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Transition Flight Modeling and Control of a Novel Tilt Tri-Rotor UAV*

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***Abstract*— A novel tilt tri-rotor unmanned aerial vehicle (UAV) which combines the vertical flight capability of a helicopter and forward flight performance of an airplane is proposed. The two front rotors are able to tilt from the vertical to the horizontal position, however, the third rotor is fixed in the aft fuselage with a small angle. The nonlinear dynamic model of tilt tri-rotor UAV is given by the equations of Newton-Euler. By banlancing flight conditions in transition mode where there are redundant controls, the transition corridor is obtained. To resolve the attitude control problem of tilt tri-rotor UAV, we proposed an improved control allocation based on the rotational speeds of three propellers and the tilt angles of two front rotors. Besides, the structure of PID controller including inter loop and outer loop is developpted. Simulation results during full envelope flight are presented, and the success of the modeling carried out as well as the control strategy designed are demonstrated.**

***Index Terms*— tilt tri-rotor UAV, dynamic model, transition corridor, control allocation.**

I. Introduction

Unmanned aerial vehicle (UAV) is an aircraft that can carry out mission without an onboard human operator. A tilt tri-rotor UAV have the ability to transition between three flight modes mainly including helicopter mode, transition mode and airplane mode [1]. In helicopter mode, the tilt tri-rotor UAV is able to vertical takeoff and landing (VTOL), so that there is no need for long runway to takeoff. In airplane mode, it has the ability to fly at high speed and efficiency. The transition mode is a transitional phase between helicopter and airplane. Owing to these three modes, tilt tri-rotor UAV enjoys many advantages compared with conventional aircraft, such as high mobility, long endurance and no site limitation. In civilian fields, it can be used in traffic monitoring, pollution control and forest fire detection, it is worth to mention that the application of tilt tri-rotor UAV to logistics sector performs an impressive role in solving the problem of terminal transport. In military applications, especially for dangerous mission, tilt tri-rotor UAV contributes to cut down risk for soldiers because no one actually has to be on board piloting it [2].

With the development of UAV, tilt tri-rotor UAV with the fuction of VTOL has attracted the attention of many research institutions from all over the world. Israel Aircraft Industries (IAI) has developed a tilt tri-rotor UAV called panther, but all related technology has never been made public. Recently, several research institutions have worked on design and control of tilt tri-rotor UAV, and have obtained some achievements. Papachristos et al. designed a tilt tri-rotor UAV which three rotors are able to tilt, and proposed an explicit model predictive control scheme [3-5]. Ozdemir et al. designed a tilt tri-rototr UAV named TURAC that has a main lift fan in the center of fuselage, a mathematical model is obtained based on CFD analysis, and many flight tests have been carried out [6,7]. Onen et al. have designed linear quadratic regulators and linear quadratic tracking controllers to control the attitude of tilt tri-rotor UAV [8]. Seo et al. proposed a tri-tilt ducted fan vehicle, and a robust quaternion feedback regulator is also designed to control attitude [9]. However, there are still few related studies in china.

In order to conduct the related research, a tilt tri-rotor UAV is designed, the prototype of it has been built and completely developed at Unmanned System laboratory, it consists of fuselage, airfoil, three propellers, horizontal stabilizer, vertical stabilizer and tilting mechanism, et al. It has total weight of about 6 kg and the max payload weight of about 1 kg, it sholuld be noted that the aircraft is powered by lithium battery. As shown in Fig. 1.

Fig. 1. Prototype of tilt tri-rotor UAV.

In this manuscript, we focus on the attitude control of aircraft in transition mode. Specifically, we proposed a tilt tri-rotor UAV with new configuration which has three brushless motors, two of them can tilt from $-90^{\circ} \sim 30^{\circ}$, and the third rotor is fixed with a small angle. The new platform has a better stability. An accurate mathematical model is obtained based on the Newton-Euler dynamic equations as well as parameter identification. In transition mode, both aerodynamic and thrust forces act on the aircraft body are very complex. To solve the problem of attitude control, a novel control allocation is developed [10]. The transition corridor is important for hover-to-cruise flight, it is acquired by balancing flight conditions. The main contribution of this work lies in the proposed novel strategy for the control of the UAV's attitude in transition mode.

