

Transport Phenomena: Diffusion (3D)

Heat : Fourier

$$\bullet \, \frac{\partial \vartheta}{\partial t} = \alpha \, \Delta \vartheta$$

Matter: Fick

$$\bullet \, \frac{\partial C}{\partial t} = \beta \, \Delta C$$

Momentum : Newton-NS

$$\bullet \frac{\partial \vec{V}}{\partial t} = \nu \, \Delta \vec{V}$$

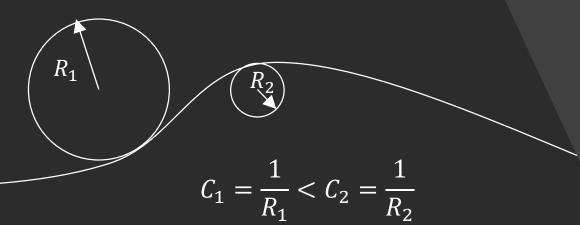
Dimensionless

$$\bullet \, \frac{\partial f}{\partial t} = \lambda \, \Delta f$$

Takahisa Okino (Oita Univ.) « <u>Correlation between Diffusion Equation and Schrödinger Equation</u> », 2013, J. Of Modern Physics, 4, 612-615.

Curvature

• For a Curve: 1D



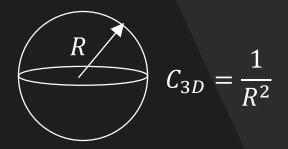
• For a Polygon: 2D

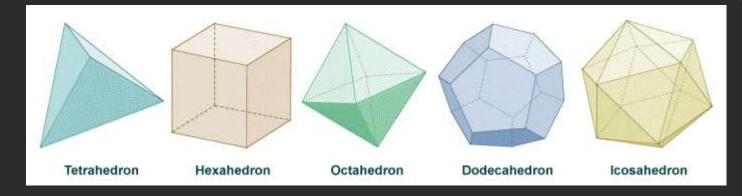
$$C_{2D} = \frac{1}{R} \qquad \qquad R$$



?

• For a polyhedra: 3D





Curvature (Delplace 2017, IJSEAS)

• 2D

$$C_{2D} = \frac{P}{2S}$$

P = Perimeter

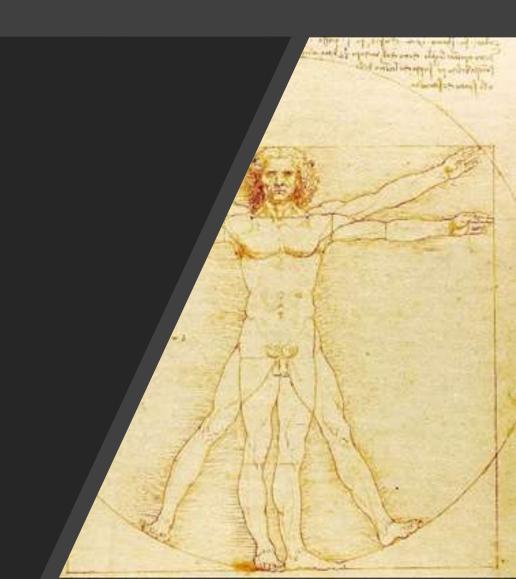
S = Surface

• 3D

$$C_{3D} = \frac{S^2}{9V^2}$$

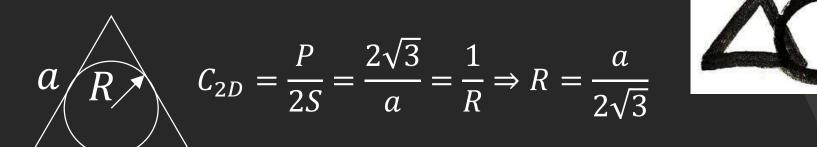
S = Surface

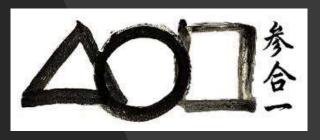
V = Volume



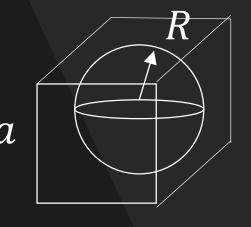
Examples For Regular Geometries

• 2D





• 3D



$$C_{3D} = \frac{S^2}{9V^2} = \frac{36a^4}{9a^6} = \frac{4}{a^2} = \frac{1}{R^2} \Rightarrow R = \frac{a}{2}$$

