SAS for Forecasting Time Series –

Part 2: The General ARIMA Model - 1

Charlie Hallahan

December 16, 2009

SAS for Forecasting Time Series”, 2nd edition by Brocklebank & Dickey, 2003

Chapter Outline*

1. Introduction

2. Prediction

3. Model Identification

4. Examples and Instructions

*These notes are based on the SAS Books by Users text “**SAS for Forecasting Time Series”, 2nd edition by Brocklebank & Dickey, 2003**

Introduction: Statistical Background

The general class of autoregressive moving average (**ARMA**) models is developed.

For each model, its **autocovariance function**, ***γ(j),*** is given.

The **estimated autocovariance function, *C(j),*** produced by **PROC ARIMA**, is then used to select an appropriate model for the data.

This step is called **model identification** in the ARIMA terminology.

Once a candidate model has been **identified** and **estimated**, the model is used to produce **forecasts**

Introduction: Terminology & Notation

Moving Average of order 1, MA(1): $Y_t = \mu + e_t - \beta e_{t-1}$

where e_t is a **white noise** (uncorrelated) sequence with mean 0 and variance σ^2.

Hence, $\text{var}(Y_t) = \gamma(0) = \sigma^2\left(1+\beta^2\right)$,

$\text{cov}(Y_t, Y_{t-1}) = \gamma(1) = E\Big(\left(e_t - \beta e_{t-1}\right)\left(e_{t-1} - \beta e_{t-2}\right)\Big) = -\beta\sigma^2$

and $\text{cov}(Y_t, Y_{t-j}) = \gamma(j) = 0$ for $j > 1$.

If we observe $\gamma(0) = 100, \gamma(1) = 40, \gamma(j) = 0$ for $j > 1$, then we can conclude that the data seems to follow an MA(1) process.

Since $\sigma^2\left(1+\beta^2\right) = 100$ and $-\beta\sigma^2$=40, we can solve for β=-0.5 and σ^2=80.

Introduction: Terminology & Notation

Dividing each $\gamma(j)$ by $\gamma(0)$ defines the **autocorrelation function** $\rho(j)$.

For an MA(1) process, $\rho(0) = 1$, $\rho(1) = -\beta/(1+\beta^2)$, and $\rho(j) = 0$ for $j > 1$.

Note that we always have $-1/2 \leq -\beta/(1+\beta^2) \leq 1/2$.

The **MA(q)** process: $Y_t = \mu + e_t - \beta_1 e_{t-1} - \ldots - \beta_q e_{t-q}$ is characterized by the fact that $\gamma(j) = \rho(j) = 0$ for $j > q$.

In terms of the backshift operator, B, we can write the MA(q) model as

$$Y_t = \mu + (I - \beta_1 B - \beta_2 B^2 - \ldots - \beta_q B^q)e_t$$

Introduction: Terminology & Notation

We can write the mixed autoregressive-moving average model **ARMA**(p, q) as

$$\left(Y_t - \mu\right) - \alpha_1\left(Y_{t-1} - \mu\right) - \ldots - \alpha_p\left(Y_{t-p} - \mu\right) = e_t - \beta_1 e_{t-1} - \ldots - \beta_q e_{t-q}$$

or $\left(I - \alpha_1 B - \ldots - \alpha_p B^p\right)\left(Y_t - \mu\right) = \left(I - \beta_1 B - \ldots - \beta_q B^q\right) e_t$

Prediction: One-Step-Ahead Predictions

The basic idea in constructing one-step-ahead predictions is to use known values when available, such as in an autoregressive model with lagged values, and set all future error terms to 0.

To illustrate the process, we'll use three simple models.

AR(1): $Y_t = \alpha Y_{t-1} + e_t$

MA(1): $Y_t = e_t - \beta e_{t-1}$

ARMA(1,1): $Y_t = \alpha Y_{t-1} + e_t - \beta e_{t-1}$

Without loss of generality, we'll assume that the mean μ is 0.

Prediction: One-Step-Ahead Predictions

$t=1,n$	$Y_t=\alpha Y_{t-1}+e_t$	$Y_t=e_t-\beta e_{t-1}$	$Y_t=\alpha Y_{t-1}+e_t-\beta e_{t-1}$
$\hat{Y}_{n+1}$	$Y_{n+1}=\alpha Y_n+e_{n+1}$	$Y_{n+1}=e_{n+1}-\beta e_n$	$Y_{n+1}=\alpha Y_n+e_{n+1}-\beta e_n$
	$\hat{Y}_{n+1}=\alpha Y_n$	$\hat{Y}_{n+1}=-\beta e_n$	$\hat{Y}_{n+1}=\alpha Y_n-\beta e_n$
	$\hat{e}_{n+1}=e_{n+1}$	$\hat{e}_{n+1}=e_{n+1}$	$\hat{e}_{n+1}=e_{n+1}$
	$\operatorname{Var}\left(\hat{e}_{n+1}\right)=\sigma^2$	$\operatorname{Var}\left(\hat{e}_{n+1}\right)=\sigma^2$	$\operatorname{Var}\left(\hat{e}_{n+1}\right)=\sigma^2$
$\hat{Y}_{n+2}$	$Y_{n+2}=\alpha Y_{n+1}+e_{n+2}$	$Y_{n+2}=e_{n+2}-\beta e_{n+1}$	$Y_{n+2}=\alpha Y_{n+1}+e_{n+2}-\beta e_{n+1}$
	$Y_{n+2}=\alpha^2 Y_n+\alpha e_{n+1}+e_{n+2}$		$Y_{n+2}=\alpha\left(\alpha Y_n+e_{n+1}-\beta e_n\right)+e_{n+2}-\beta e_{n+1}$
	$\hat{Y}_{n+2}=\alpha^2 Y_n$	$\hat{Y}_{n+2}=0$	$\hat{Y}_{n+2}=\alpha^2 Y_n-\alpha\beta e_n$
	$\hat{e}_{n+2}=\alpha e_{n+1}+e_{n+2}$	$\hat{e}_{n+2}=e_{n+2}-\beta e_{n+1}$	$\hat{e}_{n+2}=\alpha e_{n+1}+e_{n+2}-\beta e_{n+1}$
	$\operatorname{Var}\left(\hat{e}_{n+2}\right)=\left(\alpha^2+1\right)\sigma^2$	$\operatorname{Var}\left(\hat{e}_{n+2}\right)=\left(1+\beta^2\right)\sigma^2$	$\operatorname{Var}\left(\hat{e}_{n+2}\right)=\left((\alpha-\beta)^2+1\right)\sigma^2$

Prediction: One-Step-Ahead Predictions

All that remains to make the calculations on the previous page operational are to have values for the parameters in the model.

Note: the mean μ can be estimated by the sample mean.

The default method of estimation in PROC ARIMA is called **conditional least squares**.

Given expressions for each one-step-ahead error within sample, the error sum of squares is formed as a function of the unknown model parameters. Minimizing this error sum of squares leads to the **CLS** estimates.

σ^2 is estimated by dividing the residuals sum-of-squares by $n - p$, where p is the number of AR and MA parameters in the model.

Prediction: Future Predictions

The steps on the previous pages can be applied recursively to obtain forecasts into the future.

In PROC ARIMA, the forecasting method is tied to the method of estimation.

For **CLS**, to obtain forecasts, each model is re-expressed as an infinite order autoregressive process.

For example, consider the simple MA(1) process $Y_t = \mu + e_t - \beta e_{t-1}$ or $e_t = Y_t - \mu + \beta e_{t-1}$. Since $e_{t-1} = Y_{t-1} - \mu + \beta e_{t-2}$, by substitution we get $e_t = \left(Y_t - \mu\right) + \beta\left(Y_{t-1} - \mu\right) + \beta^2 e_{t-2}$.

Continuing in this way, we can write $e_t = \sum_{j=0}^{\infty} \beta^j \left(Y_{t-j} - \mu\right)$ where $\beta^j \to 0$ if $|\beta|<1$.

Alternatively, we can write $Y_t - \mu = -\sum_{j=1}^{\infty} \beta^j \left(Y_{t-j} - \mu\right) + e_t$.

Prediction: Future Predictions

Thus, a forecast of Y_t given data up to t -1 can be given by

$$\hat{Y}_t = \mu - \sum_{j=1}^{\infty} \beta^j \left(Y_{t-j} - \mu\right) + e_t.$$

Since this expression involves the infinite past, and we only have a finite amount of data back to $t = 1$, we can assume all past values beyond the first observation have the mean value μ.

This leads to the formula: $\hat{Y}_t = \mu - \sum_{j=1}^{t-1} \beta^j \left(Y_{t-j} - \mu\right)$ where estimates are substituted for unknown parameters.

Prediction: Future Predictions

The formula: $\hat{Y}_t = \mu - \sum_{j=1}^{t-1} \beta^j \left(Y_{t-j} - \mu\right)$ may not be the optimal linear combination of past values to produce a forecast of Y_t.

When maximum likelihood, **ML**, or unconditional least squares, **ULS**, are used to estimate the model parameters, an alternative forecast formula is used.

Suppose we want to minimize $E\left\{\left[\left(Y_t - \mu\right) - \phi_1\left(Y_{t-1} - \mu\right) - \phi_2\left(Y_{t-2} - \mu\right)\right]^2\right\}$ by finding ϕ_1 and ϕ_2.

Note that ϕ_1 and ϕ_2 are not model parameters. We could also consider more lags in the forecast.

Prediction: Future Predictions

Taking derivatives with respect to ϕ_1 and ϕ_2 of the expression to be minimized

leads to: $\begin{pmatrix} \gamma(0) & \gamma(1) \\ \gamma(1) & \gamma(0) \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \gamma(1) \\ \gamma(2) \end{pmatrix}.$

The solution $\hat{\phi}_1$ and $\hat{\phi}_2$ give the best linear forecast of Y_t based on Y_{t-1} and Y_{t-2}.

Using more lags just naturally extends the system of equations above.

For example, with four lags and an MA(1) process with parameter $\beta,$ we would

have: $\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix} = \begin{pmatrix} 1+\beta^2 & -\beta & 0 & 0 \\ -\beta & 1+\beta^2 & -\beta & 0 \\ 0 & -\beta & 1+\beta^2 & -\beta \\ 0 & 0 & -\beta & 1+\beta^2 \end{pmatrix} \begin{pmatrix} -\beta \\ 0 \\ 0 \\ 0 \end{pmatrix}$

For reasonably long series whose parameters are well inside the stationarity and invertibility regions (next section), the best linear combination forecast used with **ML** or **ULS** does not differ much from the truncated sum used with **CLS**.

Prediction: Future Predictions

Recall the MA(1) process $Y_t = \mu + e_t - \beta e_{t-1}$ where we assumed that $|\beta| < 1$.

As long as $|\beta| < 1$ we are able to express Y_t as a convergent AR process:

$$Y_t = \mu - \sum_{j=1}^{\infty} \beta^j \left(Y_{t-j} - \mu\right).$$

If $\beta = 1$ then the series $Y_t = \mu + e_t - 1e_{t-1}$ is said to be "*noninvertible*".

Note that we can write $Y_t = \mu + e_t - \beta e_{t-1}$ as $Y_t = \mu + (1 - \beta B)e_t$ where the polynomial $(1 - \beta B)$ has a root of $1/\beta$. So the condition $|\beta| < 1$ is equivalent to the condition that the roots of $1 - \beta B$ have magnitude greater than 1.

For a **MA**(q) process, the equivalent condition for invertibility is that the roots of $1 - \beta_1 B - \ldots - \beta_q B^q$ be outside the unit circle.

Prediction: Future Predictions

In practice, noninvertibility is rare. However, it can be inadvertantly introduced by unnecessarily taking differences.

For example, consider the "*trend stationary*" series $Y_t = \alpha_0 + \alpha_1 t + e_t$.

Then $Y_t - Y_{t-1} = \alpha_1 + e_t - e_{t-1}$, which is noninvertible.

In this case, we should just use OLS to estimate the parameters.

Model Identification: Stationarity and Invertibility

The ARMA(p,q) model $\left(1-\alpha_1 B-\alpha_2 B^2 ...-\alpha_p B^p\right)\left(Y_t-\mu\right)=\left(1-\beta_1 B-\beta_2 B^2 ...-\beta_q B^q\right)e_t$ is called **stationary** if the roots of the p^{th} degree AR polynomial are outside the unit circle and is called **invertible** if the roots of the q^{th} degree MA polynomial are outside the unit circle.

Stationarity implies that values of e_t in the distant past have a negligible effect on the current Y_t.