The rest of the paper is organised as follows. In section II, the prototype and mathematical model of tilt tri-rotor UAV are presented. The trasition corridor, control strategy and design of controller are described in section III. Section IV shows the simulation results during the complete flight process including three modes. Finally, concluding remarks are made in section V.

II. Mathematical Model

This section briefly describes the dynamic model of tilt tri-rotor UAV in transition mode, the key Parameters are acquired based on some physical measurements and parameter identification.

A. Description of the Tilt tri-rotor UAV

The tilt tri-rotor UAV has three flight modes: helicopter mode, transition mode and airplane mode. It is noted that the rear rotor is fixed with a small angle which will be explained next. In the helicopter mode, the attitude is controlled by five inputs including three rotational speeds and two tilt angles. The roll motion is achieved with the difference of thrust between two front rotors, the right rotor and the rear rotor rotate in counterclockwise direction while the left rotates in opposite direction. The rear rotor compensates the moment generated by two front rotors to stabilize the pitch angle. The yaw moment is created with the difference of tilt angle between two front rotors. During transition mode, the two main rotors tilt gradually in the forward direction, causing a gradual increase in forward speed, meanwhile, the rear rotor reduces its rotational speed with the increase of airspeed. As the speed increase, the tilt angles will reach -90°, and the axis of both front rotors will concide with the axis of the fuselage, besides, the rear rotor will stop completely. If the speed is enough for aircraft to counteract gravity, the tilt tri-rotor UAV is going to turn into airplane mode and fly like a conventional aircraft, elevator and aileron are used to control attitude. Note that the tilt angles of both front rotors are equal in the transition mode, and the platform has no rudder.

B. Equations of Motion

In this part, we focus on deriving 6 degree of freedom(DOF) nonlinear mathematical model of the tilt tri-rotor UAV. The general equations of motion for the aircraft are obtained based on Newton-Euler formulation. The schematic diagram is shown in Fig. 2.

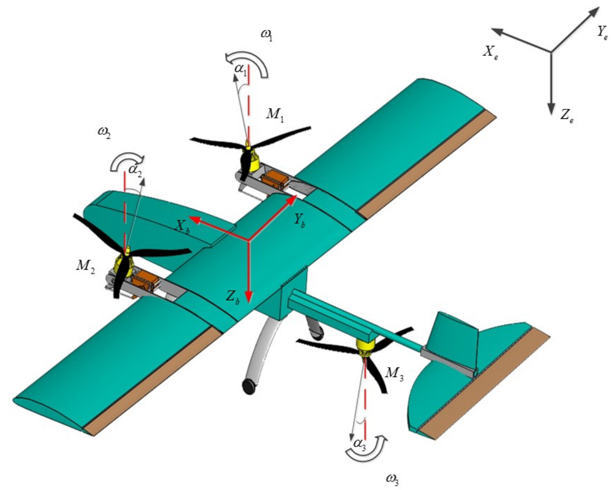

Fig. 2. Earth and body axis systems.

From the design of tilt tri-rotor UAV described above, the third rotor which is behind the centre of gravity of the aircraft is able to compensate rotation torque created by itself with a fixed setting angle. It is obvious that the small fixed setting angle of rear rotor contribute to improve the stability of the whole aircraft. The fixed setting angle can be calculated as shown below.

$$b\omega_3^2 \sin\alpha_3 r3x + d\omega_3^2 \cos\alpha_3 = 0 \tag{1}$$

Therefore

$$\alpha_3 = \arctan(-\frac{d}{br3x}) \tag{2}$$

where b and d are pull coefficient and torque coefficient of the rotor, the rear rotor rotational speed is ω_3, and α_3 represents the small fixed setting angle, $(r3x, r3y, r3z)$ represents the coordinate value of rear rotor in body frame. Rotor is a most important part of tilt tri-rotor UAV, the model of rotor is given by [11]:

$$\begin{cases} F = b\omega^2 \\ M = d\omega^2 \end{cases} \tag{3}$$

where F and M are thrust force and reactive moment opposite to the direction of rotation, from the equations above, b and d can be obtained by the least square identification method.

