# Detectability of interaural delay in high-frequency complex waveforms*

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Observers can detect interaural delays in three component stimuli produced by sinusoidally modulating the amplitude of a sinusoidal carrier. Interaural delays are detected in stimuli confined to a high-frequency region even in the presence of an intense, low-pass filtered noise. With 300-Hz modulation of a 3900-Hz carrier, detection of interaural delay is equally good when either the entire stimulus or just the envelope of the stimulus is delayed. In either case performance is as good as with a 300-Hz pure tone.

Subject Classification: 35.30.

## INTRODUCTION

The classic experiments of Rayleigh (1906) established that changes in the interaural phase of low-frequency pure tones lead to changes in the apparent location of the source of the tone; for tones of frequency greater than approximately 1500 Hz, interaural phase or time differences do not affect that apparent location of the source but interaural amplitude does. This finding for pure tones has been confirmed, extended, and refined in many subsequent papers (Stevens and Newman, 1936; Mills, 1958; Mills, 1960). A difference in the phase of a pure tone at the ears is simply another way of describing a difference in the time at which corresponding parts of the waveform arrive at each ear, and the finding of Rayleigh is conveniently summarized by the statement that the auditory system is insensitive to interaural time differences in pure tones of frequency greater than approximately 1500 Hz.

The situation with complex waveforms is somewhat different. David, Guttman, and Van Bergeijk (1959) and later Harris (1960) have shown that interaural time differences with high-pass filtered trains of clicks lead to significant changes in the apparent location of the source of the clicks. Yost, Wightman, and Green (1971) also find that the localization of trains of pulses is dependent on interaural time differences. On the other hand, the same authors have shown that the localization of a single click high-pass filtered above 2 kHz is relatively insensitive to interaural time differences. Banks and Green (1973) have confirmed this finding. David *et al.* suggested that the time information might be carried in the envelope of their stimuli—indeed, their observers were able to trade interaural time and intensity differences to centre reliably the image of short bursts of high-frequency noise even when the noise for each ear was generated from different, uncorrelated sources. The high-frequency transients used in the study of David *et al.* did not behave as low-frequency pure tones in all respects; the amount of interaural delay needed to offset a given interaural amplitude difference was larger with high-frequency transients than it was with low-frequency pure tones (Harris, 1960).

Leakey, Sayers, and Cherry (1958) and Kirikae, Nakamura, Sato, and Shitara (1971) have reported that nontransient high-frequency complex patterns will form fused images and that the image location changes with interaural delay. Leakey *et al.* reported further that the images of some high-frequency waveforms behave as if the observers were able to extract timing information from the envelope of the waveforms; with only the envelope of their waveform delayed, they obtained fusion exactly like that shown by the envelope alone.

The present experiment employed amplitude-modulated tones in an attempt to clarify the effect of interaural time differences on the apparent location of sources of high-frequency, complex stimuli. The experiments were lateralization studies and measured the ability of observers to detect interaural time differences with "pure tones" and with amplitude-modulated tones in which either the entire waveform or just the modulating waveform was delayed. Finally, detectability of interaural time information contained in the envelopes of carriers of different frequency was measured.

## I. EXPERIMENT 1

### A. Procedure

Several observers[1] participated in standard two-alternative temporal forced-choice lateralization experiments. The signals were 250-msec bursts of either a sinusoidal or complex waveform turned on and off with a linear rise and fall time[2] of 50 msec. The signals were presented binaurally at a level of 50 dB SPL over TDH-39 earphones operating in phase.

There were two observation intervals on each trial of the experiment; each interval was easily determined by the presentation of the clearly audible stimuli. In the first observation interval, the stimulus to one ear—ear 1—was delayed by $\Delta_t$ sec relative to the stimulus in the other ear—ear 2. In the second observation interval, the signal to ear 2 was delayed the same amount—$\Delta_t$ sec—relative to the signal in ear 1. On each trial the probability of the signal to the right ear being delayed in the first observation interval was 0.5 and the observer was required to indicate after each trial whether the signal to the right ear had been delayed in the first or in the second observation interval. Thus the observer heard a signal first in one location and then in another and reported the interval in which the leftmost-appearing signal had occurred.

