

Homework 2

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1 Convolutional Neural Networks

Problem A: Convolution

i.

Solution A.i:

8 filters, each with $5 \times 5 \times 3$ weights plus a bias term per filter is equal to $(8 \times 5 \times 5 \times 3) + 8 = 608$ parameters.

ii.

Solution A.ii:

$W_{out} = H_{out} = (W - F + 2P)/S + 1 = (32 - 5 + 0)/1 + 1 = 28$. Thus, the output tensor has shape $(28, 28, 8)$ where 8 is based on the number of filters.

Problem B: Pooling**i.****Solution B.i:***Calculating average using floating point numbers:*

$$\begin{bmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{bmatrix}$$

$$\begin{bmatrix} 0.5 & 1 \\ 0.25 & 0.5 \end{bmatrix}$$

$$\begin{bmatrix} 0.25 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0.5 & 0.25 \\ 1 & 0.5 \end{bmatrix}$$

ii.**Solution B.ii:***All 4 matrices have the same result:*

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

2 Recurrent Neural Networks

Problem A: LSTM

i.

Solution A.i:

Use cross entropy loss: $L_t = -y_t \log \hat{y}_t$

where $\hat{y}_t = \sigma(w_y h_t)$

$$1) \delta c_t = \frac{\partial L_t}{\partial c_t} = \underbrace{\frac{\partial L_t}{\partial \hat{y}_t} \cdot \frac{\partial \hat{y}_t}{\partial h_t}}_{(\hat{y}_t - y_t) w_y^T} \cdot \frac{\partial h_t}{\partial c_t} \quad \begin{aligned} \frac{\partial (o_t \odot \tanh(c_t))}{\partial c_t} &= o_t \odot \tanh'(c_t) \\ &= o_t (1 - \tanh^2(c_t)) \end{aligned}$$

$$\text{Thus, } \boxed{\delta c_t = (\hat{y}_t - y_t) w_y^T \cdot o_t (1 - \tanh^2(c_t))}$$

$$2) \delta o_t = \frac{\partial L_t}{\partial o_t} = \underbrace{\frac{\partial L_t}{\partial \hat{y}_t} \cdot \frac{\partial \hat{y}_t}{\partial h_t}}_{(\hat{y}_t - y_t) w_y^T} \cdot \frac{\partial h_t}{\partial o_t} \quad \tanh(c_t)$$

$$\text{Thus, } \boxed{\delta o_t = (\hat{y}_t - y_t) w_y^T \cdot \tanh(c_t)}$$

$$3) \delta i_t = \frac{\partial L_t}{\partial i_t} = \underbrace{\frac{\partial L_t}{\partial \hat{y}_t} \cdot \frac{\partial \hat{y}_t}{\partial h_t}}_{\text{from (1)}} \cdot \frac{\partial h_t}{\partial i_t} \cdot \frac{\partial c_t}{\partial i_t} \quad \tanh(w^{ex} x_t + w^{ah} h_{t-1})$$

$$\text{Thus, } \boxed{\delta i_t = (\hat{y}_t - y_t) w_y^T \cdot o_t (1 - \tanh^2(c_t)) \cdot \tanh(w^{ex} x_t + w^{ah} h_{t-1})}$$

$$4) \delta f_t = \frac{\partial L_t}{\partial f_t} = \underbrace{\frac{\partial L_t}{\partial \hat{y}_t} \cdot \frac{\partial \hat{y}_t}{\partial h_t}}_{\text{from (1)}} \cdot \frac{\partial h_t}{\partial f_t} \cdot \frac{\partial c_t}{\partial f_t} \quad c_{t-1}$$

$$\text{Thus, } \boxed{\delta f_t = (\hat{y}_t - y_t) w_y^T \cdot o_t (1 - \tanh^2(c_t)) \cdot c_{t-1}}$$

ii.

Solution A.ii:

Based on the derivations above, the gradient does not explode if the forget gates are close to 1 and the input/output gates are close to 0. Looking at the gradient of c_t , if the output gate is close to zero, the gradient approaches 0 too, which means the system remains stable and unchanged. Additionally, since the forget gate is close to 1 and input is close to 0, the cell state will keep its previous value, thereby preventing both an exploding gradient and vanishing gradient.

3 Poem Generation

Problem A: Pre-processing

i.

Solution A.i: *Your solution here*

ii.

Solution A.ii: *Your solution here*

Problem B: Model Training**i.****Solution B.i:** *Your solution here***ii.****Solution B.ii:** *Your solution here***iii.****Solution B.iii:** *Your solution here*