Thanks to the forces and moments act on the body are quite complex, its necessary to make a deep analysis. In transition mode, the forces and torques are given by:

$$\overrightarrow{F}=\overrightarrow{F_1}+\overrightarrow{F_2}+\overrightarrow{F_3}+\overrightarrow{G}+\overrightarrow{F_A} \tag{4}$$

$$\overrightarrow{M}=\overrightarrow{M_{1b}}+\overrightarrow{M_{2b}}+\overrightarrow{M_{3b}}+\overrightarrow{M_{G1}}+\overrightarrow{M_{G2}}+\overrightarrow{M_{G3}}+\overrightarrow{M_A} \tag{5}$$

in above equations, $\overrightarrow{F_1},\overrightarrow{F_2},\overrightarrow{F_3}$ are thrust generated by three rotors, $\overrightarrow{M_{1b}},\overrightarrow{M_{2b}},\overrightarrow{M_{3b}}$ not only contains the moments produced by thrust, but also includes reactive moments opposite to the direction of rotation. Most important all, $\overrightarrow{M_{G1}},\overrightarrow{M_{G2}},\overrightarrow{M_{G3}}$ are gyroscopic moments, these moments are generated when the front two motors tilt and platform's attitude change. $\overrightarrow{G}$ is gravity in body frame. $\overrightarrow{F_A},\overrightarrow{M_A}$ are created by aerodynamic that will be detail explained in next section.

Through the above analysis, the six degrees of freedom equations are proposed as follows :

$$\begin{cases}\dot{u}=vr-qw-g\sin\theta+\dfrac{F_x}{m}\\ \dot{v}=-ur+pw+g\cos\theta\sin\Phi+\dfrac{F_y}{m}\\ \dot{w}=qu-pv+g\cos\theta\cos\Phi+\dfrac{F_z}{m}\end{cases} \tag{6}$$

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix}=R_{BET}\begin{bmatrix}u\\ v\\ w\end{bmatrix} \tag{7}$$

$$\begin{cases}\dot{p}=(c_1r+c_2p)q+c_3M_x+c_4M_z\\ \dot{q}=c_5pr-c_6(p^2-r^2)+c_7M_y\\ \dot{r}=(c_8p-c_2r)q+c_4M_x+c_9M_z\\ c_1=\dfrac{(I_y-I_z)I_z-I_{xz}^2}{\Sigma},c_2=\dfrac{(I_x-I_y+I_z)I_{xz}}{\Sigma},\\ c_3=\dfrac{I_z}{\Sigma},c_4=\dfrac{I_{xz}}{\Sigma},c_5=\dfrac{I_z-I_x}{I_y},c_6=\dfrac{I_{xz}}{I_y},\\ c_7=\dfrac{1}{I_y},c_8=\dfrac{(I_x-I_y)I_x+I_{xz}^2}{\Sigma},c_9=\dfrac{I_x}{\Sigma},\\ \Sigma=I_xI_z-I_{xz}^2\end{cases} \tag{8}$$

$$\begin{bmatrix}\dot{\Phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix}=R_{BER}\begin{bmatrix}p\\ q\\ r\end{bmatrix} \tag{9}$$

where $[x,y,z]$ is the vector of the inertial position that defined on the earth frame of reference. $[u,v,w]$ is the vector of the linear velocity which is defined on the body frame of reference. p is roll angular velocity, q is pitch angular velocity, r is yaw angular velocity, these parameters are defined on the body frame of reference. Inertia moments are denoted using by I_x,I_y,I_z and I_{xz}. R_{BET} represents translation matrix, rotation matrix is expressed by R_{BER}.

C. Parameter Identification

Although the dynamic model is already built, but there are many parameters need to be identified. Because of the aerodynamic is an improtant part of the model, the aerodynamic force and moment coefficients for rigid aircraft must be obtained. The method of least-squares estimate is used to get aerodynamic coefficients [12]. To simplify the model and realize the longitudinal attitude stable control in transition mode, take the lift, drag and pitch moment into consideration. Estimation of aerodynamic parameters from flight test data requires a mathematical model of the aircraft. The model includes both the equations of motion and the equations for aerodynamic forces and moments, known as the aerodynamic model equations.