Invertibility means that we can express the current e_t as a convergent series of past values of Y_t. This is a necessary condition for producing one-step-ahead forecasts of Y_t.

Note that every moving average process is stationary.

Model Identification: Time Series Identification

A particular ARMA process can be identified by examining plots of three functions:

1. **autocorrelation function (ACF)**
2. **inverse autocorrelation function (IACF)**
3. **partial autocorrelation function (PACF)**

In **PROC ARIMA**, the **IDENTIFY** statement produces all three plots.

```
proc arima data=series;
    identify y;
run;
```

Eight example series will be generated. Plots of the above three functions for these series will serve as prototypes for an empirical series.

Model Identification: Autocovariance Function

Recall that $\gamma(j) = Cov(Y_t, Y_{t-j})$ and doesn't depend on t for a stationary series.

In the eight examples below, e_t is white noise with variance $\sigma^2 = 0$.

Series	Model	
1	$Y_t = .8Y_{t-1} + e_t,$	AR(1), $\gamma(1) > 0$
2	$Y_t = -.8Y_{t-1} + e_t,$	AR(1), $\gamma(1) < 0$
3	$Y_t = .3Y_{t-1} + .4Y_{t-2} + e_t,$	AR(2)
4	$Y_t = .3Y_{t-1} + .49Y_{t-2} + e_t,$	AR(2)
5	$Y_t = e_t + .8e_{t-1},$	MA(1)
6	$Y_t = e_t - .3e_{t-1} + .4e_{t-2},$	MA(2)
7	$Y_t = e_t,$	(white noise)
8	$Y_t = .6Y_{t-1} + e_t + .4e_{t-1},$	ARMA(1,1)

Model Identification: Autocovariance Function

AR Processes

AR(1): $Y_t - \rho Y_{t-1} = e_t$, $\gamma(j) = \rho^{|j|}\sigma^2 / \left(1-\rho^2\right)$

AR(2): $Y_t - \alpha_1 Y_{t-1} - \alpha_2 Y_{t-2} = e_t$, Start with $\gamma(0)$ and $\gamma(1)$, then note that for $j \geq 2$, we have $\gamma(j) - \alpha_1\gamma(j-1) - \alpha_2\gamma(j-2) = 0.$

The covariances may oscillate with a period depending on α_1 and α_2.

AR(p): $Y_t - \alpha_1 Y_{t-1} - \alpha_2 Y_{t-2} - \ldots - \alpha_p Y_{t-p} = e_t$, and the $\gamma(j)$ satisfy

$$\gamma(j) - \alpha_1\gamma(j-1) - \alpha_2\gamma(j-2) - \ldots - \alpha_p\gamma(j-p) = 0 \text{ for } j > p.$$

i.e., the $\gamma(j)$ satisfy the same difference equation as the series Y_t.

Model Identification: Autocovariance Function

MA Processes

MA(1): $Y_t - \mu = e_t - \beta e_{t-1}$, $\gamma(0) = \left(1+\beta^2\right)\sigma^2$, $\gamma(1) = \gamma(-1) = -\beta\sigma^2$ and $\gamma(j) = 0$ for $|j| > 1$.

MA(q): $Y_t - \mu = e_t - \beta_1 e_{t-1} - \beta_2 e_{t-2} - \ldots - \beta_q e_{t-q}$, and given $\gamma(0)$, $\gamma(1), \ldots, \gamma(q)$
then $\gamma(j) = 0$ for $|j| > q$.

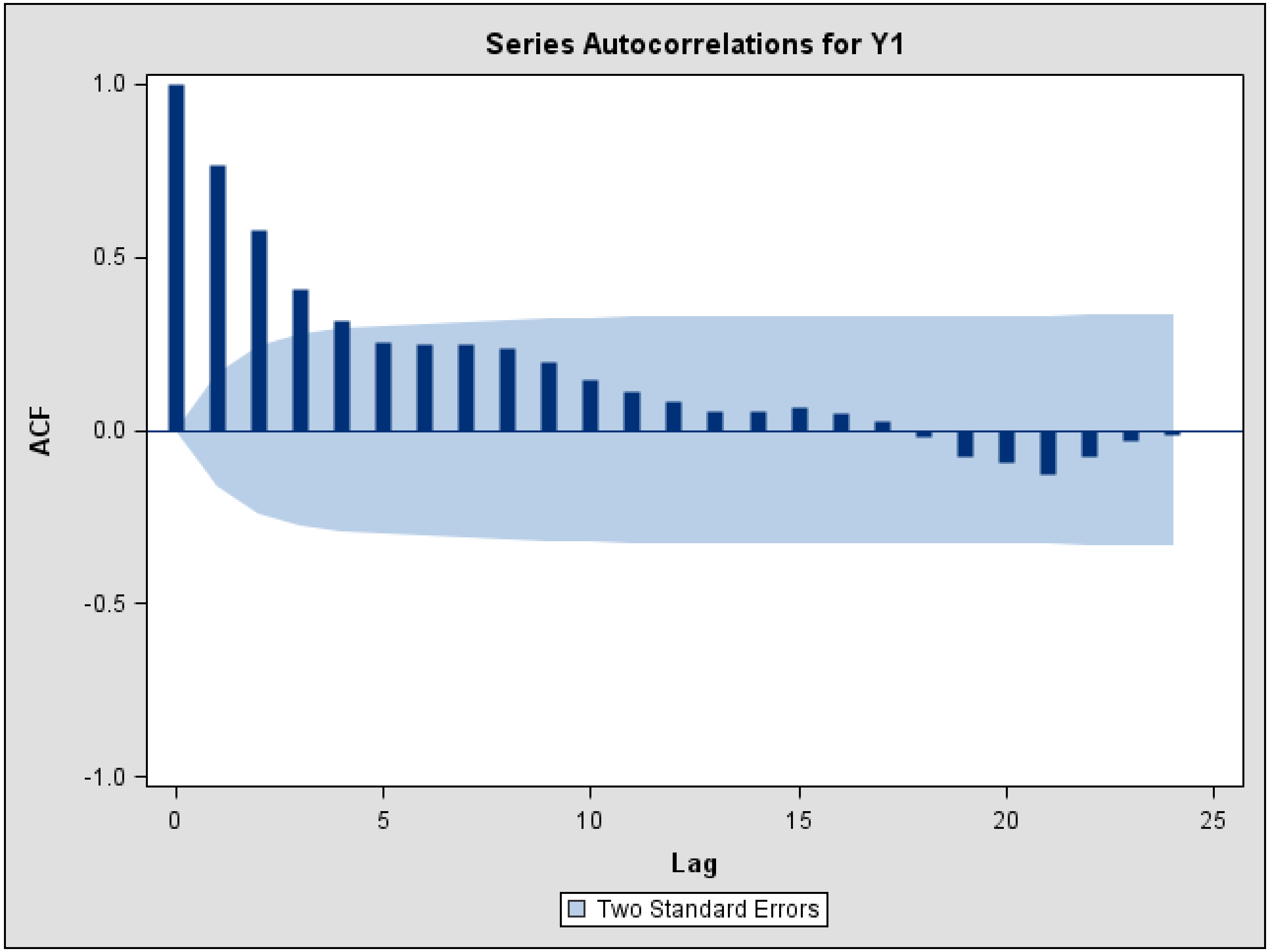
ARMA(1,1): $\left(Y_t - \mu\right) - \alpha\left(Y_{t-1} - \mu\right) = e_t - \beta e_{t-1}$, we have $|\gamma(1)| < \gamma(0)$ depending on α and β.
For $j > 1$, the pattern $\gamma(j) = \alpha\gamma(j-1)$ occurs.

ARMA(p,q): $\left(Y_t - \mu\right) - \alpha_1\left(Y_{t-1} - \mu\right) - \ldots - \alpha_p\left(Y_{t-p} - \mu\right) = e_t - \beta_1 e_{t-1} - \ldots - \beta_q e_{t-q}$,
There are r = max(p-1,q) beginning values followed by behavior characteristic of of an AR(p) process, i.e., $\gamma(j) = -\alpha_1\gamma(j-1) - \ldots - \alpha_p\gamma(j-p) = 0 \quad |j| > r$.

White noise: $Y_t = e_t$, $\gamma(j) = 0$ for $j \neq 0$.

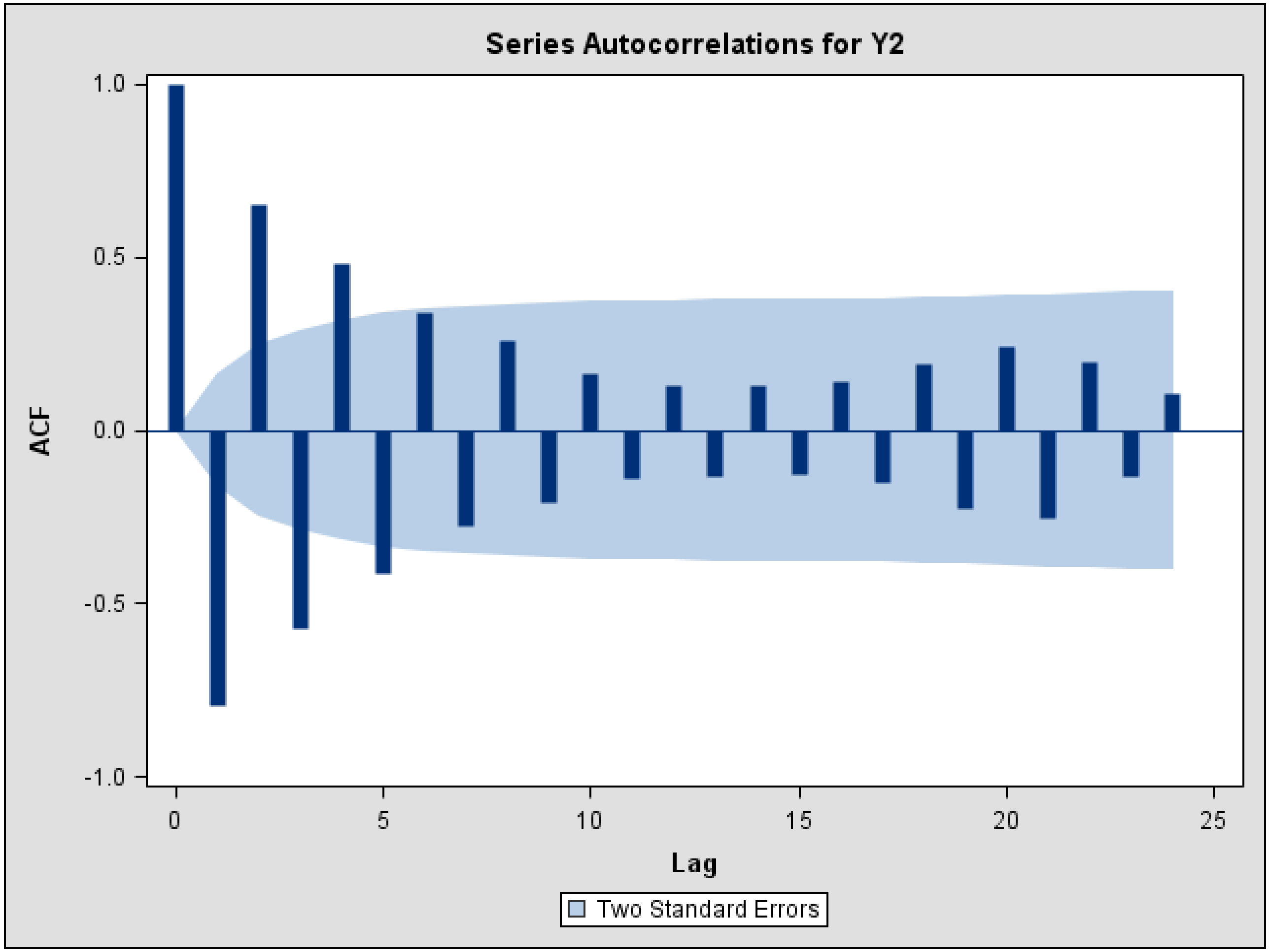
Model Identification: Autocovariance Function

