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An exploratory project expert system for eliciting correlation coefficient and sequential updating of duration estimation

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Abstract

This study proposes a framework for updating estimation of project duration in project networks. The first step of building a project expert system is to elicit the correlation coefficient of activity durations from experts' knowledge and intuition. Given the correlation coefficients elicited, the linear Bayesian approach is used to update the distribution of activity duration. In particular, by reflecting the newly observed duration of completed activities, we can update the duration of upcoming activities repeatedly throughout the entire project period. This helps keep track of the constantly changing longest duration path within the networks. Finally, it is shown that all these learning and updating schemes can be relatively easily implemented on an Excel spreadsheet, so that field managers can apply the model into real projects.

Keywords: Elicitation of correlation coefficient; Sequential updating of duration estimation; Linear Bayesian approach; Project expert system

1. Introduction

During the past five decades, since the development of the Critical Path Method (CPM) and the Program Evaluation and Review Technique (PERT), a considerable amount of theoretical research has been devoted to the area of project scheduling and control. In spite of their vast applicability and popularity, these two traditional methods contain fragile assumptions, such as a deterministic approach to activity durations in the CPM and probabilistic independence among activity durations in the PERT. Consequently these methods lack the ability to learn and adapt within a project.

Projects are, by nature, apt to be sensibly affected and delayed by uncertainties stemming from various reasons such as political, financial, logistical, and technical. It has often been said that more risks are present in Research and Development (R&D) projects, compared to projects in general due to various reasons, e.g., high uncertainty of technological feasibility. Novelty makes it difficult to estimate cost and time requirements for each activity (Manglik and Tripathy, 1988). Shen (1997) surveyed what practitioners in the Hong Kong construction industry call the major risk management actions. Among these, making proper time estimates and planning using subjective judgments are ranked as the first preventive actions, which emphasizes the importance of accurately predicting uncertainties and logical scheduling.

In the static approaches such as the CPM and the PERT, once the critical path, the longest duration path in the corresponding networks, is determined before project launching, its updating is not allowed thereafter. In practice, the critical path changes continuously, in conjunction with the change in expectation on upcoming activity duration. Most existing approaches have adopted this unrealistic assumption of probabilistic independence for the sake of simplicity and solvability of the models. However, it is not unusual to experience signal effects between activity durations in real projects. Some activities in a particular project consume similar types of resources or suffer from common environmental risks. For example, unexpected severe weather might affect several consecutive activities in the same way, i.e., dragging activity completion. By observing the rate of a former activity progression, we can estimate the rate for remaining activities. We can detect the underlying changes in projects by catching the signal effects. It must be stressed that most widely used software packages today in the area of project management, such as Microsoft Project and Primavera, do not include this type of learning. That is, they are built upon the assumption of

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statistical independence. Lack of this property in present systems has been a good motivation for this study.

A number of academic studies have focused on improving the accuracy of activity duration estimation by using the learning curve effect (Ayas, 1996; Badiru, 1995; Teplitz & Amor, 1993). However, the use of the learning curve effect is extremely limited to repetitive projects, such as apartment or condominium construction projects (The Urban Land Institute, 1995). Most projects are rather small in size, and are characterized by unique aspects (Meredith & Mantel, 1995). Therefore, the chance of learning on identical activities between two projects is very rare. The learning curve effect is a better fit for routine work, rather than activities in projects.

A more realistic model for allowing learning would be made for activities within a particular project. Such mending attempts have been made in the last decade by interrogating the assumption of probabilistic independence within a project. A PERT Belief Network, by Jenzarli (1994), combined activity duration with exogenous variables that affect activity duration in the network. Van Dorp and Duffey (1999) incorporated a factor that causes positive dependence among activity duration. Virto et al. (2002) formulated a project management problem under uncertainty with resource constraints using an augmented probability model. A full Bayesian approach was made by Covaliu and Soyer (1996; 1997) in order to update the distribution of activity durations for a small-sized project. Diaz et al. (2003) introduced the Markov Chain Monte Carlo simulation technique to model the sequential optimization problem. More recently Cho and Covaliu (2003) illustrated that a Bayesian updating together with sequential crashing decision of activity durations might decrease the expected project cost.

The first objective of this paper is to develop a project expert system in an exploratory sense that provides a framework for eliciting subjective judgment on the degree of statistical dependence between a pair of activity durations. Secondly, we aim to draw an inference about the distribution of upcoming activity duration given observation of completed ones. In the analysis, the inference is made in a linear Bayesian scheme.

