Algorithms for finding all maximal bicliques of a graph

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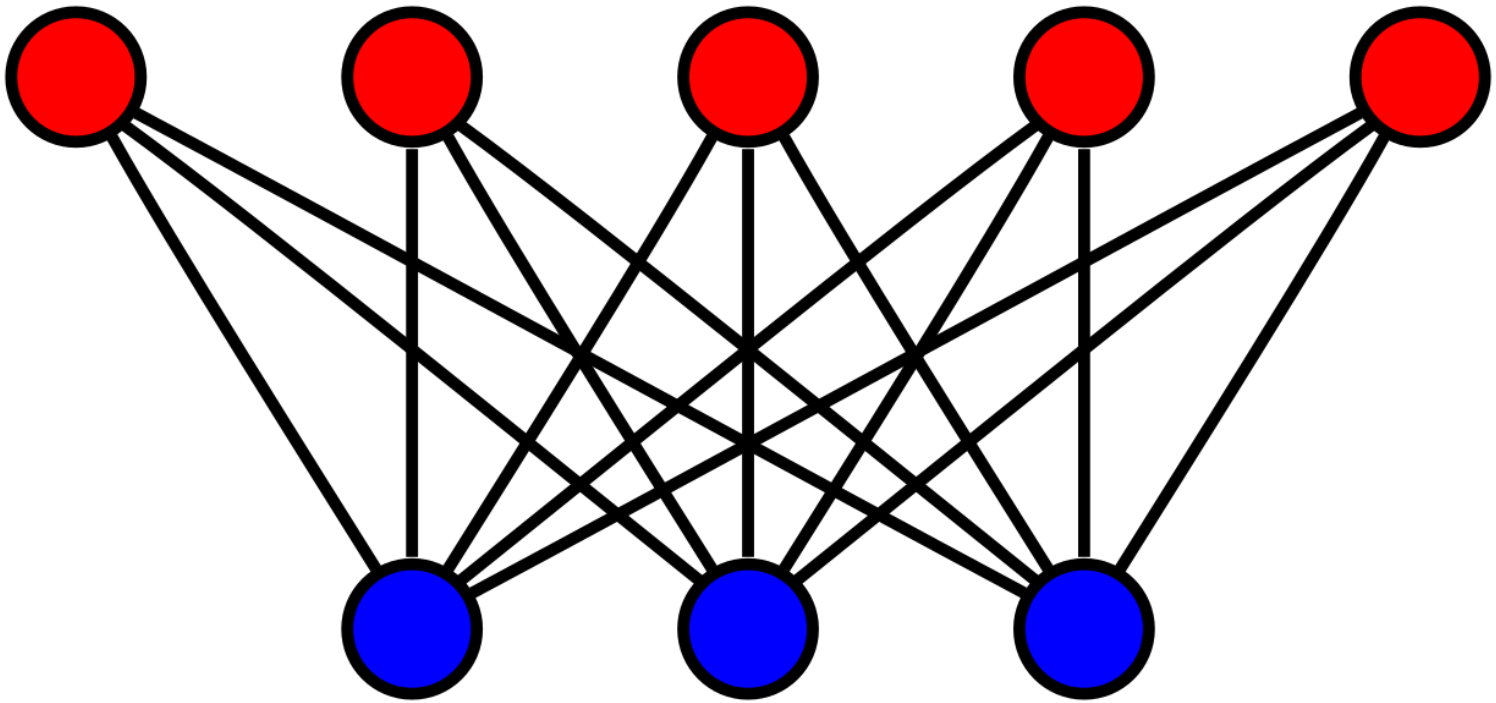
Overview

Basic Concept

So, there are graphs and they have bicliques...