$$\textbf{Series 1:}\ \ Y_t = .8Y_{t-1} + e_t$$



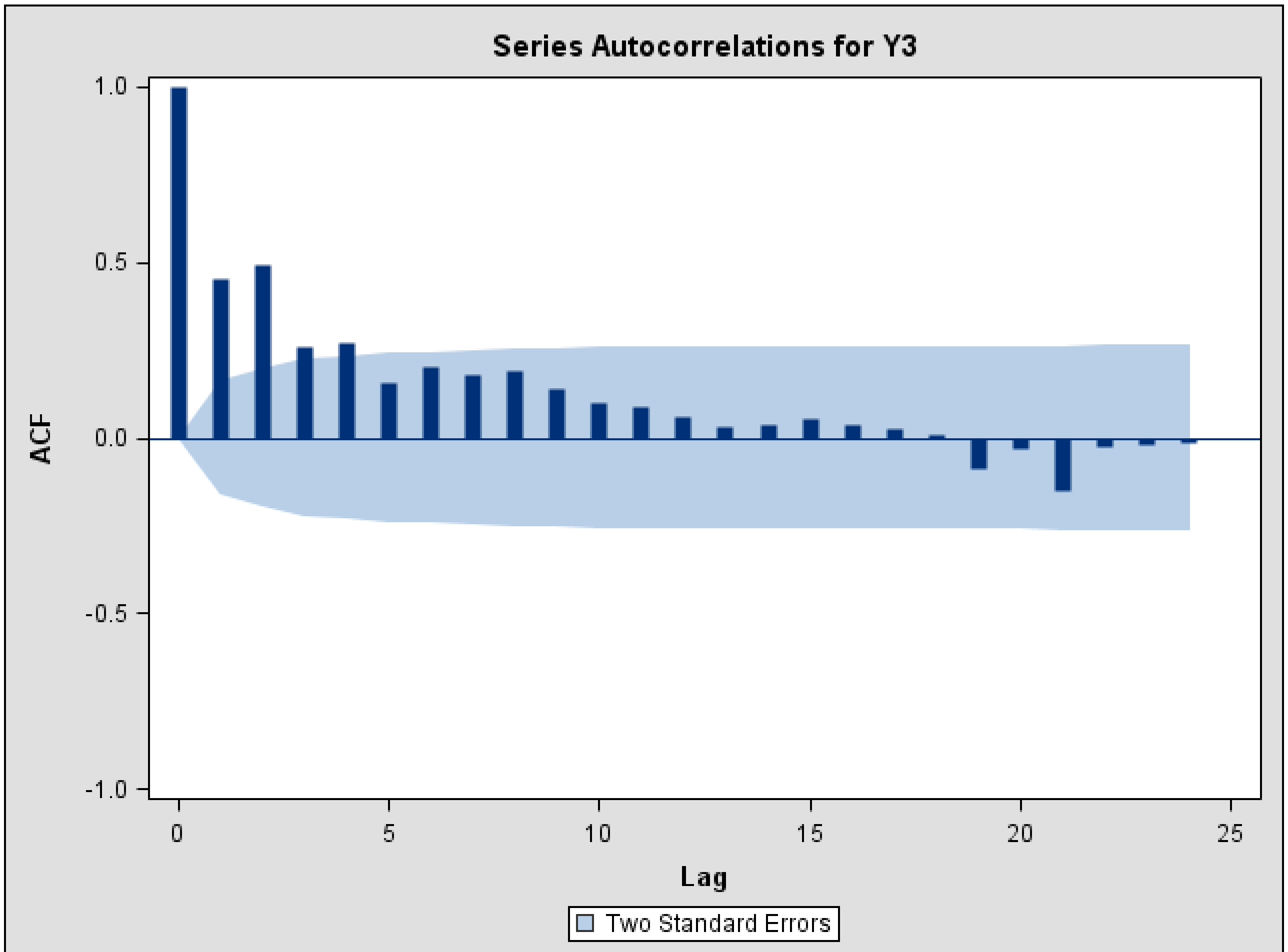
Model Identification: Autocovariance Function

$$\textbf{Series 2:}\ Y_t = -.8Y_{t-1} + e_t$$



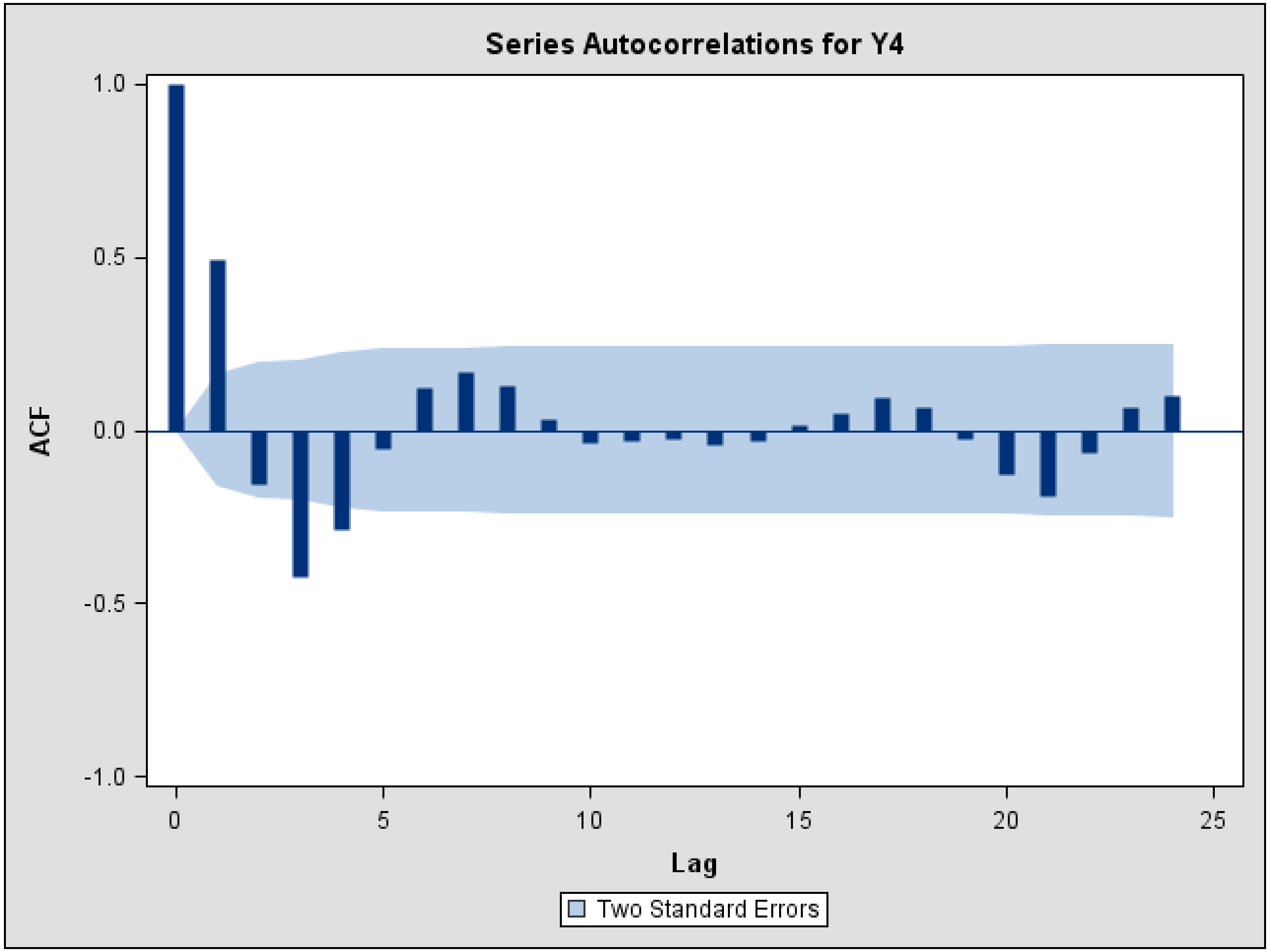
Model Identification: Autocovariance Function

$$\textbf{Series } 3: \; Y_t = .3Y_{t-1} + .4Y_{t-2} + e_t$$



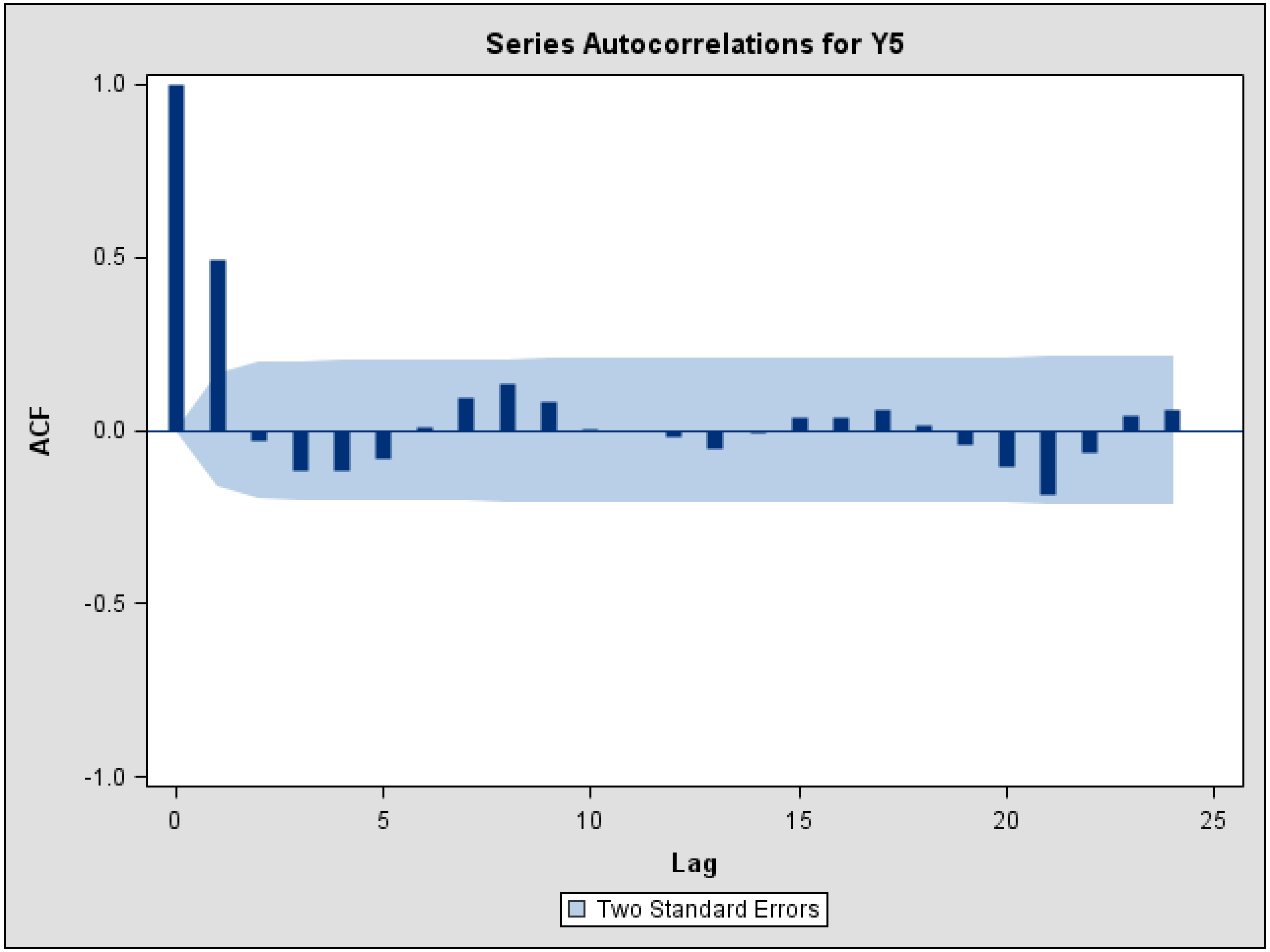
Model Identification: Autocovariance Function

