

Distributed Algorithms for Dynamic Networks

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Abstract

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1 High-Level Description of Goal

The goal of this paper is to simplify the creation of distributed algorithms for dynamic networks by demonstrating that any algorithm that works for the broadcast variant of the synchronous model with a star topology can be made to work for the broadcast variant of the synchronous model with any topology. We will do so by describing a simulation algorithm that, if run on every node in the broadcast model, will match perfectly the output of the nodes of the centrally controlled model.

2 Models

Main Execution Model

1. We are considering the broadcast variant of the synchronous model, defined with respect to a connected network topology $G=(V,E)$.
2. The broadcast variant is defined such that a given node has no knowledge of its neighbors but may send a single message per round that all of its neighbors will receive.
3. Nodes in the network have comparable unique identification numbers (UIDs) and have knowledge of their own UID.

Reference Execution Model

1. We consider the synchronous model with a star network topology.
2. A given node records its sent message and its received message for each round in a communication log.

3 Problem Definition

This paper considers the simulation problem, the problem of correctly simulating an algorithm designed for the reference model on the main model. We examine the simulation problem using three different assumptions regarding the arrival and departure of nodes in the network.

We define the main execution to be an instance of the main execution model with n processes. We define the reference execution to be an instance of the reference execution model with $n+1$ processes.

3.1 The Static Case

The static case assumes that all nodes arrive in the network at the same time and that no node may depart from the network during the execution. Because the simulation problem is defined in the synchronous model, this assumption means that round r begins and ends at the same time for each node in the network.

The simulation problem provides the algorithm the following input, algorithms a_1 and a_2 , and expects it to produce a list of communication logs. We say a given algorithm solves the simulation problem if it outputs list Y where Y is equivalent to the list of communication logs generated by the reference execution given input of algorithm a_1 to the leader and algorithm a_2 to all other nodes.

3.2 The Arrival Only Case

The arrival only case assumes that a node may arrive in the network at the beginning of any round of the main execution and no node may depart from the network during the execution. A node that arrives in the beginning of a round r in the main execution will arrive in the reference execution at the beginning of the next round of the reference execution.

3.3 The Departure Only Case

3.4 The Arrival and Departure Case

4 Algorithm

1. Run unmodified Leader Election without Network Information

- (a) Assume nodes have UIDs (which is also assumed for the distributed model)
- (b) Each process runs Terminating Synchronous Breadth First Search and the node that manages to terminate elects itself leader and tells the other nodes to terminate

2. The Simulation

- (a) Assume that each and every round of the given algorithm has a finite repetitions of a step, which is comprised of two parts
 - i. The star node sends a broadcast message to all the leaf nodes (Broadcast part)
 - ii. All leaf nodes send a receive message to the star node in response to the broadcast message (Receive part)
- (b) Each step is simulated by the leader node running a modified instance of terminating synchronous BFS
 - i. The search message sent by a parent node to its children nodes is modified to be “search” plus the broadcast message, which is dictated by the output of the input algorithm A1
 - ii. Upon receiving a search message, a node runs the input algorithm A2 on the broadcast message portion and then creates the receive message portion using the output of A2
 - iii. The done message sent by a child node to its parent node is modified to be “done” plus both the UID and the receive message of the child node as well as any done messages received by the child node
 - iv. Upon receiving a search message, a node writes the broadcast message portion to its communication log
 - v. Upon sending a done message, a node writes the receive message portion to its communication log
 - vi. The leader simulates sending a search message to itself and then simulates sending a done message back to itself
 - vii. This algorithm terminates when the leader has received done messages from all of its children and a simulated done message from itself

Static Model - All the nodes turn on at the same time

Simulation Algorithm takes one input, algorithm A, the algorithm to be simulated. Algorithm A can be broken into two distinct algorithms, A1, the algorithm run by the star process, and A2, the algorithm run by the leaf processes.