$$\begin{cases}L=\dfrac{1}{2}\rho V^2SC_L\\ D=\dfrac{1}{2}\rho V^2SC_D\\ m=\dfrac{1}{2}\rho V^2ScC_M\end{cases} \tag{10}$$

where

$$\begin{cases}C_L=C_{L0}+C_L^{\alpha}\alpha+C_L^{\delta_e}\delta_e\\ C_D=C_{D0}+C_D^{\alpha}\alpha+C_D^{\delta_e}\delta_e\\ C_M=C_{M0}+C_M^{\alpha}\alpha+C_M^{\delta_e}\delta_e\end{cases} \tag{11}$$

where L,D are aerodynamic force, M is pitch moment, C_L,C_D,C_M represent aerodynamic derivatives, α is used to represent the angle of attack, δ_e is defined as the elevator deflection, S is the wing reference area of the aircraft and c is the mean aerodynamic chord of the wing. Aerodynamic derivatives are obtained by the method of least-squares estimate based on flight experiment. The parameters of the platform are shown in table 1.

TABLE I
UAV PARAMETERS

Parameters	Value	Parameters	Value
m/kg	3.42	C_{L0}	0.3892
b	1.97×10^{-5}	C_L^{α}	2.6310
d	4.0×10^{-7}	$C_L^{\delta_e}$	0.1365
S/m^2	0.408	C_{D0}	0.1417
c/m	0.227	C_D^{α}	0.2477
$I_x/(kg\cdot m^2)$	0.22	C_{M0}	0.04142
$I_y/(kg\cdot m^2)$	0.23	C_M^{α}	-0.4935
$I_z/(kg\cdot m^2)$	0.48	$C_M^{\delta_e}$	-0.3719

III. Flight Control

In setion II, we have obtained mathematical model which is the foundation of flight control. Due to the redundant control exist in transition mode, it is difficult to achieve stable control. The design of the control strategy and transition corridor are therefore key to achieving safe flight. This section sunmmarizes the design of transition corridor and control strategy.

A. Transition Corridor

In terms of the special aircraft, transition corridor is the sticking point in designing a control system. The tilt angle schedule against airspeed in transition mode is designed based on the trim of flight conditions. Before deriving the trim of equations, the two common simplifying assumptions should be proposed. First, the aileron and elevator has no deflectin, it is necessary to explain that the platform has no rudder. Second, the angle of attack and sideslip are zero. To set up the function relation between the tilt angle and airspeed, two parts of equations are proposed as follows:

$$\begin{cases} (b\omega_2^2 c\alpha_2 - b\omega_1^2 c\alpha_1)r1y - d\omega_2^2 s\alpha_2 + d\omega_1^2 s\alpha_1 = 0 \\ -b\omega_1^2 s\alpha_1 - b\omega_2^2 s\alpha_2 - D = 0 \\ (b\omega_1^2 c\alpha_1 + b\omega_2^2 c\alpha_2)r1x + b\omega_3^2 c\alpha_3 r3x + M = 0 \end{cases} \tag{12}$$

$$\begin{cases} b\omega_1^2 c\alpha_1 + b\omega_2^2 c\alpha_2 + b\omega_3^2 c\alpha_3 + L - mg = 0 \\ \alpha_1 = \alpha_2 \end{cases} \tag{13}$$

here, c and s is symbolize cosine and sine function-s,respectively. Based on the equations above, transition corridor can be obtained in the help of the method of least square. Fig. 3 shows the result of curve fitting.

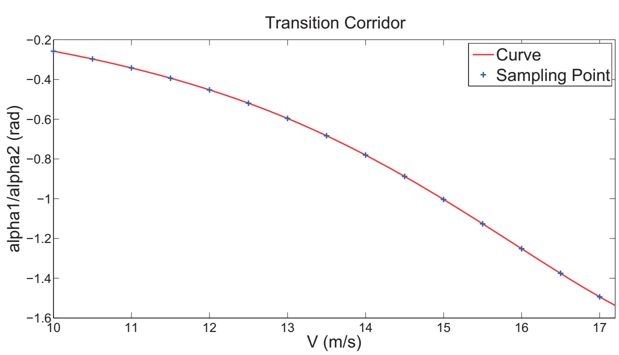

Fig. 3. Transition corridor.

Take account of the aerodynamic effect, the range of airspeed is set to 10 $\sim$ 17.2 m/s. When the airspeed is more than 10 m/s, the elevator can be used to control pitch, in addition, the gravity can be counteracted by lift with the airspeed of 17.2m/s.