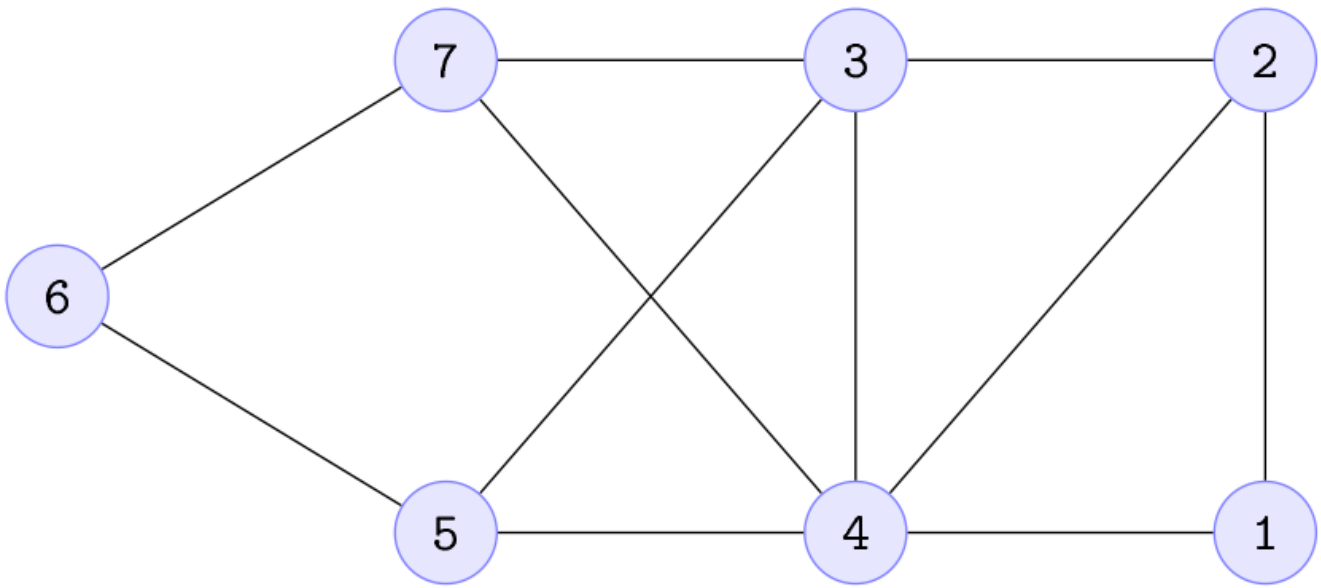


Bicliques

- Maximum Biclique Problem
- Has many applications, including clustering and m

Example

Compute all maximal bicliques of the following graph:



Algorithm LEX

- The most basic algorithm
- Relies on two biclique operations:
  - Absorption
  - Consensus Adjunction
- Main idea:
  - a seed list of bicliques is created
  - absorption applied for each biclique against every other biclique
  - consensus adjunction applied to create new bicliques
- non polynomial complexity in *both* graph size *and* the number of maximum bicliques!

Algorithm LEX

Let $B_1 = (X_1, Y_1)$ and $B_2 = (X_2, Y_2)$ be two bicliques of G. We say that B_1 *absorbs* or *contains* B_2 if $X_2 \subseteq X_1$ and $Y_2 \subseteq Y_1$, or if $X_2 \subseteq Y_1$ and $Y_2 \subseteq X_1$.

If $Y_1 \cap Y_2 \neq \emptyset$, we call $(X_1 \cup X_2, Y_1 \cap Y_2)$ one of the *consensuses* of B_1 and B_2. Similarly, each of those pairs of subsets $(X_1 \cap X_2, Y_1 \cup Y_2)$, $(Y_1 \cup X_2, X_1 \cap Y_2)$, $(X_1 \cup Y_2, Y_1 \cap X_2)$ which define bicliques (i.e., which involve two non-empty subsets) are *consensuses* of B_1 and B_2. In this way, a pair of bicliques may have 0, 1, 2, 3 or 4 consensuses.

