Human-Computer Interaction

Statistics III

Intermediate Inferential Statistics

Professor Bilge Mutlu

Today's Agenda

» *Contingency analysis*

» *Intermediate inferential statistics*

» Updated format: *Lecture* ➜ *Tutorial* ➜ *Q&A* ⟲

What about when we have nominal output variables?

	Nominal	**Categorical (2+)**	**Ordinal**	**Quantitative Discrete**	**Quantitative Non-Normal**	**Quantitative Normal**
Nominal	Chi-squared, Fisher's	Chi-squared	Chi-squared Trend, Mann-Whitney	Mann-Whitney	Mann-Whitney, log-rank†	Student's *t*
Categorical (2+)	Chi-squared	Chi-squared	Kruskal-Wallis‡	Kruskal-Wallis‡	Kruskal-Wallis‡	ANOVA††
Ordinal	Chi-squared Trend, Mann-Whitney	**	Spearman rank	Spearman rank	Spearman rank	Spearman rank, linear regression*
Quantitative Discrete	Logistic regression	**	**	Spearman rank	Spearman rank	Spearman rank, linear regression
Quantitative Non-Normal	Logistic regression	**	**	**	Plot data-Pearson, Spearman rank	Plot data-Pearson, Spearman rank & linear regression
Quantitative Normal	Logistic regression	**	**	**	Linear regression*	Pearson, linear regression

Contingency analysis

In contingency analysis, we calculate a chi-squared, X^2, statistic:

$$X^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

X^2 is the Pearson's test statistic, n is the number of observations, O_i is the observed frequency, and E_i is the expected frequency.

Data is summarized in a **contingency table** that cross-tabulates multivariate frequency distributions of variables in a matrix format.

	Reported.Gaze.Cue	
Robot	No	Yes
Geminoid	10	3
Robovie	3	10

Chi-squared test in R

```
gaze <- read.table('robot-gaze.csv', sep=",", header=TRUE)
chisq.test(table(gaze))
```

```
Pearson's Chi-squared test with Yates' continuity correction

data:  table(gaze)
X-squared = 5.5385, df = 1, p-value = 0.0186
```

Chi-squared test in JMP

Analyze > Fit X by Y

Tutorial, Q&A

Multifactorial analysis

	Nominal	**Categorical (2+)**	**Ordinal**	**Quantitative Discrete**	**Quantitative Non-Normal**	**Quantitative Normal**
Nominal	Chi-squared, Fisher's	Chi-squared	Chi-squared Trend, Mann-Whitney	Mann-Whitney	Mann-Whitney, log-rank†	Student's *t*
Categorical (2+)	Chi-squared	Chi-squared	Kruskal-Wallis‡	Kruskal-Wallis‡	Kruskal-Wallis‡	ANOVA††
Ordinal	Chi-squared Trend, Mann-Whitney	**	Spearman rank	Spearman rank	Spearman rank	Spearman rank, linear regression*
Quantitative Discrete	Logistic regression	**	**	Spearman rank	Spearman rank	Spearman rank, linear regression
Quantitative Non-Normal	Logistic regression	**	**	**	Plot data-Pearson, Spearman rank	Plot data-Pearson, Spearman rank & linear regression
Quantitative Normal	Logistic regression	**	**	**	Linear regression*	Pearson, linear regression

Consider this dataset. Can we use multiple t-tests?

Participant ID	Group	Time	Coding
Participant 01	Standard	245	0
Participant 02	Standard	236	0
Participant 03	Standard	321	0
Participant 04	Standard	212	0
Participant 05	Standard	267	0
Participant 06	Standard	334	0
Participant 07	Standard	287	0
Participant 08	Standard	259	0
Participant 09	Prediction	246	1
Participant 10	Prediction	213	1
Participant 11	Prediction	265	1
Participant 12	Prediction	189	1
Participant 13	Prediction	201	1
Participant 14	Prediction	197	1
Participant 15	Prediction	289	1
Participant 16	Prediction	224	1
Participant 17	Speech-based dictation	178	2
Participant 18	Speech-based dictation	289	2
Participant 19	Speech-based dictation	222	2
Participant 20	Speech-based dictation	189	2
Participant 21	Speech-based dictation	245	2
Participant 22	Speech-based dictation	311	2
Participant 23	Speech-based dictation	267	2
Participant 24	Speech-based dictation	197	2

$H_0 : \mu_1 = \mu_2 = \mu_3 , \alpha = .05$

3 pairwise tests: $(1 - \alpha)^3 = 0.86$

Reject H_0 when $p < 0.14$ instead of $p < 0.05$

➜ **Type I error** (reject H_0 when it is true)

What are errors in hypothesis testing?

