

单隐藏层的神经网络

[1] ▶ ML

```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd

from sklearn.linear_model import LogisticRegressionCV
from sklearn.pipeline import Pipeline
from sklearn.preprocessing import PolynomialFeatures
from sklearn.preprocessing import StandardScaler

from sklearn.metrics import mean_squared_error
from sklearn.metrics import r2_score
from math import sqrt
from datetime import datetime
```

一、定义数据集

[2] ▶ ML

```
def load_planar_dataset(m=1000):
    np.random.seed(12)
    # m = 1000 # number of samples
    t = 2 # number of type
    N = int(m/t) # number of samples per type
    D = 2 # dimensionality
    X = np.zeros((m, D))
    y = np.zeros((m, 1), dtype = 'uint8') # labels vector (0 for
red, 1 for greed)
    r = 6 # maxium ray of the flower

    for i in range(t):
        idx = range(N*i, N*(i+1))
        theta = np.linspace(i*3.12, (i+1)*3.12, N) +
np.random.randn(N)*0.2
        radius = r*np.sin(6*theta) + np.random.randn(N)*0.2
        X[idx] = np.c_[radius*np.sin(theta), radius*np.cos(theta)]
        y[idx] = i

    # X = X.T
    # y = y.T

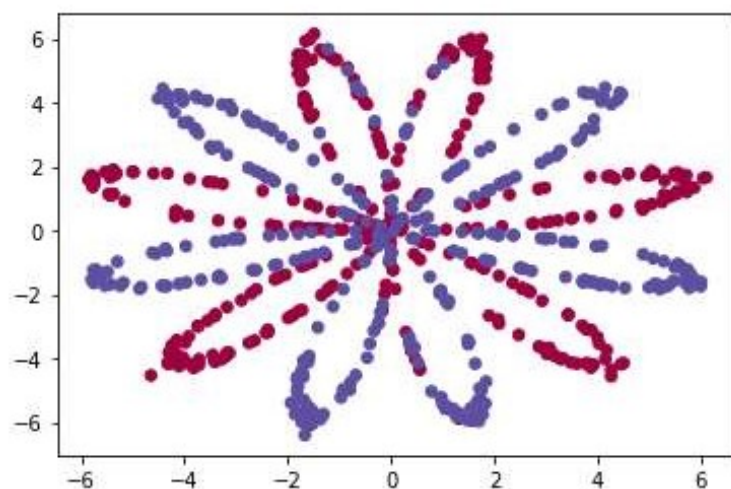
    return X, y
```

二、导出数据并可视化

[3] ▶ M4

```
m = 700
X, y = load_planar_dataset(m=m)
df = pd.DataFrame(np.hstack((X, y)))
df.to_csv("../dataAI/NeuralNetwork_Data1.txt", sep=",",
float_format="%.7f", index=False, header=None)
df.to_csv("../dataAI/NeuralNetwork_Data1.csv", sep=",",
float_format="%.7f", index=False, header=["x1", "x2", "y"])
plt.scatter(X[:, 0], X[:, 1], c=y, s=30, cmap=plt.cm.Spectral)
```

<matplotlib.collections.PathCollection at 0x10efae30>



三、用Logistic Regression分类

1. 定义绘制决策边界函数

[4] ▶ M4

```
def plot_decision_boundary(model, X, y):  
    # Set min and max values and give it some padding  
    x1_min, x1_max = X[:, 0].min(), X[:, 0].max()  
    x2_min, x2_max = X[:, 1].min(), X[:, 1].max()  
    h = 0.01  
    # Generate a grid of points with distance h between them  
    xx, yy = np.meshgrid(np.arange(x1_min, x1_max, h), np.arange  
(x2_min, x2_max, h))  
    # Predict the function value for the whole grid  
    Z = model(np.c_[xx.ravel(), yy.ravel()])  
    Z = Z.reshape(xx.shape)  
    # Plot the contour and training examples  
    plt.contourf(xx, yy, Z, cmap=plt.cm.Spectral)  
    plt.ylabel('x2')  
    plt.xlabel('x1')  
    plt.scatter(X[:, 0], X[:, 1], c=y, cmap=plt.cm.Spectral)
```

