I. Geometric Distributions.

Ex. A rep from the NFL's MKting Div randomly selects people on a random street in Kansas City, Kansas until he finds a person who attended the last home football game. Let p, the prob. that he succeeds in finding such a person, equal .2. And, let X denote the number of people he selects until he finds his first success. What is the pdf of X?

$p = P(\text{success}) = 0.2$

$1-p = P(\text{failure}) = 0.8$

Let X = # of people selected until first success.

One scenario :- $\underbrace{FFF\ldots F}_{x-1 \text{ failures}}S$

$$P(X=x) = (1-p)^{x-1}p, \quad 0 \le p \le 1, \quad x = 1, 2, \ldots$$

Def:- Assume Bernoulli trials – that is,

1. Two possible outcomes
2. Trials are ind.
3. p, the prob of success, remains the same from trial to trial

Let X = # of trials to first success.

Then, the pdf of X is

$$f(x) = P(X = x) = (1-p)^{x-1} p$$

{Geometric Dist}

$0 \le p \le 1$

$x = 1, \ldots$

$$E X = \frac{1}{p}$$

$$V X = \frac{1-p}{p^2}$$

Ex:- Look at the example we started with.

What is the prob. that the marketing rep must select 4 people before he finds one who attended the last home football game.

$$P(X = 4) = 0.8^3 \times 0.2 = 10.24\%$$

What is the prob. that the marketing rep. must select more than 6 people before he finds one who attended the last football game?.

$$P(X > 6) = 1 - P(X \leq 6)$$

$$P(X \leq 6) = P(1) + P(2) + \cdots + P(6)$$

$$= 0.2 + 0.8 \times 0.2 + \cdots + 0.8^5 \times 0.2$$

$$= 0.2\left[1 + 0.8 + 0.8^2 + 0.8^3 + 0.8^4 + 0.8^5\right]$$

$$= 0.7378$$

$$\therefore P(X > 6) = 1 - 0.7378$$

$$\approx 26.2\%$$

Expected # of people before the rep finds his first "success"?.

$$EX = \frac{1}{0.2} = 5$$

The Negative Binomial Distribution (4)

A rep from the NFL's mkting div. randomly selects people on a random street in Kansas City, Kansas until he finds a person who attended the last home football game. Let p, the prob. that he succeeds in finding such a person, equal 0.20. Now, Let X denote the # of people he selects until he finds $r=3$ who attended the last home football game. What is the prob. that $X=10$?

$p = P(\text{succ}) = 0.20$ $\quad 1-p = P(\text{fails}) = 0.80$

ex. an outcome

$$\{1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\}\ 10$$
$$F\ F\ \underset{S}{\uparrow}\ F\ F\ F\ \underset{S}{\uparrow}\ F\ F\ S$$

Prob. of this event

7 failures

$$P(X=10) = \binom{9}{2}(0.8)^{7}(0.2)^{3}$$

$$P(X=x) = \binom{x-1}{r-1}(1-p)^{x-r}(p)^{r}$$

$x = r, r+1, \ldots$

Def:- Assume bernoulli trials
- 1. 2 possible outcomes
- 2. Ind. trials
- 3. Prob of success, p, remains the same from one trial to the next.

Let X denote the # of trials until the r^{th} success. Then

$$f(x) = P(X=x) = \binom{x-1}{r-1} (1-p)^{x-r} p^r$$

$$0 \le p \le 1 \; ; \quad x = r, r+1, r+2, \ldots$$

$$EX = \frac{r}{p} \qquad VX = \frac{r(1-p)}{p^2}$$

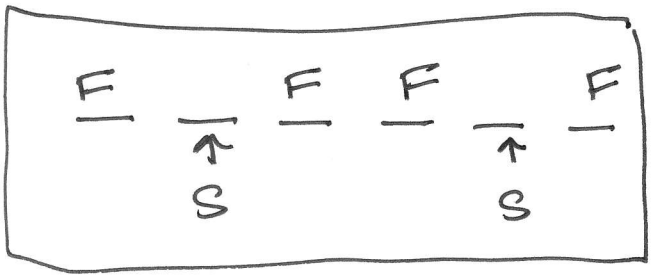
ex. An oil Co. conducts a geological study that indicates that an exploratory oil well should have a 20% chance of striking oil. What is the probability that the first strike comes on the 3rd well drilled?

$$P(X=3) = \binom{3-1}{1-1} (1-p)^{3-1} p^1$$

$$= \binom{2}{0} \cdot .8^2 \cdot .2$$

$$= .8^2 \cdot .2 = .128 = 12.8\%$$

what is the prob that the 3rd strike comes on the 7th well drilled?



$$\binom{7-1}{3-1} \times .8^{4} \times .2^{3}$$

$$= \binom{6}{2} \times .8^{4} \times .2^{3}$$

$$= 0.049$$

$$\simeq 5\%$$

$$EX = \frac{r}{p} = \frac{3}{.20} = 15$$

what does this mean.

$$VX = \frac{r(1-p)}{p^2} = \frac{3(0.80)}{.20^2} = 60$$