Type I error: Rejecting H_0 when it is true

Type II error: Accepting H_0 when it is false

Type III error: Correctly rejecting

	H_0 Is true	**H_1 Is true**
Fail to reject H_0	*Right decision*	*Wrong decision* **Type II error** (False negative)
Reject H_0	Wrong decision **Type I error** (False positive)	*Right decision*

Analysis of Variance (ANOVA)

Definition: Analysis of variance (ANOVA) is a collection of statistical models and their associated estimation procedures (such as the "variation" among and between groups) used to analyze the differences among group means in a sample.[1]

Procedures:

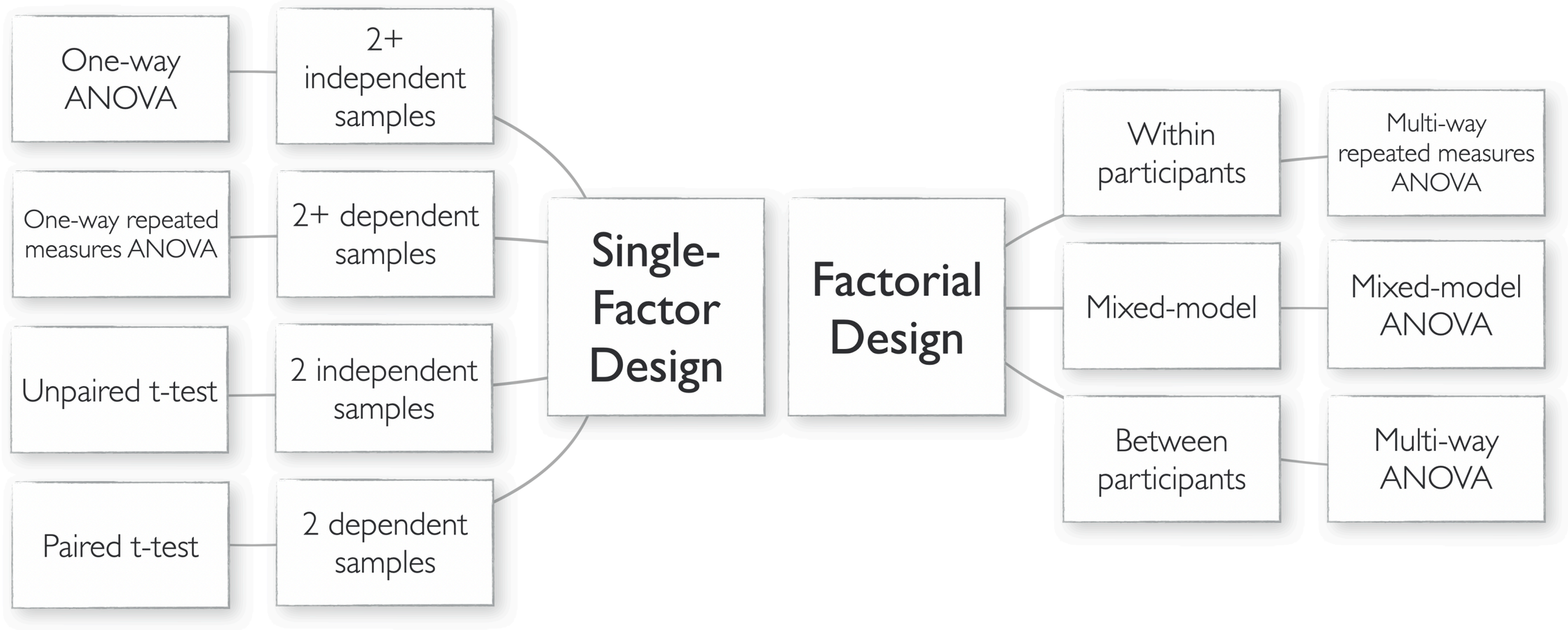
1. One-way (single factor)
2. Two-way (two factors)
3. Multi-way (multiple factors)

Models:

1. Fixed effects (between)
2. Random effects (within)
3. Mixed effects (mixed)

[1] Wikipedia: ANOVA

How do we choose among these procedures?



How do we conduct ANOVA?