2. 线性逻辑回归分类

[5] ▶ M4

```
logi_reg = LogisticRegressionCV(cv=3)
logi_reg.fit(X, y.ravel())
```

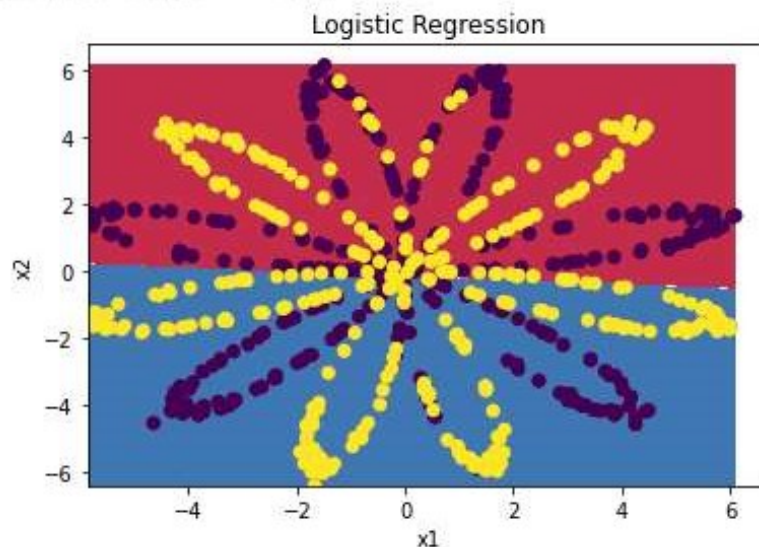
LogisticRegressionCV(cv=3)

[6] ▶ ML

```
plot_decision_boundary(lambda x:logi_reg.predict(x), X, y)
plt.title("Logistic Regression")
plt.scatter(X[:, 0], X[:, 1], c=y)

y_predict = logi_reg.predict(X)
train_rmse_score = sqrt(mean_squared_error(y, y_predict))
train_r2_score = r2_score(y, y_predict)
print("所有数据集上得分: {:.7f} -- {:.7f}".format(train_rmse_score,
train_r2_score))
```

所有数据集上得分: 0.6358347 -- -0.6171429



3. 多项式逻辑回归

[9] ▶ M4

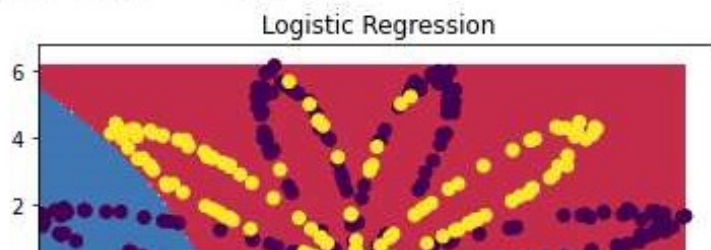
```
poly_logi_reg = Pipeline([
    ("multi_feature", PolynomialFeatures(degree=3)),
    ("std_scaler", StandardScaler()),
    ("logi_reg", LogisticRegressionCV(cv=3))
])
poly_logi_reg.fit(X, y)

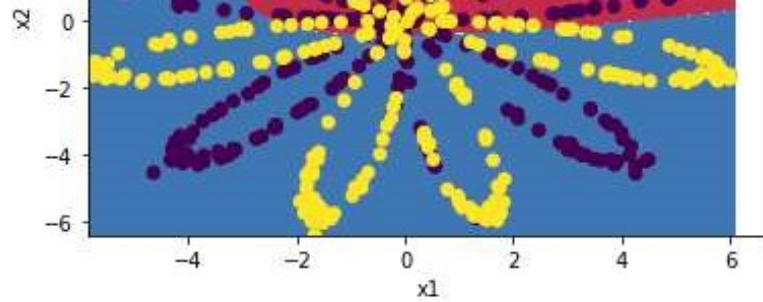
plot_decision_boundary(lambda x: poly_logi_reg.predict(x), X, y)
plt.title("Logistic Regression")

plt.scatter(X[:, 0], X[:, 1], c=y)

y_predict = poly_logi_reg.predict(X)
train_rmse_score = sqrt(mean_squared_error(y, y_predict))
train_r2_score = r2_score(y, y_predict)
print("所有数据集上得分: {:.7f} -- {:.7f}".format(train_rmse_score,
    train_r2_score))
```