2. Nature of dependence in projects

As pointed out earlier, most existing approaches have assumed activity durations to be independent of each other. As a result, early completion or delay of preceding activity affects only the starting and finishing point of time for succeeding activities, but not their duration. This paper asserts that activity durations might be correlated with each other and thus the duration of a preceding activity might reveal valuable information about the durations of succeeding activities. We relax such an unrealistic assumption of independence based on two reasons: resource sharing and common environmental risk factors.

First, it is common for some activities of a project to share resources. Resources usually include material, labor, equipment, money, and utility. Among these, human resource might be a good candidate for yielding a statistical dependence on certain activities. Suppose a newly-organized task force team is supposed to carry out a group of activities in a serial way. If it took the team more time than expected for the preceding activity, we should be able to predict that activities that follow would take longer than originally expected. Presumably the cause of overall delay might be low labor productivity, resulting from various factors such as knowledge, experience, and expertise.

Second, activity durations might be affected by common environmental risk factors such as weather, geographical characteristics, and financial risks. According to the American Association of State Highway Officials (1973), weather tremendously affects construction of highway bridges on almost all areas such as concrete masonry, bridge railing, painting, waterproofing, expansion joints, and so on. Weather is a composite of various factors such as humidity, temperature, shade, and stability of air flow. Another risk factor would be, e.g., financial risk including the change in various interest rates, foreign exchange rate, real estate prices, and more importantly people's expectations regarding these rates.

3. Elicitation of correlation coefficient

Statistical dependence between activity durations in a particular project can be adequately captured by the quantitative framework of a project expert system proposed in this study. The project manager usually estimates the probability distribution for activity duration either subjectively or by integrating field expert opinions. If they follow the traditional PERT paradigm, they need to estimate three PERT beta time estimates for each activity duration: pessimistic time, most-likely time, and optimistic time. If a normal distribution is more suitable, they must provide mean time and variance estimates for each activity duration. Here we have two major tasks. One is to determine the duration of each activity, which is given directly from project manager or field experts. The other is to elicit the degree of statistical dependence from the estimates provided by the first task.

In this study it is assumed that activity duration follows a normal distribution. The marginal parameters, mean and standard deviation, must be assessed either by one expert or by a group of experts who have experienced similar projects before and thus possess a strong quantitative feeling toward these quantities. If the parameters are directly assessed from an expert, there will be no room for conflict. If multiple experts participate in assessing such parameters, simple averaging or weighted averaging method would be adequate for aggregating estimates, depending on the level of knowledge and skills of each expert. It is reasonable to assign a bigger weight on people of higher expertise. Assessing mean duration and standard deviation for each activity seems to be straight forward and trivial, compared to assessing a correlation coefficient.

In cases where historical data are available, a sample correlation coefficient can be easily calculated. In others where no such data are available, e.g., in projects, the degree of association between two variables must be extracted subjectively. A correlation coefficient is a synonym for the strength of association. Directly assessing a correlation coefficient has been recognized to be very difficult even for people in the quantitative analysis field. The best that people can do is to judge the existence of dependence or any type of association between two quantities. Like the derivation of utility or other psychological factors in management problems, we have elicited the quantity of our interest indirectly by measuring the decision-maker's attitude toward another system which is naturally easier to quantify and evaluate. For example, subjective probability for a certain event is derived using a betting, and utility is derived by a lottery-type questionnaire. The problem of assessing subjective probabilities and expectations was dealt with in many previous studies (Kendall, 1955; Savage, 1971; Lindley, Tversky & Brown, 1979).

In this study, we attempted to draw a correlation coefficient based on the idea by Gokhale and Press (1982). For two random quantities associated with each other, a bivariate normal density is assumed. We need to introduce a conditional probability called concordant. For variables X and Y having two independently observed pairs, (X_1, Y_1) and (X_2, Y_2), the concordance probability is:

$$\text{C\_Pr} = \Pr(Y_2 > Y_1 | X_2 > X_1). \tag{1}$$

The concordance probability is a monotone increasing function of correlation coefficient as illustrated in Fig. 1. Since there exists a one-to-one relationship between concordance probability and correlation coefficient, we can estimate the correlation coefficient inversely if the concordance probability is obtainable from Eq. (1). The shape of the figure is quite intuitive. For a perfectly positive correlation, the two quantities always move in the same direction. For a perfectly negative correlation, they never move in the same direction. For a zero correlation, there is a half chance that they move in the same direction. For the other range, they behave in a non-linear fashion, as shown in the figure.