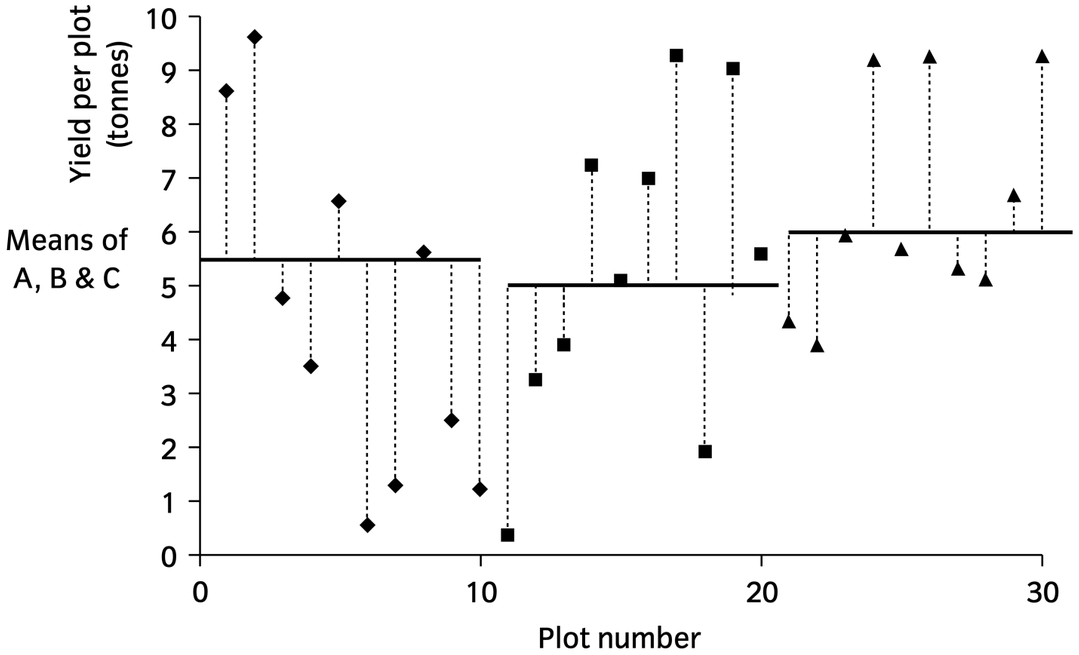
We calculate the *F*-statistic.

$$F = \frac{\sigma_{explained}}{\sigma_{unexplained}} = \frac{SS_{treatment}/(k-1)}{SS_{error}/(n-k)}$$

$$F = \frac{\sum n_i (M_i - \sum (Mi/k))^2/(k-1)}{\sum\sum (X_{it} - M_i)^2/(n-k)}$$

k: number of populations

n: sample size



One-way ANOVA in R

```
model = aov(Time~Group,data=data)
summary(model)
```

```
             Df Sum Sq Mean Sq F value Pr(>F)
Group         2   7842    3921   2.174  0.139
Residuals    21  37880    1804
```

One-way ANOVA in JMP

Analyze > Fit X by Y

Oneway Anova

Summary of Fit

Rsquare	0.171518
Adj Rsquare	0.092615
Root Mean Square Error	42.47149
Mean of Response	245.125
Observations (or Sum Wgts)	24

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Group	2	7842.250	3921.13	2.1738	0.1387
Error	21	37880.375	1803.83		
C. Total	23	45722.625			

Means for Oneway Anova

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
Prediction	8	228.000	15.016	196.77	259.23
Speech-based dictation	8	237.250	15.016	206.02	268.48
Standard	8	270.125	15.016	238.90	301.35

Std Error uses a pooled estimate of error variance

Are we done?

The ANOVA analysis only told us whether the *methods* had a significant effect on *time*, not which method is more effective.

We can make two types of *pairwise* comparisons:

1. *A priori* comparisons (planned contrasts)

   $H_0: \mu_1 = \mu_2; H_1: \mu_1 > \mu_2$

2. *Post hoc* comparisons (exploratory pairwise tests)

   Test μ_1 vs μ_2, μ_1 vs μ_3, μ_2 vs μ_3

A priori comparisons in R

```
levels(data$Group)
comparison = c(1,-1,0)
mat = cbind(comparison)
contrasts(data$Group) <- mat
model = aov(Time~Group, data= data)
summary.aov(model, split = list(Group=list("mu1 vs mu2"=1)))
```

```
                    Df Sum Sq Mean Sq F value Pr(>F)
Group                2   7842    3921   2.174  0.139
  Group: mu1 vs mu2  1    342     342   0.190  0.668
Residuals           21  37880    1804
```

A prior comparisons in JMP

Compare Means > Each pair, Student's t

Means Comparisons

Comparisons for each pair using Student's t

Confidence Quantile

t	Alpha
2.07961	0.05

LSD Threshold Matrix

Abs(Dif)-LSD

	Standard	Speech-based dictation	Prediction
Standard	-44.162	-11.287	-2.037
Speech-based dictation	-11.287	-44.162	-34.912
Prediction	-2.037	-34.912	-44.162

Positive values show pairs of means that are significantly different.