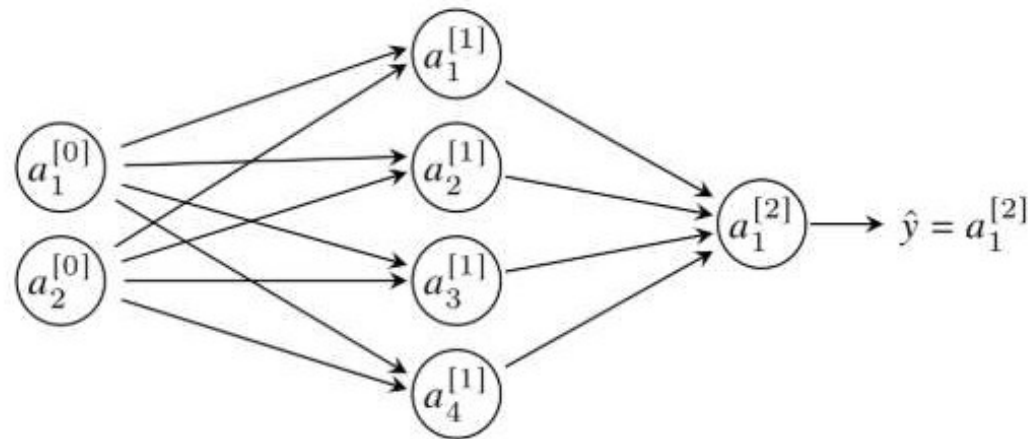
所有数据集上得分: 0.6633250 -- -0.7600000





四、Neural Network with One Hidden Layer

1. 2-Layer NN模型



2. NN算法步骤

- 确定结构，即输入层、隐藏层及输出层的单元数
- 初始化权矩阵 $W^{[l]}$ 、 $b^{[l]}$, $l = 1, 2, \dots, L$
- 利用正向传播过程，计算预测值 y_{pred} 及损失函数 $L(\hat{y}, y)$
- 利用反向传播过程，计算导数 $dW^{[2]}$ 、 $db^{[2]}$ 、 $dW^{[1]}$ 、 $db^{[1]}$
- 利用梯度法更新参数 W^2 、 $b^{[2]}$ 、 $W^{[1]}$ 和 $b^{[1]}$

3. 定义结构

[10] ▶ ML

```
# define neural network layer sizes
def neural_network_layer_sizes(X, Y):
    ...

    Arguments:
        X -- input dataset of shape(feature size, number of
examples)
        y -- labels of shape (output size, number of examples)

    Returns:
```



```

    n_x -- The size of input layer
    n_h -- The size of hidden layer
    n_y -- The size of output layer
    ...
n_x = X.shape[0]
n_h = 4
n_y = Y.shape[0]

return (n_x, n_h, n_y)

```

4. 初始化参数

[11] ▶ ▶≡ ML

```

# define initialize parameters
def initialize_parameters(n_x, n_h, n_y):
    ...

    Argument:
        n_x -- The size of input layer
        n_h -- The size of hidden layer
        n_y -- The size of output layer

    Returns:
        W1 -- weight matrix for layer 1 of shape (n_h, n_x)
        b1 -- bias vector for layer 1 of shape(n_h, 1)
        W2 -- weight matrix for layer 2 of shape (n_y, n_h)

```

```

    W2 -- weight matrix for layer 2 of shape (n_y, n_h)
    b2 -- bias vector for layer 2 of shape(n_y, 1)
'''

np.random.seed(12)

W1 = np.random.randn(n_h, n_x)*0.01
b1 = np.zeros((n_h, 1))
W2 = np.random.randn(n_y, n_h)*0.01
b2 = np.zeros((n_y, 1))

assert(W1.shape == (n_h, n_x))
assert(b1.shape == (n_h, 1))
assert(W2.shape == (n_y, n_h))
assert(b2.shape == (n_y, 1))

parameters = {"W1": W1, "b1": b1, "W2": W2, "b2": b2}

return parameters

