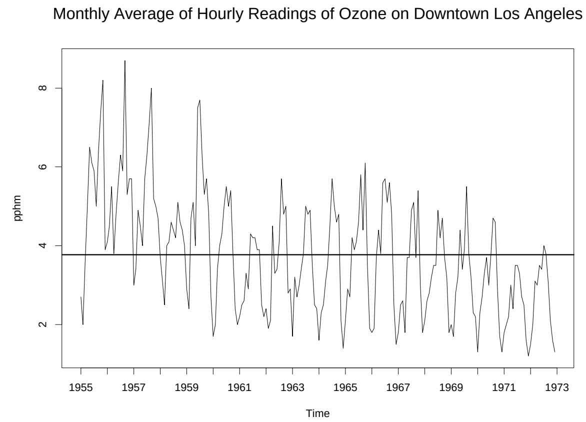
Handout 9

Intervention Models

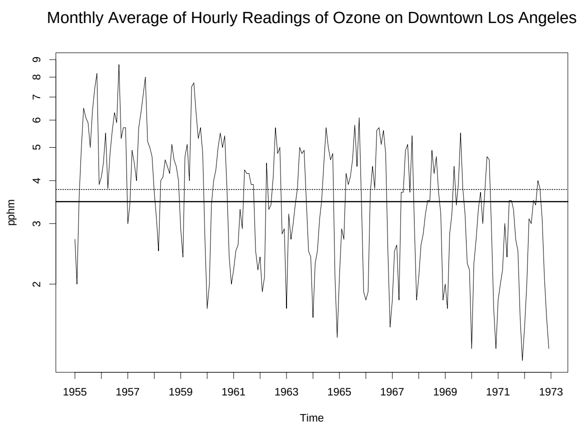
Class notes for Statistics 451: Applied Time Series
Iowa State University

March 29, 2006
18h 20min

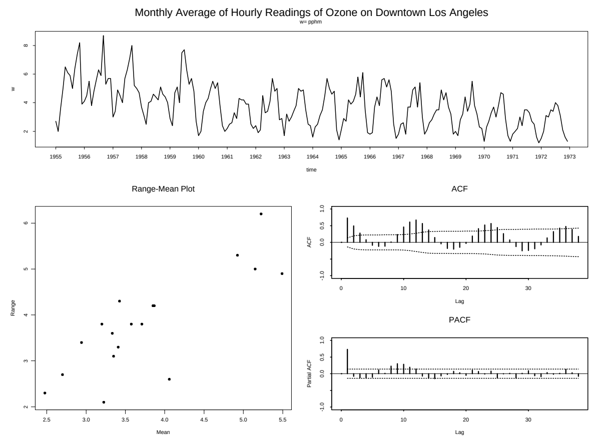
Los Angeles Ozone Data Monthly Averages 1955-1972



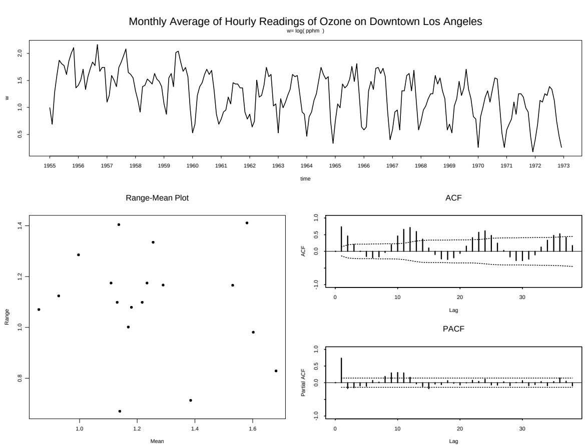
Los Angeles Ozone Data Monthly Averages 1955-1972