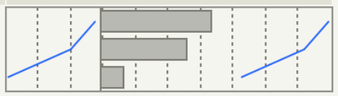
Connecting Letters Report

Level		Mean
Standard	A	270.12500
Speech-based dictation	A	237.25000
Prediction	A	228.00000

Levels not connected by same letter are significantly different.

Ordered Differences Report

Level	- Level	Difference	Std Err Dif	Lower CL	Upper CL	p-Value
Standard	Prediction	42.12500	21.23574	-2.0371	86.28715	0.0605
Standard	Speech-based dictation	32.87500	21.23574	-11.2871	77.03715	0.1365
Speech-based dictation	Prediction	9.25000	21.23574	-34.9121	53.41215	0.6676



Post hoc comparison in R

```
TukeyHSD(model)
```

```
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = Time ~ Group, data = data)

$Group
                                        diff       lwr      upr     p adj
Speech-based dictation-Prediction      9.250 -44.27619 62.77619 0.9011856
Standard-Prediction                   42.125 -11.40119 95.65119 0.1409733
Standard-Speech-based dictation       32.875 -20.65119 86.40119 0.2896872
```

Post hoc comparison in JMP

Compare Means > All Pairs, Tukey HSD

Comparisons for all pairs using Tukey-Kramer HSD

Confidence Quantile

q*	Alpha
2.52057	0.05

HSD Threshold Matrix

Abs(Dif)-HSD

	Standard	Speech-based dictation	Prediction
Standard	-53.526	-20.651	-11.401
Speech-based dictation	-20.651	-53.526	-44.276
Prediction	-11.401	-44.276	-53.526

Positive values show pairs of means that are significantly different.

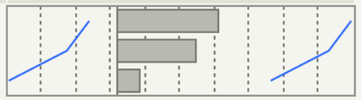
Connecting Letters Report

Level		Mean
Standard	A	270.12500
Speech-based dictation	A	237.25000
Prediction	A	228.00000

Levels not connected by same letter are significantly different.

Ordered Differences Report

Level	- Level	Difference	Std Err Dif	Lower CL	Upper CL	p-Value
Standard	Prediction	42.12500	21.23574	-11.4012	95.65119	0.1410
Standard	Speech-based dictation	32.87500	21.23574	-20.6512	86.40119	0.2897
Speech-based dictation	Prediction	9.25000	21.23574	-44.2762	62.77619	0.9012



What if we had a within-participants design?

Participant ID	Group	Time	Coding
Participant 01	Standard	245	0
Participant 01	Prediction	246	1
Participant 01	Speech-based dictation	178	2
Participant 02	Standard	236	0
Participant 02	Prediction	213	1
Participant 02	Speech-based dictation	289	2
Participant 03	Standard	321	0
Participant 03	Prediction	265	1
Participant 03	Speech-based dictation	222	2
Participant 04	Standard	212	0
Participant 04	Prediction	189	1
Participant 04	Speech-based dictation	189	2
Participant 05	Standard	267	0
Participant 05	Prediction	201	1
Participant 05	Speech-based dictation	245	2
Participant 06	Standard	334	0
Participant 06	Prediction	197	1
Participant 06	Speech-based dictation	311	2
Participant 07	Standard	287	0
Participant 07	Prediction	289	1
Participant 07	Speech-based dictation	267	2
Participant 08	Standard	259	0
Participant 08	Prediction	224	1
Participant 08	Speech-based dictation	197	2