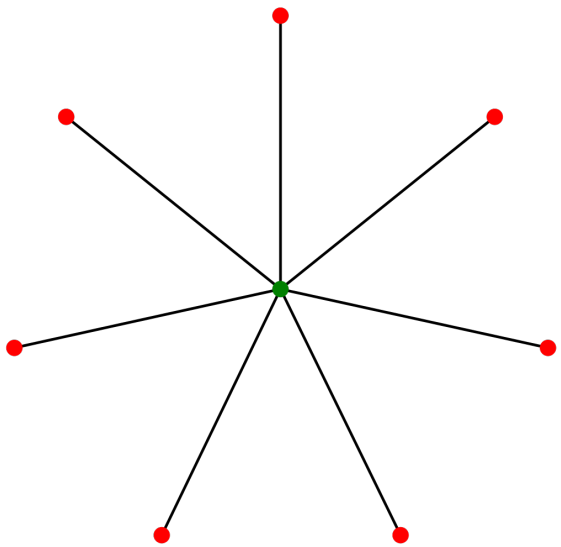
Algorithm MBEA

(not implemented yet)

Algorithm MICA

Modular input consensus algorithm (MICA)

- **Modular consensus**: sequence of operations performed on each pair of bicliques in the repeat loop.
- **Input**: starting with a special input set of maximal bicliques that covers the stars of the graph.



Algorithm MICA

The algorithm has two steps (n is the number of vertices):

- **Start** with a list C_0 of at most n maximal bicliques that covers the stars of the graph. Let $C := C_0$.
- **Repeat**: For every pair of distinct bicliques $B_1 \in C_0$ and $B_2 \in C$, if B_1 and B_2 have a consensus B_3 which is not absorbed by any member of C, then extend B_3 to a maximal biclique B_4, and add B_4 to C.

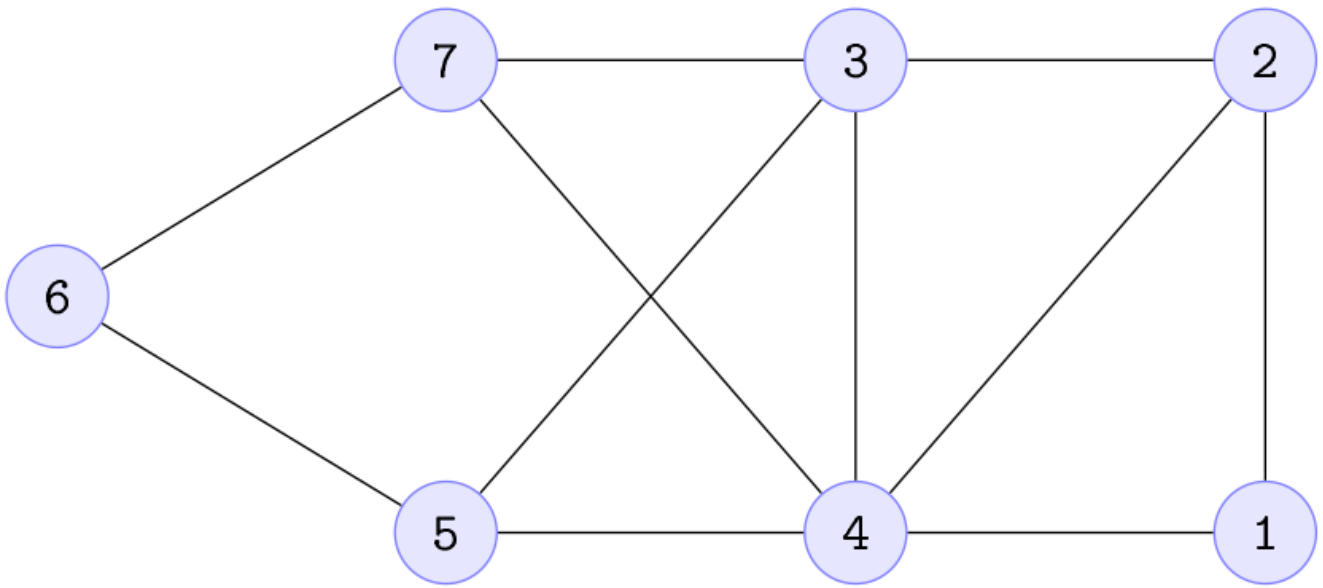
Algorithm MICA

Some subtantial improvements for implementation:

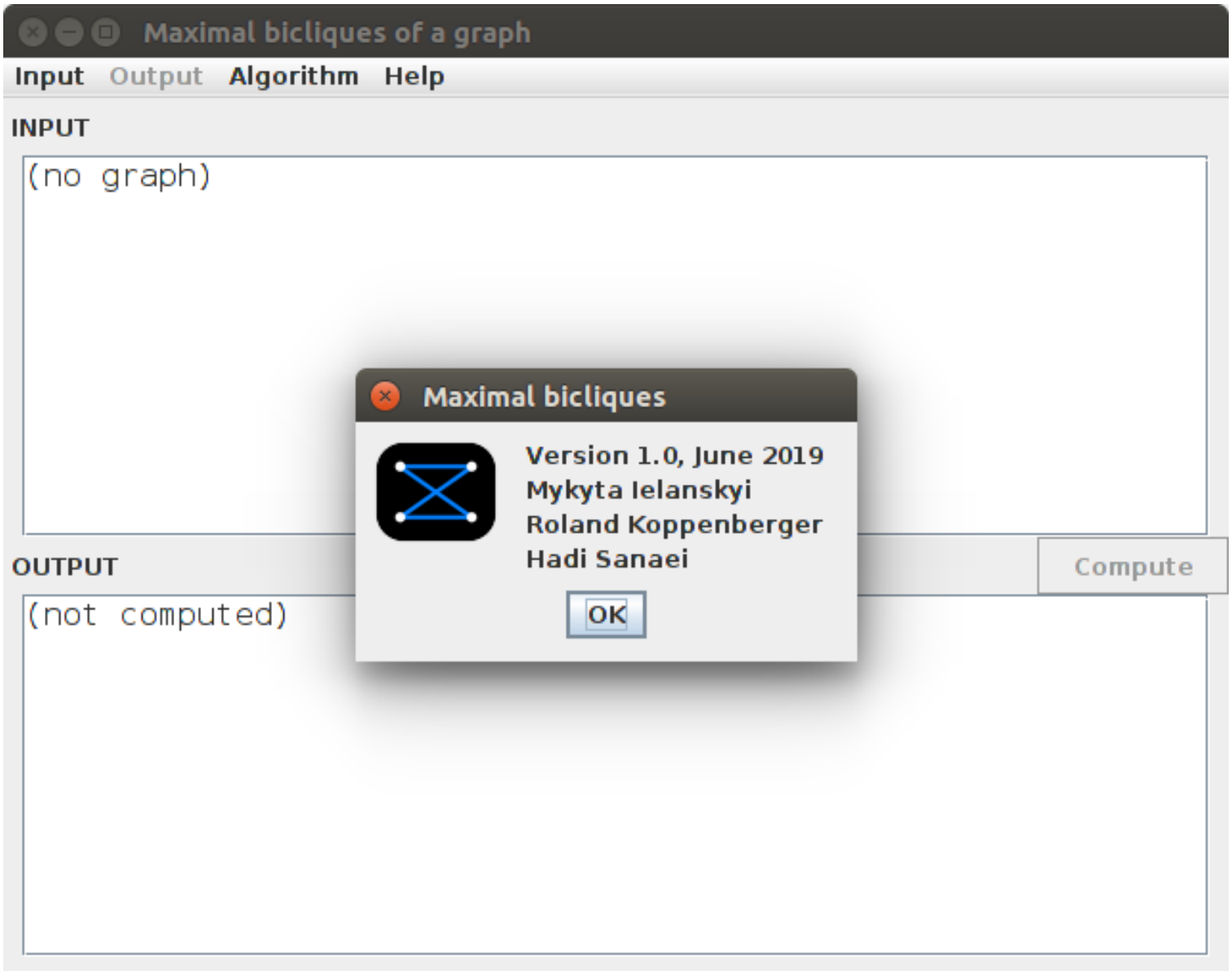
- Many consensus adjunction operations produce the same output.
- $X \subset \mathit{Vertices}$: $\Gamma(X) :=$ set of vertices adjacent to each vertex in X. For all $X \subset \mathit{Vertices}$: $(\Gamma^2(X), \Gamma(X))$ is a maximal biclique.
- Proceed in stages, consider only newly added bicliques in each stage.
- Time complexity: incrementally polynomial, i.e. time for computation of next new maximal biclique is bounded polynomially in number of vertices and number of maximal bicliques computed so far.