Laplacian and Curvature

Self-Evident in 1D

$$\Delta f = \frac{\partial^2 f}{\partial x^2} = f''(x)$$

• Less evident in 3D

$$\Delta f = k \ C(\bar{f} - f)$$

Cube
$$\Delta f = \frac{24}{a^2} \left(\bar{f} - f \right) = 6 C \left(\bar{f} - f \right)$$

Physics is Curvature

• Transport Phenomena
$$\frac{\partial \theta}{\partial t} = \alpha \ k \ C \ (\bar{\theta} - \theta)$$

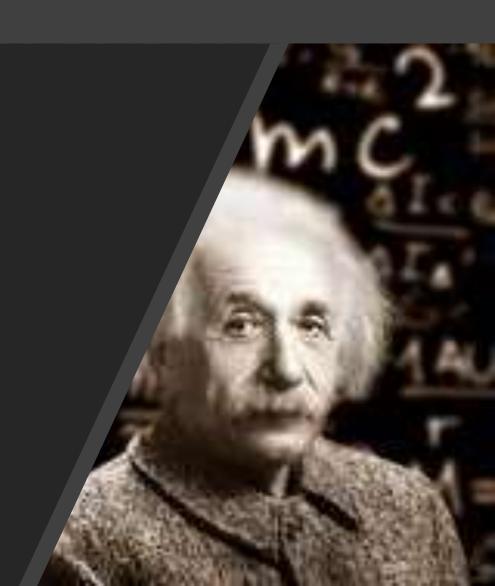
$$\frac{\partial C}{\partial t} = \beta k C (\bar{C} - C)$$

$$\frac{\partial \vec{V}}{\partial t} = v k C (\bar{V} - \vec{V})$$

Gravity

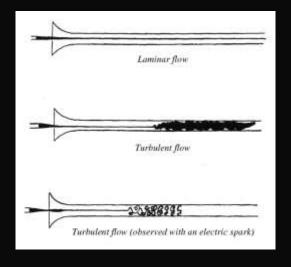
Einstein: Energy Density proportional to Space-Time CURVATURE

CURVATURE is a Major Quantity for Physics Models

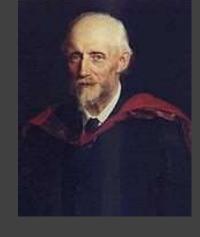


In Fluid Mechanics

• 2D

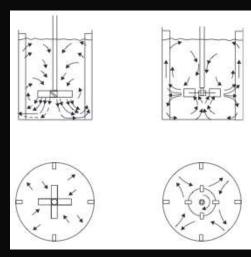


Tubular Flow



Osborne Reynolds 1842-1912





Mixing

Reynolds Number and Curvature

• 2D : Flow in Ducts

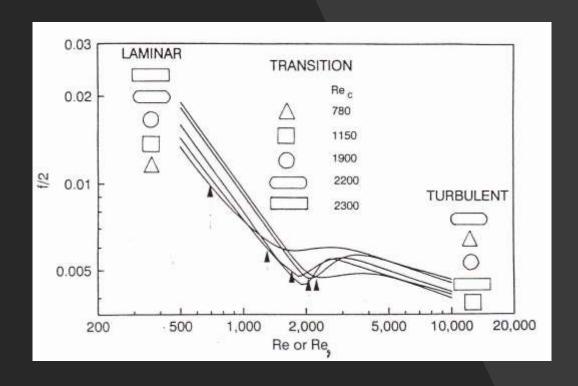
$$Re = \frac{\rho \overline{V} D_h}{\eta} = \frac{\overline{V} D_h}{\nu} = \frac{\overline{V} 2 R_h}{\nu} = \frac{2\overline{V}}{\nu} \cdot \frac{1}{C_h} = \frac{C_f}{C_h}$$