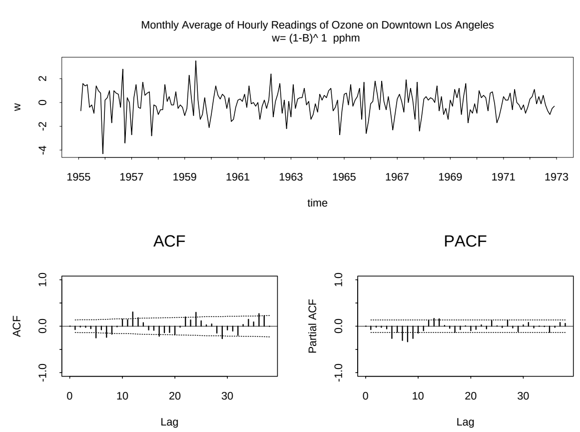
Graphical Output from Function `iden` for the Ozone Data with No Differencing



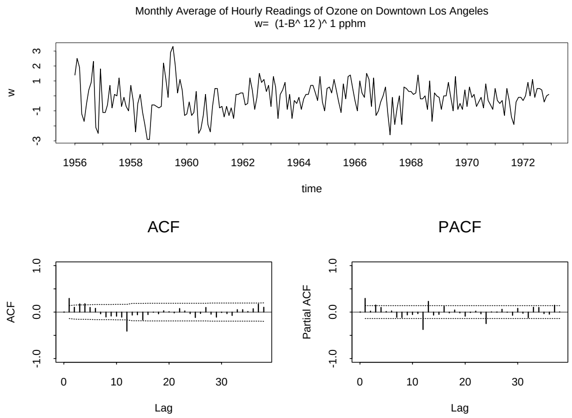
Graphical Output from Function `iden` for the Log Ozone Data with No Differencing



Graphical Output from Function `iden` for the Ozone Data with 1 Regular Difference



Graphical Output from Function `iden` for the Ozone Data with 1 Seasonal Difference



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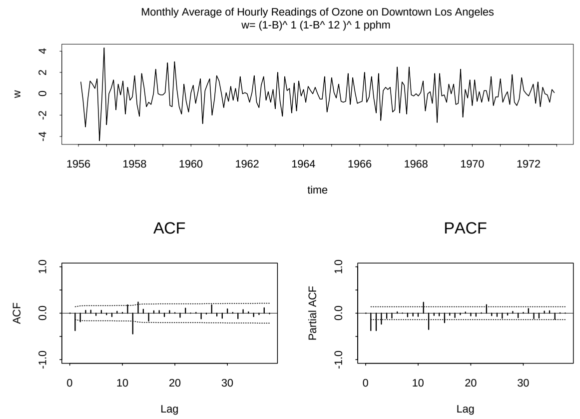
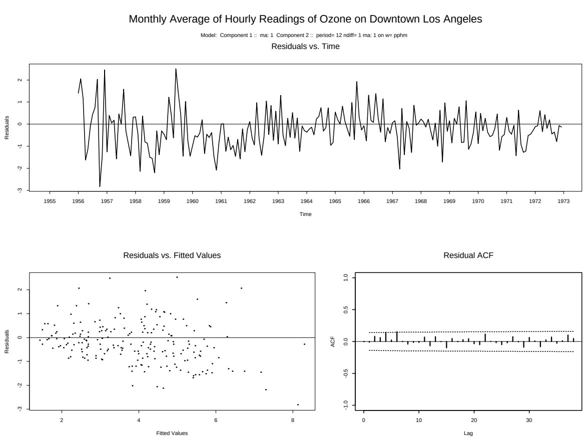
Graphical Output from Function `iden` for the Ozone Data with 1 Regular and 1 Seasonal Difference



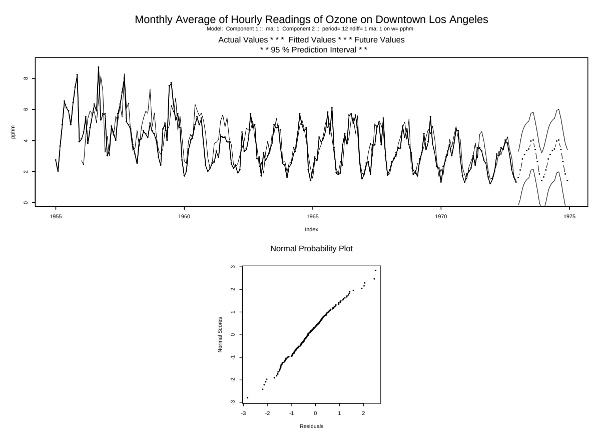
9 - 8

Graphical Output from Function `esti` for the Ozone Data SARIMA$(0,0,1)(0,1,1)_{12}$ Model, Part 1