$$\textbf{Series 4:}\ Y_t = .7Y_{t-1} - .49Y_{t-2} + e_t$$



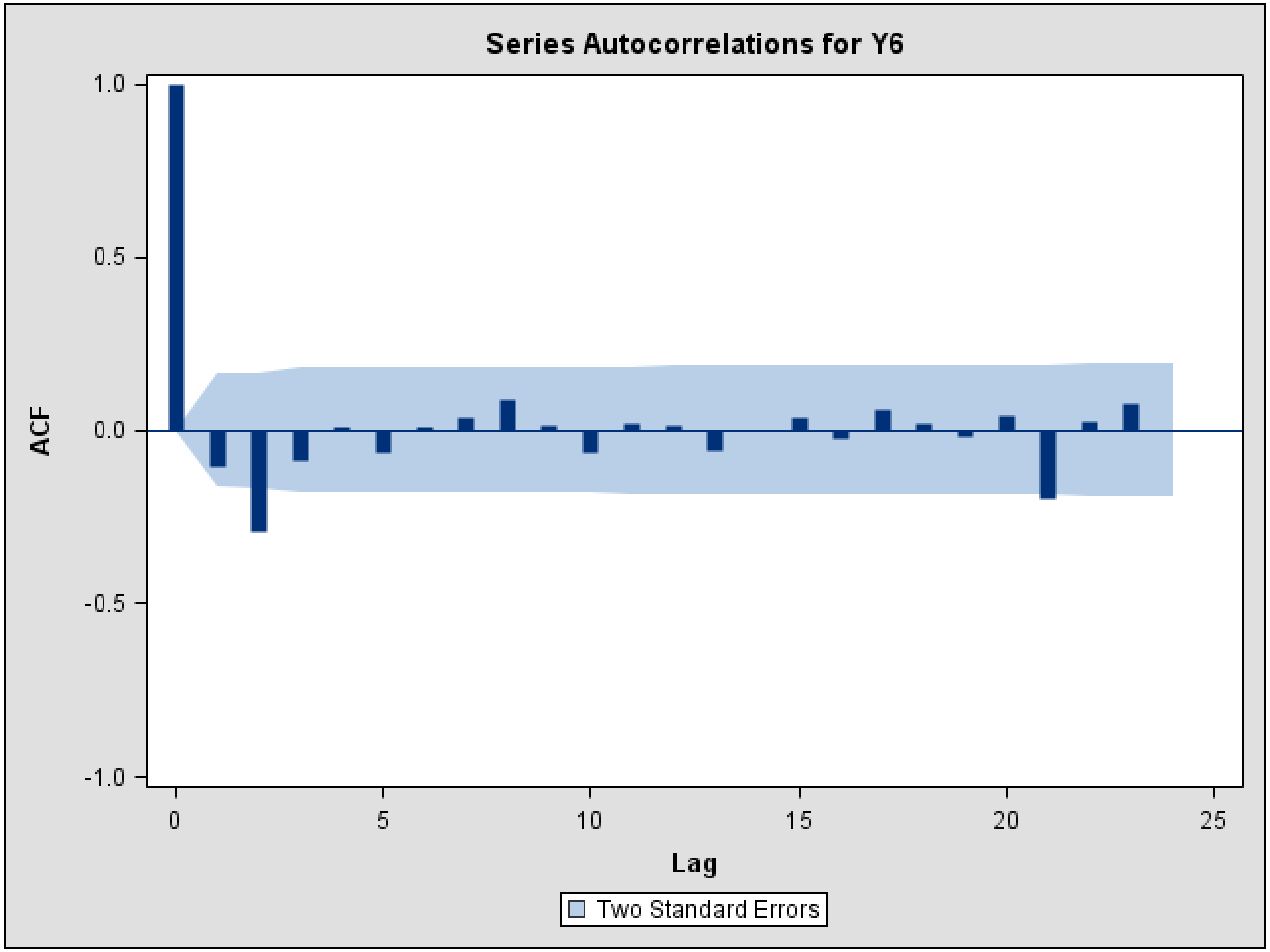
Model Identification: Autocovariance Function

$$\textbf{Series } 5: \; Y_t = e_t + .8e_{t-1}$$



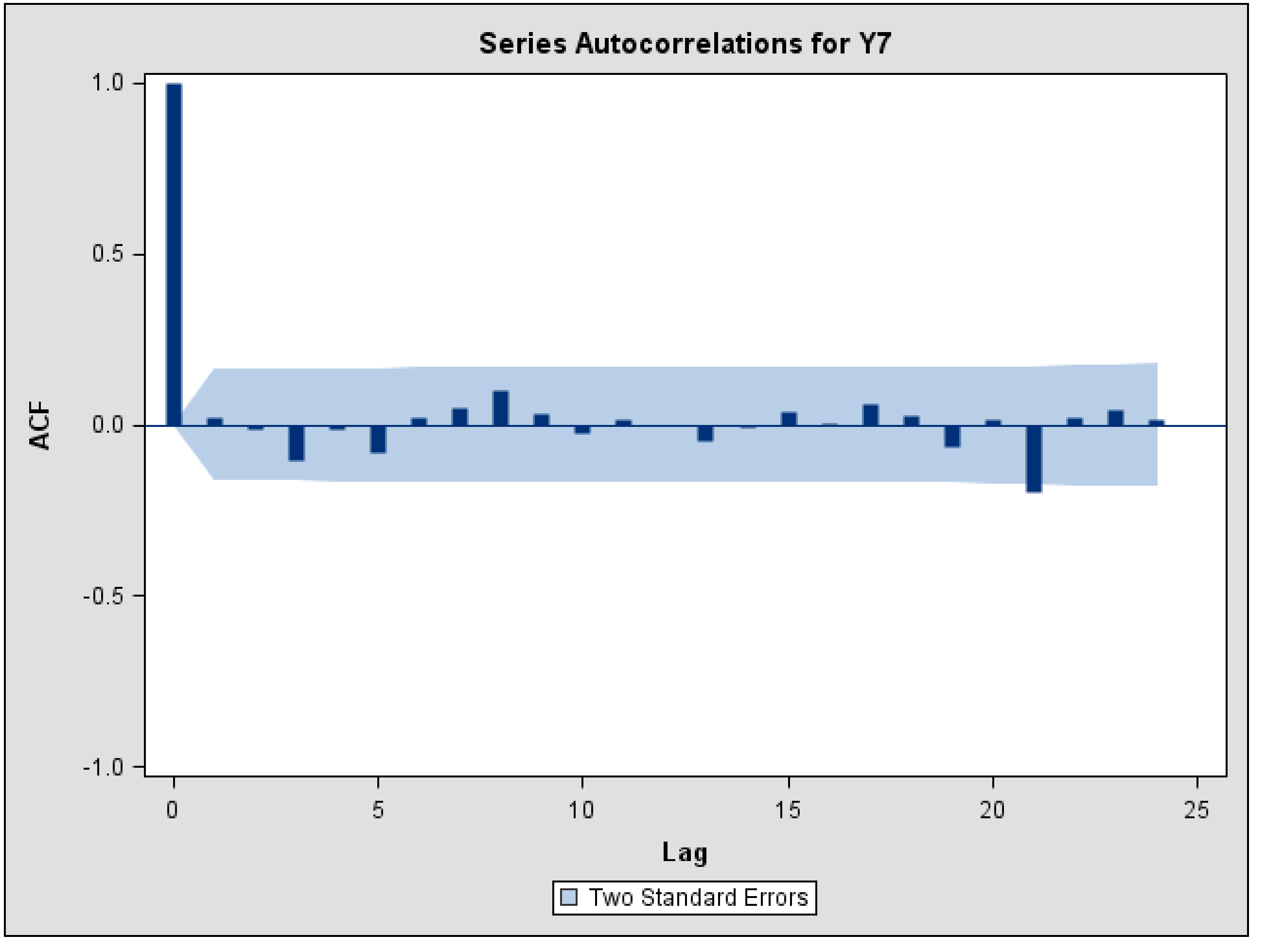
Model Identification: Autocovariance Function

$$\textbf{Series 6}: \; Y_t = e_t - .3e_{t-1} - .4e_{t-2}$$



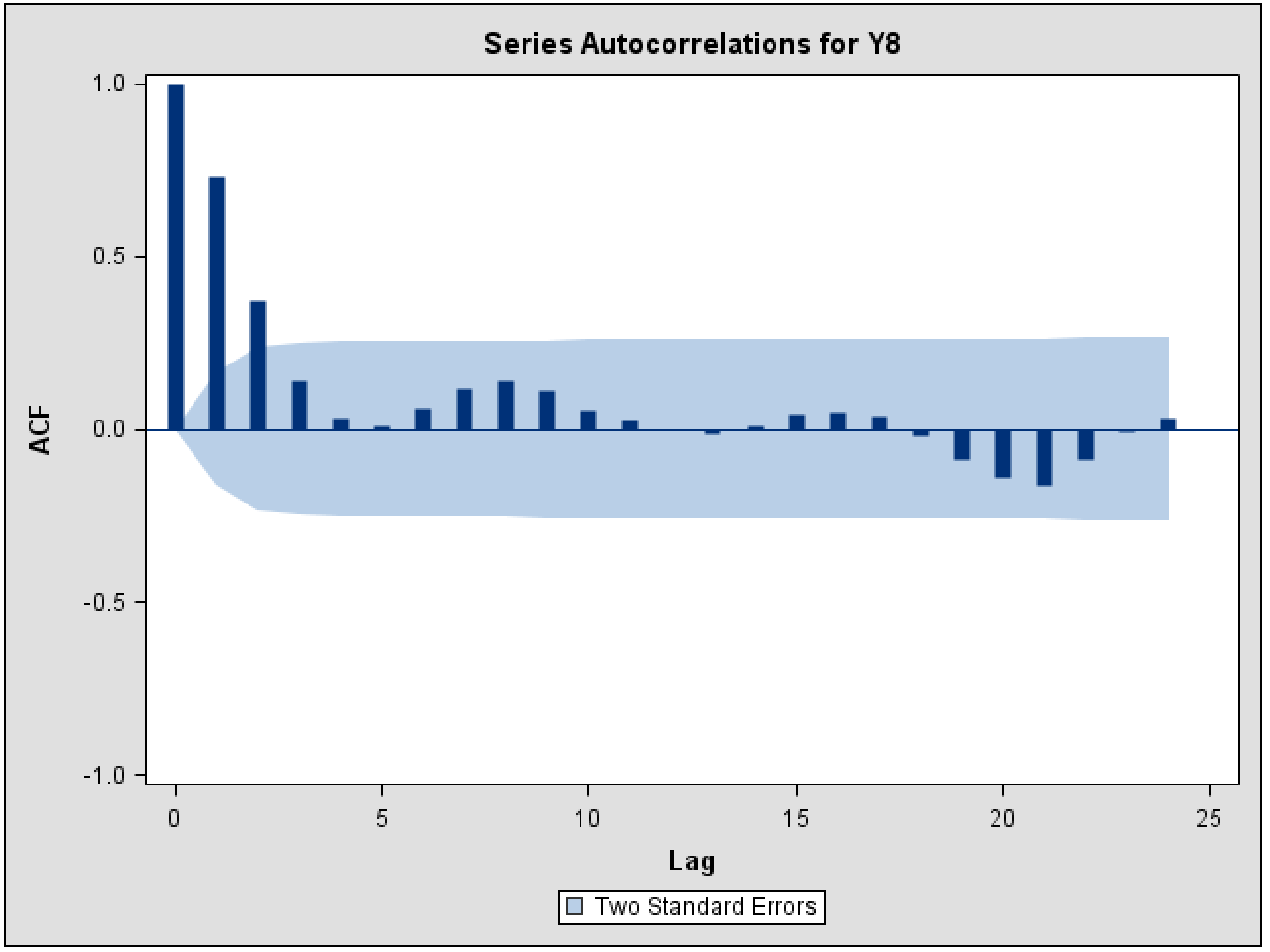
Model Identification: Autocovariance Function

$$\textbf{Series } 7: \; Y_t = e_t$$



Model Identification: Autocovariance Function

$$\textbf{Series 8}: \; Y_t = .6Y_{t-1} + e_t + .4e_{t-1}$$



Model Identification: Autocovariance Function

Note: The SAS code to create the 8 series can be downloaded from the SAS website. I used ODS graphics for the ACF plots. The code to do this is given below.

```
%macro ACFPlot(series);
        ods graphics on;
        ods html;
        ods select SeriesACFPlot;
        *ods trace on;
        proc arima data=ffc2010.series plots(unpack);
                identify var = &series;
        quit;
        ods html close;
        ods graphics off;
        *ods trace off;
%mend;

%ACFPlot(y1);
%ACFPlot(y2);  etc
```

Model Identification: Partial Autocovariance Function

The partial autocorrelation function, **PACF**, is defined as follows:

Given a time series Y_t and a positive integer j, , regress Y_t on $Y_{t-1}, Y_{t-2}, \quad Y_{t-j}$.

The estimated coefficient b_j on $Y_{t\text{-}j}$ is the value of PACF(j) and is denoted $\hat{\pi}_j$.

