An Improved Artificial Potential Field Method for Path Planning and Formation Control of the Multi-UAV Systems

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***Abstract*—Path planning and formation control are both challenging and critical issues in robotics, which involve computing an optimal path from the initial position to target while keeping the desired formation. This brief studies the path planning and formation control problem for multiple unmanned aerial vehicles (multi-UAVs) in 3-D constrained space. Considering the local minimum of artificial potential function (APF), an effective improved artificial potential function (IAPF) based path planning approach is proposed for the multi-UAV systems. By introducing a rotating potential field, the UAVs can escape from the common local minimum and oscillations efficiently. Afterwards, by using the leader-follower model, a formation controller based on potential function method is developed to ensure that the follower UAVs keep the desired angles and distances with the leader, and a Lyapunov function is designed to analyze the closed-system stability. Finally, simulation studies under different environmental constraints confirm the efficiency of the proposed approaches in addressing the path planning and formation control issues in 3-D space.**

***Index Terms*—Multi-UAV system, path planning, formation control, artificial potential field.**

I. Introduction

MULTIPLE unmanned aerial vehicles (Multi-UAVs) system refers to multiple UAVs complete tasks such as area search, combat, and environmental data collection [1]. In recent decades, the multi-UAVs systems have attached remarkable attention due to the wide applications [2]. The path planning and the formation control are both challenging and critical issues, and have important roles in the multi-UAVs system [3]–[5]. Specifically, formation control aims to achieve a prescribed geometric shape, by controlling the relative distances and angles of the inter-agents in complex constrained environment [6]. Currently, researchers have done a lot of works on formation control, such as leader follower model [7], virtual structure [8], behavior-based [9], etc. Leader follower model is a popular approach due to its simplicity and stability [10]. In practice, to successfully implement formation control approaches, many issues need to be considered, such as the obstacle and collision avoidance.

Artificial potential field (APF) method is characterized by the simpler controller and less parameters design, and has been widely used for real-time formation control [11], collision and obstacle avoidance [12], and so on. The essence of APF method is to design a potential field, in which the agent is attracted by the target and repelled by obstacles or neighboring agents. Most existing formation control methods merely consider an obstacle-free or simple obstacle environment. Lu *et al.* [13] put forward a potential field based formation control approach for a group of agents, which enables the agents to achieve formation control by regulating their relative positions. Combining the flocking with distance-based shape control, a steepest descent formation control method was proposed for the multi-agent system to achieve formation control and share of a common velocity [14]. Unfortunately, the obstacle avoidance was not considered.

For multi-UAV formation control, the collision and obstacle avoidance are big challenges due to the uncontrollability and complexity. To solve this problem, the APF methods are usually considered since its concise mathematical description and simplicity [15]. However, the goals non-reachable with obstacle nearby (GNRON) and the common situations of local minimum are natural limitations inherent in the APF methods. Wen *et al.* [5] combined the leader-follower formation approach and APF method to address formation control problem with obstacle avoidance. However, they did not consider the local minimum of the potential function in complex environment. To solve the local minimum, considering the agent dimensions, a repulsive potential function was proposed by [16], in which a modified artificial goal-technique and virtual-obstacle method are developed to avoid obstacle cavity. The proposed scheme ensures that the agent escapes from the local minimum in both static and dynamic obstacles environment. By using the well-known

Manuscript received May 19, 2021; revised August 4, 2021; accepted September 12, 2021. Date of publication September 15, 2021; date of current version March 15, 2022. This work was supportd by the National Natural Science Foundation of China under Grant 61720106010. This brief was recommended by Associate Editor J. Wu. *(Corresponding author: Yuanqing Xia.)*

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Color versions of one or more figures in this article are available at https://doi.org/10.1109/TCSII.2021.3112787.

Digital Object Identifier 10.1109/TCSII.2021.3112787

particle swarm optimization (PSO) algorithm, a modified APF approach is presented to solve the local minimum and improve the quality of path planning [17]. However, the previous methods only analyzed the local minimum of single agent, while ignoring the formation control and collision avoidance. Considering simultaneously the local minimum and formation control problems, in our previous work [18], a virtual spring and target point guidance method is proposed to achieve the path planning of multi-agent formation in complex environment.

