Martin Irungu
Fall 2020 September 14, 2020

Stochastic Methods Lab
Homework 1

Problem 2.
An individual who plans to retire in 40 years has decided to put an amount A in the bank at the beginning of each of the next 480 months, after which she will withdraw 2000 $ at the beginning of each of the following 360 months. Assuming a nominal yearly interest rate of 2% compounded monthly, how large does A need to be? (Note: You can either solve this exercise with a python program or by hand. The answer is a bit more than 700 $.)

Solution. Another way of stating this is that she wants to by a 2000 $ 30-year general annuity in 40 years. The present value of a general annuity due paying W $ with m payments per year for n years is

$$PV = W\frac{1-(1+\frac{r}{m})^{-mn}}{\frac{r}{m}}.$$

This should be set equal to the future value of an existing cashflow of m payments of A $ per year for n' years.

$$FV = A\frac{(1+\frac{r}{m})^{mn'}-1}{\frac{r}{m}}.$$

Setting $PV = FV$ and solving for A :

$$A = W\frac{1-(1+\frac{r}{m})^{-mn}}{(1+\frac{r}{m})^{mn'}-1}$$

Substituting the given values $W = 2000, m = 12, r = 0.02, n = 30, n' = 40$ yields

$$\underline{A = 736.75\,\$}$$