22077303

MATHEMATICS
STANDARD LEVEL
PAPER 1

Monday 7 May 2007 (afternoon)

1 hour 30 minutes

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working. Working may be continued below the lines, if necessary.

1. Consider the infinite geometric sequence $25, 5, 1, 0.2, \ldots$.

    (a)   Find the common ratio.

    (b)   Find

        (i)   the 10th term;

        (ii)   an expression for the nth term.

    (c)   Find the sum of the infinite sequence.

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2.   Consider the events A and B, where $P(A) = \dfrac{2}{5}$, $P(B') = \dfrac{1}{4}$ and $P(A \cup B) = \dfrac{7}{8}$.

(a)   Write down $P(B)$.

(b)   Find $P(A \cap B)$.

(c)   Find $P(A | B)$.

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3. In the triangle PQR, $PR = 5$ cm, $QR = 4$ cm and $PQ = 6$ cm.

Calculate

(a) the size of $P\hat{Q}R$;

(b) the area of triangle PQR.

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4. The cumulative frequency graph below shows the heights of 120 girls in a school.
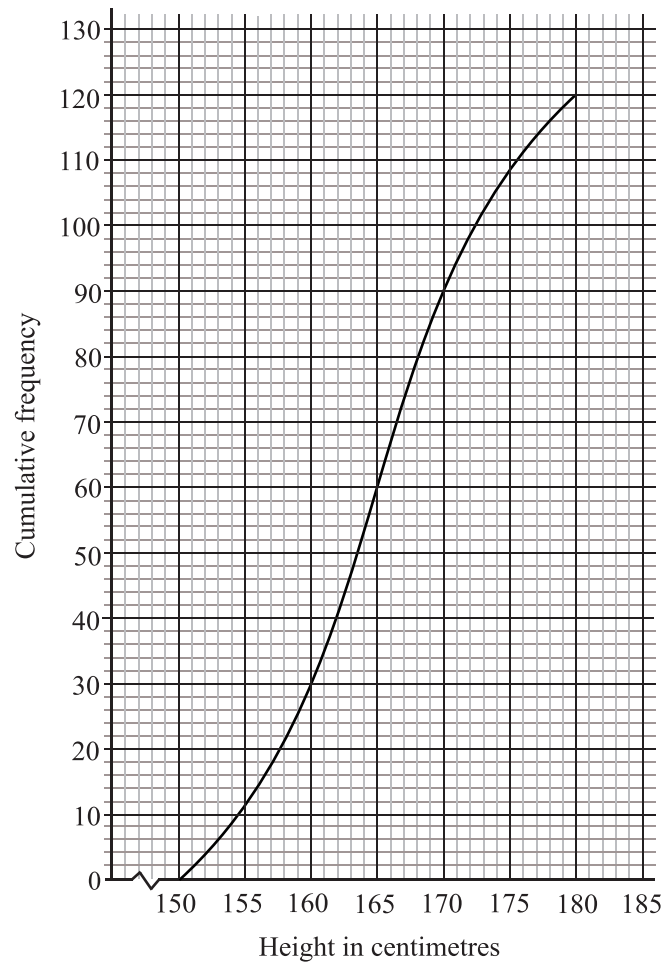
(a) Using the graph

    (i) write down the median;

    (ii) find the interquartile range.

(b) Given that 60 % of the girls are taller than a cm, find the value of a.

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Turn over

5.   Given that $p = \log_a 5$, $q = \log_a 2$, express the following in terms of p and/or q.

   (a) $\log_a 10$

   (b) $\log_a 8$

   (c) $\log_a 2.5$

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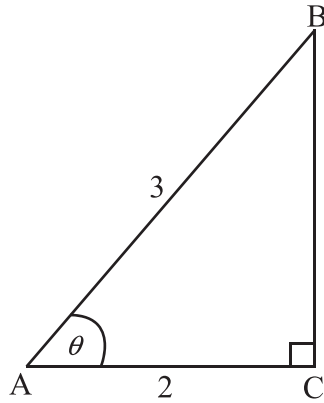
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6. The following diagram shows a triangle ABC, where $\hat{ACB}$ is $90°$, $AB = 3$, $AC = 2$ and $\hat{BAC}$ is θ.

(a) Show that $\sin\theta = \dfrac{\sqrt{5}}{3}$.

(b) Show that $\sin 2\theta = \dfrac{4\sqrt{5}}{9}$.

(c) Find the **exact** value of $\cos 2\theta$.

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7. The functions $f(x)$ and $g(x)$ are defined by $f(x) = e^x$ and $g(x) = \ln(1+2x)$.

(a) Write down $f^{-1}(x)$.