B. Control Strategy

Transition corridor is the foundation of control. In the next part, we proposed control allocation based on transition corridor. The strategy of control in transition mode is the most important part for stable flight. It is obvious that the altitude and attitude are the key factors for stable transition. So that the aim of control is to stablize the altitude and attitude in transition.

The whole flight is divided into three stages based on flight mode. In the first stage, tilt tri-rotor UAV takes off and then accelerates to 10 m/s in helicopter mode. After the airspeed achieves 10 m/s, the aircraft enters second stage, as the speed is increased, the wings start generating more lift force, while the need for vertical thrust is decreased, the need for horizontal thrust is increased. Thus, the two front rotors shall be tilted forward, and it contributes to increase speed. It may be observed from Fig. 3 that the tilt angles will reach $-90°$ when the airspeed is 17.2 m/s. Once the the axis of both front rotors concide with the axis of the fuselage, the aircraft enters third stage and then flies horizontally like a twin-engine fixed wing aircraft. The whole control strategy is shown in Fig. 4.

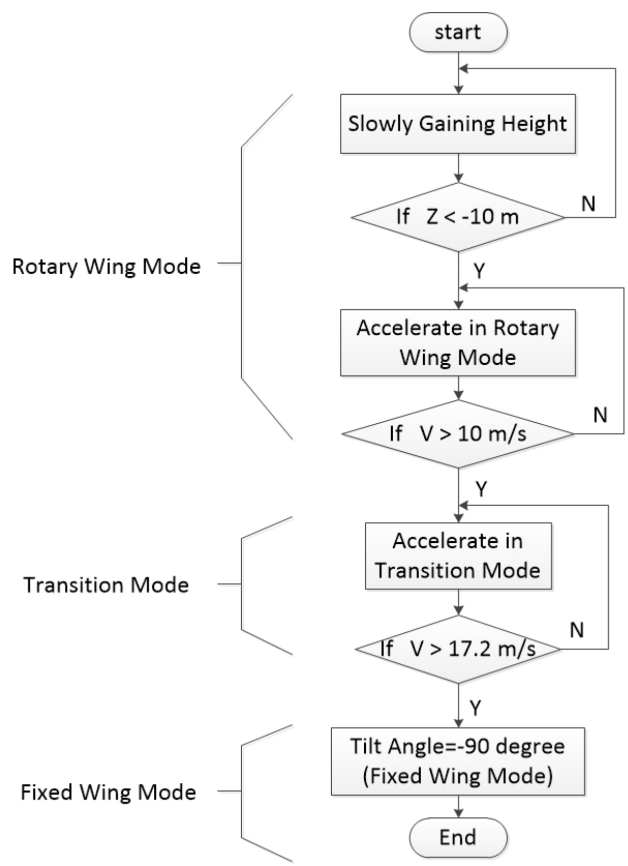

Fig. 4. Control strategy.

The aircraft dynamics is an over-actuated system, thus, the actuator inputs requires an appropriate allocation. The forces and moments generated by control allocation are called virtual inputs and the actuator inputs of the system are called actual inputs[13]. In terms of controller, taking airspeed V, altitude Z and Euler angles Φ, θ, ψ as inputs, in response, the virtual torque R, P, N are as outputs. Before propose the equations of control allocation, it should be remembered that all the angles are defined by right hand rule. The allocation strategy contains two parts. For the first part, the rotational angular velocities of three rotors can be determined from equations below:

$$\begin{cases} R = (b\omega_2^2 c\alpha_2 - b\omega_1^2 c\alpha_1)r1y - d\omega_2^2 s\alpha_2 + d\omega_1^2 s\alpha_1 \\ N = -b\omega_1^2 s\alpha_1 - b\omega_2^2 s\alpha_2 - D \\ 0 = (b\omega_1^2 c\alpha_1 + b\omega_2^2 c\alpha_2)r1x + b\omega_3^2 c\alpha_3 r3x \end{cases} \tag{14}$$

For the second part, the pitch of the aircraft is controlled by elevator. The virtual torque P is given by controller, so that the deflection of elevator can be determined by P , as shown in follows:

$$\begin{cases} P = \dfrac{1}{2}\rho V^2 S c C_M \\ C_M = C_{M0} + C_M^{\alpha}\alpha + C_M^{\delta_e}\delta_e \end{cases} \tag{15}$$

where α is the angle of attack, besides, the deflection of elevator is represented by δ_e. So far, the control allocation has been described in detail. The control allocation simplified the complex aircraft dynamics to a single body with force and moment as its inputs. This simplification contributes to design a simple but efficient PID controller for the aircraft. The general structure of the controller consist of two parts inner loop and outer loop. The complete closed-loop control system is depicted in Fig. 5.