Example

Again: *Compute all maximal bicliques of the following graph:*



GUI


Maximal bicliques of a graph
Input
Output
Algorithm
Help
INPUT
(no graph)
OUTPUT
(not computed)
Compute
Maximal bicliques
Version 1.0, June 2019
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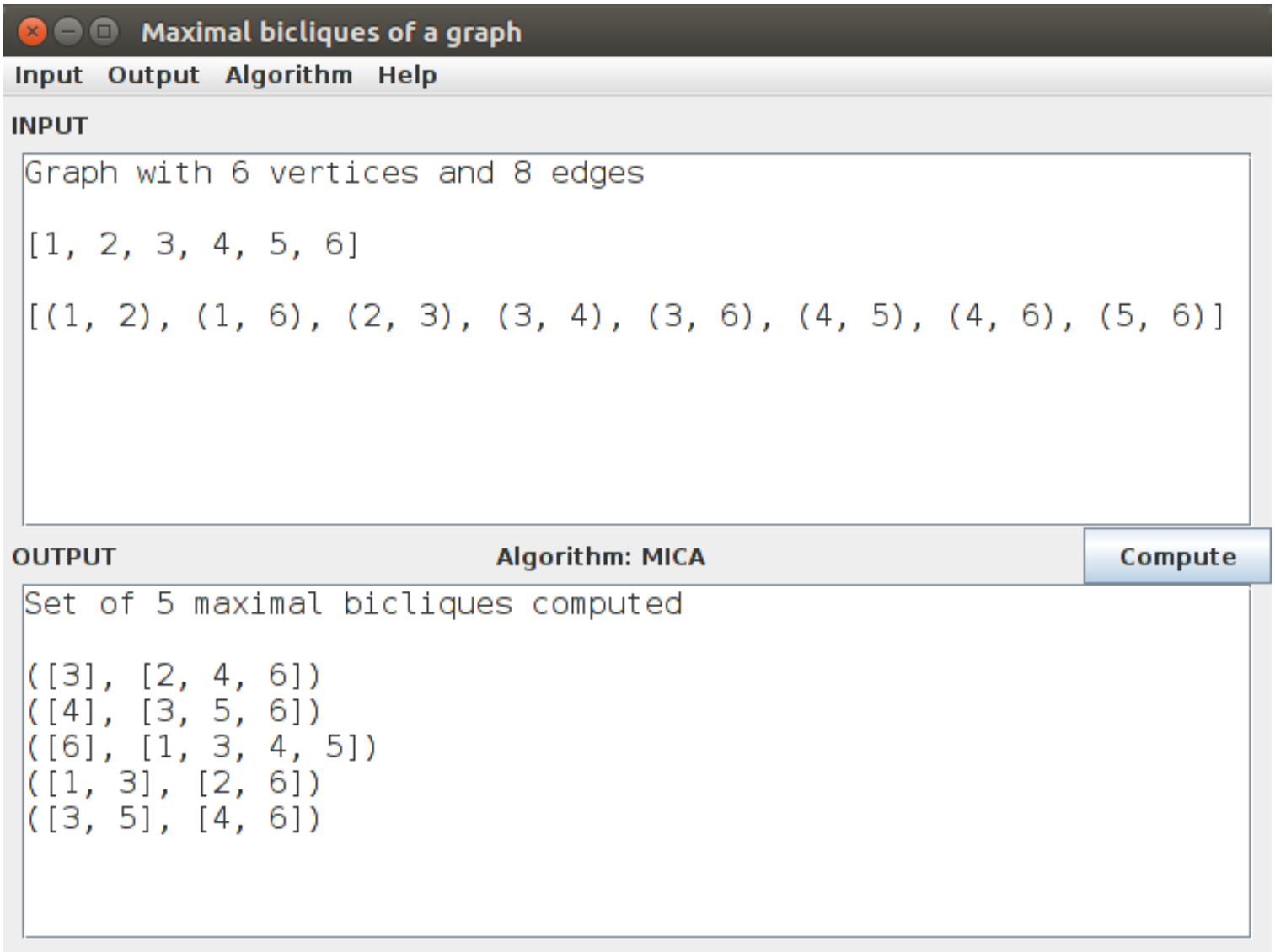
OK