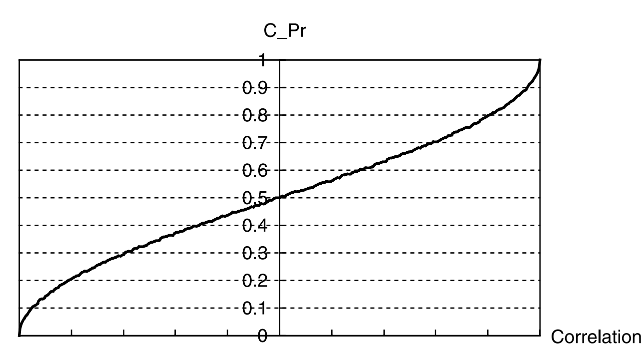

Fig. 1. Relationships of correlation coefficient and concordance probability.

3.1. Application of Concordance Probability into Project Estimation

For a pair of activities, we suggest the following three steps to successfully elicit the correlation coefficient of the two. For two activities, e.g., A and B, the eliciting steps are as follows.

Step 1. Determine the mean duration and the standard deviation for each activity.

If an expert answers the above parameters, they can be used directly. Provided that multiple experts do the answering, an averaging process must be employed, either simple averaging or weighted averaging.

Step 2. Ask the following questions.

"Does each pair of activities share human resources in a substantial way, or is it influenced by a common environmental risk? If so, do you have a strong feeling about the existence of dependence or association between the two activities?"

The answer is dichotomous. "Yes" means proceeding to Step 3. Otherwise, the correlation coefficient should be estimated as zero. When multiple experts do not answer unanimously, the majority rule might be a good determination.

Step 3. Elicit correlation coefficient from the concordance probability.

If a single expert is available, it is appropriate to ask the following question.

"In what fraction of the cases would you expect that the duration of activity B will be longer than its expected duration, given that the duration of activity A is longer than its expected duration?"

The above question is equivalent to asking the fraction of $(T_B > \mu_B | T_A > \mu_A)$, which represents the probability to be concordant. For example, if the fraction answered is 7/8, then the correlation coefficient of T_A and T_B is inversely estimated as 0.92 from Fig. 1.

In another situation when multiple experts exist, the concordance probability can be assessed in a different way. Suppose five experts have provided the mean duration and the standard deviation in Step 1. For eliciting a correlation

Table 1
Computation of concordance probability

pair (i, j)	$\mu_{A(j)} > \mu_{A(i)}$	$\mu_{B(j)} > \mu_{B(i)}$
1, 2	Yes	Yes
1, 3	Yes	Yes
1, 4	Yes	Yes
1, 5	Yes	Yes
2, 3	Yes	No
2, 4	Yes	Yes
2, 5	Yes	Yes
3, 4	Yes	Yes
3, 5	No	N/A
4, 5	No	N/A

coefficient, only the mean durations are used. Let the mean durations of activities A and B (unit: days) from five experts 1, 2, 3, 4, and 5 be as follows.

Expectation by expert 1: $(\mu_{\mathbf{A}(1)}, \mu_{B(1)}) = (8, 16)$
Expectation by expert 2: $(\mu_{\mathbf{A}(2)}, \mu_{B(2)}) = (10, 21)$
Expectation by expert 3: $(\mu_{\mathbf{A}(3)}, \mu_{B(3)}) = (12, 19)$
Expectation by expert 4: $(\mu_{\mathbf{A}(4)}, \mu_{B(4)}) = (13, 23)$
Expectation by expert 5: $(\mu_{\mathbf{A}(5)}, \mu_{B(5)}) = (11, 22)$

From the above data, we can compute the fraction $\mu_{B(j)}$, $\mu_{B(i)}$ of all pairs $\{(\mu_{\mathbf{A}(i)}, \mu_{B(i)}), (\mu_{A(j)}, \mu_{B(j)})\}$ in which $\mu_{\mathbf{A}(j)}$, $\mu_{A(i)}$ where $i, j = 1, 2, 3, 4$, and 5 $(i \neq j)$. Since we take two at a time from five pairs, the number of possible combinations is ten. Table 1 displays the computing processes. The concordance probability is:

$$\text{C\_Pr} = \Pr\left(\mu_{B(j)} > \mu_{B(i)} \middle| \mu_{A(j)} > \mu_{A(i)}\right) = 7/8.$$

Again, from Fig. 1, the correlation coefficient must be assessed as 0.92.