Two hundred observations were made at each of several delays in order to determine the function relating

  

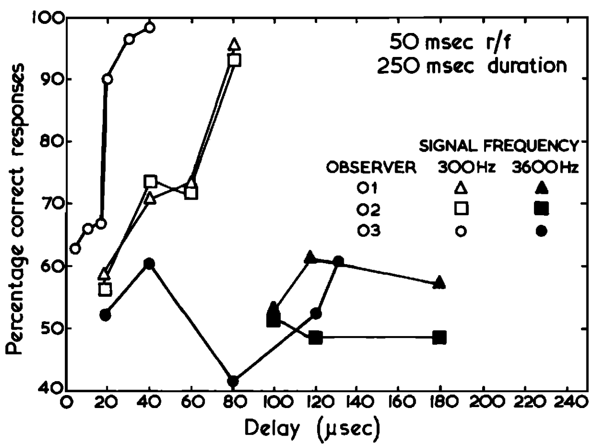


FIG. 1. The percentage of correct lateralization judgments as a function of the interaural delay in microseconds. The signals were 250-msec bursts of either a 300- or 3600-Hz sinusoid with a 50-msec rise-and-fall time. Each data point is based on 200 judgments by each of three observers.

the percentage of correct responses to interaural delay. In the first experiment, "pure tone" stimuli of two different signal frequencies—300 or 3600 Hz—were used. The results for three observers are shown in Fig. 1 and might have been predicted from the usual finding that interaural delays produce measurable changes in the apparent location of pure tones of frequency less than approximately 1500 Hz but not with tones of frequency greater than that (Rayleigh, 1906). There is considerable variablity among observers; in order to achieve 75% correct responses, Observers 1 and 2 require three times the delay needed by Observer 3 to achieve the same performance level.

In the next experiments, complex rather than "pure tone" stimuli were used. The waveforms were produced by sinusoidally modulating the amplitude of a sinusoid to produce stimuli, $s(t)$, of the form given by

$$s(t) = (1 + m\cos 2\pi f_m t)\cos 2\pi f_c t. \tag{1}$$

The overall amplitude of the waveform is unspecified in Eq. 1, but the relative amplitudes of the three components of the stimulus are determined by the depth of modulation—the factor $m$. In the following experiments, unless otherwise noted, $m$ was equal to 1, so that a three-component signal was produced in which the components at frequencies $f_c \pm f_m$ Hz were half the amplitude of the carrier at frequency $f_c$ Hz.

The delayed waveform is given by

$$s(t - \Delta_t) = 1 + m\cos[2\pi f_m(t - \Delta_t)]\cos[2\pi f_c(t - \Delta_t)], \tag{2}$$

where $\Delta_t$ is the delay in seconds. The relative amplitudes and frequencies of the three components of the stimulus are not altered by the delay. Each component is simply delayed by the same amount, $-\Delta_t$. The equal delay at all frequencies produces a linear phase shift, that is, one proportional to the frequency of the component shifted.

The carrier and modulating frequencies were chosen so that none of the three components of the complex fell below 1800 Hz. The amplitude-modulated signals lasted 250 msec and were turned on and off with 50-msec rise-and-fall time as was the case with the "pure tone" signals.

The amplitude-modulated waveform was generated using a standard multiplier as a modulator. Care was taken to ensure that power at the modulation frequency, $f_m$, nominally not present in the waveform, was at least 40 dB below the level of any of the three appropriate components at $f_c$ and $f_c \pm f_m$ Hz. The complex waveform was presented at an overall level of 50 dB SPL.

In addition to the complex waveforms, a low-frequency masking noise was continuously present. The noise, identical at each earphone, was limited to the frequency band below 600 Hz and had a Spectrum Level of 50 dB in that band.

The carrier frequency in all the experiments about to be described was harmonically related to the modulating frequency so that the three components of the stimulus were three successive harmonics of the modulation frequency. No attempt was made, however, to lock the frequency or initial phase of the oscillators generating the carrier and modulating signal. As in the pure tone case, delays were produced by adding an Ad-Yu variable delay line (model 8 01 D1) to the channel for one ear or the other. The difference in gain at each frequency was much less than 0.5 dB over the frequency range spanned by the signals.

The observer's ability to detect interaural delays in complex signals was measured in the standard two-alternative forced-choice paradigm previously described.