```

5. 正向预测 (Forward Propagation)

1) 迭代公式

对于给定的样本 $x^{(i)}$ ，正向过程如下：

- 正向计算过程：

- 分量格式
$$\begin{cases} z^{[1](i)} &= W^{[1]}x^{(i)} + b^{[1]} \\ a^{[1](i)} &= g^{[1]}(z^{[1](i)}) \\ z^{[2](i)} &= W^{[2]}a^{[1](i)} + b^{[2]} \\ a^{[2](i)} &= \sigma(z^{[2](i)}) = \hat{y}^{(i)} \end{cases}$$

- 向量格式
$$\begin{cases} Z^{[1]} &= W^{[1]}X + b^{[1]} \\ A^{[1]} &= g^{[1]}(Z^{[1]}) \\ Z^{[2]} &= W^{[2]}A^{[1]} + b^{[2]} \\ A^{[2]} &= \sigma(Z^{[2]}) = \hat{y} \end{cases}$$

- 计算预测值：


$$y_p^{(i)} = \begin{cases} 1 & \text{if } \hat{y}^{(i)} \geq 0.5 \\ 0 & \text{otherwise} \end{cases}$$

- 成本函数：

$$J = -\frac{1}{m} \sum_{i=0}^m \left(y^{(i)} \log(\hat{y}^{(i)}) + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)}) \right)$$

2) 算法的实现

- 定义激活函数


[12] ▶  ML

```
def sigmoid(Z):  
    return 1/(1+np.exp(-Z))
```

[13] ▶  ML

```
def ReLU(Z):  
    return np.maximum(0, Z)
```

- 计算预测值

[14] ▶  ML

```
def forward_propagation(X, parameters):  
    """  
    Arguments:  
        X -- input data of size (n_x, m)  
        parameters -- python dictionary containing the initialize  
        parameters  
  
    Returns:  
        A2 -- The sigmoid output of the second activation
```



```

        cache -- a dictionary containing "Z1", "A1", "Z2" and "A2"
        """

        W1 = parameters["W1"]
        b1 = parameters["b1"]
        W2 = parameters["W2"]
        b2 = parameters["b2"]

        # FP
        Z1 = np.dot(W1, X) + b1
        # A1 = np.tanh(Z1)
        A1 = ReLU(Z1)
        Z2 = np.dot(W2, A1) + b2
        A2 = sigmoid(Z2)

        assert(A2.shape == (b2.shape[0], X.shape[1]))

        cache = {"Z1": Z1, "A1": A1, "Z2": Z2, "A2": A2}

        return A2, cache

```

- 成本函数

[15] ▶ ML

```

def J(A2, y, parameters):
    m = y.shape[1]

```

```

m = y.shape[1]

cost = -1/m*np.sum(y*np.log(A2)+(1-y)*np.log(1-A2))
cost = np.squeeze(cost)

assert(isinstance(cost, float))

return cost

```

6. 反向计算导数 (Backward Propagation)

()

1) 迭代公式

- 分量格式

$$\begin{cases}
 dz^{[2](i)} = a^{[2](i)} - y^{(i)} \\
 dW^{[2]} = dz^{[2](i)} a^{[1](i)T} \\
 db^{[2]} = dz^{[2](i)} \\
 dz^{[1](i)} = W^{[2]T} dz^{[2](i)} * g^{[1]'}(z^{[1]}) \\
 dW^{[1]} = dz^{[1](i)} a^{[0](i)T} \quad (a^{[0](i)} = x^{(i)}) \\
 db^{[1]} = dz^{[1](i)}
 \end{cases}$$

- 向量格式

$$\left\{ \begin{array}{lcl} dZ^{[2]} & = & A^{[2]} - y \\ dW^{[2]} & = & \frac{1}{m} dZ^{[2]} A^{[1]T} \\ db^{[2]} & = & \frac{1}{m} np.sum(dZ^{[2]}, axis = 1, keepdims = True) \\ dZ^{[1]} & = & W^{[2]T} dZ^{[2]} * g^{[1]'}(Z^{[1]}) \\ dW^{[1]} & = & \frac{1}{m} dZ^{[1](i)} A^{[0]T} \quad (A^{[0]} = X) \\ db^{[1]} & = & \frac{1}{m} np.sum(dZ^{[1]}, axis = 1, keepdims = True) \end{array} \right.$$

