Martin Irungu
Fall 2020 21.10.2020

Stochastic Methods Lab
Final Project

Problem 2. Traversing the Tree Diagonally
Write a short summary of the following topic in your own words based on the references provided.

Solution. In class, the binomial option pricing model was introduced, and in Homework 2, option prices were computed by backward induction. This involved evaluating every node in the binomial tree, with one `for` loop nested in another, resulting in $1 + \cdots + N = O(N^2)$ evaluations. It turns out that for American options, many of these evaluations are unnecessary.
We focus on American puts (A parity formula exists allowing one to extend this to American calls) and make two observations regarding the binomial tree:

1. If both an up and a down move in the tree result in early exercise, then the current node must also be early exercise (consider the continuous time model).

2. If a node is non-exercise, then all nodes on the same horizontal level are non-exercise. The put's value can only increase with longer time to maturity at the same stock price.

We may start evaluating at the diagonal whose terminal value is just above the strike price on the last step, sweeping the diagonals, and the tree below the strike price must consist entirely of early exercise nodes by our first observation. If in a diagonal sweep we reach an early exercise node, we could simply jump to the root node of that tree using the given probabilities, so skip any further computations along a diagonal when an early exercise node is reached and move to the next one. The process stops when a diagonal D entirely consisting of non-early exercise nodes is reached. First note that one cannot go from the root of the tree to a node where the option is exercised without crossing D. The option value equals the sum of the discounted option values of the nodes on D, each multiplied by the probability that the stock price hits the diagonal for the first time at that node. Since the option values on D have already been computed (any node being evaluated has had its successors already evaluated), this part only takes $O(N)$ evaluations.