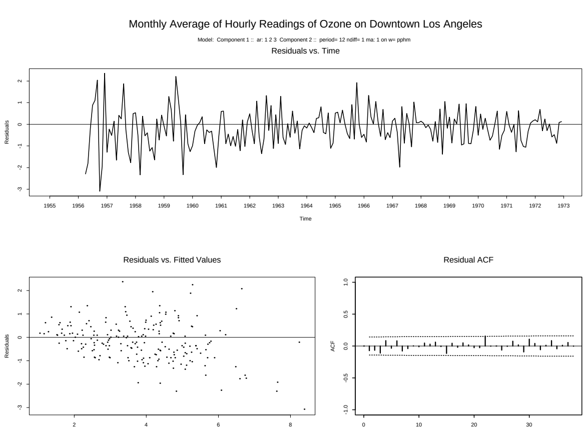
9 - 9

Graphical Output from Function `esti` for the Ozone Data SARIMA$(0,0,1)(0,1,1)_{12}$ Model, Part 2



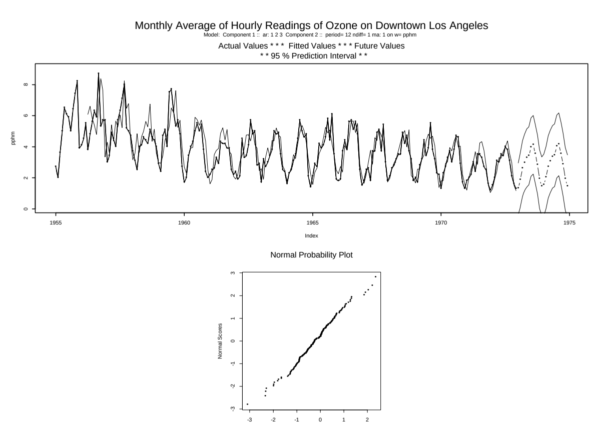
9 - 10

Graphical Output from Function `esti` for the Ozone Data SARIMA$(3,0,0)(0,1,1)_{12}$ Model, Part 1



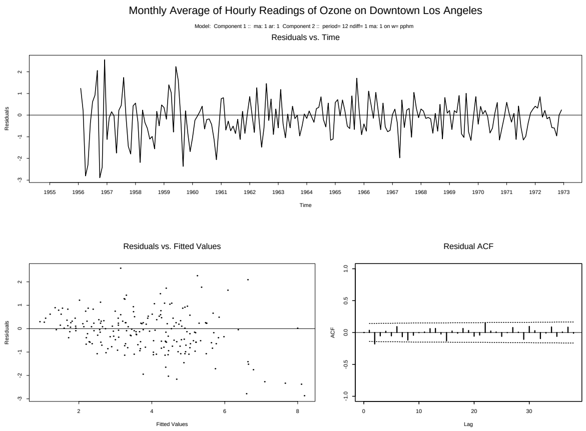
9 - 11

Graphical Output from Function `esti` for the Ozone Data SARIMA$(3,0,0)(0,1,1)_{12}$ Model, Part 2