- Tips:

- To compute $dZ^{[1]}$, you'll need to compute $g^{[1]'}(Z^{[1]})$.
- If $g^{[1]}(\cdot)$ is the tanh function and let $a = g^{[1]}$, then $g^{[1]'} = 1 - a^2$.
- If $g^{[1]}(\cdot)$ is the sigmoid function, then $g^{[1]'} = a(1 - a)$, where $a = g^{[1]}$.
- If $g^{[1]}(\cdot)$ is the ReLU function, then $g^{[1]'} = 1(x > 0)$, where $a = g^{\{1\}}$.

2) 代码实现

[16] ▶ ML

```
def backward_propagation(X, y, parameters, cache):
```

```
    """
```

```
    Arguments:
```

```
    X -- input data of size (n_x, m)
```

```
    y -- "true" label vector of shape(n_y, m)
```

parameters -- python dictionary containing the initialize parameters
cache -- a dictionary containing "Z1", "A1", "Z2" and "A2"

Returns:

grads -- python dictionary containing the gradients with respect to different parameters

"""

m = X.shape[1] # number of samples

W1 = parameters["W1"]

W2 = parameters["W2"]

A1 = cache["A1"]

A2 = cache["A2"]

BP

dZ2 = A2 - y

dW2 = 1 / m * np.dot(dZ2, A1.T)

db2 = 1 / m * np.sum(dZ2, axis=1, keepdims=True)

dZ1 = np.dot(W2.T, dZ2) * (1 - np.power(A1, 2))

dW1 = 1 / m * np.dot(dZ1, X.T)

db1 = 1 / m * np.sum(dZ1, axis=1, keepdims=True)

return

grads = {"dW1": dW1, "db1": db1, "dW2": dW2, "db2": db2}

return grads

7. 更新参数

General gradient descent rule: , where η is the learning rate and θ represents a parameter.

```
[17] ▶ M4
def update_parameters(parameters, grads, learning_rate = 1.2):
    """
    Updates parameters using the gradient descent update rule
    given above

    Arguments:
    parameters -- python dictionary containing your parameters
    grads -- python dictionary containing your gradients

    Returns:
    parameters -- python dictionary containing your updated
    parameters
    """

    # old paremeters
    W1 = parameters["W1"]
```

```

b1 = parameters["b1"]
W2 = parameters["W2"]
b2 = parameters["b2"]

# gradient values
dW1 = grads["dW1"]
db1 = grads["db1"]
dW2 = grads["dW2"]
db2 = grads["db2"]

# update parameters
W1 = W1 - learning_rate * dW1
b1 = b1 - learning_rate * db1
W2 = W2 - learning_rate * dW2
b2 = b2 - learning_rate * db2

# return
parameters = {"W1": W1, "b1": b1, "W2": W2, "b2": b2}

return parameters

```

8. 算法整合

[18] ▶ ML

```
def neural_network_model(X, y, n_h, learning_rate = 0.9
```

```

def neural_network_model(X, y, n_h, learning_rate = 0.01,
max_iterations = 10000, err_torlance = 1e-12, print_cost = False):
    """
    Arguments:
    X -- dataset of shape (2, number of examples)
    y -- labels of shape (1, number of examples)
    n_h -- size of the hidden layer
    num_iterations -- Number of iterations in gradient descent
loop
    print_cost -- if True, print the cost every 1000 iterations

    Returns:
    parameters -- parameters learnt by the model and then can be
used to predict.
    """
    tic = datetime.now()

    np.random.seed(12)
    # layer of neural network
    n_x = neural_network_layer_sizes(X, y)[0]
    n_y = neural_network_layer_sizes(X, y)[2]

    # initialize parameters
    parameters = initialize_parameters(n_x, n_h, n_y)
    W1 = parameters["W1"]
    b1 = parameters["b1"]
    W2 = parameters["W2"]
    b2 = parameters["b2"]

