CSE 151B Deep Learning
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Homework 1

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1 Supervised Learning

Problem A: Feature Representation

Solution A:

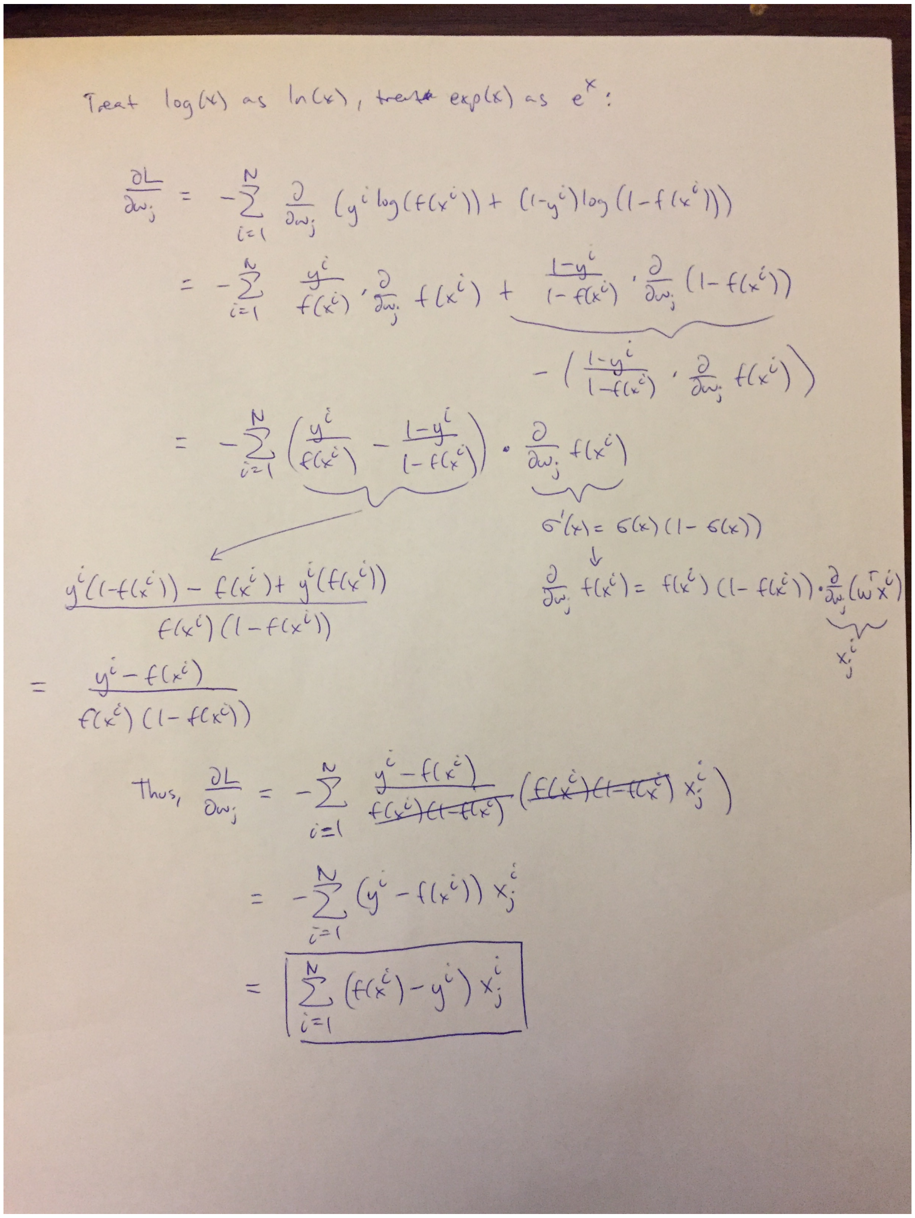
The matrix representing the four commit messages is as follows:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Based on the bag-of-words representation, each row in the matrix is a feature vector for the corresponding sentence. In each vector, there is a feature per word in the dictionary ($bug, fix, correct, error, wrong$)*, and the binary value for each feature represents whether the word is present in the sentence.*