We conduct a *repeated-measures* or *random-effects* one-way ANOVA

Within-participants one-way ANOVA in R

```
model = aov(Time~Group+Error(Participant.ID/Group), data= data)
summary(model)

Error: Participant.ID
          Df Sum Sq Mean Sq F value Pr(>F)
Residuals  7  19113    2730

Error: Participant.ID:Group
          Df Sum Sq Mean Sq F value Pr(>F)
Group      2   7842    3921   2.925 0.0868 .
Residuals 14  18767    1341
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

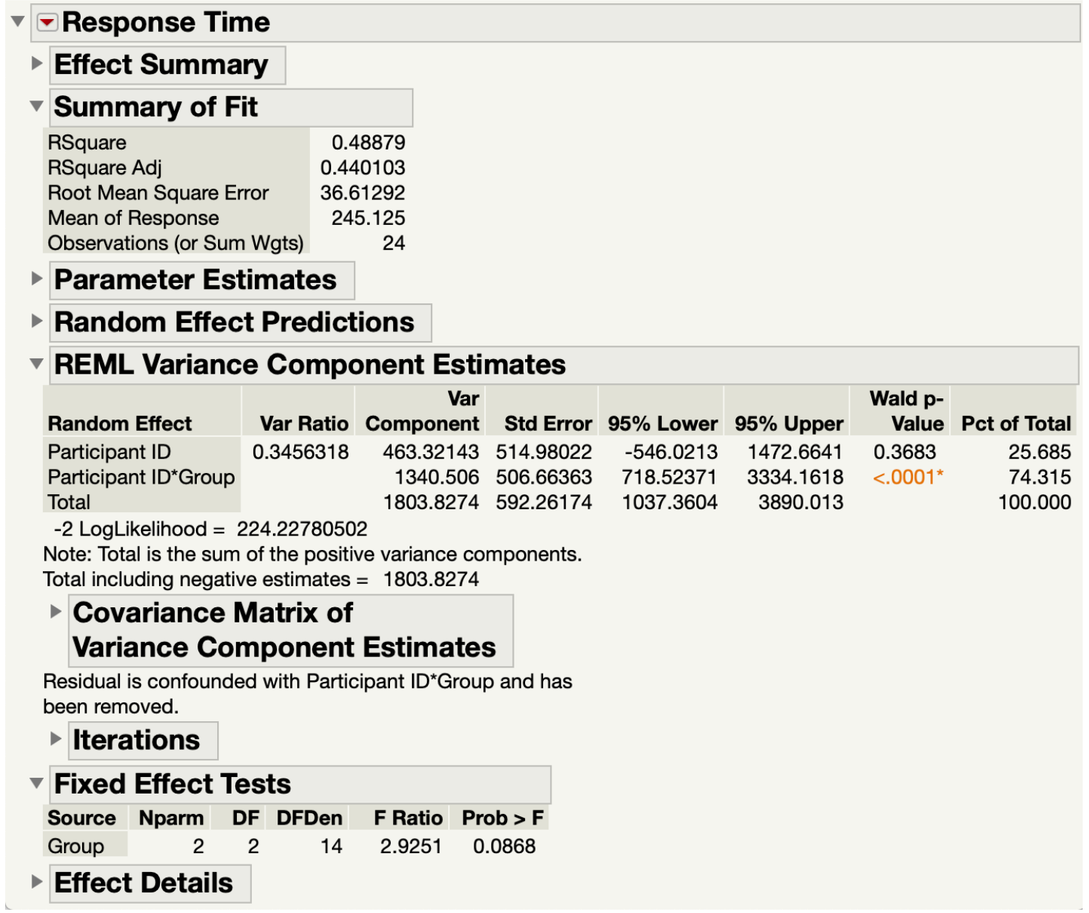
Within-participants one-way ANOVA in JMP

Using the Full Factorial Repeated Measures ANOVA Add-In:

Add-ins > Repeated Measures > Full-Factorial Design (Mixed Effects)

For additional options (e.g., comparisons):

Launch Dialog > Emphasis: Effect Leverage



Response Time

Effect Summary

Summary of Fit

RSquare	0.48879
RSquare Adj	0.440103
Root Mean Square Error	36.61292
Mean of Response	245.125
Observations (or Sum Wgts)	24

Parameter Estimates

Random Effect Predictions

REML Variance Component Estimates

Random Effect	Var Ratio	Var Component	Std Error	95% Lower	95% Upper	Wald p-Value	Pct of Total
Participant ID	0.3456318	463.32143	514.98022	-546.0213	1472.6641	0.3683	25.685
Participant ID*Group		1340.506	506.66363	718.52371	3334.1618	<.0001*	74.315
Total		1803.8274	592.26174	1037.3604	3890.013		100.000

-2 LogLikelihood = 224.22780502

Note: Total is the sum of the positive variance components.

Total including negative estimates = 1803.8274

Covariance Matrix of Variance Component Estimates

Residual is confounded with Participant ID*Group and has been removed.

Iterations

Fixed Effect Tests

Source	Nparm	DF	DFDen	F Ratio	Prob > F
Group	2	2	14	2.9251	0.0868

Effect Details

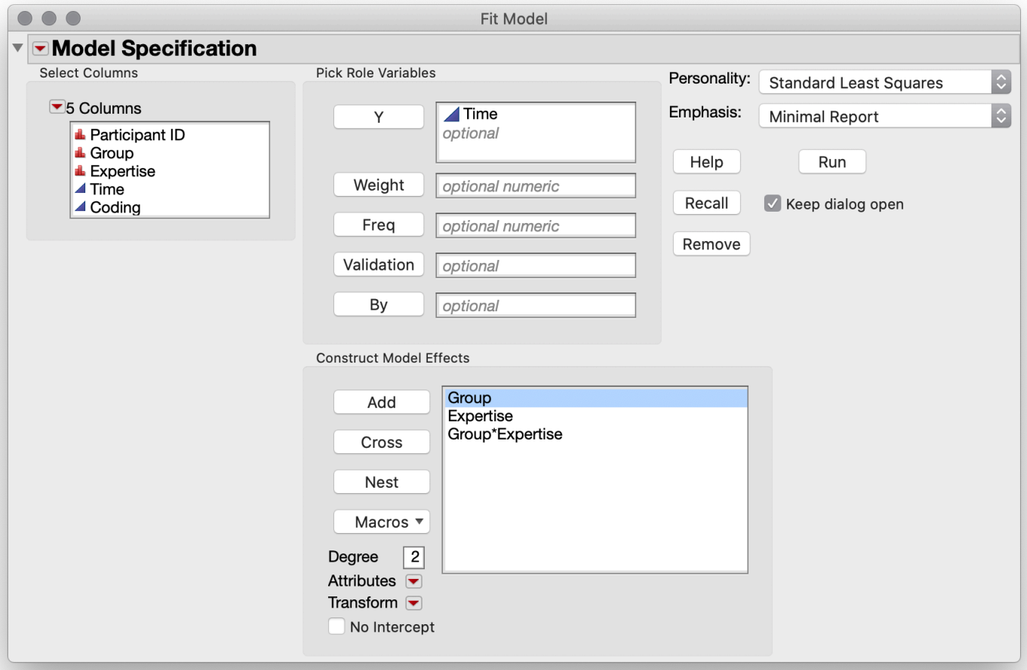
Between-participants two-way ANOVA in R