```



```

cost_value_list = list(np.zeros((max_iterations, 1)))
last_cost = 0
# loop (using gradient descent)
for i in range(max_iterations):
    # 1. Forward Propagation
    A2, cache = forward_propagation(X, parameters)
    # 2. Compute Cost Function
    cost = J(A2, y, parameters)
    # 3. Backward Propagation
    grads = backward_propagation(X, y, parameters, cache)
    # 4. Update Parameters
    parameters = update_parameters(parameters, grads,
learning_rate=learning_rate)
    # 5. Print Cost Values
    if print_cost and i % 1000 == 0:
        print
        ("\n=====")
        print("Cost value after iteration {:d}: {:.f}".format
(i, cost))

    # 6. Exit Control
    cost_value_list[i] = cost
    if np.abs(cost-last_cost) < err_torlance:
        cost_value_list = cost_value_list[0:i]
        break
    last_cost = cost

    toc = datetime.now()

```



```

    toc = datetime.now()
    delta = toc - tic
    # print("NN Model Train Time: {:.f} S.".format
(delta.total_seconds()))
    # return optimum parameters
    return parameters, cost_value_list, delta.total_seconds()

```

9. 模型测试

[19] ▶ ⌵ ML

```

def nn_model_test(learning_rate=1.2):
    parameters, cost_list = neural_network_model(X.T, y.T, n_h=4,
learning_rate=learning_rate, max_iterations=10000, err_torlance
= 1e-12, print_cost=True)
    print("W1 = " + str(parameters["W1"]))
    print("b1 = " + str(parameters["b1"]))
    print("W2 = " + str(parameters["W2"]))
    print("b2 = " + str(parameters["b2"]))

    plt.plot(range(len(cost_list)), cost_list)

```

[21] ▶ ⌵ ML

```
nn_model_test(2.1)
```

```

=====
Cost value after iteration 0: 0.693077

```

```
=====
Cost value after iteration 1000: 0.440976

=====
Cost value after iteration 2000: 0.437611

=====
Cost value after iteration 3000: 0.435636

=====
Cost value after iteration 4000: 0.434290

=====
```

10. 预测

```
[22] ▶ ▶ ML
def predict(X, parameters):
    """
    Using the learned parameters, predicts a class for each
    example in X

    Arguments:
    parameters -- python dictionary containing your parameters
    X -- input data of size (n_x, m)

    Returns
```


```

        predictions -- vector of predictions of our model (red: 0 /
blue: 1)
        """

    A2, cache = forward_propagation(X, parameters)
    predictions = np.array(A2>=0.5, dtype=int)

    return predictions

```

[23] ▶  ML

```

def predict_test_case():
    np.random.seed(12)
    X_test = np.random.randn(2, 3)
    parameters = {'W1': np.array([[ -0.00615039,  0.0169021 ],
                                   [ -0.02311792,  0.03137121],
                                   [ -0.0169217 , -0.01752545],
                                   [  0.00935436, -0.05018221]]),
                  'W2': np.array([[ -0.0104319 , -0.04019007,  0.01607211,
                                   0.04440255]]),
                  'b1': np.array([[ -8.97523455e-07],
                                   [  8.15562092e-06],
                                   [  6.04810633e-07],
                                   [ -2.54560700e-06]]),
                  'b2': np.array([[ 9.14954378e-05]])}
    return X_test, parameters

```

[24] ▶  ML

```

X_test, parameters_test = predict_test_case()

```



```
predictions = predict(X_test, parameters_test)
print("Predict mean: {:.f}".format(np.mean(predictions)))
```