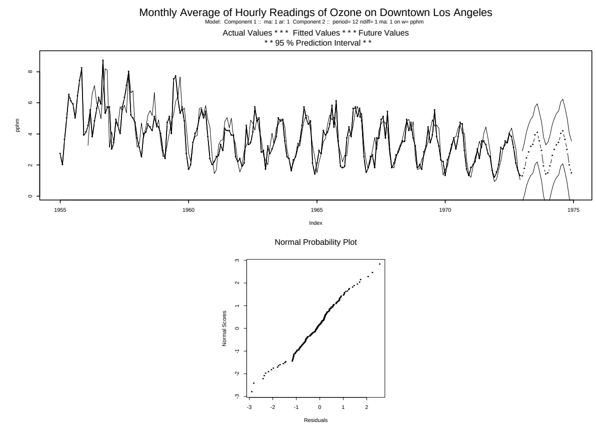


9 - 12

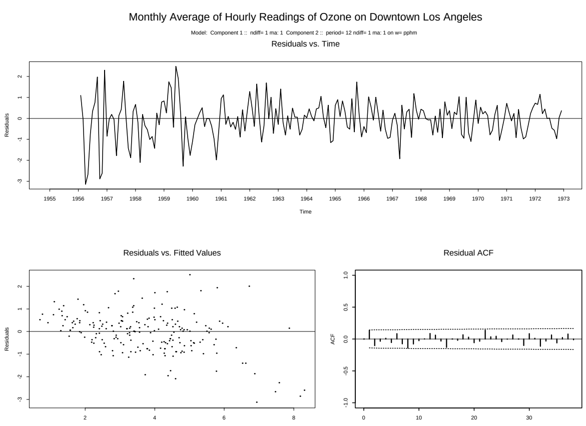
Graphical Output from Function `esti` for the Ozone Data SARIMA$(1,0,1)(0,1,1)_{12}$ Model, Part 1



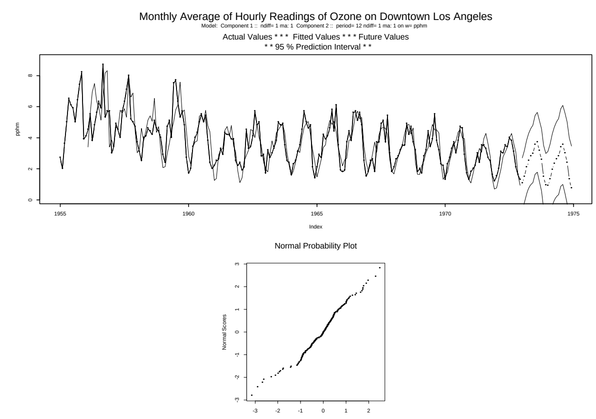
Graphical Output from Function `esti` for the Ozone Data SARIMA$(1,0,1)(0,1,1)_{12}$ Model, Part 2



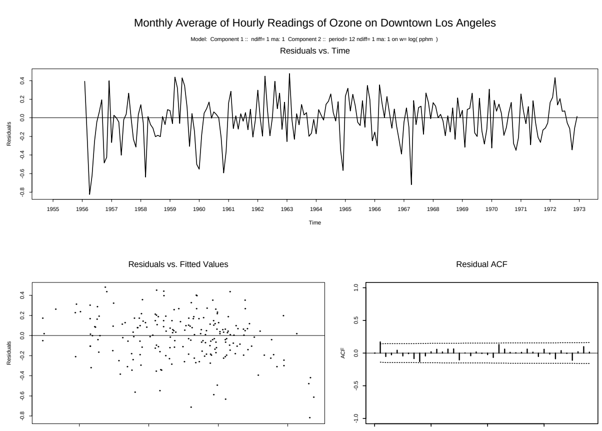
Graphical Output from Function `esti` for the Ozone Data SARIMA$(0,1,1)(0,1,1)_{12}$ Model, Part 1



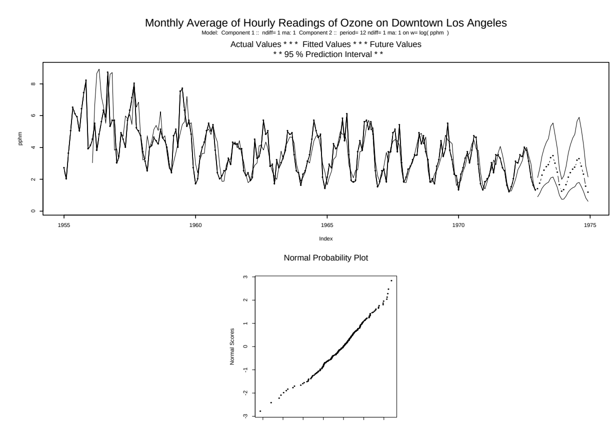
Graphical Output from Function `esti` for the Ozone Data SARIMA$(0,1,1)(0,1,1)_{12}$ Model, Part 2



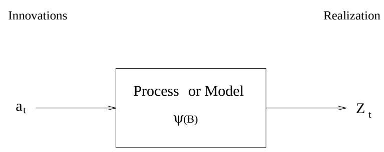
Graphical Output from Function `esti` for the Ozone Data with Log Transformation and SARIMA$(0,1,1)(0,1,1)_{12}$ Model, Part 1