## B. Results and discussion

Figure 2 shows the results of this experiment for Observer 1. The percentage of correct responses in 200 trials is plotted on the ordinate against interaural delay on the abscissa. The different curves were generated with different carrier frequencies all modulated at 300 Hz. Lateralization performance was best with

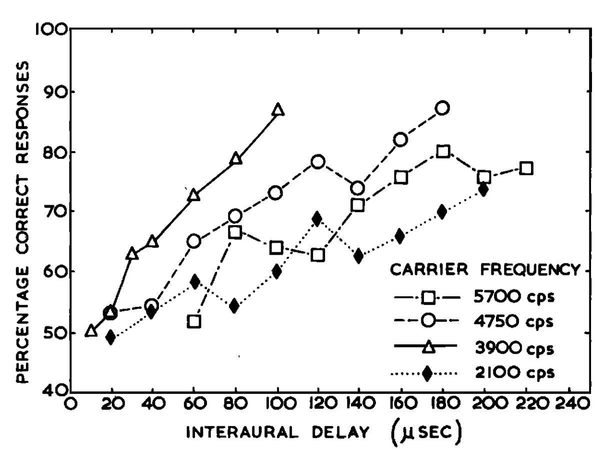


FIG. 2. The percentage of correct lateralization judgments as a function of the interaural delay in microseconds. The signals were 100% amplitude-modulated sinusoidal carriers at the carrier frequency shown. The modulation was also sinusoidal at 300 Hz. The entire signal waveform was delayed. Data points are based on 200 judgments by Observer 1.

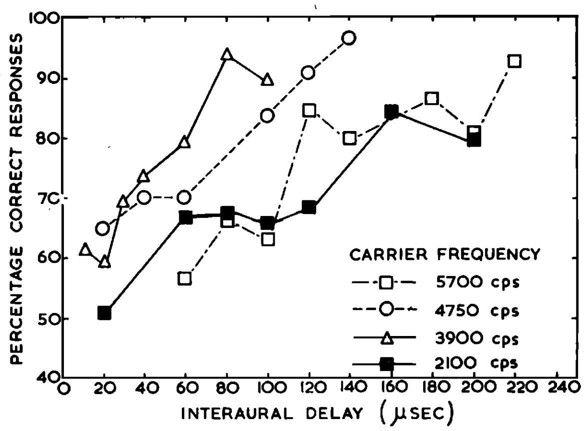


FIG. 3. The percentage of correct lateralization judgments as a function of the interaural delay in microseconds. The signals were 100% amplitude-modulated sinusoidal carriers at the carrier frequency shown. The modulation was also sinusoidal at 300 Hz. The entire signal waveform was delayed. Data points are based on 200 judgments by Observer 2.

the 3900-Hz carrier frequency. At this carrier frequency, Observer 1 could correctly detect delays of about 65 μsec 75% of the time. Observer 2, for whom similar data are shown in Fig. 3, achieved the same performance level with a 50-μsec delay. These delays are very close to those required by observers to achieve 75% correct lateralization with a 300-Hz "pure tone."

With higher and lower carrier frequencies the psychometric functions became shallower, that is, the rate of improvement in performance with increasing interaural delay becomes less. The effect of carrier frequency on the slope of the functions is large, so that at carrier frequencies of either 2100 or 5700 Hz observers require delays of approximately 200 μsec to achieve 75% correct responses. Thus, for a 300-Hz modulation frequency, there appears to be a broad maximum in the relation between lateralization performance and carrier frequency with a maximum between 2 and 4 kHz. Figure 4 shows the results for Observer 3. The lateralization performance of this observer was much better than that of the other observers—indeed, no measurable decrement in performance appeared until a 7500-Hz carrier frequency was used. This observer's performance with pure tones was also better than that of the other observers. There is considerable variability among observers.

The results of this experiment indicate that observers can reliably lateralize complex high-frequency tones on the basis of interaural delay alone. Similar delays produced the same level of performance with both the amplitude-modulated waveform and with pure tones of the same frequency as the amplitude modulation; lateralization of any component of the complex waveform on the basis of interaural delay, if it were at all possible, would require much greater delays than those required with the complete complex. This finding tends to support the hypothesis that observers somehow extract interaural timing information from the envelope of the complex waveform. In order to test this hypothesis further, lateralization was measured for waveforms in which only the envelope was delayed.

## II. EXPERIMENT 2

### A. Procedure

The same two-alternative forced-choice lateralization task described in the previous experiment was used. The amplitude, rise-and-fall time, and duration of the stimuli were all as before. The same 600-Hz low-pass filtered noise was again present at the same mean Spectrum Level of 50 dB. The observers' task was the same except that only the envelope of the amplitude modulated waveform, not the entire waveform, was delayed.