Predict mean: 0.666667

11. 绘制决策边界及模型评价

[25] ▶ ML

```
parameters, cost_list, training_time = neural_network_model(X.T,
y.T, n_h=17, learning_rate=5.2, max_iterations=10000,
err_torlance = 1e-12, print_cost = False)

plot_decision_boundary(lambda x:predict(x.T, parameters), X, y)
plt.title("Neural Network Classification")

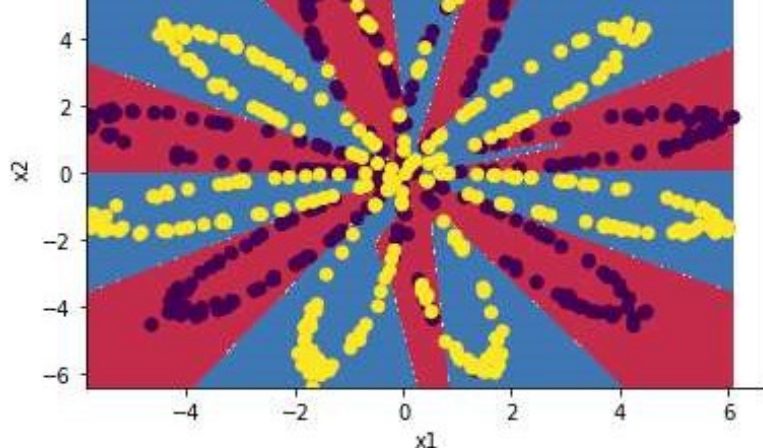
plt.scatter(X[:, 0], X[:, 1], c=y)

y_predict = predict(X.T, parameters)
rmse_score = sqrt(mean_squared_error(y.T, y_predict))
accuracy_score = np.sum(np.array(y.T==y_predict, dtype=int)) / len
(y)
print("所有数据集上得分: {:.7f} -- {:.7f}, 耗时{:.f}秒!".format
(rmse_score, accuracy_score, training_time))
```

所有数据集上得分: 0.2420153 -- 0.9414286, 耗时6.830964秒!

Neural Network Classification





12. 不同隐藏层节点的比较

[32] ▶ M4

```
plt.figure(figsize=(16, 32))
hidden_layer_sizes = [1, 2, 3, 4, 5, 7, 10, 15, 20, 50, 78, 99]
for i, n_h in enumerate(hidden_layer_sizes):
    # 决策边界
    plt.subplot(len(hidden_layer_sizes), 2, i*2+1)
    plt.title('The Size Hidden Layer: %d' % n_h)
    parameters, cost_list, training_time = neural_network_model
    (X.T, y.T, n_h=n_h, learning_rate=0.9, max_iterations=10000,
    err_torlance = 1e-12, print_cost = False)
    plot_decision_boundary(lambda x:predict(x.T, parameters), X,
    y)

    plt.scatter(X[:, 0], X[:, 1], c=y)
```

函数值

```
plt.subplot(len(hidden_layer_sizes), 2, i*2+2)
plt.title('The Cost Function value descent')
plt.plot(range(len(cost_list)), cost_list)
```

```
y_predict = predict(X.T, parameters)
rmse_score = sqrt(mean_squared_error(y.T, y_predict))
accuracy_score = np.sum(np.array(y.T==y_predict, dtype=int)) /
len(y)
```

```
print("模型 (Hidden Units = {:d}) 得分: {:.7f} -- {:.7f}, 耗时  
{:f}秒!".format(n_h, rmse_score, accuracy_score, training_time))
```

模型 (Hidden Units = 1) 得分: 0.7071068 -- 0.5000000, 耗时1.514948秒!
模型 (Hidden Units = 2) 得分: 0.7071068 -- 0.5000000, 耗时1.274590秒!
模型 (Hidden Units = 3) 得分: 0.7071068 -- 0.5000000, 耗时1.513126秒!
模型 (Hidden Units = 4) 得分: 0.7071068 -- 0.5000000, 耗时1.619667秒!
模型 (Hidden Units = 5) 得分: 0.7071068 -- 0.5000000, 耗时1.735398秒!
模型 (Hidden Units = 7) 得分: 0.7071068 -- 0.5000000, 耗时1.992669秒!
模型 (Hidden Units = 10) 得分: 0.7071068 -- 0.5000000, 耗时2.357605秒!
模型 (Hidden Units = 15) 得分: 0.7071068 -- 0.5000000, 耗时2.919191秒!
模型 (Hidden Units = 20) 得分: 0.7071068 -- 0.5000000, 耗时3.554785秒!
模型 (Hidden Units = 50) 得分: 0.7071068 -- 0.5000000, 耗时9.611463秒!
模型 (Hidden Units = 78) 得分: 0.7071068 -- 0.5000000, 耗时14.464670秒!
模型 (Hidden Units = 99) 得分: 0.7071068 -- 0.5000000, 耗时18.351490秒!