(b) (i) Find $(f \circ g)(x)$.

(ii) Find $(f \circ g)^{-1}(x)$.

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8. (a) Write down the inverse of the matrix $A = \begin{pmatrix} 1 & -3 & 0 \\ 2 & 0 & 1 \\ 4 & 1 & 3 \end{pmatrix}$.

(b) Hence or otherwise solve

$$x - 3y = 1$$
$$2x + z = 2$$
$$4x + y + 3z = -1$$

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9. The velocity v of a particle at time t is given by $v = e^{-2t} + 12t$. The displacement of the particle at time t is s. Given that $s = 2$ when $t = 0$, express s in terms of t.

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10. The heights of boys at a particular school follow a normal distribution with a standard deviation of 5 cm. The probability of a boy being shorter than 153 cm is 0.705.

(a)   Calculate the mean height of the boys.

(b)   Find the probability of a boy being taller than 156 cm.

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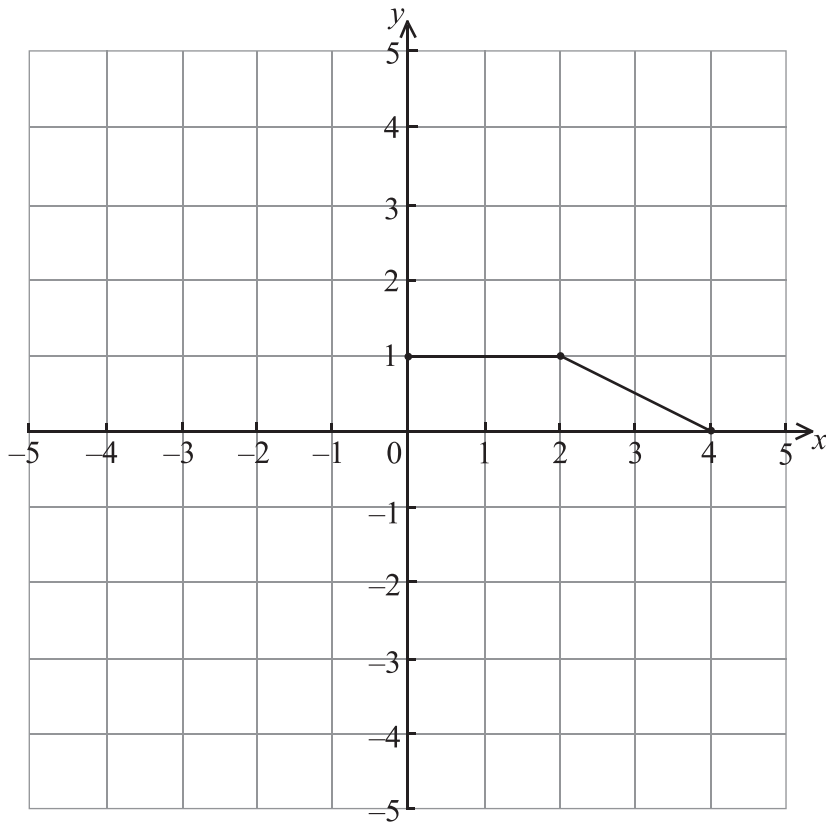
Turn over

11. The graph of the function $y = f(x)$, $0 \leq x \leq 4$, is shown below.

(a) Write down the value of

(i) $f'(1)$;

(ii) $f'(3)$.