The envelope delay was produced by inserting the delay line into the channel leading to the appropriate input of the modulator. The waveform produced by delaying the envelope, $s\Delta(t)$, is given by

$$s_{\Delta}(t)=\{1+m\cos[2\pi f_m(t-\Delta_t)]\}\cdot\cos 2\pi f_c t\,, \tag{3}$$

where $\Delta_t$ is the envelope delay in seconds. As in the case in which the entire waveform is delayed, the relative amplitudes and spectral location of the three components of the stimulus are not altered; further, the phase of the component at the carrier frequency remains unchanged. The upper frequency sideband, at frequency $f_c+f_m$ Hz, is delayed by an amount $\Delta_{t_u}$ given by

$$\Delta_{t_u}=\Delta_t[f_m/(f_c+f_m)], \tag{4}$$

where $\Delta_t$ is the envelope delay and $f_m$ and $f_c$ are the modulation and carrier frequencies, respectively. Equation 4 shows that the delay of the upper sideband is proportional to the envelope delay but reduced, in the case of the harmonic complex used here, by the reciprocal of the harmonic number of that component.

The lower-frequency sideband is not delayed at all. Rather it is advanced by an amount proportional to the envelope delay but reduced by the reciprocal of the harmonic number of that component. The effect of the envelope delay of $\Delta_t$ sec on the three components of the

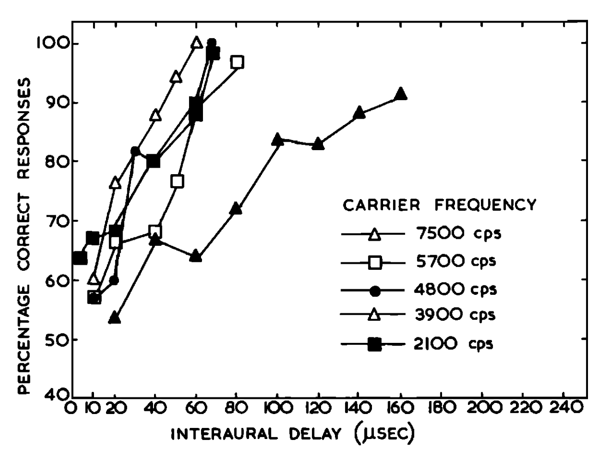


FIG. 4. The percentage of correct lateralization judgments as a function of the interaural delay in microseconds. The signals were 100% amplitude-modulated sinusoidal carriers at the carrier frequency shown. The modulation was also sinusoidal at 300 Hz. The entire signal waveform was delayed. Data points are based on 200 judgments by Observer 3.

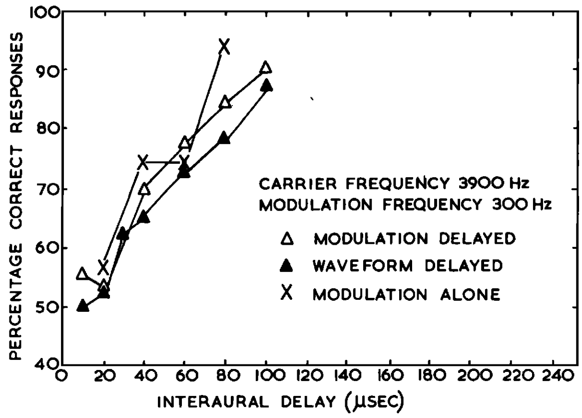


FIG. 5. The percentage of correct lateralization judgments with three different types of signal as a function of the interaural delay in microseconds. The **X**'s indicate data from the case in which a 300-Hz "pure tone" was the stimulus, the closed symbols a 3900-Hz carrier with 100% sinusoidal amplitude modulation at 300 Hz for which the entire waveform was delayed, the open symbols the same amplitude modulated waveform in which only the envelope was delayed. Data points are based on 200 judgments by Observer 1.

stimulus may be summarized as a phase shift, $\Delta\phi(f)$, having a constant term and a term linearly dependent on frequency. The phase shift for a component at frequency $f$ is given by

$$\Delta\phi = -2\pi\Delta_t f + 2\pi\Delta_t f_c. \qquad (5)$$

## B. Results and discussion

Figures 5 and 6 show as open symbols the percentage of correct responses as a function of interaural delay for the case in which only the envelope of the waveform was delayed. Comparison of these data with the functions obtained with the entire waveform delayed (closed symbols) and with a "pure tone" of the modulation frequency (indicated by **X**'s) shows that observers detect delays in the envelope of an amplitude-modulated waveform as well as they detect delays at the modulation frequency itself—at least for a 300-Hz modulation of a 3900-Hz carrier. These results are similar to those reported by Young and Carhart (1973).