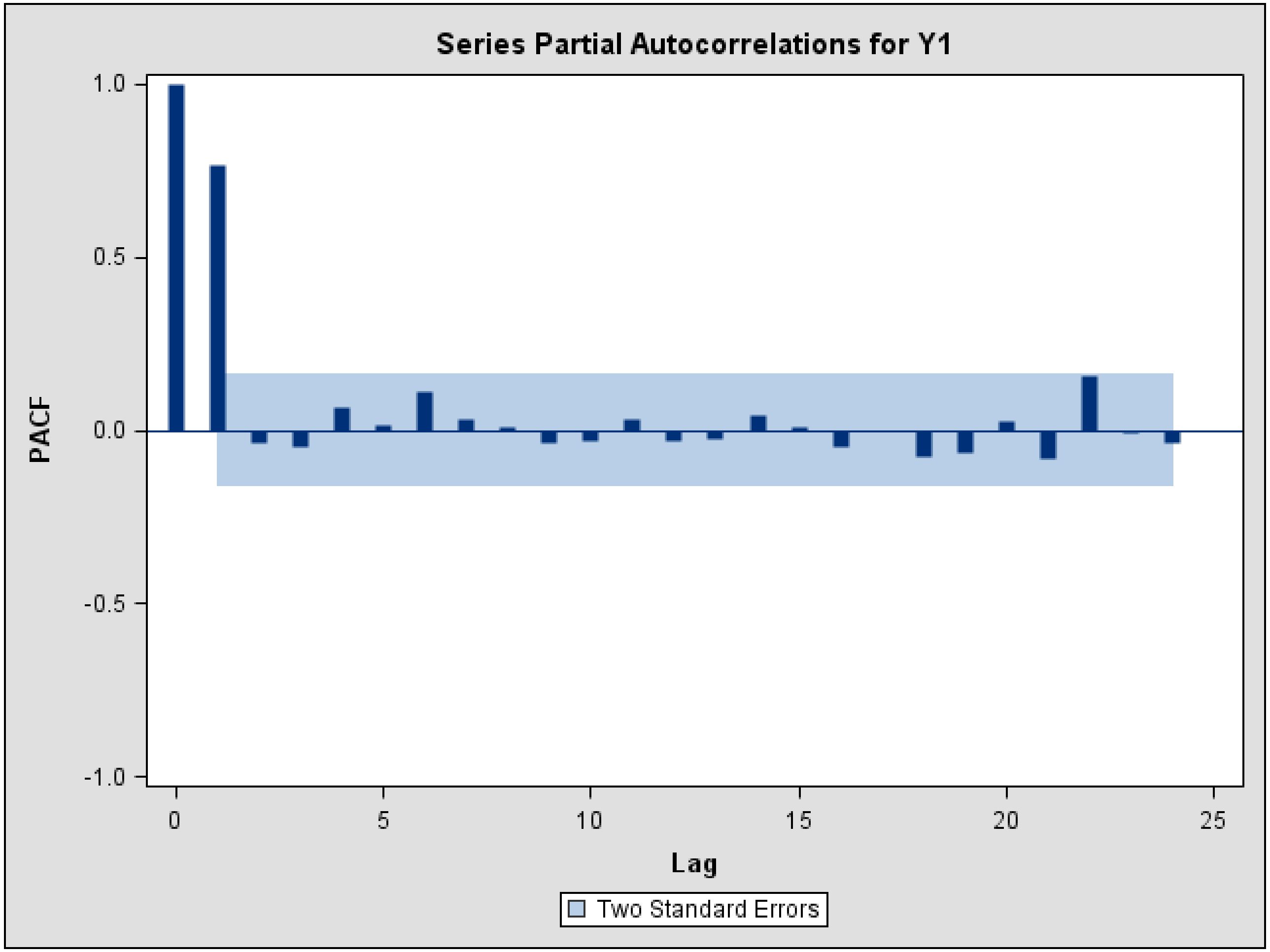
Note that for an AR(p) process, the theoretical value of PACF(j) = 0 for $j > p$.

For a moving average process MA(q), the PACF does not equal 0 beyond some fixed value.

Note the duality of the behavior of the ACF function for a MA-process and the PACF function for an AR-process. Each becomes zero at some point.

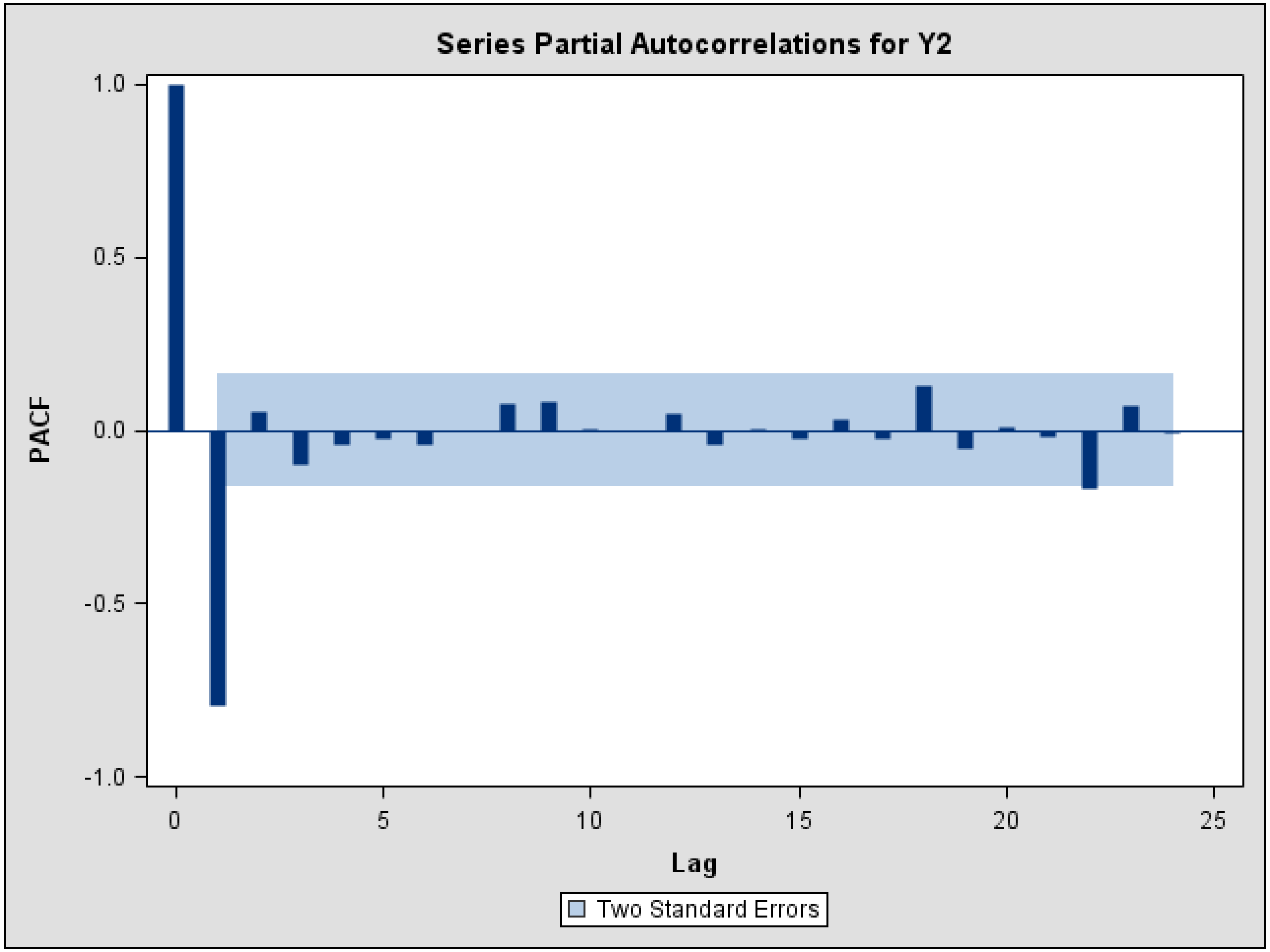
Model Identification: Partial Autocovariance Function

$$\textbf{Series 1}: \; Y_t = .8Y_{t-1} + e_t$$



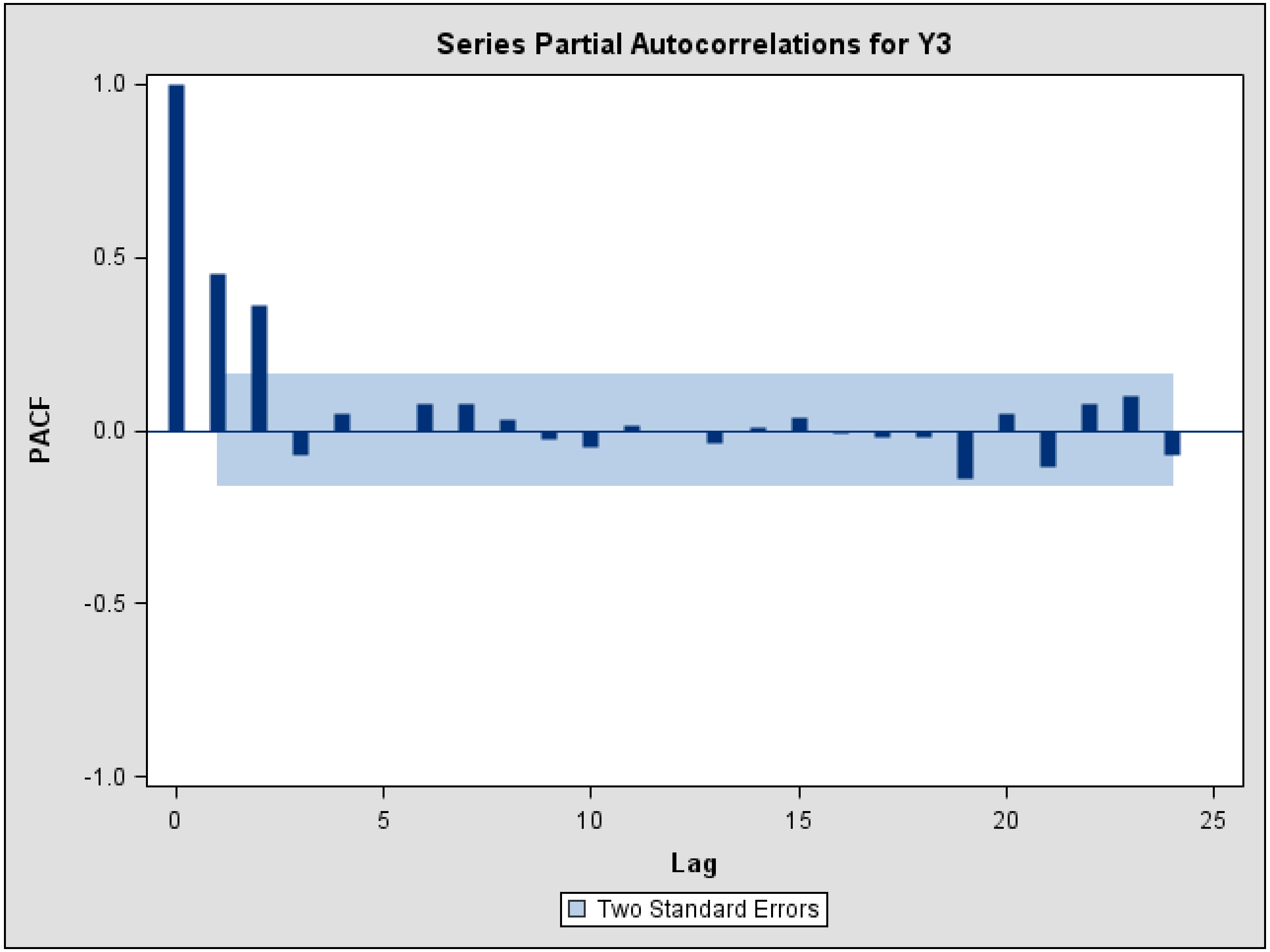
Model Identification: Partial Autocovariance Function