In this brief, considering the issues of collision and obstacle avoidance, local minimum and formation control, a path planning and formation approach based on IAPF method is put forward for the multi-UAVs system. The innovations are list as follows:

1) On the basis work in [19], we extend the motion situational awareness map (MSAM) to 3-dimensional space, model the irregular obstacles as a regular ball-type obstacle, which can greatly reduce the computational complexity of the APF.
2) Aiming at solving the local minimum and oscillations problems of the APF, a novel rotating potential field around the obstacles is put forward, which is perpendicular to the repulsive potential field from the obstacle to ensure that the resultant force of UAV is not equal to zero.
3) A novel IAPF formation and path planning control approach is developed to ensure that the UAVs calculate an optimal path to the target in the predefined formation. Based on the Laypunov stability theorem, it is shown that all UAVs can converge to the target area, and the resultant force gradually tends to zero.

The structure of this brief is as follows. We describe the preliminaries and problem statement in Section II. In Section III, the formation and path planning control strategy based on an IAPF are proposed. The effectiveness of proposed strategies is illustrated by simulation results in Section IV. This brief is concluded in Section V.

II. Preliminaries and Problem Statement

A. UAV Dynamics

For brevity, we assume that the UAV is a point mass, regardless of the size and the shape of an UAV involving in the formation control [20]. Consider a group of n UAVs, where $q_i(t)$ represents the position of UAV A_i at time t. For simplicity, the dynamics of $A_i (i \in \{1, 2, \ldots, n\})$ is modeled as a particle [21].

$$\begin{cases} \dot{q}_i = v_i \\ \dot{v}_i = u_i \end{cases} \tag{1}$$

where $v_i \in \mathbb{R}$ is velocity and u_i is control input of A_i. It is assumed that the group UAVs is a homogeneous swarm, which is composed of the UAVs with the same characteristics (e.g., sensors, size and interaction ability). The agents are able to estimate their positions and share information with neighbors in the global coordinate system. The environment (spatial constraints) is uniform (time invariant) in the task.

B. Graph Theory

This section briefly introduces some basic concepts of graph theory. For a group of N UAVs in $\mathbb{R}^3$, with the location of i-th UAV denoted by q_i. Denote d_{ij} and d_{des} as the real distance and desired distance between UAVs A_i and A_j, respectively. The information exchange among UAVs can be modeled by a directed graph $\mathcal{G} = (\nu, \varepsilon)$, where $\nu = \{1, 2, \ldots, n\}$ is a list of vertices, one corresponding to each UAV, and $\varepsilon \subseteq \{(i, j) : i, j \in \nu, i \neq j\}$ is a list of agent pairs (edges of the graph). An edge $(i, j) \in \varepsilon$ in the graph means UAVs A_i and A_j can exchange information. For any pair of distinct agents, if there exists a directed path, then graph $\mathcal{G}$ is said to be connected [22].

The adjacency matrix is given by $I = [a_{ij}] \in \mathbb{R}^{n\times n}$ with $a_{ij} \neq 0 \Leftrightarrow (i, j) \in \varepsilon$. For graph $\mathcal{G}$, $a_{ij} = a_{ji}$, and the adjacency matrix I is symmetric. The set of neighbors of A_i is defined by

$$N_i = \left\{j \in \nu : a_{ij} \neq 0\right\} = \{j \in \nu : (i, j) \in \varepsilon\} \tag{2}$$