Graphical Output from Function `esti` for the Ozone Data with Log Transformation and SARIMA$(0,1,1)(0,1,1)_{12}$ Model, Part 2



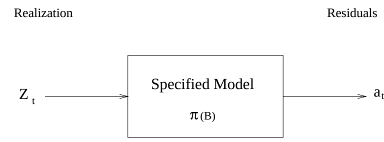
Innovation to Realization Filter



Model:

$$
\begin{aligned}
(1-\mathrm{B}^{12})Z_t &= (1-\theta_1\mathrm{B})(1-\Theta_1\mathrm{B}^{12})a_t \\
Z_t &= \frac{(1-\theta_1\mathrm{B})(1-\Theta_1\mathrm{B}^{12})}{(1-\mathrm{B}^{12})}a_t = \psi(\mathrm{B})a_t \\
Z_t &= a_t + \psi_1 a_{t-1} + \psi_2 a_{t-2} + \cdots
\end{aligned}
$$

Realization to Residual Filter



Model:

$$
\begin{aligned}
(1-\mathrm{B}^{12})Z_t &= (1-\theta_1\mathrm{B})(1-\Theta_1\mathrm{B}^{12})a_t \\
a_t &= \frac{(1-\mathrm{B}^{12})}{(1-\theta_1\mathrm{B})(1-\Theta_1\mathrm{B}^{12})}Z_t = \pi(\mathrm{B})Z_t
\end{aligned}
$$

$$
a_t = Z_t - \pi_1 Z_{t-1} - \pi_2 Z_{t-2} - \cdots
$$

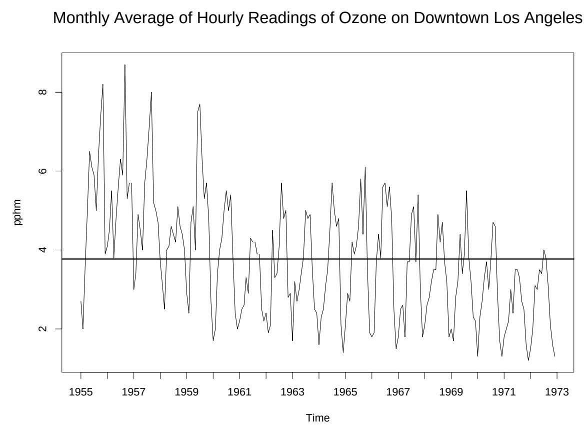
Time Series Intervention Modeling

Include regression (usually dummy variables) terms to account for permanent or temporary changes in the process.

Some Applications:

- Labor strike
- Onset of the "energy crisis."
- Political party in power
- Effect of promotional events on sales
- Modeling aberration in data
  - ▶ Outliers (accommodation)
  - ▶ Missing observations (imputation)

Los Angeles Ozone Data Monthly Averages 1955-1972



Changes in the Ozone Process

- Golden State Freeway opened January 1960
- Starting in 1966 all new cars had to have air pollution controls

$$
\begin{aligned}
I_{1t} &= \begin{cases} 0, & t < \text{January 1960}, \\ 1, & t \geq \text{January 1960}, \end{cases} \\
I_{2t} &= \begin{cases} 1, & \text{June-October (warm months) beginning 1966}, \\ 0, & \text{otherwise}, \end{cases} \\
I_{3t} &= \begin{cases} 1, & \text{November-May (cool months) beginning 1966}, \\ 0, & \text{otherwise}, \end{cases}
\end{aligned}
$$

Possible Input Variables for Intervention Analysis

- Step function beginning at time T

$$
S_t^{(T)} = \begin{cases} 0, & t < T, \\ 1, & t \geq T, \end{cases}
$$

Example with $T = 10$

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$S_t^{(10)}$	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1

- Impulse (or pulse) function at time T

$$
P_t^{(T)} = (1-\mathrm{B})S_t^{(T)} = S_t^{(T)} - S_{t-1}^{(T)} = \begin{cases} 1, & t = T, \\ 0, & \text{otherwise}, \end{cases}
$$