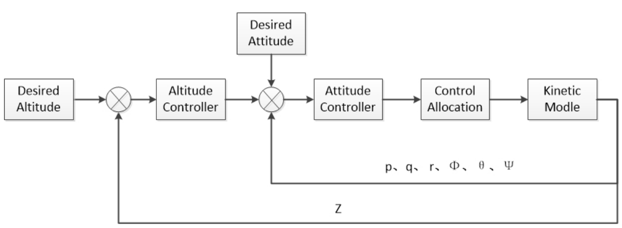

Fig. 5. Aircraft control loop.

The inner loop is used to control attitude and the outer loop is used for altitude control. The difference between feedback value and desired input is calculated as the control input. The attitude controller computes the desired virtual inputs, so the desired actual inputs can be generated by control allocation, a closed control system is developed through a series of calculations. In this section, the transition corridor and control allocatin are developed, besides, the structure of PID controller is proposed, in order to verify the rationality and validity of the algorithm, a complete simulation test should be carried out.

IV. Simulation Research

A dynamic model is built in section II, moreover, control allocation and transition corridor are given by section III, so that, a simulation need to be completed in this section. Using the nonlinear equations of motion derived earlier and the allocation strategy above, a mathematical model is created and implemented on MATLAB Simulink. To simplify the design of controller and improve the stability of flight, two control modules are used to control the aircraft in different flight stages, As shown in Fig. 6. PID controller is designed to achieve stable flight, simulation results show that the controller is of good robustness, and its control effectiveness is satisfactory, table 2 represents all attitude and position PID controller's gain values.

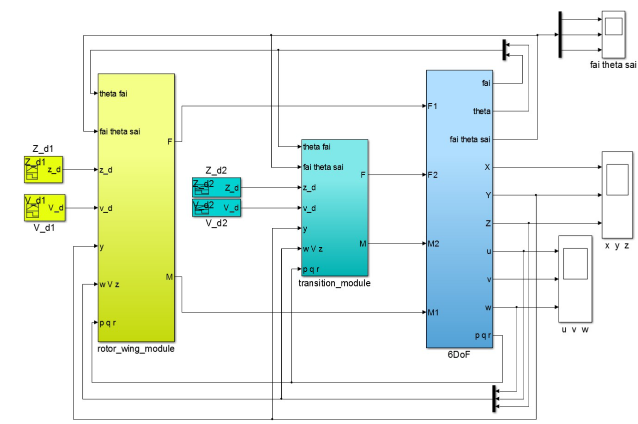

Fig. 6. Simulink model of tilt tri-rotor UAV.

TABLE II
PID CONTROLLER'S GAIN VALUES

Controller	Gains	V	Y	Z	Roll	Pitch	Yaw
Helicopter	k_p	0.3	5	3	100	60	8
	k_i	0.1	0.2	0.1	18	0.1	1
	k_d	0.1	2	0.3	0	0.2	0
Transition	k_p	10	0.2	0.08	4	20	0.2
	k_i	0.01	0.02	0.04	0.01	2	0.02
	k_d	0	0.1	0.005	0	0.1	0

The simulation is divided into three stages that corresponding to three flight modes. Simulations are performed on 6-DoF nonlinear mathematical model and complete flight (hover-transition-cruise) of the tilt tri-rotor UAV is simulated for about 50 seconds. It is worth to mentioned that the control is transferred from helicopter module to transition module in 25 seconds. The flight process has been described in Fig. 4. Simulation results are shown in Fig.7, 8, 9 and 10.