C. Leader-Follower Formation Model

The leader-follower model is widely used for multi-UAV formation control [10]. As shown in Fig. 1, A_l is assigned as the leader UAV, which has the influence on the motion of the follower UAVs $A_{fi} (i = 1, 2, \ldots, n-1)$. d_{li} is defined as the desired distance between A_l and A_{fi}. We define $q = (x, y, z, \theta, \delta)^T$ as the attitude of each UAV in 3-D space. θ is the deflection angle from the orientation of the UAV to x axis, and δ is the pitch angle from the orientation of the UAV to z axis. We have the following equation

$$\begin{bmatrix} x_{fi}(t) \\ y_{fi}(t) \\ z_{fi}(t) \end{bmatrix} = \begin{bmatrix} x_l(t) - d_{li}\cos\theta_{lfi}(t) \\ y_l(t) - d_{li}\sin\theta_{lfi}(t) \\ z_l(t) - d_{li}\sin\delta_{lfi}(t) \end{bmatrix} \tag{3}$$

where θ_{lfi} represents the bearing angle of from A_l and A_{fi} to x axis, δ_{lfi} denotes the bearing angle of from A_l and A_{fi} to z axis.

Then the kinematic of the follower are expressed as

$$\begin{bmatrix} \dot{x}_{fi} \\ \dot{y}_{fi} \\ \dot{z}_{fi} \\ \dot{\theta}_{fi} \\ \dot{\delta}_{fi} \end{bmatrix} = \begin{bmatrix} v_{fi}\cos\theta_{fi} \\ v_{fi}\sin\theta_{fi} \\ v_{fi}\sin\delta_{fi} \\ w_{fi}^{\theta} \\ w_{fi}^{\delta} \end{bmatrix} \tag{4}$$

where $\theta_{fi} = \arctan(y_{fi}(t) - y_{fi}(t-1))/(x_{fi}(t) - x_{fi}(t-1))$, $\delta_{fi} = \arctan(z_{fi}(t) - z_{fi}(t-1))/(x_{fi}(t) - x_{fi}(t-1))$.

III. The Proposed Method

A. Multi-UAV Formation Control

Based on the leader-follower model, a potential function based formation control law is developed to achieve the desired formation for a group of UAVs. The follower UAVs are supposed to keep the desired distances and angles with the leader based on the formation control law.

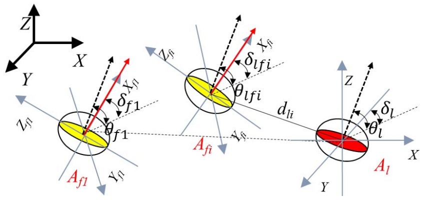


Fig. 1. The leader-follower model.

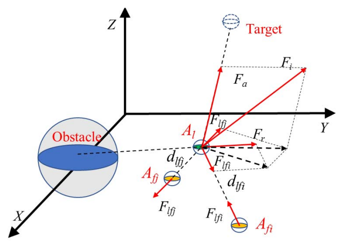


Fig. 2. The force analysis of the UAVs.

We define that U_{ij} is the potential function between UAVs A_i and A_j

$$U_{ij}(q_i, q_j) = k_a\left(f_{ij}(q_i, q_j) + \frac{d_{des}^2}{f_{ij}(q_i, q_j)}\right) \tag{5}$$

where k_a is a positive coefficient, $f_{ij}(q_i, q_j) = \|q_i - q_j\|$ denotes the distance of A_i and A_j.

The potential field force F_{ij} between A_i and A_j is defined as the negative gradient of potential field U_{ij}

$$\begin{aligned} F_{ij}(q_i, q_j) &= -\nabla_{q_i} U_{ij}(q_i, q_j) \\ &= -k_a\left(1 - \frac{d_{des}^2}{f_{ij}(q_i, q_j)^2}\right)\nabla_{q_i} f_{ij}(q_i, q_j) \end{aligned} \tag{6}$$

The resultant force of all UAVs in the formation is

$$F_{all} = F_{ij} I. \tag{7}$$

Remark 1: It is shown from (6) that if the actual distance d_{ij} between A_i and A_j is larger than the desired distance d_{des}, an attractive force will be generated to pull the two UAVs to approach each other. However, if d_{ij} is smaller than d_{des}, a repulsive force will be produced to push the two UAVs apart away. Furthermore, when d_{ij} in (6) close to zero, the repulsive force will become infinite, which has the ability to ensure that the UAVs avoid collisions during the interactive movement. While the attractive force will guarantee that the system keep the desired formation. Moreover, the resultant force F_{all} of the formation is an important indicator for evaluating the formation maintenance effect, and the smaller of F_{all}, the better performance of the formation maintenance is achieved.