Example with $T = 10$

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$P_t^{(10)}$	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0

Possible Input Variables for Intervention Analysis

- Step function beginning at time T

$$S_t^{(T)} = \begin{cases} 0, & t < T, \\ 1, & t \geq T, \end{cases}$$

Example with $T = 10$

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$S_t^{(10)}$	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1

- Ramp function beginning at time T

$$R_t^{(T)} = \frac{S_t^{(T)}}{(1-\mathrm{B})} = R_{t-1}^{(T)} + S_t^{(T)} = \begin{cases} t - T + 1, & t \geq T, \\ 0, & \text{otherwise}, \end{cases}$$

Example with $T = 10$

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$R_t^{(10)}$	0	0	0	0	0	0	0	0	0	1	2	3	4	5	6

Response to Different Kinds of Inputs

Input	Model Term $\omega_1\mathrm{B}$	$\frac{\omega_1\mathrm{B}}{(1-\mathrm{B})}$	$\frac{\omega_1\mathrm{B}}{(1-\delta_1\mathrm{B})}$
$P_t^{(T)}$	Pulse ω_1	Step ω_1	$100\delta_1$ percent increase (decay) from ω_1 to original level
$S_t^{(T)}$	Step ω_1	Ramp ω_1 per unit time	$100\delta_1$ percent increase (decay) from ω_1 to $\frac{\omega_1}{(1-\delta_1)}$
$R_t^{(T)}$	Ramp ω_1 per unit time		

With $0 \leq \delta_1 \leq 1$, if ω_1 is positive, we have exponential (percent) increase or linear ramp-up. If ω_1 is negative, we have exponential (percent) decrease or linear ramp-down.

Strategy for Identifying Intervention Models

- Look at plot of the time series realization and check times of the external events.
- Fit a univariate ARIMA/SARIMA model to the data. Examine residuals, especially around the time of the external events; use as a preliminary noise model.
- Consider the underlying mechanisms related to the event and their possible effect.
- Identify appropriate terms to describe the effect of the intervention.

Seasonal (Preliminary Noise) Model for the Ozone Data

$$\begin{aligned}(1-\mathrm{B}^{12})Z_t &= (1-\theta_1\mathrm{B})(1-\Theta_1\mathrm{B}^{12})\,a_t \\ Z_t &= \frac{(1-\theta_1\mathrm{B})(1-\Theta_1\mathrm{B}^{12})}{(1-\mathrm{B}^{12})}\,a_t = \psi_N(\mathrm{B})\,a_t = N_t\end{aligned}$$

Unscrambled model

$$Z_t = Z_{t-12} - \theta_1 a_{t-1} - \Theta_1 a_{t-12} + \theta_1\Theta_1 a_{t-13} + a_t$$

Dummy Variables and "Integrated" Dummy Variables

t	I_{1t}	I_{2t}	I_{3t}	$I_{2t}^* = \frac{I_{2t}}{(1-\mathrm{B}^{12})}$	$I_{3t}^* = \frac{I_{3t}}{(1-\mathrm{B}^{12})}$	
1	0	0	0	0	0	
2	0	0	0	0	0	
⋮	⋮	⋮	⋮	⋮	⋮	
60	0	0	0	0	0	freeway opens
61	1	0	0	0	0	
62	1	0	0	0	0	
⋮	⋮	⋮	⋮	⋮	⋮	
137	1	0	0	0	0	automobile
138	1	1	0	1	0	emissions
139	1	1	0	1	0	law passed
140	1	1	0	1	0	
141	1	1	0	1	0	
142	1	1	0	1	0	
143	1	0	1	0	1	
144	1	0	1	0	1	

Continued

Dummy Variables and "Integrated" Dummy Variables

t	I_{1t}	I_{2t}	I_{3t}	$I_{2t}^* = \frac{I_{2t}}{1-\mathrm{B}^{12}}$	$I_{3t}^* = \frac{I_{3t}}{1-\mathrm{B}^{12}}$	
145	1	0	1	0	1	
146	1	0	1	0	1	
147	1	0	1	0	1	
148	1	0	1	0	1	
149	1	0	1	0	1	
150	1	1	0	2	0	
151	1	1	0	2	0	
152	1	1	0	2	0	
⋮	⋮	⋮	⋮	⋮	⋮	
216	1	0	1	0	7	last observation
217	1	0	1	0	7	dummy variables
218	1	0	1	0	7	for forecasting
⋮	⋮	⋮	⋮	⋮	⋮	
236	1	1	0	9	0	
237	1	1	0	9	0	
238	1	1	0	9	0	
239	1	0	1	0	9	
240	1	0	1	0	9	