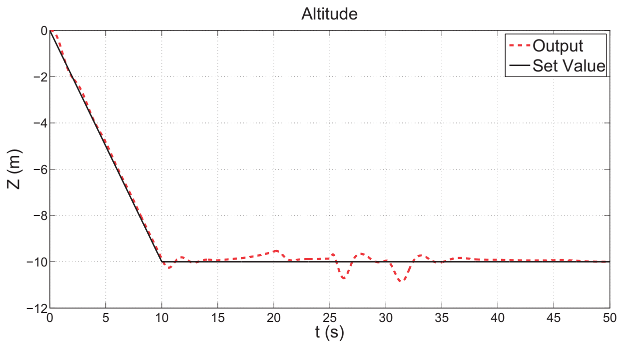

Fig. 7. The change of altitude during the flight simulation.

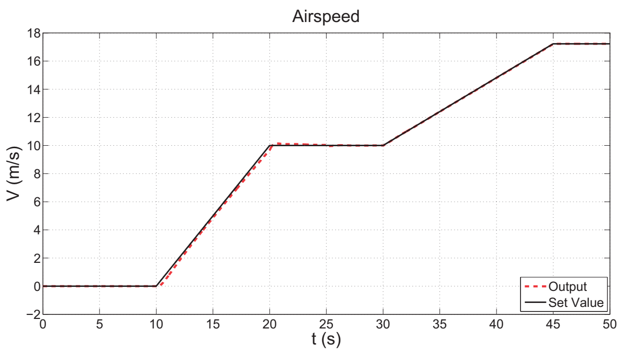

Fig. 8. The change of airspeed during the flight simulation.

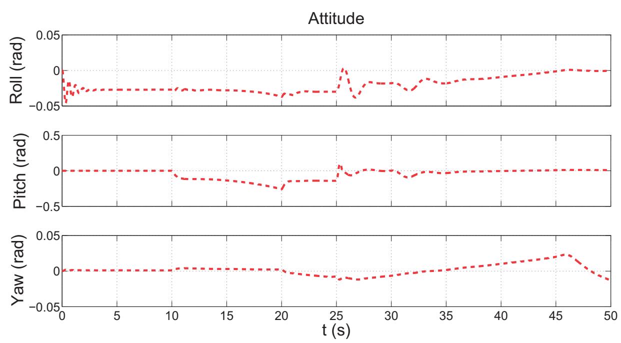

Fig. 9. The change of attitude during the flight simulation.

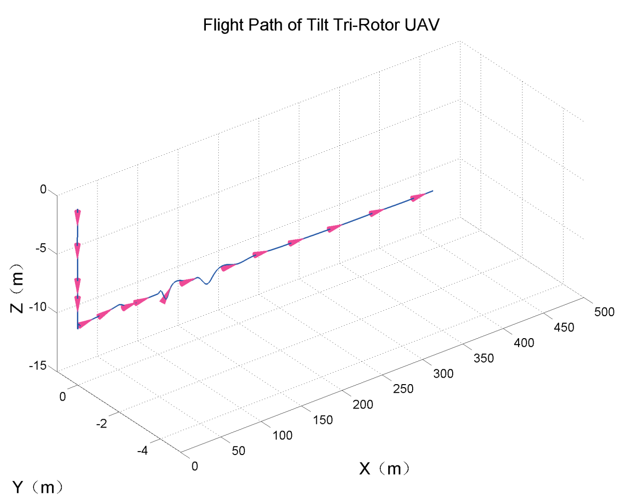

Fig. 10. The flight path of tilt tri-rotor UAV.

Altitude change is shown in Fig. 7, the altitude control shows good performance. Note that the curve has a small fluctuation which is generated by the start of transition. It may be observed from Fig. 8 that the speed tracking is excellent over the entire flight envelope. Fig. 9 shows the change of attitude, the pitch angle is used for accelerating in helicopter mode, so that the pitch angle is negative from 10 to 25 seconds. From the figures above, it is observed that a clear success has been achieved by the utilization of the dynamic allocation and the control strategy.

V. Conclusion

In this study, a nonlinear mathematical model that includes the aerodynamic coefficients of the airframe is developed. Based on this mathematical model, the transition corridor of the aircraft is derived. A novel control allocation for the stable control of aircraft in transition mode is proposed. Finally, the simulation results of complete flight are presented, and confirmed the effectiveness of the proposed transition corridor and control allocation.

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