4. Sequential updating model

We describe how the distribution of duration of upcoming activities can be sequentially updated in the linear Bayesian scheme. This is followed by the illustration of implementing a project expert system on a spreadsheet.

4.1. Linear Bayesian model

Reluctance by some people to use the Bayesian approach might come from a computational load, including calculus and integration on joint probability density functions. To relieve such computational complexity required in the full Bayesian scheme, while maintaining superior statistical inference, a few studies have introduced a linear Bayesian method (Farrow et al, 1997; Farrow, 1998). A notable merit of the linear Bayesian approach is that only the first two moments, i.e., mean and variance, of the distribution of a random quantity are used in the updating processes. Neither calculus nor integration on density functions is necessary. Thus, project professionals can use this approach with a minimal level of arithmetic.

We formulate the Bayesian updating scheme using the linear Bayes' theorem by Hartigan (1969) in particular. Notation $E[\cdot]$ represents the expectation of a quantity and $V[\cdot]$ for its variance. For two activity durations T_A and T_B, a linear equation is set for the expectation of T_A given T_B such that $E[T_A|T_B] = cT_B + d$ where c and d are chosen to minimize the variance of the equation. Then, the conditional variance of T_B given T_A, $V[T_B|T_A]$, can be directly calculated from the combination of marginal variance and present data variance.

$$\frac{1}{V[T_B|T_A]} = \frac{1}{V[T_B]} + \frac{c^2}{V[T_A|T_B]} \tag{2}$$

Next, the conditional expectation of T_B given T_A, is obtained by weighted-averaging its marginal mean and present data where the weights coming from Eq. (2) such that:

$$E[T_B|T_A] = E[T_B]\frac{\frac{1}{V[T_B]}}{\frac{1}{V[T_B|T_A]}} + \left(\frac{T_A - d}{c}\right)\frac{\frac{c^2}{V[T_A|T_B]}}{\frac{1}{V[T_B|T_A]}} \tag{3}$$

In this study, learning is defined such that we can update the first two moments of a distribution of the duration of upcoming activities given observation of the duration of completed activities.

The following example will illustrate how learning occurs given the data about activity duration (unit: days), following a bivariate normal density, and the correlation coefficient of T_A and T_B:

$$T_A \sim N(25, 4^2), \qquad T_B \sim N(30, 5^2), \qquad \rho_{AB} = 0.6. \tag{4}$$

The conditional mean and variance of T_B given T_A can be computed using Eqs. (2) and (3):

$$T_B|T_A \sim N\left(11.25 + 0.75T_A, 4^2\right).$$

For example, if activity A has taken 29 days, one standard deviation above its marginal mean, then the conditional expected duration of activity B given T_A is updated as 33 days, which is greater than the marginal mean. If activity A has taken shorter than expected, e.g., 21 days, one standard deviation below its marginal mean, then the conditional expected duration of activity B given T_A is updated as 27 days, less than the marginal mean. Learning with respect to expectation is apparently identified. The longer activity A took, the longer activity B tends to take.

The result of a two-way sensitivity analysis is illustrated in Fig. 2 for the conditional expected duration of activity B, as a function of the duration of activity A and the correlation coefficient of T_A and T_B. For the range where the correlation coefficient is positive, the more T_A increases from the marginal expectation of 25 days, the more the conditional expected duration of activity B given T_A increases from the

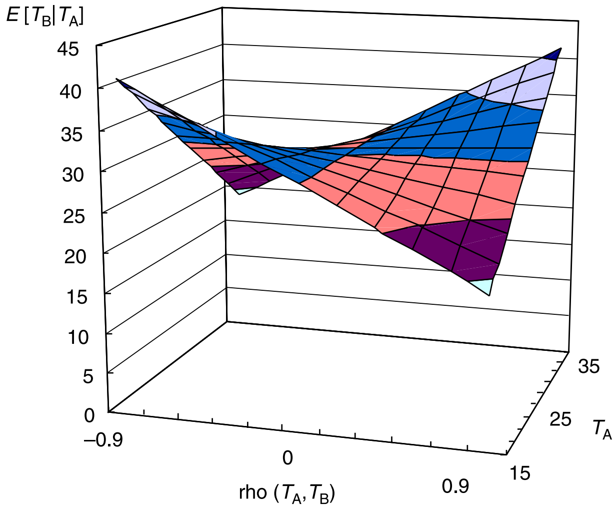

Fig. 2. Conditional expected duration of activity B given the duration of activity A and the correlation coefficient.