```
model = aov(Time~Group*Expertise, data=data)
summary(model)
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Group	2	7842	3921	2.175	0.143
Expertise	1	1395	1395	0.774	0.391
Group:Expertise	2	4030	2015	1.117	0.349
Residuals	18	32455	1803		

Between-participants two-way ANOVA in JMP

Analyze > Fit Model



Oneway Anova

Summary of Fit

Rsquare	0.171518
Adj Rsquare	0.092615
Root Mean Square Error	42.47149
Mean of Response	245.125
Observations (or Sum Wgts)	24

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Group	2	7842.250	3921.13	2.1738	0.1387
Error	21	37880.375	1803.83		
C. Total	23	45722.625			

Means for Oneway Anova

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
Prediction	8	228.000	15.016	196.77	259.23
Speech-based dictation	8	237.250	15.016	206.02	268.48
Standard	8	270.125	15.016	238.90	301.35

Std Error uses a pooled estimate of error variance

Within-participants two-way ANOVA in R

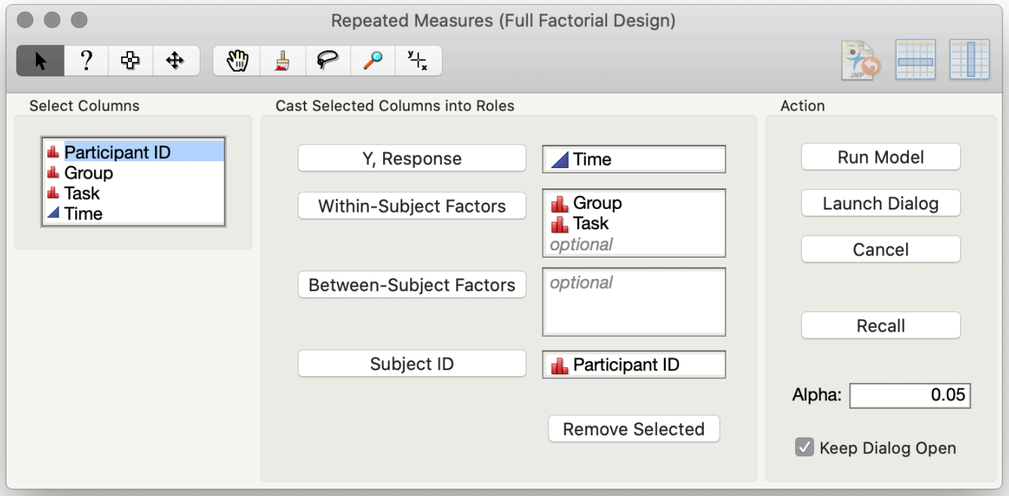
```
model = aov(Time~(Group*Task)+Error(Participant.ID/(Group*Task)), data= data)
summary(model)
```