$$\textbf{Series } 2: \; Y_t = -.8Y_{t-1} + e_t$$



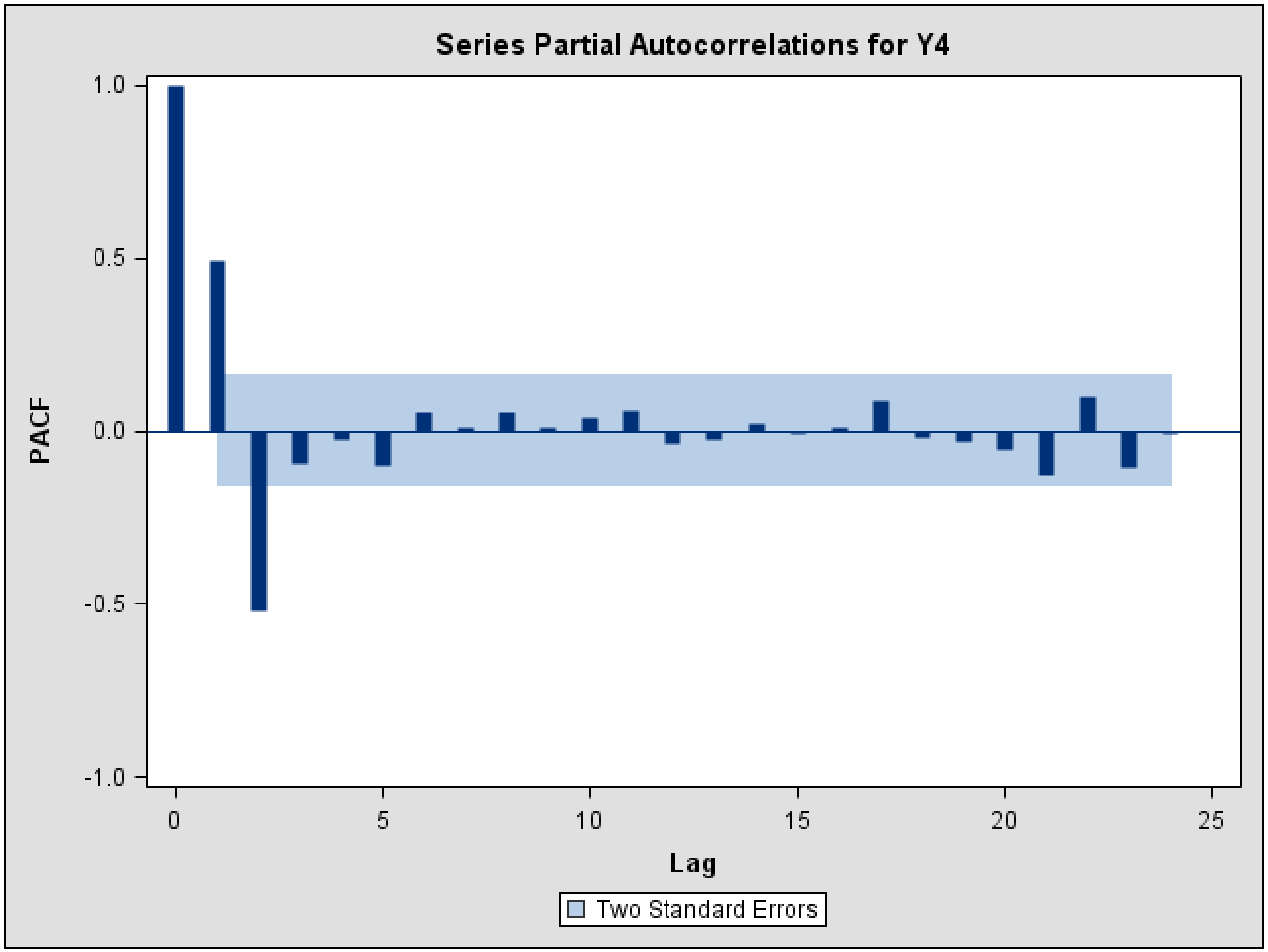
Model Identification: Partial Autocovariance Function

$$\textbf{Series } 3: \; Y_t = .3Y_{t-1} + .4Y_{t-2} + e_t$$



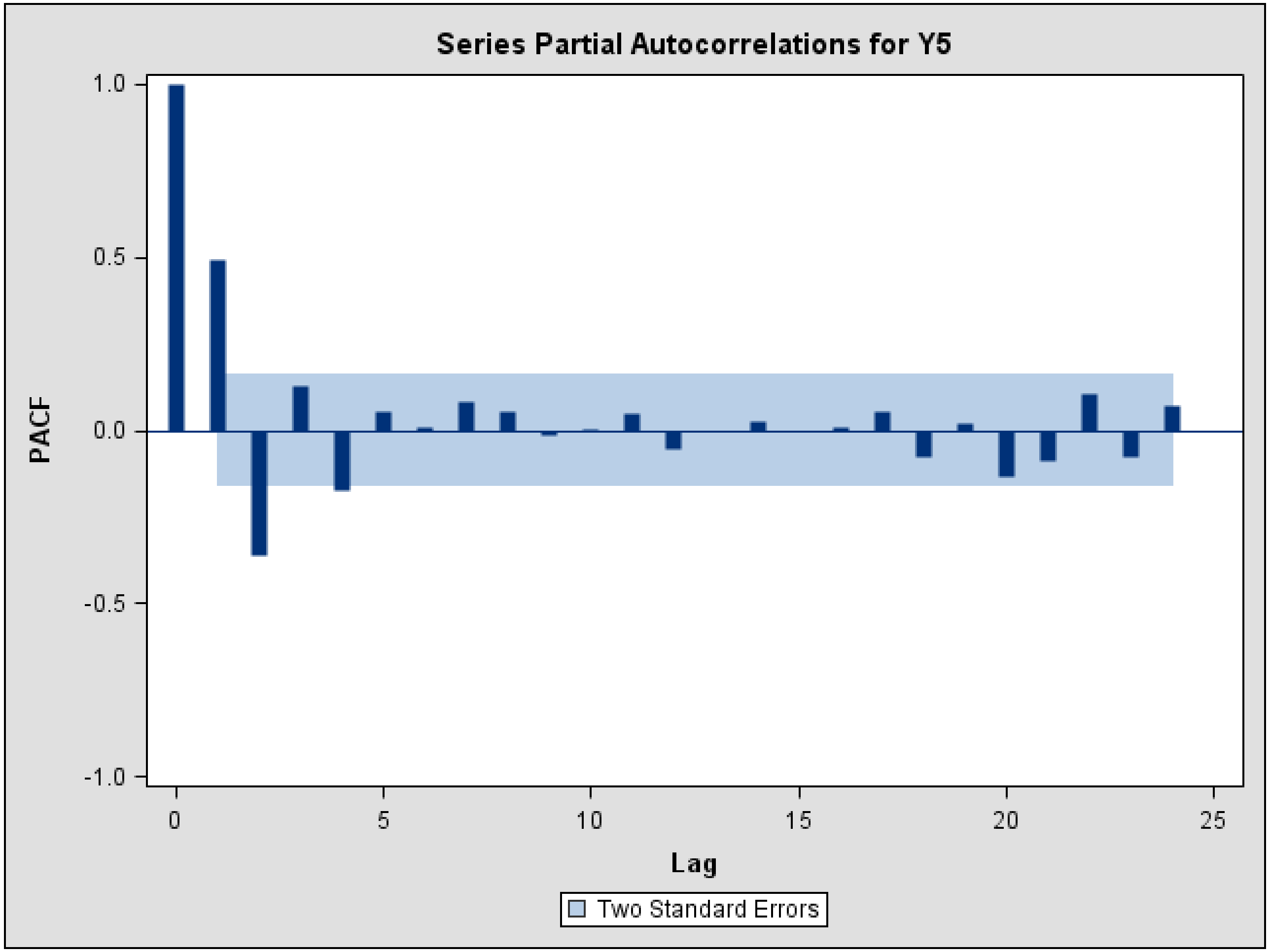
Model Identification: Partial Autocovariance Function

$$\textbf{Series 4:}\ Y_t = .7Y_{t-1} - .49Y_{t-2} + e_t$$



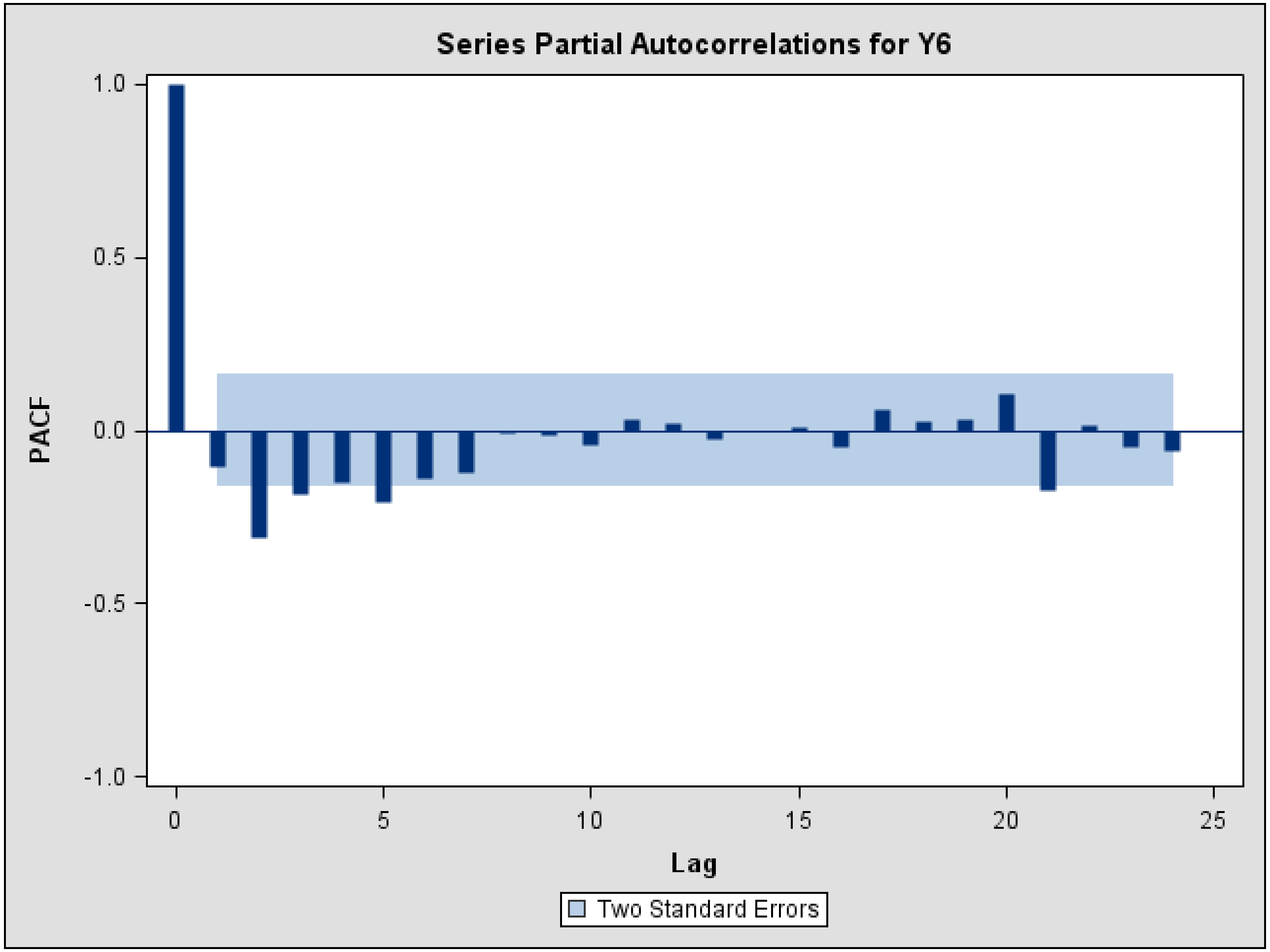
Model Identification: Partial Autocovariance Function