GUI

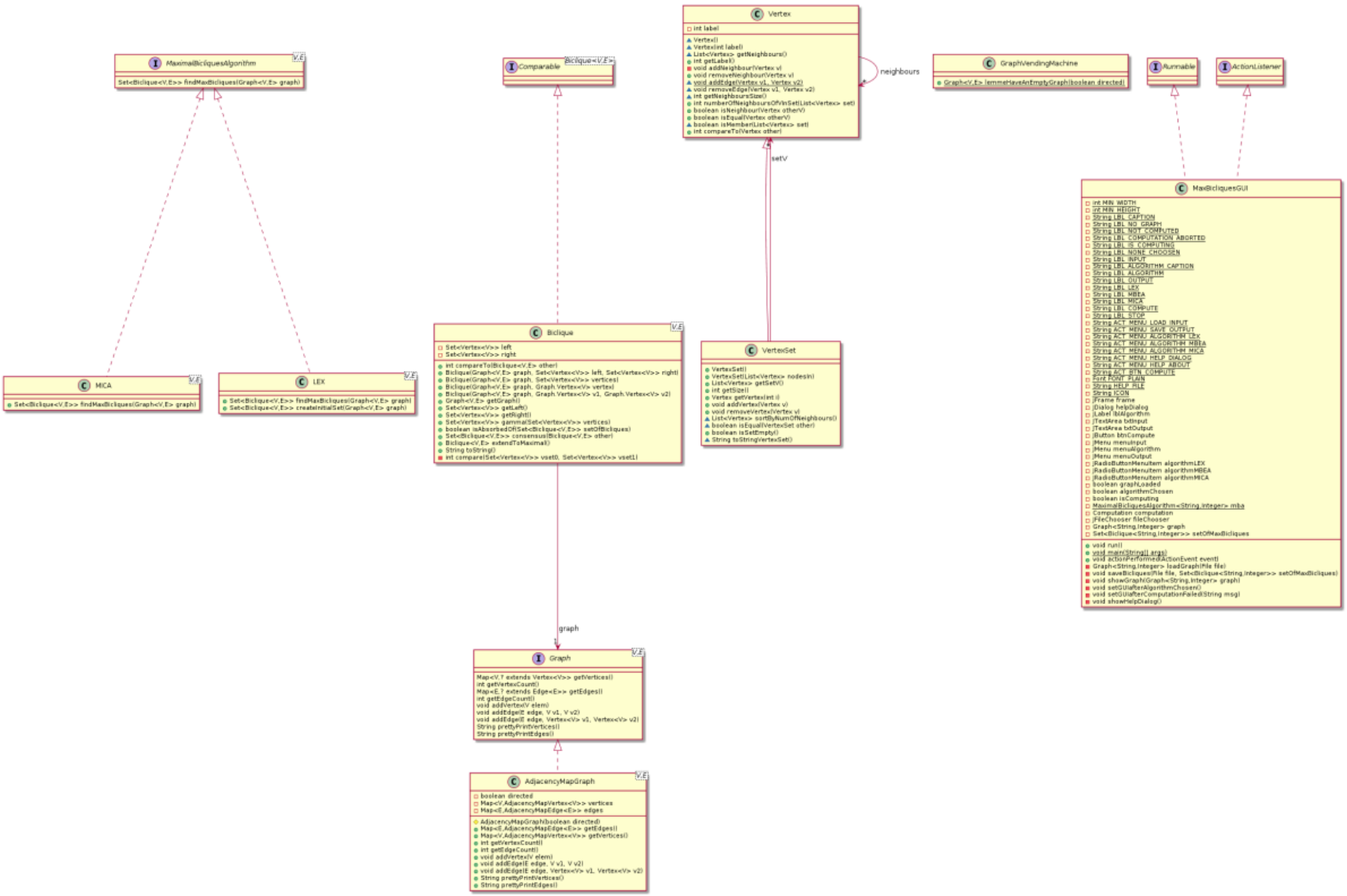
What can one do with the graphical user interface?