marginal expectation of 30 days. The more decreases from 25 days, the more the conditional expectation of activity B given T_A decreases from 30 days. These tendencies become stronger for an increasing correlation coefficient. For the range of negative correlation coefficient, consistent patterns are identified. Lastly, if the correlation coefficient is zero, the conditional expected duration of activity B given T_A remains unchanged at 30 days regardless of T_A.

It is also worthwhile to examine another property of the proposed model with respect to risks associated with the estimation. If Eq. (2) is carefully scrutinized, it is obvious that the conditional variance becomes smaller than the marginal variance. Briefly, precision is 1 over variance. A greater precision implies less uncertainty about the corresponding distribution. Since the conditional precision is the sum of the marginal precision and the present data precision, as seen in Eq. (2), the conditional precision is always greater than the marginal precision. Given the same marginal parameters, if the correlation coefficient is 0.2, the conditional distribution of T_B given T_A follows N($23.75+0.25T_A$, 4.89^2). For the correlation coefficient of 0.4, it follows N($17.5+0.5T_A$, 4.58^2, and for 0.8, N($5.0+1.0T_A$, 3.00^2). In conclusion, an increasing correlation coefficient results in a smaller variance for the conditional distribution, which implies forecasting will be more accurate. The effect of learning in terms of uncertainty reduction becomes greater for a project consisting of highly correlated activities.

4.2. Implementing project expert system on spreadsheet

The proposed linear Bayesian model can be implemented using a spreadsheet modeling tool, such as Excel, as shown in Fig. 3. A simple example of Project Expert System (PES) is demonstrated using condition (4) in Section 4.1. A cell surrounded with thick solid lines (—) is used for an input cell, while a cell with grid lines (- - -) is used for an output. “u_ex.” denotes an unconditional expectation, “c_ex.” for a conditional expectation, and “obs.” for an observation.

Before any observations are noted, the expected project duration is determined by the unconditional expected duration of activities. In (a) of Fig. 3, it is shown as 55 days. Upon observing the duration of activity A, e.g., 28 days in (b) of Fig. 3, the conditional duration of activity B is calculated as 32.25 days, using Eqs. (2) and (3). Thereby, the expected project duration is updated as 60.25 days. For these computations, only a minimal level of algebraic functions of Excel is used. Of course, it is desirable for the PES to keep adding customized input and output functions in the future, depending on the needs of the users. One of the most challenging duties for the project team would be monitoring, on a real time basis, the probability of completing the project within a certain point in time. If the committed project delivery time is 65 days, by plugging this value into the input cell, the probability is computed as 88.20%, using the “NORMDIST” function in Excel.

5. Application example

The kitchen appliance project from Stevensons (1993) was used for demonstrating how the proposed PES can be implemented. The project consists of 13 activities, and activity descriptions, durations, and precedence relationships are summarized in Table 2. For implementing the PES, two major tasks include eliciting correlation coefficients and developing a sequential updating model.

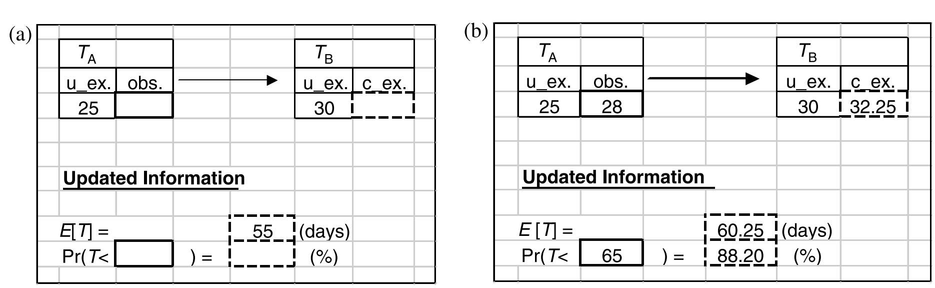

Fig. 3. Simple user interface example of project expert system (a) Before observation (b) After observation and/or feeding question.