$$\textbf{Series}\,5: \; Y_t = e_t + .8e_{t-1}$$



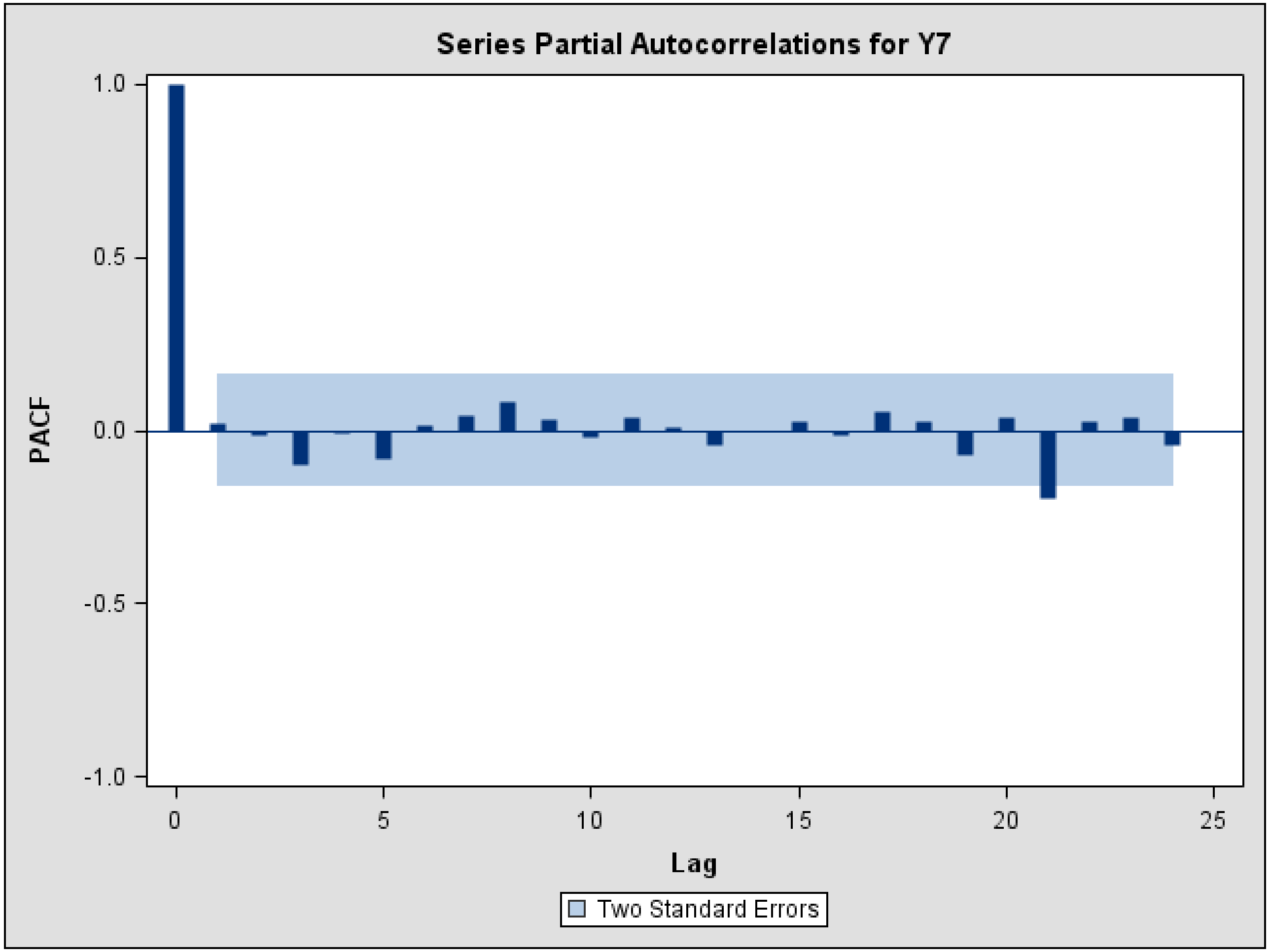
Model Identification: Partial Autocovariance Function

$$\textbf{Series}\ 6: \ Y_t = e_t - .3e_{t-1} - .4e_{t-2}$$



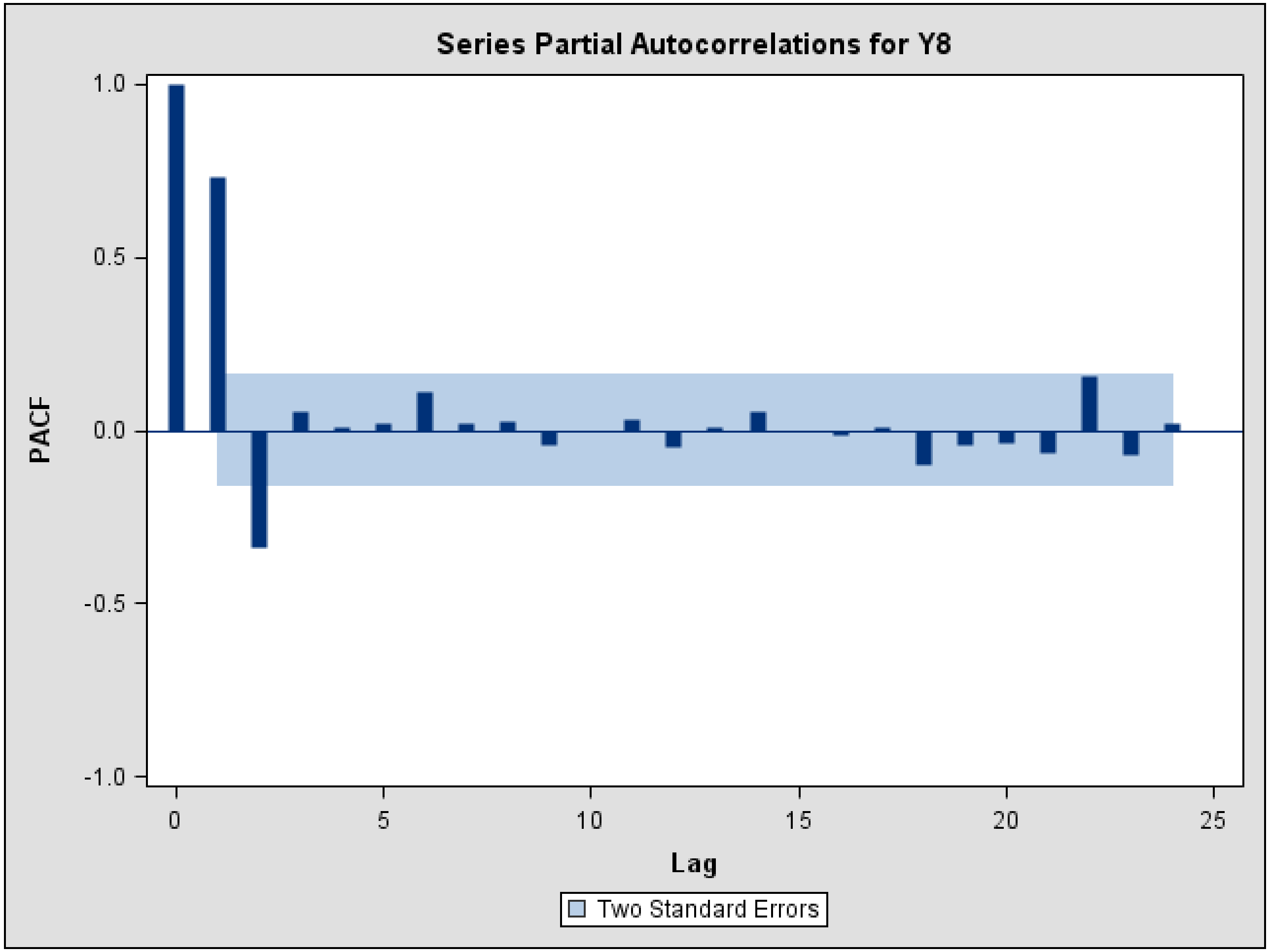
Model Identification: Partial Autocovariance Function

$$\textbf{Series 7}: \; Y_t = e_t$$



Model Identification: Partial Autocovariance Function

$$\textbf{Series 8: } Y_t = .6Y_{t-1} + e_t + .4e_{t-1}$$



Model Identification: Chi-Square Check of Residuals

If a time series has been correctly identified, then its residuals should be **white noise**.

A white noise series has a **flat ACF**, i.e., $\gamma(j) = 0$ for all $j > 0$.

The **Ljung-Box chi-square test** can be used to see if series is white noise or not.

The test statistic is $n(n+2)\sum_{j=1}^{k} r^2(j)/(n-j)$ and tests the null hypothesis that the first k autocorrelations of a series are zero.

PROC ARIMA calculates this statistic for the residuals of an estimated model.

Examples and Instructions

We'll now go through the identification steps for the 8 generated series of 150 observations each that were previously introduced.

```
Proc ARIMA data = series;
        identify var = Y1 nlag = 10;
run;
```

The default number of lags is 24 autocorrelations.

Other options:

noprint: suppresses printout. Useful when estimating a series that has already been identified.

center: subtracts the series mean from each observation.

Examples and Instructions

IDENTIFY Statement for series 1 - 8

```
Proc ARIMA data = series;
        identify var = Y1 nlag = 10;
        identify var = Y2 nlag = 10;
        etc.
run;
```

Examples and Instructions: Series 1

```
                         Identifying Series 1

                          The ARIMA Procedure

                        Name of Variable = Y1

                  Mean of Working Series    -0.83571
                  Standard Deviation        1.610893
                  Number of Observations         150
```

Autocorrelations

```
Lag    Covariance    Correlation    -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1      Std Error

  0      2.594976        1.00000    |                    |********************|            0
  1      1.993518        0.76822    |                .   |***************     |     0.081650
  2      1.493601        0.57557    |              .     |************        |     0.120563
  3      1.063870        0.40997    |             .      |********            |     0.137669
  4      0.819993        0.31599    |             .      |******              |     0.145581
  5      0.652487        0.25144    |             .      |*****.              |     0.150084
  6      0.644574        0.24839    |             .      |*****.              |     0.152866
  7      0.637198        0.24555    |             .      |*****.              |     0.155534
  8      0.609458        0.23486    |             .      |*****.              |     0.158097
  9      0.504567        0.19444    |             .      |**** .              |     0.160406
 10      0.372414        0.14351    |             .      |***  .              |     0.161970

                                 "." marks two standard errors
```

Examples and Instructions: Series 1

Partial Autocorrelations

```
 Lag    Correlation    -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1

   1        0.76822    |                 .  |***************    |
   2       -0.03560    |                 . *|  .                |
   3       -0.05030    |                 . *|  .                |
   4        0.06531    |                 .  |* .                |
   5        0.01599    |                 .  |  .                |
   6        0.11264    |                 .  |**.                |
   7        0.02880    |                 .  |* .                |
   8        0.00625    |                 .  |  .                |
   9       -0.03668    |                 . *|  .                |
  10       -0.03308    |                 . *|  .                |
```

Autocorrelation Check for White Noise

To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	**202.72**	**6**	**<.0001**	0.768	0.576	0.410	0.316	0.251	0.248

Note that the **ACF** and **PACF** both have the requisite pattern for an **AR*(p)*** process, i.e., the ACF declines exponentially and the PACF cuts off after lag 1, implying an **AR(1)** process. The chi-square test strongly rejects that Y1 is white noise.

Examples and Instructions: Series 2

```
                         Identifying Series 2

                         The ARIMA Procedure

                         Name of Variable = Y2

                Mean of Working Series    -0.07304
                Standard Deviation        1.740946
                Number of Observations         150


                          Autocorrelations

Lag    Covariance    Correlation    -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1     Std Error

  0      3.030893        1.00000    |                    |********************|             0
  1     -2.414067        -.79649    |    ****************|  .                 |      0.081650
  2      1.981819        0.65387    |               .    |*************       |      0.122985
  3     -1.735348        -.57255    |         ***********|     .              |      0.144312
  4      1.454755        0.47998    |              .     |**********          |      0.158735
  5     -1.242813        -.41005    |            ********|      .             |      0.168132
  6      1.023028        0.33753    |             .      |*******             |      0.174672
  7     -0.844730        -.27871    |             .******|      .             |      0.178968
  8      0.790137        0.26069    |             .      |***** .             |      0.181838
  9     -0.623423        -.20569    |             .  ****|      .             |      0.184313
 10      0.494691        0.16322    |             .      |***   .             |      0.185837

                   "." marks two standard errors
```

Examples and Instructions: Series 2

```
                    Partial Autocorrelations

 Lag    Correlation    -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1

   1       -0.79649    |    ****************|  .                 |
   2        0.05329    |                 .  |* .                 |
   3       -0.10166    |                 .**|  .                 |
   4       -0.03975    |                 . *|  .                 |
   5       -0.02340    |                 .  |  .                 |
   6       -0.04177    |                 . *|  .                 |
   7       -0.00308    |                 .  |  .                 |
   8        0.07566    |                 .  |**.                 |
   9        0.08029    |                 .  |**.                 |
  10        0.00344    |                 .  |  .                 |
```

Autocorrelation Check for White Noise

To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	**294.24**	**6**	**<.0001**	-0.796	0.654	-0.573	0.480	-0.410	0.338

Note that the **ACF** and **PACF** both have the requisite pattern for an **AR*(p)*** process, i.e., the ACF declines exponentially and the PACF cuts off after lag 1, implying an **AR(1)** process. The chi-square test strongly rejects that Y2 is white noise.