Participant ID	Group	Task	Time
Participant 01	Standard	Complex	285
Participant 01	Prediction	Complex	160
Participant 01	Speech-based dictation	Complex	201
Participant 01	Standard	Simple	272
Participant 01	Prediction	Simple	191
Participant 01	Speech-based dictation	Simple	161
Participant 02	Standard	Complex	189
Participant 02	Prediction	Complex	250
Participant 02	Speech-based dictation	Complex	178
Participant 02	Standard	Simple	247
Participant 02	Prediction	Simple	288
Participant 02	Speech-based dictation	Simple	180
Participant 03	Standard	Complex	233
Participant 03	Prediction	Complex	285
Participant 03	Speech-based dictation	Complex	225
Participant 03	Standard	Simple	200
Participant 03	Prediction	Simple	202
Participant 03	Speech-based dictation	Simple	162

```
Error: Participant.ID
          Df Sum Sq Mean Sq F value Pr(>F)
Residuals  7   7224    1032

Error: Participant.ID:Group
          Df Sum Sq Mean Sq F value Pr(>F)
Group      2   1650   825.2   0.345  0.714
Residuals 14  33441  2388.6

Error: Participant.ID:Task
          Df Sum Sq Mean Sq F value Pr(>F)
Task       1    341   341.3   0.119   0.74
Residuals  7  20055  2865.0

Error: Participant.ID:Group:Task
           Df Sum Sq Mean Sq F value Pr(>F)
Group:Task  2   1845   922.5   0.644   0.54
Residuals  14  20053  1432.3
```

Within-participants two-way ANOVA in JMP

Add-ins > Repeated Measures > Full-Factorial Design (Mixed Effects)



Summary of Fit

RSquare	0.397171
RSquare Adj	0.325405
Root Mean Square Error	37.84614
Mean of Response	216.625
Observations (or Sum Wgts)	48

Parameter Estimates

Term	Estimate	Std Error	DFDen	t Ratio	Prob>|t|
Intercept	216.625	4.636889	7	46.72	<.0001*
Group[Prediction]	1.6875	9.976255	14	0.17	0.8681
Group[Speech-based dictation]	-7.875	9.976255	14	-0.79	0.4431
Task[Complex]	2.6666667	7.725769	7	0.35	0.7401
Group[Prediction]*Task[Complex]	-2.229167	7.725311	14	-0.29	0.7772
Group[Speech-based dictation]*Task[Complex]	8.4583333	7.725311	14	1.09	0.2920

Random Effect Predictions

REML Variance Component Estimates

Random Effect	Var Ratio	Var Component	Std Error	95% Lower	95% Upper	Wald p-Value	Pct of Total
Participant ID	-0.324559	-464.875	323.07858	-1098.097	168.34739	0.1502	0.000
Participant ID*Group	0.3338216	478.14286	526.34376	-553.4719	1509.7577	0.3637	20.022
Participant ID*Task	0.3334123	477.55655	541.42698	-583.6208	1538.7339	0.3778	19.998
Participant ID*Group*Task		1432.3304	541.36999	767.74244	3562.5512	<.0001*	59.980
Total		2388.0298	687.37698	1457.7831	4611.3193		100.000

-2 LogLikelihood = 455.15548778
Note: Total is the sum of the positive variance components.
Total including negative estimates = 1923.1548

Covariance Matrix of Variance Component Estimates

Residual is confounded with Participant ID*Group*Task and has been removed.

Iterations

Fixed Effect Tests

Source	Nparm	DF	DFDen	F Ratio	Prob > F
Group	2	2	14	0.3455	0.7138
Task	1	1	7	0.1191	0.7401
Group*Task	2	2	14	0.6441	0.5400