Table 2
Data about the kitchen appliance project (duration unit: days)

Activity	Description	Immediate predecessors	$E[T_i]$	$V[T_i]$
A	Select and order equipment	–	20	5^2
B	Receive equipment from supplier	A	32	8^2
C	Install and set up equipment	A	15	3^2
D	Finalize bill of materials	B	14	4^2
E	Order component parts	C	24	7^2
F	Receive component parts	E	12	3^2
G	First production run	D, F	20	6^2
H	Finalize marketing plan	–	28	6^2
I	Produce magazine ads	H	30	6^2
J	Script for TV ads	H	21	5^2
K	Produce TV ads	J	30	3^2
L	Begin ad campaign	I, K	30	6^2
M	Ship product to customers	G, L	15	3^2

First, we explain the processes of eliciting correlation coefficients. Step 1 is finished since the mean durations and the standard deviations of activities are assumed to be collected and displayed in Table 2. Step 2 involves asking questions about the existence of statistical dependence or association between the pairs of activities. As seen in Fig. 4, 78 pairs exist for 13 activities. For each pair, the project team is supposed to determine whether there is a strong reason to believe that the two activities are statistically associated. In Fig. 4, e.g., eight pairs are assumed to be given “y” which stands for “yes”. This means that in Step 3, eight concordance probabilities must be computed. One of two computing techniques, proposed in Section 3, can be applied, depending on the number of experts who participated in the estimation processes. Finally, using the inverse function in Fig. 1, the correlation coefficients for the eight pairs are elicited.

For ease and speed of the computation process, it would be a good approach to build a matching table for a concordance probability and a correlation coefficient, wherein the correlations are calibrated in two decimal points, resulting in 200 intervals for the range of −1.00 and +1.00. For example, if the concordance probability computed lies between 0.7610 (including lower bound) and 0.7672 (excluding upper bound), then the correlation coefficient will be assigned as 0.74, and for the concordance probability between 0.7672 (including lower bound) and 0.7708 (excluding upper bound), the correlation coefficient will be 0.75.

	A	B	C	D	E	F	G	H	I	J	K	L	M
A		y											
B													
C				y									
D							y						
E									y				
F													y
G													
H									y				
I												y	
J													
K												y	
L													
M													

Fig. 4. Matrix for asking statistical dependence for the pairs of activities.

Once the correlation coefficients are available, we are ready to build up a sequential updating model. By linking Table 2 and Fig. 4 together, the linear Bayesian model can be constructed. Only for those activities whose statistical dependence is confirmed, the conditional mean and variance are computed using Eqs. (2) and (3). The ultimate user interface of the PES should look similar to the one on the spreadsheet, as illustrated in Fig. 5. Four paths exist given the precedence relationships. For activities without predecessors, two kinds of cells should be prepared: a cell for an unconditional expected duration and an input cell for an observed duration. For ending activities, a cell for an unconditional expected duration and an output cell for a conditional expected duration are necessary. For other activities lying in the middle of networks, it is appropriate to supply all three types of cells, even for unconditioned activities. For such unconditioned activities, activity J in this analysis, the conditional expected duration is automatically set to equal the unconditional one. It would be more economical to do so rather than distinguishing activities into two categories of having zero or non-zero correlations, which may result in a complex algorithm without a substantial gain. The initial user interface would look like Fig. 5.

The expected duration of project is estimated as 106 days by adding the unconditional expectations of the critical path activities. As the observed durations are substituted for the input cells upon the completion of each activity, the PES will start to substitute the conditional expected duration for activities that are conditioned and also update the expected project duration. The PES predicts the probability of completing the project by a certain day if inquired.

By the way, according to the evaluation criteria for the systems suggested by Liebowitz (1986), the proposed PES may be satisfactory in the following concerns: ability to update, ease of use, hardware, cost-effectiveness, quality of decision, and design time. Thus, we believe that the PES in this study, although not reaching a highly professional level, pioneered a new way of learning and updating processes in the area of project estimation.