Reynolds Number is the Ratio of Two Curvatures

For a Pipe of Circular Cross-Section

$$C = \frac{1}{R}$$
 and $\bar{V} = \frac{Q}{\pi R^2}$

$$Re = \frac{2}{\pi \nu} . C. Q$$



For a given Flow Rate Q and given kinematic viscosity ν Reynolds Number is Proportional to Cross-Section Curvature C

Reynolds Number and Curvature

• 2D : Flow in Ducts

For a duct of square cross-section

$$Re = \frac{1}{2\nu}.C.Q$$

For a duct of equilateral triangular cross-section

$$Re = \frac{2}{3\sqrt{3}\nu}.C.Q$$

For a given flow-rate Q and a given kinematic viscosity ν Reynolds number is proportional to cross-section curvature C

Reynolds Number and Curvature

• 3D : Mixing

$$Re_R = \frac{\rho ND^2}{\eta} = \frac{ND^2}{\nu} = \frac{4NR^2}{\nu} = \frac{4N}{\nu} \cdot \frac{1}{C} = \frac{C_f}{C}$$

Rotational Reynolds Number is the Ratio of two Curvatures

Considering that $\frac{4N}{\nu}$ is the 3D Flow Curvature and $\bar{\dot{\gamma}}=K_S$. N we have $\bar{\dot{\gamma}}=\frac{K_S}{4}$. C_f . ν

Proportionality between velocity gradient and curvature is a major general relationship!

Heat Transfer

Increasing S enhances heat exchange

According to : $C_{3D} = \frac{S^2}{9V^2}$ - you increase S without changing V.

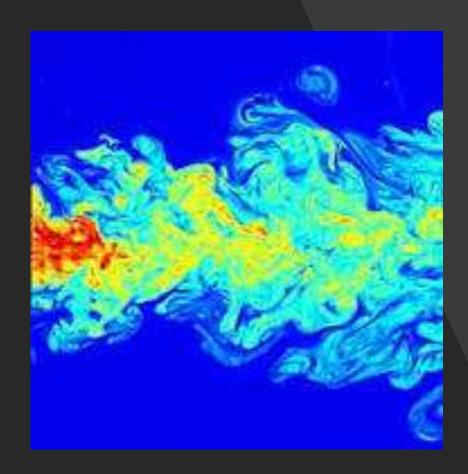
In agreement with "Hairy Ball" Mathematical Model.



Heat Transfer

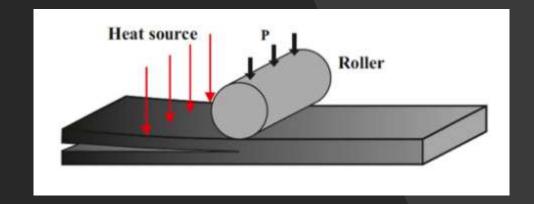
Turbulence enhances heat transfer

Flow « Fractalization » increase S without changing V.

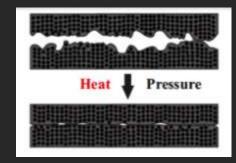


Heat & Momentum Transfers: Composites Consolidation (Prof. Chinesta and Dr Abisset-Chavanne)

Both Heat and Pressure applied to composites sheets.



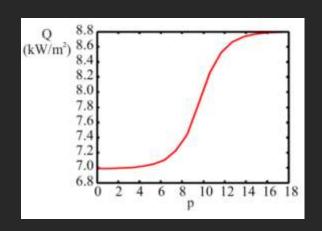
Interfaces irregularities disappear due to both heat and pressure giving polymer molecules rearrangement.

