30% smaller for women


Men
Women
0.75
0.50
0.25
0.00
Pr(X=x)
25
50
75
25
50
75
Age
Class
Class 1
Class 2
Class 3
Class 4

Even more (re)freshing:



# Problems you will encounter when doing latent class analysis

(and some solutions)

# Some problems

- Local maxima
- Boundary solutions
- Non-identification

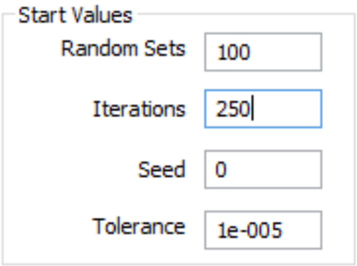
# Problem: Local maxima

**Problem**: there may be different sets of "ML" parameter estimates with different L-squared values we want the solution with lowest L-squared (highest log-likelihood)

**Solution**: multiple sets of starting values

```
poLCA(cbind(Y1, Y2, Y3)~1, antireli, nclass=2, nrep=100)

Model 1: llik = -3199.02 ... best llik = -3199.02
Model 2: llik = -3359.311 ... best llik = -3199.02
Model 3: llik = -2847.671 ... best llik = -2847.671
Model 4: llik = -2775.077 ... best llik = -2775.077
Model 5: llik = -2810.694 ... best llik = -2775.077
....
```



# Problem: boundary solutions

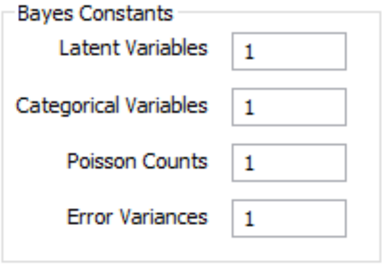
**Problem**: estimated probability becomes zero/one, or logit parameters extremely large negative/positive

Example:

```
$badge
          Pr(1)  Pr(2)
class 1:  0.8640 0.1360
class 2:  0.1021 0.8979
class 3:  0.4204 0.5796
class 4:  0.0000 1.0000
```

**Solutions**:

1. Not really a problem, just ignore it;
2. Use priors to smooth the estimates
3. Fix the offending probabilities to zero (classical)



# Problem: non-identification

- Different sets of parameter estimates yield the same value of L-squared and LL value: estimates are not unique
- Necessary condition DF>=0, but not sufficient
- Detection: running the model with different sets of starting values or, formally, checking whether rank of the Jacobian matrix equals the number of free parameters
- “Well-known” example: 3-cluster model for 4 dichotomous indicators

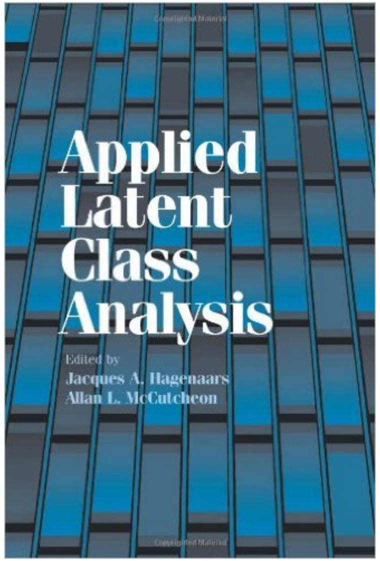
# What we did not cover

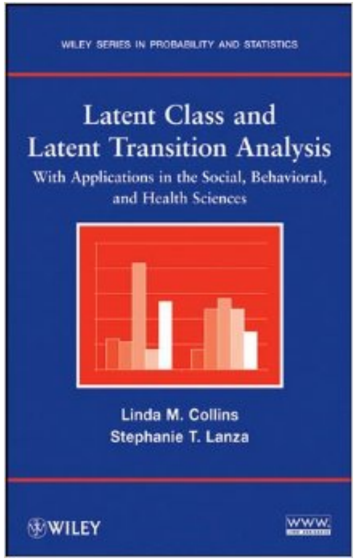
- 1 step versus 3 step modeling
- Ordinal, continuous, mixed type indicators
- Hidden Markov (“latent transition”) models
- Mixture regression

# What we did cover

- Latent class “cluster” analysis
- Model formulation, different parameterizations
- Model interpretation, profile
- Model fit evaluation: global, local, and substantive
- Classification
- Common problems with LCM and their solutions

# Further study

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poLCA: An R Package for Polytomous Variable Latent Class Analysis

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# Thank you for your attention!

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