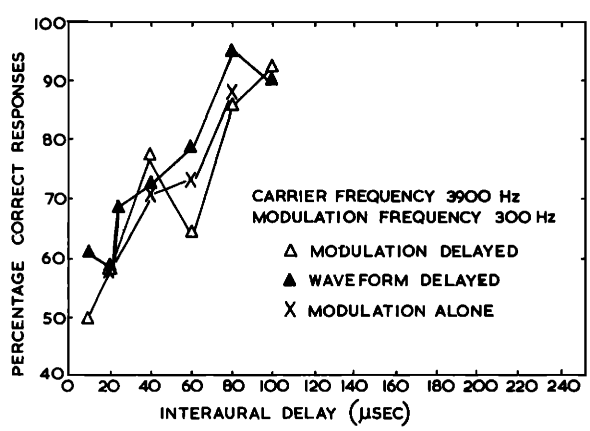


FIG. 6. The percentage of correct lateralization judgments with three different types of signal as a function of the interaural delay in microseconds. The **X**'s indicate data from the case in which a 300-Hz "pure tone" was the stimulus, the closed symbols a 3900-Hz carrier with 100% sinusoidal amplitude modulation at 300 Hz for which the entire waveform was delayed the open symbols, the same amplitude-modulated waveform in which only the envelope was delayed. Data points are based on 200 judgments by Observer 2.

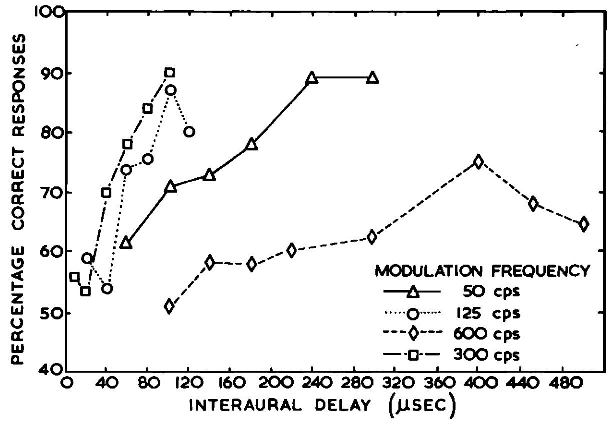


FIG. 7. The percentage of correct lateralization judgments based on delays in the envelope of an amplitude-modulated waveform as a function of the interaural delay in microseconds. The carrier frequency was 3900 Hz in all cases and the modulation was at the frequency indicated in the figure. Data points are based on 200 judgments by Observer 1.

Figure 7 shows the effect of varying modulation frequency on the lateralization performance of Observer 1 when the carrier frequency is fixed at 3900 Hz. Modulation frequencies of greater or less than 300 Hz lead to poorer performance. With 600-Hz modulation, for example, Observer 1 barely achieves 75% correct responses with 400-$\mu$sec interaural delay. The performance of the observer detecting interaural delays in a 600-Hz pure tone is much better; an interaural delay of only 30 $\mu$sec leads to 75% correct responses. Similar data for Observer 2 are shown in Fig. 8.

A different comparison may be made in Fig. 9, which shows lateralization performance of Observer 3 as a

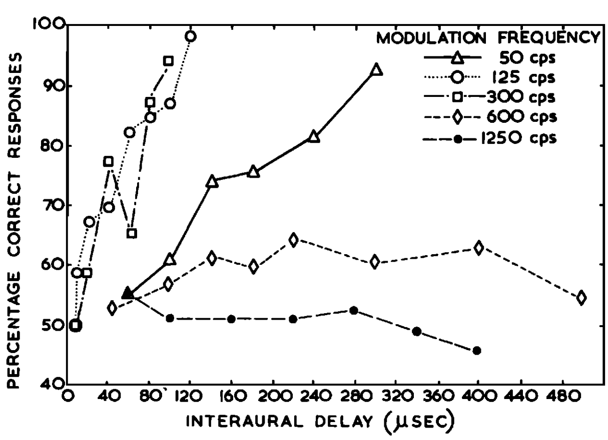


FIG. 8. The percentage of correct lateralization judgments based on delays in the envelope of an amplitude-modulated waveform as a function of the interaural delay in microseconds. The carrier frequency was 3900 Hz in all cases and the modulation was at the frequency indicated in the figure. Data points are based on 200 judgments by Observer 2.