Member Variables - maxID (UID); parent (UID); totalChildren (int); child-Count (int); wait (int); Message has a root (a round, the UID); a id of the sender (UID); a type search, choose, done; a receiver (UID), defaults to NULL;

Dynamic Addition Model - Nodes turn on at various times but they don't turn off. The leader elected will be the node with the highest UID out of all of the nodes that turned on at round 1

Variables - maxRoot - (a round, the UID); parent (UID); totalChildren (int); childCount (int); wait (int); Message has a root (a round, the UID); a id of the sender (UID); a type search, choose, done; a receiver (UID);

5 Analysis

Lemma 5.1. *For the given network, a node will eventually set leader to true and no more than one node will have leader equal to true at the beginning of any round r .*

Proof. One node will eventually set leader to true (Lemma ??). No more than one node will have leader set to true at any point (Lemma ??). \square

Definition 1. *Let u_{max} be the ID of the process with the maximum UID in the network. Let BFS instance b_i refer to an instance of the terminating breadth-first search protocol initiated by process with ID i .*

Lemma 5.2. *For every round r , at most one node has leader = true at the beginning of round r .*

Proof. A node with ID i will only set leader = true if the BFS instance b_i terminates. A BFS tree b_i will terminate only if i equals u_{max} (Lemma ??). Only the process with ID u_{max} , will set leader = true. \square

Lemma 5.3. *A BFS instance b_i will only terminate if i equals u_{max} .*

Proof. Termination of a BFS instance b_j requires all other processes in the network to send a done message to b_j . Given BFS instance b_j where $j < u_{max}$, there is at least one process, the process with ID u_{max} , that will never reply done to b_j . Therefore, b_j will never terminate. \square

Lemma 5.4. *One node will eventually set its variable leader to true.*

Proof. A BFS instance b_i will eventually terminate if every node in the network runs b_i . Every node in the network will eventually run $b_{u_{max}}$ so $b_{u_{max}}$ will eventually terminate and the process with ID u_{max} will set leader = true. \square

Definition 2. *For a given node, define r_t as the consecutive rounds of the main execution that directly map to round t of the reference execution. For a given node and any round t in the reference execution, r_t exists (Lemma ??). For a given node a in the reference execution, define s_a as the node in the main execution that simulates a .*

Lemma 5.5. *By induction, the main execution correctly simulates the reference execution for any round q .*

Proof. By Induction.

Base Case $q = 0$: Before round 0, the main execution has correctly simulated the reference execution because all the communication logs are empty and therefore equivalent.

Inductive Hypothesis: Suppose the theorem holds for all values of q up to k .

Inductive Step: Let $q = k + 1$. In round k of the reference execution, the star node sends message m to every child node and records ‘sent m ’ in its log. In the main execution, u_{max} broadcasts m and records ‘sent m ’ in its log. In the reference execution, every child node receives m and records ‘received m ’ in its log. In the main execution, the children of u_{max} receive m , record ‘received m ’ in its log, and broadcast m to their children. Eventually, every node in the main execution network receives m and records ‘received m ’ in its log.

In the reference execution, a given child node a sends message m_1 to the star node and records ‘sent m_1 ’ in its log. The star node receives m_1 and records ‘received m_1 ’ in its log. In the main execution, a eventually broadcasts m_1 to its parent and records sent m_1 in its log. a ’s parent receives m_1 , which is broadcasted up the network until it eventually reaches u_{max} , which records ‘received m_1 ’ in its log.

After round $k + 1$, the list of the logs of the reference execution will be equivalent to that of the main execution. So the theorem holds for $q = k + 1$. By the principle of mathematical induction, the theorem holds for all rounds in the execution. □

Lemma 5.6. *For a given node and any round t in the reference execution, r_t exists.*

Proof. For the star node in the reference execution, the simulation of one round of its execution is defined as beginning when u_{max} broadcasts its message and ending when u_{max} has received messages from all of its children. For any given child node a in the reference execution, the simulation of one round of its execution is defined as between when a broadcasts its message and when a receives the leader’s message from its parent. □

```

initVariables() ;
for round 1 ... r do
    for each message m in Inbox do
        if m.maxID > maxID then
            | updateMaxRoot() ;
        end
        if m.maxID == maxID then
            if m.type == choose AND receiver == myUID then
                | childCount++ ;
                | totalChildren++ ;
            end
            if m.type == done AND receiver == myUID then
                | childCount- ;
                if childCount == 0 then
                    | sendDoneMsg( ) ;
                end
            end
        end
        if m.maxID < maxID then
            | msg = (type=search, sender=myUID, maxID=maxID) ;
            | Outbox.enqueue(msg) ;
        end
    end
    if wait != 0 AND childCount == 0 then
        | wait- ;
        if wait == 0 then
            | sendDoneMsg( ) ;
        end
    end
    for each message m in Outbox do
        | broadcast(m);
    end
    myRound++ ;
end