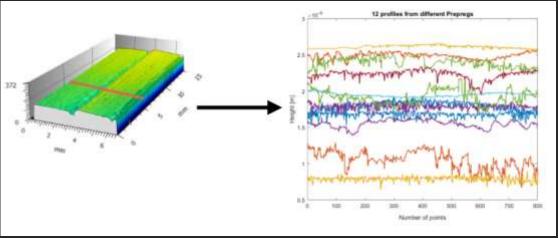


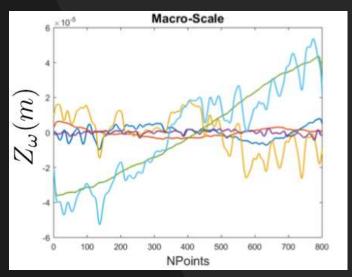
You increase the number of spikes with the **Same contact area**!

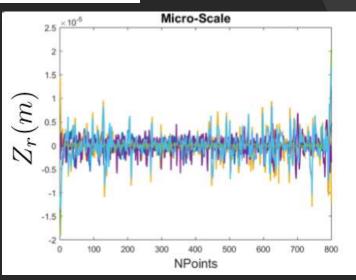
Higher is **Curvature** (spikes number) for the same area, higher is the thermal flux Q with a behaviour typical of percolation physics.

F.Chinesta et al., "High-resolution thermal analysis at thermoplastic pre-impregnated composite interfaces", 2015, Composite Interfaces, 22/8, 767-777

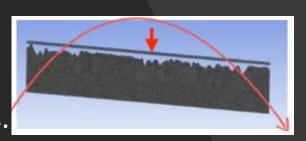








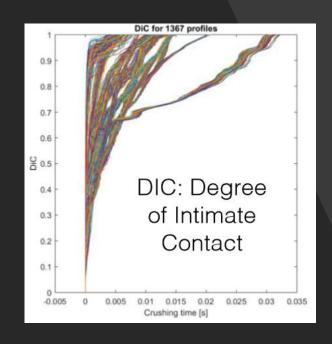
Huge Number of Crushing Experiments to Measure the Time Needed to Reach Intimate Contact Between Composite Sheets.



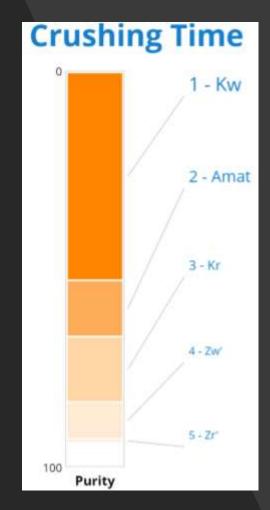
Five Variables Tested using ESI Software MINESET:

- Macroscopic Slope : First Derivative.
- Microscopic Slope: First Derivative.
- Macroscopic Curvature : Second Derivative.
- Microscopic Curvature: Second Derivative.
- Area.

For Crushing Time Determination



- Curvature Appears to be The Key Parameter for Heat and Momentum Transfers.
- In Real Surfaces with Complex (Multi-Scale) Roughness Distributions, Macro-Scale Curvature appeared (as intuitively expected) the Dominant Parameter.
- A Deep-Learning Model using Neural Networks was Created and Trained using Experimental Results.
 Results gave <u>Curvature as the Dominant Parameter!</u>



Huge Perspectives

- Process Improvement: Optimal design of surfaces for processing.
- Surfaces Design: adapted interfaces for particular properties.
- New Processes : 3D printing.
- Optimal Process Control : using curvature sensors.
- Actives Surfaces : adapting their geometry (curvature) to reach specific properties.

Conclusion

- Physics is governed by Curvature
- Curvature is a Major Parameter of all Physics Models.
- Thank You