- Load a graph from a text file.
- Select an algorithm for computation.
- Compute the set of maximal bicliques.
- Save the result of the computation in a text file.

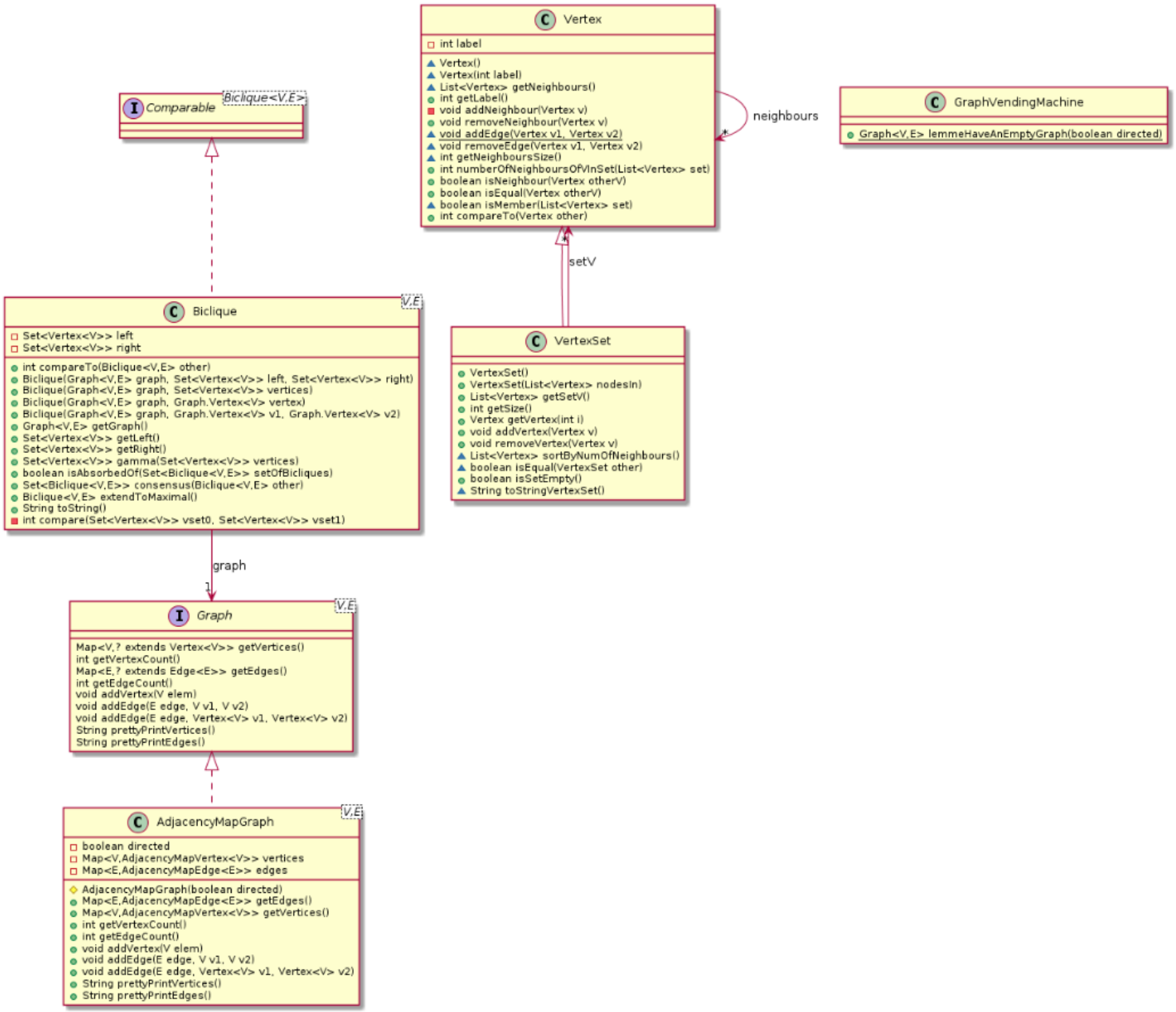
GUI


Maximal bicliques of a graph
Input Output Algorithm Help
INPUT
Graph with 6 vertices and 8 edges
[1, 2, 3, 4, 5, 6]
[(1, 2), (1, 6), (2, 3), (3, 4), (3, 6), (4, 5), (4, 6), (5, 6)]