Two-way mixed-effects ANOVA in R

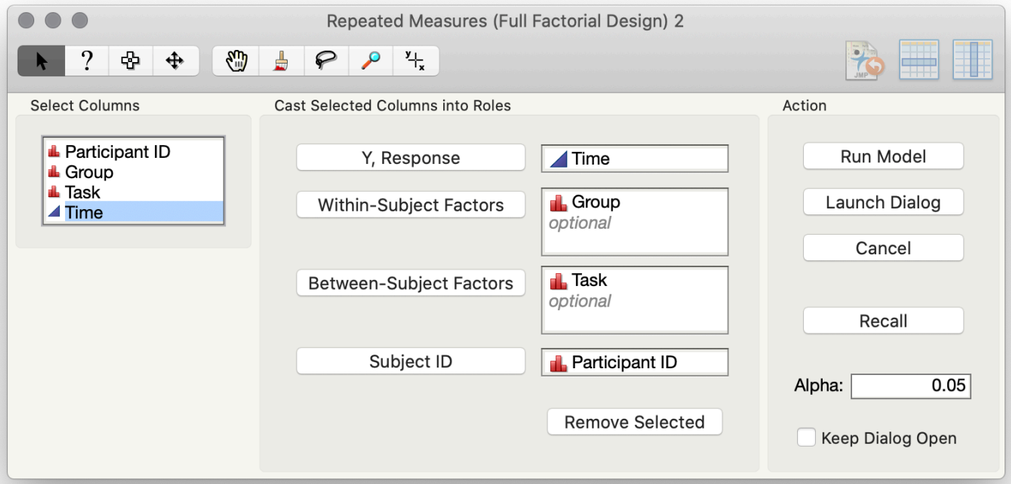
```
model = aov(Time~(Group*Task)+Error(Participant.ID/Group)+Task,data=data)
summary(model)
```

Participant ID	Group	Task	Time
Participant 01	Standard	Complex	285
Participant 01	Prediction	Complex	160
Participant 01	Speech-based dictation	Complex	201
Participant 02	Standard	Simple	272
Participant 02	Prediction	Simple	191
Participant 02	Speech-based dictation	Simple	161
Participant 03	Standard	Complex	189
Participant 03	Prediction	Complex	250
Participant 03	Speech-based dictation	Complex	178
Participant 04	Standard	Simple	247
Participant 04	Prediction	Simple	288
Participant 04	Speech-based dictation	Simple	180
Participant 05	Standard	Complex	233
Participant 05	Prediction	Complex	285
Participant 05	Speech-based dictation	Complex	225
Participant 06	Standard	Simple	200
Participant 06	Prediction	Simple	202
Participant 06	Speech-based dictation	Simple	162

```
Error: Participant.ID
          Df Sum Sq Mean Sq F value Pr(>F)
Task       1    341   341.3   0.175  0.682
Residuals 14  27279  1948.5

Error: Participant.ID:Group
           Df Sum Sq Mean Sq F value Pr(>F)
Group       2   1650   825.2   0.432  0.654
Group:Task  2   1845   922.5   0.483  0.622
Residuals  28  53493  1910.5
```

Two-way mixed-effects ANOVA in JMP

Add-ins > Repeated Measures > Full-Factorial Design (Mixed Effects)



Summary of Fit

RSquare	0.057814
RSquare Adj	-0.05435
Root Mean Square Error	43.70896
Mean of Response	216.625
Observations (or Sum Wgts)	48

Parameter Estimates

Term	Estimate	Std Error	DFDen	t Ratio	Prob>|t|
Intercept	216.625	6.371352	14	34.00	<.0001*
Task[Complex]	2.6666667	6.371352	14	0.42	0.6819
Group[Prediction]	1.6875	8.922054	28	0.19	0.8513
Group[Speech-based dictation]	-7.875	8.922054	28	-0.88	0.3849
Task[Complex]*Group[Prediction]	-2.229167	8.922054	28	-0.25	0.8045
Task[Complex]*Group[Speech-based dictation]	8.4583333	8.922054	28	0.95	0.3512

Random Effect Predictions

REML Variance Component Estimates

Random Effect	Var Ratio	Var Component	Std Error	95% Lower	95% Upper	Wald p-Value	Pct of Total
Participant ID[Task]	0.0066379	12.681548	298.71885	-572.7966	598.15973	0.9661	0.659
Participant ID*Group[Task]		1910.4732	510.59544	1203.1556	3494.4955	<.0001*	99.341
Total		1923.1548	419.68502	1307.4704	3106.8671		100.000

-2 LogLikelihood = 457.81133323

Note: Total is the sum of the positive variance components.

Total including negative estimates = 1923.1548

Covariance Matrix of Variance Component Estimates

Residual is confounded with Participant ID*Group[Task] and has been removed.

Iterations

Fixed Effect Tests

Source	Nparm	DF	DFDen	F Ratio	Prob > F
Task	1	1	14	0.1752	0.6819
Group	2	2	28	0.4319	0.6535
Task*Group	2	2	28	0.4829	0.6221