B. Multi-UAV Path Planning Control

In this section, considering the local minimum and oscillation problems of the traditional APF algorithm, a novel IAPF algorithm is developed for the multi-UAV system path planning in the 3-D constrained space. As shown in Fig. 2, A_l will suffer repulsive force F_r from the obstacle and attractive force F_a from the target. Moreover, A_l also suffers the forces F_{lfi} and F_{lfj} from A_i and A_j, respectively. Finally, A_l is supposed to move in the direction of resultant force F_i, the same goes for the followers A_{fi} and A_{fj}.

According to the APF [23], a collision-free path is calculated along the direction of the declining potential function. Therefore, the APF is defined as

$$U = U_a + \sum_{i=1}^{m} U_r + U_{ij} \tag{8}$$

where U_a is the attractive potential; U_r represents the repulsive potential, m is the number of the obstacles.

The artificial force is defined as the negative gradient of APF, which is given by

$$\begin{aligned} F &= -\nabla U \\ &= -\nabla U_a - \nabla \sum_{i=1}^{m} U_r - \nabla U_{ij} \\ &= F_a + \sum_{i=1}^{m} F_r + F_{ij} \end{aligned} \tag{9}$$

The attractive potential field function from the target is defined as follows

$$U_a(q) = \frac{1}{2} k_t f_t^2(q, q_t) \tag{10}$$

where k_t is a positive coefficient, $q_t = (x_t, y_t, z_t)^T$ denotes the target location. $f_t(q, q_t) = \|q - q_t\|$ is the Eucslidean distance from A_i to the target.

The attractive force is defined as the negative gradient of the attractive potential field function U_a

$$F_a(q) = -\nabla U_a(q) = -k_t f_t(q, q_t) \tag{11}$$

When the obstacles are far away from the UAVs, in order to prevent the obstacles from affecting the motion of the UAVs, the following repulsive potential function is defined by Khatib [23]

$$U_r(q) = \begin{cases} \frac{1}{2} k_r \left(\frac{1}{f_o(q, q_o)} - \frac{1}{P_o}\right)^2, & f_o(q, q_o) \le P_o \\ 0, & f_o(q, q_o) > P_o \end{cases} \tag{12}$$

where k_r is a positive coefficient, $f_o(q, q_o) = \|q - q_o\|$ represents the distance of the UAV and obstacle, q_o is the coordinates of the obstacle, and P_o denotes the maximum impact range of obstacles.

Similar to the attractive force, the repulsive force is defined as the negative gradient of the repulsive potential function U_r.

$$\begin{aligned} F_r(q) &= -\nabla U_r(q) \\ &= \begin{cases} -k_r\left(\frac{1}{f_o(q, q_o)} - \frac{1}{P_o}\right)\frac{\nabla f_o(q, q_o)}{f_o^2(q, q_o)}, \\ \quad f_o(q, q_o) \le P_o \\ 0, \quad f_o(q, q_o) > P_o \end{cases} \end{aligned} \tag{13}$$

Although the traditional APF has a good performance in path planning control, it has local minimum and oscillations problem, which can be represented as

$$\|F\| = \left\| F_a + \sum_{i=1}^{m} F_r + F_{ij} \right\| \le \eta \tag{14}$$

where η is an arbitrarily small positive scaling factor, (14) means that when the attractive force and repulsive force is equivalent and collinear reverse, it will cause the UAVs to be trapped in local minimum.

To solve the local minimum, we introduce a rotating potential field U_e around the obstacles, whereby a rotating potential force F_e is produced to solve the local minimum and oscillation problems of UAVs, and the U_e is defined as

$$U_e(q) = \begin{cases} \frac{1}{2}k_e f_e^2(q, q_o), & f_e(q, q_o) \leq P_o \\ 0, & f_e(q, q_o) > P_o \end{cases} \tag{15}$$

where k_e is a positive coefficient.