6. Concluding remarks

It is often said that projects today have been plagued with increasing uncertainty as the environmental changes surrounding business have become more unpredictable with a shorter cycle, e.g., due to accelerating speed of technological advancement. Therefore, it would be worthwhile to develop more realistic project estimation techniques

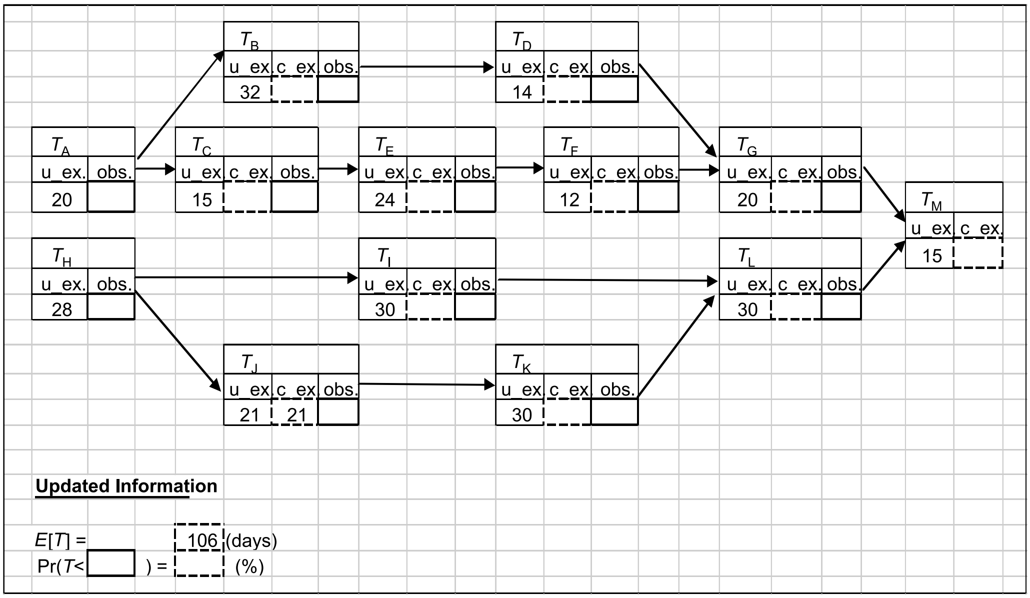

Fig. 5. Project Expert System for the kitchen appliance project.

that allow learning from experience. The main contributions of this study can be summarized into three aspects.

An important contribution of this study to be noted is that the method of eliciting people's knowledge, statistical dependence among activity durations in particular, has been proposed as the first procedure for building an expert system for projects. Since even sophisticated experts in the field of quantitative analysis seem to have difficulty assessing correlation coefficient directly, it is essential to build a template for eliciting it indirectly. With the prepared questionnaire for the project managers and field experts, it is easier for them to comprehend the meaning of a concordance probability and translate it into a correlation coefficient.

Secondly, the potential effects of Bayesian learning on estimation are significant. Learning about activities from observation within a particular project is essential, since projects are, by nature, non-repetitive. In other words, we gain very little advantage using statistical inference from traditional learning system such as, e.g., learning curve effect. This study has shown that the linear Bayesian scheme, in fact, has a substantial impact on estimating the duration of upcoming activities and reducing uncertainties associated with estimation. Although resource planning does not belong in the scope of this study, more precise forecasting based on coherent inference might influence projects in many fields to be managed more efficiently, resulting in significant saving of resources.

Lastly, this study suggested how the spreadsheet modeling for the PES, in an exploratory sense, should be implemented. We term it "exploratory" because current software and decision support systems for project management did not include the knowledge elicitation and sequential learning proposed in this study. We have just initiated a few user interfaces that a newly designed PES should contain, and thereby the proposed spreadsheet models may look a little rough in a professional perspective. However, the practical contribution in the area of project management techniques must be emphasized in light of the fact that the PES proposed is easy to understand and implement using the Excel spreadsheet, which is readily available today.

Future directions of this study might take two possible avenues. One of the limitations of this study is that the PES proposed has dealt with the estimation side only. In order to be a complete expert system for projects, the PES should also contain a decision-making module. Decision-making in the context of project management generally implies the resource allocation plan. Monitoring constantly changing critical path activities makes us revise resource shifting. In other words, it must continuously control what activities to be controlled more, i.e., more resources must be reallocated, and others with less resources. For example, those upcoming activities with an increased expected duration must be allocated more resources compared to the original plan, in order to meet the project deadline and to minimize the total project cost. The other extension of this study is to enhance the quality of PES through embodying more desirable properties and designing more user-friendly interfaces. Also, it may be possible for the PES to be professionally programmed as an add-in module for currently used software or systems.

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