Examples and Instructions: Series 3

```
                         Identifying Series 3

                        Name of Variable = Y3

                  Mean of Working Series    -0.55064
                  Standard Deviation        1.237272
                  Number of Observations         150


                           Autocorrelations

Lag  Covariance  Correlation  -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1    Std Error

  0    1.530842      1.00000  |                    |*******************|            0
  1    0.693513      0.45303  |                 .  |*********          |     0.081650
  2    0.756838      0.49439  |                .   |**********         |     0.096970
  3    0.395653      0.25845  |               .    |*****              |     0.112526
  4    0.417928      0.27301  |               .    |*****              |     0.116416
  5    0.243252      0.15890  |               .    |*** .              |     0.120609
  6    0.311005      0.20316  |               .    |****.              |     0.121997
  7    0.274850      0.17954  |               .    |****.              |     0.124232
  8    0.295125      0.19279  |               .    |****.              |     0.125950
  9    0.212710      0.13895  |               .    |*** .              |     0.127902
 10    0.154864      0.10116  |               .    |**  .              |     0.128904

                    "." marks two standard errors
```

Examples and Instructions: Series 3

Partial Autocorrelations

```
Lag    Correlation    -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1

  1       0.45303     |                  .  |*********          |
  2       0.36383     |                  .  |*******            |
  3      -0.07085     |                  . *|  .                |
  4       0.04953     |                  .  |* .                |
  5      -0.00276     |                  .  |  .                |
  6       0.07517     |                  .  |**.                |
  7       0.07687     |                  .  |**.                |
  8       0.03254     |                  .  |* .                |
  9      -0.02622     |                  . *|  .                |
 10      -0.04933     |                  . *|  .                |
```

Autocorrelation Check for White Noise

To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	**101.56**	**6**	**<.0001**	0.453	0.494	0.258	0.273	0.159	0.203

Note that the **ACF** and **PACF** both have the requisite pattern for an **AR*(p)*** process, i.e., the ACF declines exponentially and the PACF cuts off after lag 2, implying an **AR(2)** process. The chi-square test strongly rejects that Y3 is white noise.

Examples and Instructions: Series 4

```
                              Identifying Series 4

                              Name of Variable = Y4

                        Mean of Working Series    -0.21583
                        Standard Deviation        1.381192
                        Number of Observations         150


                                 Autocorrelations

Lag    Covariance    Correlation    -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1      Std Error

  0      1.907692        1.00000    |                    |********************|             0
  1      0.935589        0.49043    |                 .  |**********          |      0.081650
  2     -0.297975        -.15620    |                .***|   .                |      0.099366
  3     -0.810601        -.42491    |            ********|   .                |      0.100990
  4     -0.546360        -.28640    |              ******|   .                |      0.112278
  5     -0.106682        -.05592    |               .   *|    .               |      0.117047
  6      0.237817        0.12466    |               .    |**  .               |      0.117225
  7      0.324887        0.17030    |               .    |*** .               |      0.118105
  8      0.241111        0.12639    |               .    |*** .               |      0.119731
  9      0.055065        0.02886    |               .    |*   .               |      0.120617
 10     -0.073198        -.03837    |               .   *|    .               |      0.120663

                                "." marks two standard errors
```

Examples and Instructions: Series 4

Partial Autocorrelations

```
Lag    Correlation    -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1

  1        0.49043    |                 .  |**********          |
  2       -0.52236    |          **********|  .                 |
  3       -0.09437    |                 .**|  .                 |
  4       -0.02632    |                 . *|  .                 |
  5       -0.09683    |                 .**|  .                 |
  6        0.05711    |                 .  |* .                 |
  7        0.00687    |                 .  |  .                 |
  8        0.05190    |                 .  |* .                 |
  9        0.00919    |                 .  |  .                 |
 10        0.03505    |                 .  |* .                 |
```

Autocorrelation Check for White Noise

To Lag	Chi-Square	DF	Pr > ChiSq	--------------------Autocorrelations--------------------					
6	**84.33**	**6**	**<.0001**	0.490	-0.156	-0.425	-0.286	-0.056	0.125

Note that the **ACF** and **PACF** both have the requisite pattern for an **AR*(p)*** process, i.e., the ACF declines exponentially and the PACF cuts off after lag 2, implying an **AR(2)** process. The chi-square test strongly rejects that Y4 is white noise.

Examples and Instructions: Series 5

```
                         Identifying Series 5

                      Name of Variable = Y5

                Mean of Working Series    -0.30048
                Standard Deviation        1.316518
                Number of Observations         150


                           Autocorrelations

Lag    Covariance    Correlation    -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1      Std Error

  0      1.733219        1.00000    |                    |********************|              0
  1      0.852275        0.49173    |                 .  |**********          |       0.081650
  2     -0.055217        -.03186    |                .  *|   .                |       0.099452
  3     -0.200380        -.11561    |                . **|   .                |       0.099520
  4     -0.203287        -.11729    |                . **|   .                |       0.100411
  5     -0.144763        -.08352    |                . **|   .                |       0.101320
  6      0.011068        0.00639    |                .   |   .                |       0.101778
  7      0.163554        0.09436    |                .   |** .                |       0.101781
  8      0.234861        0.13551    |                .   |***.                |       0.102363
  9      0.141452        0.08161    |                .   |** .                |       0.103551
 10     0.0013709        0.00079    |                .   |   .                |       0.103979

                    "." marks two standard errors
```

Examples and Instructions: Series 5

Partial Autocorrelations

```
Lag    Correlation    -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1

  1        0.49173    |                 .  |**********          |
  2       -0.36093    |             *******|  .                 |
  3        0.12615    |                 .  |***                 |
  4       -0.17086    |                 ***|  .                 |
  5        0.05473    |                 .  |* .                 |
  6        0.00681    |                 .  |  .                 |
  7        0.08204    |                 .  |**.                 |
  8        0.05577    |                 .  |* .                 |
  9       -0.01303    |                 .  |  .                 |
 10        0.00517    |                 .  |  .                 |
```

Autocorrelation Check for White Noise

To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	42.48	6	<.0001	0.492	-0.032	-0.116	-0.117	-0.084	0.006

Note that the **ACF** and **PACF** both have the requisite pattern for an **MA*(q)*** process, i.e., the PACF declines exponentially and the ACF cuts off after lag 1, implying an **MA(1)** process. The chi-square test strongly rejects that Y5 is white noise.

Examples and Instructions: Series 6

```
                              Identifying Series 6

                             Name of Variable = Y6

                       Mean of Working Series    -0.04253
                       Standard Deviation        1.143359
                       Number of Observations         150


                                 Autocorrelations

Lag    Covariance    Correlation    -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1      Std Error

  0      1.307271        1.00000    |                   |********************|             0
  1     -0.137276        -.10501    |                .**|  .                 |      0.081650
  2     -0.385340        -.29477    |             ******|  .                 |      0.082545
  3     -0.118515        -.09066    |               . **|   .                |      0.089287
  4     0.0083104        0.00636    |               .   |   .                |      0.089899
  5     -0.084843        -.06490    |               .  *|   .                |      0.089902
  6      0.011812        0.00904    |               .   |   .                |      0.090214
  7      0.045677        0.03494    |               .   |*  .                |      0.090220
  8      0.119262        0.09123    |               .   |** .                |      0.090310
  9      0.018882        0.01444    |               .   |   .                |      0.090922
 10     -0.083572        -.06393    |               .  *|   .                |      0.090937

                             "." marks two standard errors
```

Examples and Instructions: Series 6

```
                          Partial Autocorrelations

  Lag     Correlation    -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1

    1       -0.10501     |                 .**|  .                 |
    2       -0.30920     |              ******|  .                 |
    3       -0.18297     |                ****|  .                 |
    4       -0.14923     |                 ***|  .                 |
    5       -0.20878     |                ****|  .                 |
    6       -0.13688     |                 ***|  .                 |
    7       -0.12493     |                 .**|  .                 |
    8       -0.00862     |                 .  |  .                 |
    9       -0.01255     |                 .  |  .                 |
   10       -0.04518     |                 . *|  .                 |
```

Autocorrelation Check for White Noise

To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	**17.03**	**6**	**0.0092**	-0.105	-0.295	-0.091	0.006	-0.065	0.009

Note that the **ACF** and **PACF** both have the requisite pattern for an **MA*(q)*** process, i.e., the PACF declines exponentially and the ACF cuts off after lag 2, implying an **MA(2)** process. The chi-square test strongly rejects that Y6 is white noise.

Examples and Instructions: Series 7

```
                         Identifying Series 7

                        Name of Variable = Y7

                   Mean of Working Series    -0.15762
                   Standard Deviation        1.023007
                   Number of Observations         150


                            Autocorrelations

Lag   Covariance   Correlation   -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1     Std Error

  0     1.046543       1.00000   |                    |********************|           0
  1     0.019680       0.01880   |                 .  |  .                 |    0.081650
  2    -0.012715       -.01215   |                 .  |  .                 |    0.081679
  3    -0.107313       -.10254   |                 .**|  .                 |    0.081691
  4    -0.012754       -.01219   |                 .  |  .                 |    0.082544
  5    -0.085250       -.08146   |                 .**|  .                 |    0.082556
  6     0.023489       0.02244   |                 .  |  .                 |    0.083090
  7     0.048176       0.04603   |                 .  |* .                 |    0.083131
  8     0.106544       0.10181   |                 .  |**.                 |    0.083300
  9     0.033337       0.03185   |                 .  |* .                 |    0.084126
 10    -0.026272       -.02510   |                 . *|  .                 |    0.084206

                    "." marks two standard errors
```

Examples and Instructions: Series 7

Partial Autocorrelations

```
 Lag    Correlation    -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1

   1        0.01880    |                 .  |  .                 |
   2       -0.01251    |                 .  |  .                 |
   3       -0.10213    |                 .**|  .                 |
   4       -0.00867    |                 .  |  .                 |
   5       -0.08435    |                 .**|  .                 |
   6        0.01474    |                 .  |  .                 |
   7        0.04159    |                 .  |* .                 |
   8        0.08562    |                 .  |**.                 |
   9        0.03364    |                 .  |* .                 |
  10       -0.02114    |                 .  |  .                 |
```

Autocorrelation Check for White Noise

To Lag	Chi-Square	DF	Pr > ChiSq	--------------------Autocorrelations--------------------					
6	**2.85**	**6**	**0.8269**	0.019	-0.012	-0.103	-0.012	-0.081	0.022

Note that the **ACF** and **PACF** both have the requisite pattern for white noise process, i.e., the ACF and the PACF cuts off after lag 0, implying a white noise process. The chi-square test strongly accepts that Y7 is white noise.

Examples and Instructions: Series 8

```
                          Identifying Series 8

                         Name of Variable = Y8

                  Mean of Working Series    -0.57405
                  Standard Deviation        1.591833
                  Number of Observations         150


                             Autocorrelations

Lag    Covariance    Correlation    -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1      Std Error

  0      2.533932        1.00000    |                    |********************|              0
  1      1.848193        0.72938    |                 .  |***************     |       0.081650
  2      0.946216        0.37342    |               .    |*******             |       0.117303
  3      0.352595        0.13915    |               .    |***  .              |       0.124976
  4      0.086093        0.03398    |               .    |*    .              |       0.126005
  5      0.025473        0.01005    |               .    |     .              |       0.126066
  6      0.150883        0.05955    |               .    |*    .              |       0.126071
  7      0.295444        0.11659    |               .    |**   .              |       0.126259
  8      0.359400        0.14183    |               .    |***  .              |       0.126975
  9      0.279258        0.11021    |               .    |**   .              |       0.128026
 10      0.142827        0.05637    |               .    |*    .              |       0.128657

                       "." marks two standard errors
```

Examples and Instructions: Series 8

Partial Autocorrelations

```
Lag    Correlation    -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1

  1        0.72938    |                 .  |***************     |
  2       -0.33883    |           *******|  .                 |
  3        0.05222    |                 .  |* .                 |
  4        0.00753    |                 .  |  .                 |
  5        0.01918    |                 .  |  .                 |
  6        0.11087    |                 .  |**.                 |
  7        0.02107    |                 .  |  .                 |
  8        0.02566    |                 .  |* .                 |
  9       -0.03977    |                 . *|  .                 |
 10       -0.00138    |                 .  |  .                 |
```

Autocorrelation Check for White Noise

To Lag	Chi-Square	DF	Pr > ChiSq	--------------------Autocorrelations--------------------					
6	**106.65**	**6**	**<.0001**	0.729	0.373	0.139	0.034	0.010	0.060

A mixed process is harder to identify. In this example, which is actually an **ARMA(1,1)** process, the ACF appears to decline exponentially and the PACF appears to cut off after 2 lags. In theory, both should decline exponentially. The chi-square test strongly rejects that Y8 is white noise.