```

Algorithm 1: Simulation Algorithm for Static Model

```

myRound == 0 ;
maxID = myUID ;
message m = (type=search, sender=myUID, maxID=maxID) ;

```

Algorithm 2: initVariables method

```

maxID = m.maxID;
parent = m.sender;
childCount = totalChildren = 0; msg1 = (type=choose, sender=myUID,
maxID=maxID, receiver=m.sender) ;
Outbox.enqueue(msg1) ;
msg2 = (type=search, sender=myUID, maxID=maxID) ;
Outbox.enqueue(msg2) ;
wait = 3 ;

```

Algorithm 3: updateMaxRoot method

```

msg = ( type=done, sender=myUID, maxID=maxID ) ;
Outbox.enqueue( msg ) ;

```

Algorithm 4: sendDoneMsg method

```

for each action in A1 do
  if leader == true then
    message = (type=r.action, sender=myUID) ;
    broadcast ( message ) ;
    msg = ( response to message ) ;
    commLog.write (response to message) ;
  end
  for each round r do
    for each message m in Inbox do
      if m.sender == parent then
        forwardMsgToChildren( ) ;
      end
      if m.receiver == myUID then
        add m to msg ;
        childCount- ;
        if childCount == 0 then
          Outbox.enqueue(msg) ;
        end
      end
    end
    for each message m in Outbox do
      broadcast(m) ;
    end
  end
end
end

```

Algorithm 5: Static Simulation Algorithm

```

msg = (response to m, receiver = parent) ;
commLog.write(response to m) ;
if totalChildren == 0 then
    | Outbox.enqueue(msg) ;
else
    | forwardMsg = (type = m.type, sender=myUID) ;
    | Outbox.enqueue(forwardMsg) ;
end

```

Algorithm 6: forwardMsgToChildren


```

initVariables() ;
for round 1...r do
    for each message m in Inbox do
        if m.root > maxRoot then
            | updateMaxRoot() ;
        end
        if m.root == maxRoot then
            if m.type == choose AND receiver == myUID then
                | childCount++ ;
                | totalChildren++ ;
            end
            if m.type == done AND receiver == myUID then
                | childCount- ;
                | if childCount == 0 then
                    | | sendDoneMsg() ;
                end
            end
        end
        if m.root < maxRoot then
            | msg = (type=search, sender=myUID, root=(r=maxRoot.r+1,
            | id=maxRoot.id) ) ;
            | Outbox.enqueue(msg) ;
        end
    end
    if wait != 0 AND childCount == 0 then
        | wait- ;
        | if wait == 0 then
            | | sendDoneMsg() ;
        end
    end
    for each message m in Outbox do
        | broadcast(m);
    end
    myRound++ ;
    maxRoot = (r=maxRoot.r+1, id=maxRoot.id) ;
end

```

Algorithm 7: Simulation Algorithm for the Dynamic Addition Model

```

myRound == 0 ;
maxRoot = (r=myRound, sender=myUID) ;
message m = (type=search, id=myUID, root=maxRoot) ;

```

Algorithm 8: initVariables method for Dynamic Addition Model

```

maxRoot = m.root;
parent = m.sender;
childCount = totalChildren = 0; msg1 = (type=choose, sender=myUID,
root=(r=maxRoot.r+1, id=maxRoot.id), receiver=m.id) ;
Outbox.enqueue(msg1) ;
msg2 = (type=search, sender=myUID, root=(r=maxRoot.r+1,
id=maxRoot.id) ) ;
Outbox.enqueue(msg2) ;
wait = 3 ;

```

Algorithm 9: updateMaxRoot method for Dynamic Addition Model

```

msg = ( type=done, sender=myUID, root=(r=maxRoot.r+1,
id=maxRoot.id), receiver=parent ) ;
Outbox.enqueue( msg ) ;

```

Algorithm 10: sendDoneMsg method for Dynamic Addition Model