Seasonal Intervention Model for the Ozone Data

$$
\begin{aligned}
Z_t &= \omega_1 I_{1t} + \frac{\omega_2}{(1-\mathrm{B}^{12})} I_{2t} + \frac{\omega_3}{(1-\mathrm{B}^{12})} I_{3t} + \frac{(1-\theta_1\mathrm{B})(1-\Theta_1\mathrm{B}^{12})}{(1-\mathrm{B}^{12})} a_t \\
Z_t &= \omega_1 I_{1t} + \frac{\omega_2}{(1-\mathrm{B}^{12})} I_{2t} + \frac{\omega_3}{(1-\mathrm{B}^{12})} I_{3t} + N_t
\end{aligned}
$$

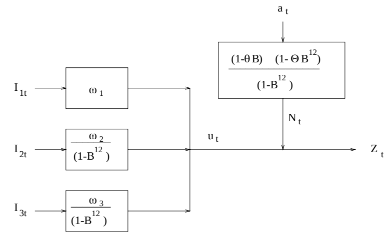
Unscrambled model

$$
(1-\mathrm{B}^{12})Z_t = \omega_1(1-\mathrm{B}^{12})I_{1t} + \omega_2 I_{2t} + \omega_3 I_{3t} + (1-\theta_1\mathrm{B})(1-\Theta_1\mathrm{B}^{12})\,a_t
$$

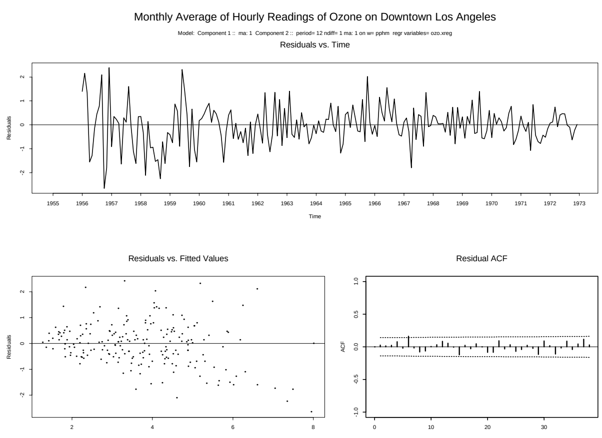
$$
Z_t = Z_{t-12} + \omega_1(I_{1t} - I_{1t-12}) + \omega_2 I_{2t} + \omega_3 I_{3t} - \theta_1 a_{t-1} - \Theta_1 a_{t-12} + \theta_1\Theta_1 a_{t-13} + a_t
$$

Ozone Intervention Model

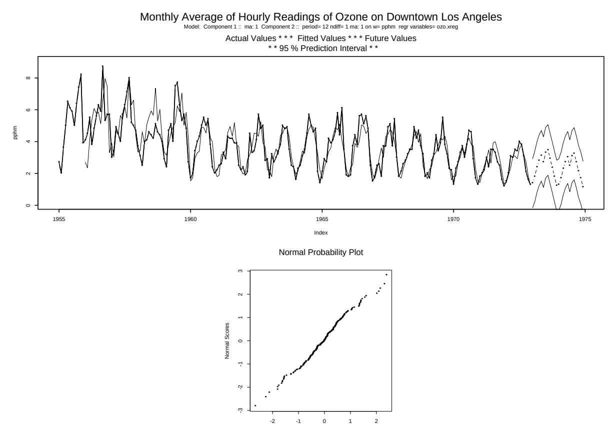
$$
\begin{aligned}
Z_t &= \omega_1 I_{1t} + \frac{\omega_2}{(1-\mathrm{B}^{12})} I_{2t} + \frac{\omega_3}{(1-\mathrm{B}^{12})} I_{3t} + \frac{(1-\theta_1\mathrm{B})(1-\Theta_1\mathrm{B}^{12})}{(1-\mathrm{B}^{12})} a_t \\
Z_t &= \omega_1 I_{1t} + \frac{\omega_2}{(1-\mathrm{B}^{12})} I_{2t} + \frac{\omega_3}{(1-\mathrm{B}^{12})} I_{3t} + N_t
\end{aligned}
$$