The rotating potential force F_e is the negative gradient of U_e

$$F_e(q) = \begin{cases} -k_e f_e(q, q_o), & f_e(q, q_o) \leq P_o \\ 0, & f_e(q, q_o) > P_o \end{cases} \tag{16}$$

It is worth mention that U_e is none-zero only when the UAV is found to stop at a local minimum, which is produced by the obstacles within the largest impact distance, and the direction of U_e is perpendicular to U_r of the obstacles.

Inspired by [24], the multi-UAV formation and path planning controller is designed as follows:

$$u_i = F_a + \sum_{i=1}^{m} F_r + \sum_{j=1, i\neq j}^{n} F_{ij} + F_e - k_d \dot{q}_i \tag{17}$$

where k_d is a positive control gain used for damping.

C. Stability and Convergence Analysis

Theorem 1: Consider the multi-UAV dynamic system (1). Assume that the control signals are bounded, the control law (17) can enable the $\dot{q}_i$ and F_i asymptotically converge to zero as $t \to 0$ for UAV $A_i (i = 1, 2, \ldots, n)$.

Proof: Consider the Lyapunov-like function

$$U(q) = \sum_{i=1}^{n} \left[U_a(q_i) + \sum_{i=1}^{m} F_r + \sum_{j=1, i\neq j}^{n} U_{ij}(q_i, q_j) + U_e(q_i) + \frac{1}{2}\dot{q}_i^T \dot{q}_i \right] \tag{18}$$

From Eqs. (5), (10), (12), and (15), $U_a(q_i)$, $U_r(q_i)$, $U_{ij}(q_i, q_j)$, and $U_e(q_i)$ are no-negative, therefore, $U(q) \geq 0$.

Differentiating (18) with respect to time yields:

$$\begin{aligned} \dot{U}(q) &= \sum_{i=1}^{n} \dot{q}_i^T \left[\nabla_{q_i} U_a(q_i) + \nabla_{q_i} \sum_{i=1}^{m} U_r(q_i) + \sum_{j=1, i\neq j}^{n} \nabla_{q_i} U_{ij}(q_i, q_j) + \nabla_{q_i} U_e(q_i) + \ddot{q}_i \right] \\ &= \sum_{i=1}^{n} \dot{q}_i^T \left[-F_a - \sum_{i=1}^{m} F_r - \sum_{j=1, i\neq j}^{n} F_{ij} - F_e + \ddot{q}_i \right] \end{aligned} \tag{19}$$

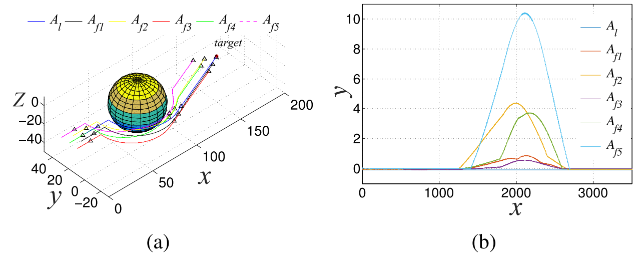


Fig. 3. Case of local minimum: (a) Trajectories of six UAVs formation path planning and (b) trajectory tracking errors of follower UAVs.

Substituting the control law (17) into (1), we have

$$\ddot{q}_i = F_a + \sum_{i=1}^{m} F_r + \sum_{j=1, i\neq j}^{n} F_{ij} + F_e - k_d \dot{q}_i \tag{20}$$

Then, substituting (20) into (19), we have

$$\dot{U}(q) = -k_d \sum_{i=1}^{n} \dot{q}_i^T \dot{q}_i \leq 0 \tag{21}$$

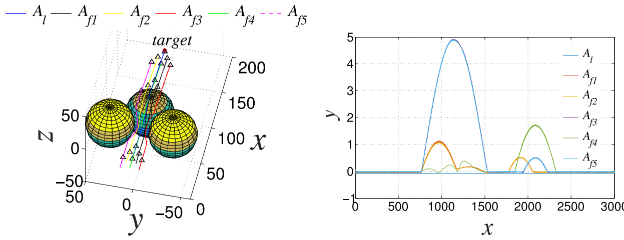
Hence, $\dot{q}_i \to 0$ as $t \to 0$ for UAV $A_i (i = 1, 2, \ldots, n)$. As for the leader UAV, $F_l \neq 0$ means that the leader has not reached the target; for the follower UAVs, $F_{fi} \neq 0$ means that the followers will try to keep the desired distance and angle with the leader. Therefore, under the assumption that the target region is always obstacle free, the implementation of the control law (17) can guarantee that all the UAVs move toward the target and stop at equlibrium state, while keeping the desired formation. The proof is complete. ■

