Semi-Log Model

- The slope coefficient measures the relative change in Y for a given absolute change in the value of the explanatory variable (t).

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Semi-Log Model

- Using calculus:

$$
\begin{aligned}
b_2 &= \frac{\partial \ln Y}{\partial t} \\
&= \left(\frac{1}{Y}\right)\left(\frac{\partial Y}{\partial t}\right) \\
&= \frac{\frac{\partial Y}{Y}}{\partial t} \\
&= \frac{\text{relative change in Y}}{\text{absolute change in t}}
\end{aligned}
$$

If we multiply the relative change in Y by 100, we get the percentage change or growth rate in Y for an absolute change in t.

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Log (GDP) 1969-83

- Log(Real GDP) = 6.9636 + 0.0269t
- se (.0151) (.0017)
- R^2 = .95
  - GDP grew at the rate of .0269 per year, or at 2.69 percent per year.
  - Take the antilog of 6.9636 to show that at the beginning of 1969 the estimated real GDP was about 1057 billions of dollars, i.e. at t = 0

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Compound Rate of Growth

- The slope coefficient measures the instantaneous rate of growth
- How do we get r --the compound growth rate?
  - b_2= ln (1 + r)
  - antilog (b_2)= (1 + r)
  - So r = antilog (b_2) - 1
  - So r = antilog(.0269) -1
    - r= 1.0273 - 1 = .0273

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Linear-Trend Model

- Trend model regresses Y on time.
  - $Y_t = b_1 + b_2 t + e_t$
  - This model shows whether GNP is increasing or decreasing over time
  - The model does not give the rate of growth.
  - If $b_2 > 0$, then an upward trend.
  - If $b_2 < 0$, then a downward trend.

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Linear-Trend Example

- GNP = 1040.11 + 34.998t
- se (18.85) (2.07) $R^2 = .95$
  - GNP is increasing at the absolute amount of $35 billion per year.
  - There is a statistically significant upward trend.
- Growth model measures relative performance
- Trend model measures absolute performance

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3. Lin-Log Models

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Lin-Log Model

- The dependent variable is linear, but the explanatory variable is in log form.
  - Used in situations for example where the rate of growth of the money supply affects GNP.

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Lin-Log Example

- $GNP = b_1 + b_2 \ln M + e$
  - The slope coefficient is dGNP/dlnM
    - It measures the absolute change in GNP for a relative change in M.
- If b_2 is 2000, a unit increase in the log of the money supply increases GNP by $2000 billion.
  - Alternatively, a 1% increase in the money supply increases GNP by 2000/100 = $20 billion.

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Lin-Log Example

- In this case, we need to divide by 100 since we are changing the money supply change from a relative change to a percentage change.

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4. Functional Form Summary

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Data

- GNP and money supply over the period 1973-87 in the U.S.
- GNP in billions of dollars = Y
  - Mean = 2791.47
- M2 in billions of dollars = X
  - Mean = 1755.67

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Log-linear model

- lnY = 0.5531 + 0.9882lnX
- How interpret?
  - The slope coefficient is dlnY/dlnX
    - i.e. relative change in Y / relative change in X
  - For a 1% increase in the money supply, the average value of GNP increases by .9882% (almost 1%)

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Log-linear model

- The slope coefficient is also an elasticity:

$$b_2 = \frac{\Delta(\ln Y)}{\Delta(\ln X)} = \frac{\Delta Y * X}{\Delta X * Y}$$

- For intercept: Y = antilog b1.
  - This is the average GNP when lnX = 0.

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Log-Lin Model

- lnY = 6.8616 + 0.00057X
- How interpret?
  - The slope coefficient is dlnY/dX
    - i.e. relative change in Y / absolute change in X
  - For a billion dollars rise in the money supply, the log of GNP rises by .00057 per year.
    - To make a %, multiply by 100: GNP rises by 0.057% per year.

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Log-Lin Model

- How to convert to an elasticity?
  - The slope coefficient is:

$$b_2 = \frac{\partial \ln Y}{\partial X}$$

$$= \left(\frac{1}{Y}\right)\left(\frac{\partial Y}{\partial X}\right)$$

To get an elasticity multiply by $\overline{X}$

$.00057(1755.67) = 1.0007$

So a 1% increase in the money supply leads to a 1.0007% increase in GNP

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Lin-Log Model

- Y = -16329.0 + 2584.8lnX
- How interpret?
  - The slope coefficient is dY/dlnX
    - i.e. absolute change in Y/ relative change in X
  - A unit increase in the log of the money supply increases GNP by 2584.8 billion dollars.
    - If money supply rises by 1%, GNP rises by $26 billion dollars.