OUTPUT
Algorithm: MICA
Compute
Set of 5 maximal bicliques computed
([3], [2, 4, 6])
([4], [3, 5, 6])
([6], [1, 3, 4, 5])
([1, 3], [2, 6])
([3, 5], [4, 6])

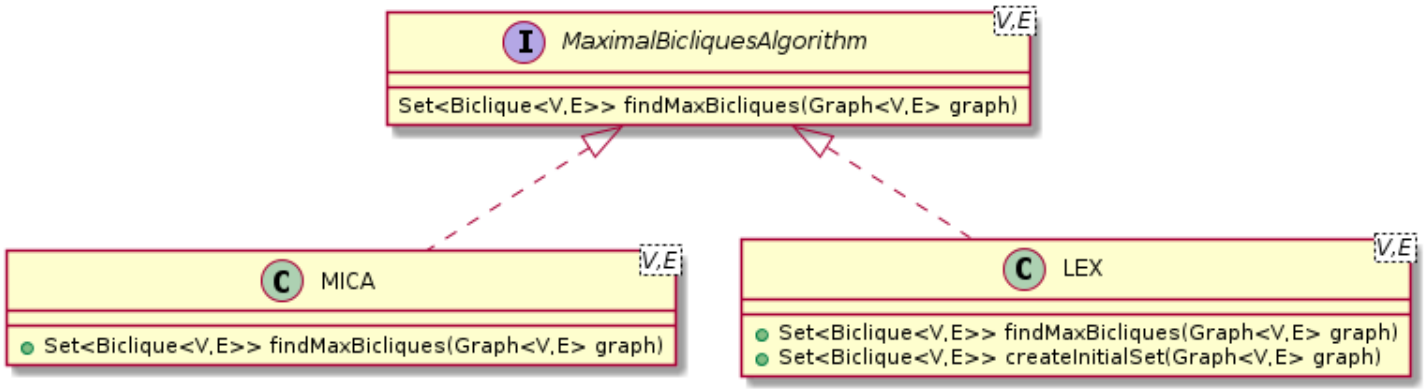
package bicliques



package bicliques.graphs



package bicliques.algorithms



package bicliques.ui

Practical demonstration

We conclude with some practical computations.