IV. Simulation Results

To prove the effectiveness of the proposed strategy, we depict the simulation results of the path planning and formation control schemes applied to 6 UAVs. The parameters of the potential function are designed as $k_a = 10$, $k_t = 1$, and $k_r = 500$. In the formation, the desired distances are set to $d_{lf1} = d_{lf2} = 10$, $d_{lf3} = d_{lf5} = 20$, and $d_{lf4} = 10\sqrt{3}$, the desired bearing angles are given as $\theta_{lf1} = \theta_{lf3} = \frac{\pi}{3}$, $\theta_{lf2} = \theta_{lf5} = -\frac{\pi}{3}$, and $\theta_{lf4} = 0$.

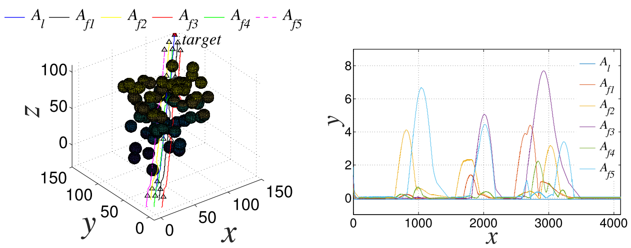
Figure 3(a) shows the trajectories of six UAVs path planning in case of a common local minimum in 3-D constrained environment. The coordinates of the leader, center of the obstacle and the target are in a straight line. It can be seen from Eq. (14) that the leader has a non-target local minimum under the traditional APF [23]. From Fig. 3(a), the leader UAV can escape from the local minimum and avoid the obstacle, the followers try to avoid the obstacle while keeping the desired distances and angles with the leader. Fig. 3(b) depicts the trajectory tracking errors of the follower UAVs A_{f1}-A_{f5}. Although the trajectory tracking errors oscillate during the obstacle avoidance, they converge to zero with the completion of obstacle avoidance.

Figure 4(a) depicts the trajectories of six UAVs path planning in a complex environment. From Fig. 4(a), it indicates that the proposed method can plan a collision-free and safely path to target, and the UAVs are moving in the desired formation while avoiding the obstacles. Fig. 4(b) shows the trajectory tracking errors of the followers, although the trajectory



(a) (b)

Fig. 4. Case of complex static obstacle: (a) Trajectories of six UAVs formation path planning, (b) trajectory tracking errors of follower UAVs.



(a) (b)

Fig. 5. Case of random distribution obstacle environment: (a) Trajectories of six UAVs formation path planning and (b) trajectory tracking errors of follower UAVs.

tracking errors oscillate during the obstacle avoidance, they converge to zero with the completion of obstacle avoidance.

Figure 5(a) shows the trajectories of six UAVs to avoid the random distribution obstacles environment. The trajectory tracking errors of the followers A_{f1}-A_{f5} are shown in Fig. 5(b). Fig. 5(a)-5(b) show that the proposed approaches can achieve the path planning without any collisions and local minimums while keeping the desired formation in complex environment effectively.

V. Conclusion

In this brief, we present a novel IAPF method for a multi-UAV system to solve the path planning and formation control in 3-D constrained space. To overcome the common local minimum problem of the APF, a rotating potential function is proposed, which can effectively prevents the agent from falling into a local minimum. Afterwards, a potential function-based formation and path planning controller is developed to drive all the UAVs to calculate a collision free path from the initial position to the target while keeping the desired formation. Then a Lyapunov function is designed to analyze the stability of the system. Finally, we layout several common situations, and the simulation results have demonstrated the effectiveness of the proposed approaches in achieving path planning and formation control for a multi-UAV system.

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