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Lin-Log Model

- How to convert to an elasticity?
  - The slope coefficient is:

$$b_2 = \frac{\partial \mathrm{Y}}{\partial \ln \mathrm{X}}$$

$$= \left(\frac{\mathrm{X}}{1}\right)\left(\frac{\partial \mathrm{Y}}{\partial \mathrm{X}}\right)$$

To get an elasticity divide by $\overline{\mathrm{Y}}$

$$2584.8/279\,1.47 = .9260$$

A 1% increase in the money supply leads to a .9260% increase in GNP

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Linear Model

- $Y = 101.20 + 1.5323X$
- How interpret?
  - The slope coefficient is dY/dX
    - i.e. absolute change in Y/ absolute change in X
  - For a $1 billion increase in the money supply increases GNP by $1.5323 billion dollars.

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Linear Model

- How to convert to an elasticity?
  - The slope coefficient is: dY/dX
    - Multiply this coefficient by Xbar/Ybar
      - 1.5323 (1755.67/2791.47) = .9637
  - A 1% increase in the money supply leads to a .9637% increase in GNP

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Monetarist Hypothesis

- Can test the monetarist hypothesis with double log model
  - 1% increase in money supply leads to a 1% increase in GNP
    - A t-test reveals that coefficient not different from 1.

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Summary

- Models are similar:
  - Elasticities are similar.
  - R^2 are similar
    - Can only compare same similar dependent variables
  - All t values are significant
- Not much to choose among models.
  - Depends on issue-elasticity, growth, absolute change, etc.

dmacpher@coss.fsu.edu

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5. Reciprocal Model

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Reciprocal Model

- $Y = b_1 + b_2(1/X) + e$
  - Model is linear in the parameters, but nonlinear in the variables
  - As X increases,
    - The term 1/X approaches 0
    - Y approaches the limiting value of b_1.

dmacpher@coss.fsu.edu

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Fixed Cost Example

- Average fixed cost of production declines continuously as output increases:
  - Fixed cost is spread over a larger and larger number of units and eventually becomes asymptotic.

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Phillips Curve Example

- Sometimes the Philips curve is expressed as a reciprocal model
  - $Y = b_1 + b_2(1/X) + e$
    - Y = rate of change of money wages (inflation)
    - X = unemployment rate.

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Phillips Curve Example

- The curve is steeper above the natural unemployment rate than below.
  - Wages rise faster for a unit change in unemployment if the unemployment rate is below the natural rate of unemployment than if it is above.

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Phillips Curve Example

- Suppose we fit this model to data.
  - Using UK data 1950-66.
    - Y = -1.4282 + 8.7243 1/X
    - se (2.068) (2.848)
  - This shows that the wage floor is -1.43%
    - As the unemployment rate increases indefinitely, the % decrease in wages will not be more than 1.43 percent per year.

dmacpher@coss.fsu.edu

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6. Polynomial Regression Models

dmacpher@coss.fsu.edu

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Polynomial Model

- These are models relating to cost and production functions
- Ex: Long run average cost and output
- LRAC curve is a U-shaped curve.
  - Capture by a quadratic function (second degree polynomial):
  - $LRAC = b_1 + b_2Q + b_3Q^2$

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Polynomial Model

- In stochastic form:
  - $LRAC = b_1 + b_2Q + b_3Q^2 + e$
- We can estimate LRAC by OLS.
- Q and Q^2 are correlated
  - They are not linearly correlated so do not violate the assumptions of CLRM.

dmacpher@coss.fsu.edu

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S & L Example

- Use data for 86 S&Ls for 1975.
  - Output Q is measured as total assets
  - LRAC is measured as average operating expenses as % of total assets
- Results:
  - $LRAC = 2.38 -.615Q + .054\ Q^2$

dmacpher@coss.fsu.edu

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S & L Example

- This estimated function is U-shaped.
- Its point of minimum average cost if reached when total assets reach $569 billions:
  - dLAC/dQ = -.615 + 2(.054) Q
  - Set equal to 0
    - -.615 + .108 Q = 0
    - Q = .615/.108 = 569

dmacpher@coss.fsu.edu

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S & L Example

- This is used by regulators to decide whether mergers are in the public interest and also by managers to decide on efficient scale.
- It turns out that most S&Ls had substantially less than $74 in assets, so mergers or growth ok.

dmacpher@coss.fsu.edu

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END OF CHAPTER 6

dmacpher@coss.fsu.edu

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