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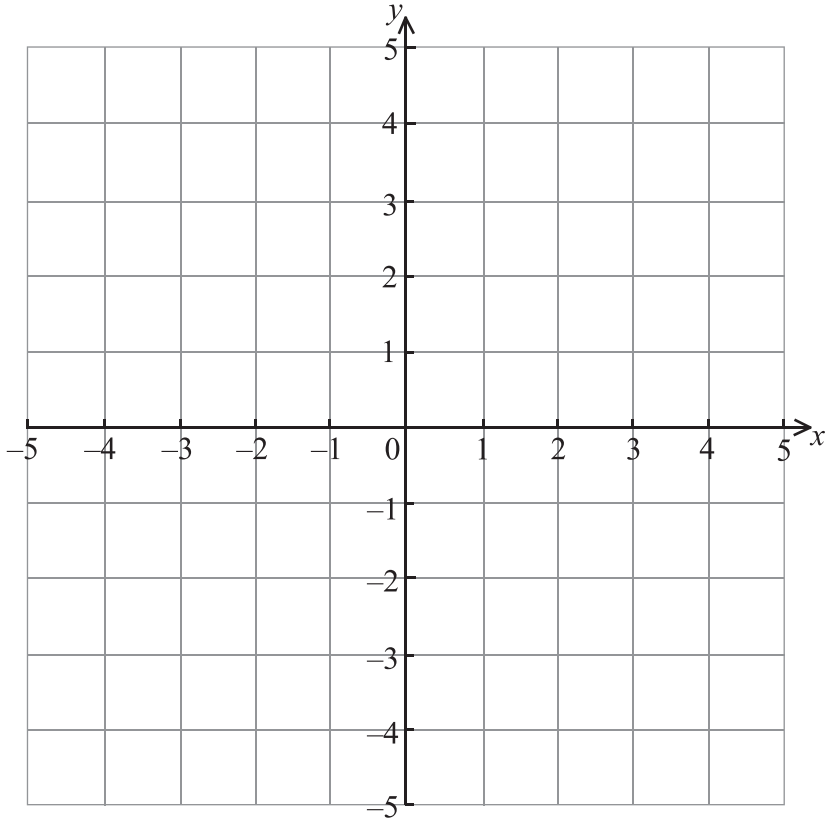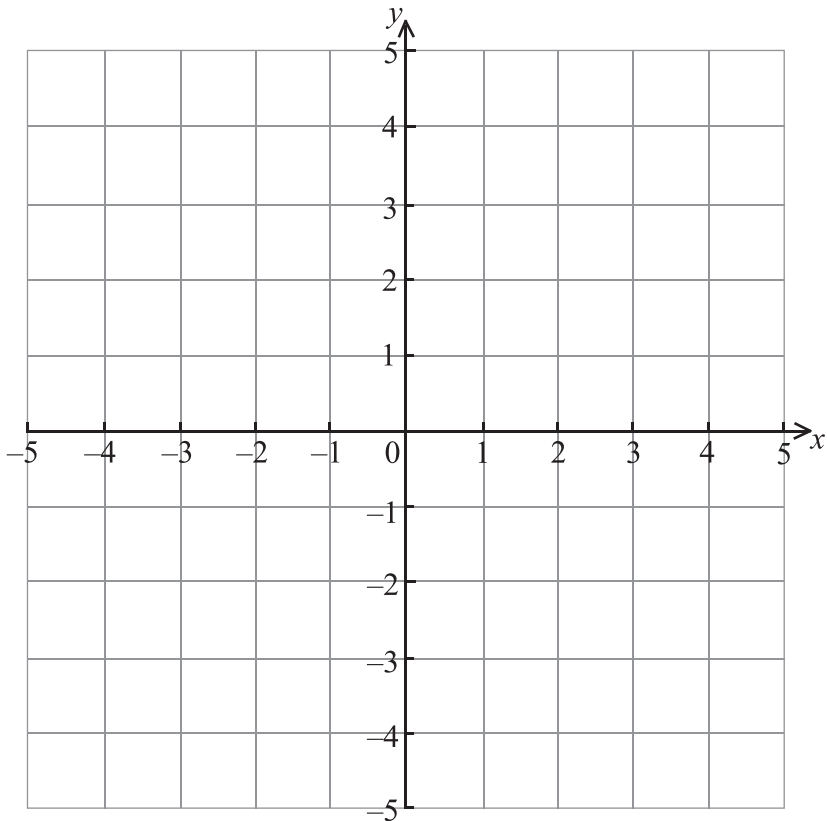
(This question continues on the following page)

(Question 11 continued)

(b)   On the diagram below, draw the graph of $y = 3f(-x)$.

(c)   On the diagram below, draw the graph of $y = f(2x)$.

Turn over

12. Consider the expansion of the expression $(x^3 - 3x)^6$.

(a) Write down the number of terms in this expansion.

(b) Find the term in x^{12}.

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13. Let $f(x) = x^3 - 3x^2 - 24x + 1$.

The tangents to the curve of f at the points P and Q are parallel to the x-axis, where P is to the left of Q.

(a) Calculate the coordinates of P and of Q.

Let N_1 and N_2 be the normals to the curve at P and Q respectively.

(b) Write down the coordinates of the points where

(i) the tangent at P intersects N_2;

(ii) the tangent at Q intersects N_1.

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14. It is given that $\int_1^3 f(x)\,dx = 5$.

(a) Write down $\int_1^3 2f(x)\,dx$.

(b) Find the value of $\int_1^3 (3x^2 + f(x))\,dx$.

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15. (a) Given that $(2^x)^2 + (2^x) - 12$ can be written as $(2^x + a)(2^x + b)$, where $a, b \in \mathbb{Z}$, find the value of a and of b.

(b) Hence find the **exact** solution of the equation $(2^x)^2 + (2^x) - 12 = 0$, and explain why there is only one solution.

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