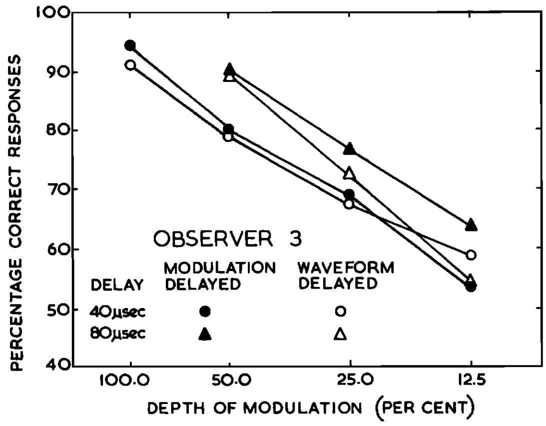


FIG. 9. The percentage of correct lateralization judgments of a 3900-Hz carrier sinusoidally modulated at 300 Hz as a function of the depth of modulation (on a logarithmic scale). Two different interaural delays, 40 μsec (circular symbols) and 80 μsec (triangular symbols) were used. Open symbols represent data from the condition in which the entire waveform was delayed, filled symbols when only the envelope was delayed. Data points are based on 200 judgments by Observer 3.

function of the depth of modulation. The waveform was produced by modulating a 3900-Hz carrier at a rate of 300 Hz. The open symbols represent data taken with the entire waveform delayed (as in Experiment 1) and the filled symbols represent data from the condition in which only the envelope of the waveform was delayed. Circles symbolize the results for 40-μsec delay and triangles those for 80-μsec delay. Performance drops roughly linearly with the logarithm of the depth of modulation. When only the modulation is delayed, the difference in performance between 40- and 80-μsec delay remains similar at each modulation depth, indicating that the psychometric functions relating performance to delay for different depths of modulation are probably parallel for this observer. When the entire waveform is delayed, on the other hand, the data suggest that the psychometric functions become shallower and that the observer cannot judge laterality at small modulation depths without very large delays.

It is clear from Figs. 5–8 that, within certain limits, observers are able to lateralize sound on the basis of delays in the envelope of an amplitude-modulated signal. Lateralization performance in some cases is virtually identical with that in the case where the entire waveform is delayed and clearly much better than the observer could achieve with any single component of the waveform. In the final experiment, I attempted to determine whether envelope delays in different spectral regions might also be useful to Observers trying to lateralize sound.

## III. EXPERIMENT 3

### A. Procedure

The same two-alternative forced-choice lateralization task was used and only the envelope of the waveform was delayed. The experiment was thus like Experiment 2 in that the interaural delay information was carried in the envelope of the waveform at each ear. In Experiment 2, however, the same carrier frequency—generated by a single oscillator—was used in each waveform. In the present experiment, different carrier frequencies were used in each ear. The two different carriers, each modulated at the same frequency, were lead separetely to the ears. Carrier frequencies both above and below 3900 Hz were used, but the modulation frequency was kept at 300 Hz. The lower frequency carrier was always presented to the observer's left ear and all carriers were integer multiples (harmonics) of the modulation frequency. The observers could easily detect differences in the waveforms with different carriers if they removed one or the other earphone, but the simultaneous presentation of the waveforms to each ear seemed always to produce the impression of a single, if somewhat diffuse, sound source.

### B. Results and discussion

Figure 10 shows the percentage of correct responses obtained in 200 trials as a function of the interaural delay. The frequency of the carrier at each ear is different (except for a single case shown for comparison) and the data are from Observer 1. Figure 11 shows similar data for Observer 2. For both observers, lateralization performance is worse whenever the carrier frequencies at the two ears are different.

The rate at which lateralization performance deteriorates with increasing separation of the carrier frequencies is rather faster than might be expected simply from the way in which lateralization performance deteriorates with increasing carrier frequency in the case in which the carriers have the same frequency.