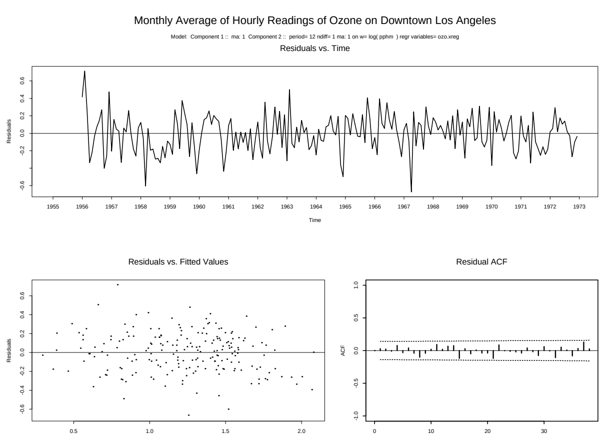
Graphical Output from Function `esti` for the Ozone Data with Intervention $\mathrm{SARIMA}(0,1,1)(0,1,1)_{12}$ Model, Part 1



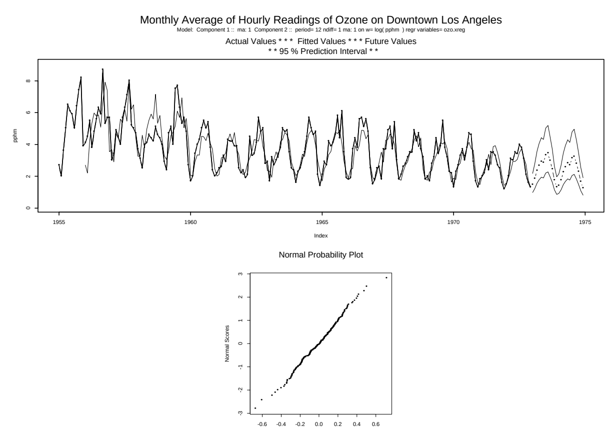
Graphical Output from Function `esti` for the Ozone Data with Intervention $\mathrm{SARIMA}(0,1,1)(0,1,1)_{12}$ Model, Part 2



Graphical Output from Function `esti` for the Ozone Data with Log Transformation and Intervention $\mathrm{SARIMA}(0,1,1)(0,1,1)_{12}$ Model, Part 1



Graphical Output from Function `esti` for the Ozone Data with Log Transformation and Intervention $\mathrm{SARIMA}(0,1,1)(0,1,1)_{12}$ Model, Part 2



Unscrambled Seasonal Intervention Models

Overall:

$$Z_t = Z_{t-12} + \omega_1(I_{1t} - I_{1t-12}) + \omega_2 I_{2t} + \omega_3 I_{3t} - \theta_1 a_{t-1} - \Theta_1 a_{t-12} + \theta_1\Theta_1 a_{t-13} + a_t$$

Before 1960 and between 1961 and 1966

$$Z_t = Z_{t-12} - \theta_1 a_{t-1} - \Theta_1 a_{t-12} + \theta_1\Theta_1 a_{t-13} + a_t$$

In 1960

$$Z_t = Z_{t-12} + \omega_1 - \theta_1 a_{t-1} - \Theta_1 a_{t-12} + \theta_1\Theta_1 a_{t-13} + a_t$$

All warm months after 1966

$$Z_t = Z_{t-12} + \omega_2 - \theta_1 a_{t-1} - \Theta_1 a_{t-12} + \theta_1\Theta_1 a_{t-13} + a_t$$

All cool months after 1966

$$Z_t = Z_{t-12} + \omega_3 - \theta_1 a_{t-1} - \Theta_1 a_{t-12} + \theta_1\Theta_1 a_{t-13} + a_t$$