Problem B: Logistic Regression

Solution B:



Treat $\log(x)$ as $\ln(x)$, ~~treat~~ $\exp(x)$ as e^x:

$$\frac{\partial L}{\partial w_j} = -\sum_{i=1}^{N} \frac{\partial}{\partial w_j}\left(y^i \log(f(x^i)) + (1-y^i)\log(1-f(x^i))\right)$$

$$= -\sum_{i=1}^{N} \frac{y^i}{f(x^i)} \cdot \frac{\partial}{\partial w_j} f(x^i) + \underbrace{\frac{1-y^i}{1-f(x^i)} \cdot \frac{\partial}{\partial w_j}(1-f(x^i))}_{-\left(\frac{1-y^i}{1-f(x^i)} \cdot \frac{\partial}{\partial w_j} f(x^i)\right)}$$

$$= -\sum_{i=1}^{N} \underbrace{\left(\frac{y^i}{f(x^i)} - \frac{1-y^i}{1-f(x^i)}\right)} \cdot \underbrace{\frac{\partial}{\partial w_j} f(x^i)}$$

$$\sigma'(x) = \sigma(x)(1-\sigma(x))$$

$$\downarrow$$

$$\frac{\partial}{\partial w_j} f(x^i) = f(x^i)(1-f(x^i)) \cdot \underbrace{\frac{\partial}{\partial w_j}(w^T x^i)}_{x_j^i}$$

$$\frac{y^i(1-f(x^i)) - f(x^i) + y^i(f(x^i))}{f(x^i)(1-f(x^i))}$$

$$= \frac{y^i - f(x^i)}{f(x^i)(1-f(x^i))}$$

Thus, $$\frac{\partial L}{\partial w_j} = -\sum_{i=1}^{N} \frac{y^i - f(x^i)}{\cancel{f(x^i)(1-f(x^i)})}\left(\cancel{f(x^i)(1-f(x^i)}\, x_j^i\right)$$

$$= -\sum_{i=1}^{N} (y^i - f(x^i))\, x_j^i$$

$$= \boxed{\sum_{i=1}^{N} (f(x^i) - y^i)\, x_j^i}$$

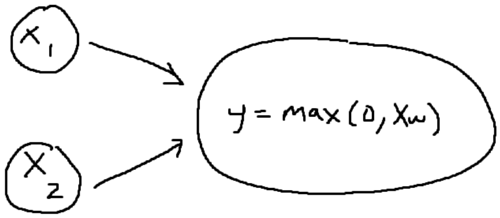
2 Multi-Layer Perceptron

Problem A: Function Approximation

i. OR

Solution A.i:

The drawing below shows the fully connected network, with the max *operation representing the ReLU unit. Based on inputs* $x_1 \in \{0,1\}$ *and* $x_2 \in \{0,1\}$*, the following architecture will output* 0 *given* $X = [0,0]$*, and it will output a value greater than or equal to* 1 *given any other combination, thus satisfying the constraints with minimum layers.*



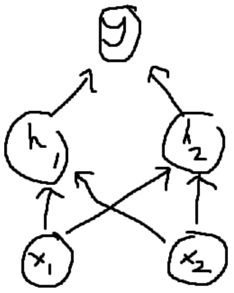
$$X = [x_1 \;\; x_2]$$

$$w = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

ii. XOR

Solution A.ii:

At a minimum, two fully-connected layers are necessary in order to compute XOR. This is because XOR is not linearly separable, so it must be represented based on a combination of other operations. In a minimal form (in terms of number of layers), XOR can be represented as $(x_1 + x_2)(x_1x_2)'$*. Thus, the first layer would compute the OR and NAND operations, and the second layer would combine these outputs with an AND operation. An example of the XOR network could be the following.*



network: $f(x; W, c, w, b) =$

$$w^T \cdot \max\{0, W^T x + c\} + b$$

where $W = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $c = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$, $w = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, $b = 0$

Thus, given $X = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$, $y = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$