Figures 12 and 13 show the effects of different modulation frequencies for both observers in one of the cases in which the carrier frequencies differ in the two ears. The carrier frequencies are 3900 and 4800 Hz and modulation frequencies ranging from 50 to 1250 Hz were used. There are large discrepancies in the results obtained with different observers; Observer 1, for example, is best with 300-Hz modulation. His performance becomes increasingly worse as the frequency of modula-

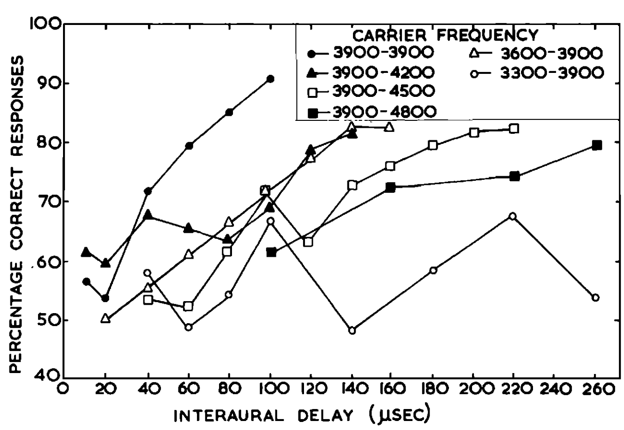


FIG. 10. The percentage of correct lateralization judgment of sinusoidal carriers 100% amplitude modulated at 300 Hz as a function of interaural delay. Only the envelope of the waveform was delayed and different carrier frequencies were used at each ear. Data points are based on 200 judgments by Observer 1.

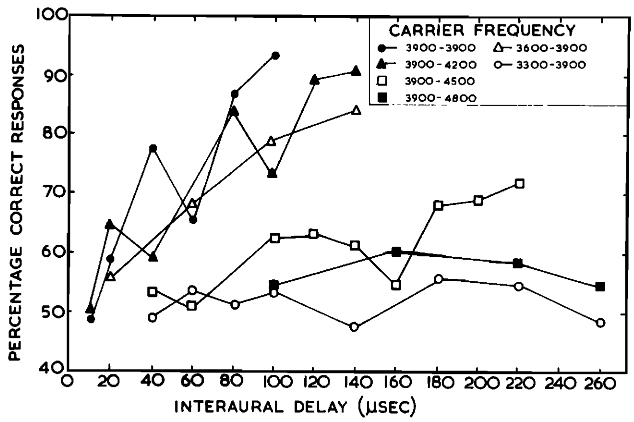


FIG. 11. The percentage of correct lateralization judgments of sinusoidal carriers 100% amplitude modulated at 300 Hz as a function of interaural delay. Only the envelope of the waveform was delayed and different carrier frequencies were used at each ear. Data points are based on 200 judgments by Observer 2.

tion becomes greater or less than 300 Hz. Observer 2, on the other hand, barely achieves 60% correct response as 300-Hz modulation and is very much better with 125-Hz modulation. At frequencies above and below 125-Hz modulation, her performance is worse. The data from both observers indicates their inability to lateralize signals with high modulation frequencies.

## IV. SUMMARY DISCUSSION

(1) The results of this experiment support the contention of Leakey *et al.* (1958) and of David *et al.* (1959) that observers are able to use temporal information carried in the envelope of high-frequency waveforms.

(2) Time information may be derived not only from the envelope of brief transients but also, with certain limits on the carrier and modulation frequencies, from amplitude-modulated waveforms.

(3) The detectability of interaural delays in the envelope

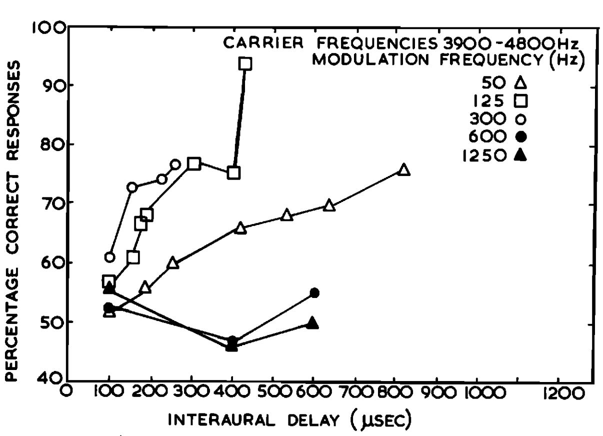


FIG. 12. The percentage of correct lateralization judgments of *sinusoidal* carriers 100% amplitude modulated, at five modulation frequencies, as a function of interaural delay. Only the envelope of the waveform was delayed and different carriers were used at each ear. Data points are based on 200 judgments by Observer 1.

