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Stochastic Methods Lab
Homework 4

Problem 2.
Consider two portfolios: A) You buy one call and sell one put, for the same stock with price S at time 0 and $S(T)$ at expiration T, and with same strike price K. B) You buy one stock and borrow bonds worth K at time T . Then use a "no-arbitrage argument" to derive a relationship between the prices of European calls and puts. The resulting formula is called the "put-call parity".

Solution. Let C and P be the call and put price respectively, with $A(t), B(t)$ the values at time t of the portfolios A and B respectively. At time T portfolio A will be worth

$$\begin{aligned} A(T) &= \max(S(T) - K, 0) - \max(K - S(T), 0) \\ &= \max(S(T) - K, 0) + \min(S(T) - K, 0) \\ &= S(T) - K. \end{aligned}$$

Here we have used the facts

$$\max(a, b) = -\min(-a, -b)$$

and

$$a + b = \max(a, b) + \min(a, b).$$

Clearly portfolio B is also worth $S(T) - K$. By no-arbitrage, if two portfolios have the same value in time T they have the same value now, and so:

$$\underline{C - P = S - Ke^{-rT}}$$