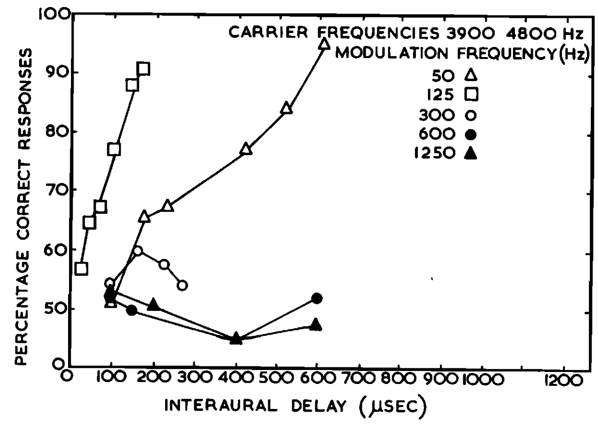


FIG. 13. The percentage of correct lateralization judgments of *sinusoidal* carriers 100% amplitude modulated, at five modulation frequencies, as a function of interaural delay. Only the envelope of the waveform was delayed and different carriers were used at each ear. Data points are based on 200 judgments by Observer 2.

of a 3900-Hz sinusoidal carrier sinusoidally modulated 100% at a frequency of 300 Hz is as good as the detectability of interaural delays in 300-Hz pure tone. With a 600-Hz modulation frequency, however, the detectability of interaural delay is less than that for a 600-Hz pure tone. Too few experiments have been done to establish whether there are different optimal modulation frequencies at each carrier frequency.

(4) Lateralization performance is possible with depths of modulation less than 100%, although the detectability of interaural time differences decreases with decrease in the depth of modulation. In this respect, the behavior of the observer differs from that of one type of single unit in the rat cochlear nucleus in response to amplitude-modulated signals (Møller 1972). The units reported by Møller responded to sound bursts with a single precisely timed discharge provided there was a silent period between bursts of sound. Other units in the cochlear nucleus of rat, however, showed the modulation frequency in the cycle histogram with modulation of only 10%. The maximum "amplitude" with which the modulation appeared in this unit was curiously at 300 Hz. The carrier frequency was 16.9 kHz.

(5) Differences in the phase of the envelope of amplitude-modulated stimuli can be detected when the carrier frequencies are different at the two ears, although performance is best when the carrier frequencies are identical.

It should be noted, however, that the waveforms producing the best performance (a) were within the existence region of the tonal residue (Ritsma, 1962); (b) were comprised of harmonically related components; (c) had the same apparent frequency at each carrier frequency; and (d) had the same apparent frequency at any envelope delay [because the envelope delay produces a linear plus constant phase shift that does not lead to changes in the apparent frequency of the complex (de Boer, 1961)]. Experiments with uncorrelated carriers, i.e., inharmonic residues, are clearly required to determine the importance of interaural short-term correlation (Leakey *et al.*, 1958).

(6) It would be interesting to know if lateralization performance deteriorates when the stimuli at each ear have different apparent frequency; the results of the experiments reported here allow exclusion of the possiblity that lateralization of high-frequency complex tones is based on timing information carried in the "residue tone," since lateralization is possible with complexes with sufficiently low modulation frequencies to produce no "residue tone."

(7) That time information is carried in low-frequency difference tones may be excluded by virtue of the fact that high-intensity low-frequency masking noise would have masked any aural-harmonic components below 600 Hz. Distortion products of frequency $kf_1 - (k-1)f_2$ having frequency below 1500 Hz have negligible amplitude at the intensity of the signals used here. The fact that low-frequency masking noise does not disrupt the lateralization of certain high-frequency waveforms indicates that the conclusion of Yost *et al.* is incorrect; low-frequency information in the waveform is not required for lateralization. Had Yost *et al.* measured lateralization with, say, a train of 64 pulses in the presence of low-pass noise, they would, in all likelihood, have reached the same conclusion.

(8) The findings of Yost *et al.* (1972) indicate that signal duration is an important determinant of the detectability of interaural time differences in pulse trains; no attempt has been made to explore this variable, but Ricard and Hafter (1973) find that duration affects low-frequency tones with short 2.5-msec rise times only when the duration is shorter than some critical value between 50 and 150 msec.

(9) The experimental results support the general model of binaural interaction proposed by David *et al.* (1959), and provide some indication of the limitation of an hypothetical "envelope extractor," the output of which must preserve some measure of envelope phase.

## ACKNOWLEDGMENTS

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[1]I am Observer 1.

[2]Informal experiments indicated that the percentage of correct detections of interaural delays with pure tones was roughly a negative exponetial function of the rise/fall time of the 250-msec burst used, reaching an asymptote of constant performance between 20 and 40 msec. 50-msec rise/fall time in these experiments was chosen to be approximatley the shortest rise/fall time at the assymptotic performance level.

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