NORTHWESTERN UNIVERSITY

SCHOOL OF CONTINUING STUDIES

Handout: Problem Set #7 Solutions

PREDICT 401: Introduction to Statistical Analysis

These problem sets are meant to allow you to practice and check the accuracy of your work. Please do not review the solutions until you have finalized your work. Although these problem sets are not submitted and graded, treat them as if they were. It is to your great benefit to work on and even struggle with the problem sets. Looking at the solutions before finalizing your work will, quite simply, make for a less meaningful learning experience.

1. You are looking for research funding for the upcoming year. You have two options:

Strategy 1: Submit three proposals for funding. Each has a 0.4 likelihood of being accepted. Each proposal would take 1/3 of your time. Assume that the likelihood of getting any proposal funded is independent of whether any other is funded.

Strategy 2: Submit one proposal for funding. It has a 0.6 likelihood of being funded. It would take 100% of your time.

a. List all the possible outcomes from Strategy 1. What is the probability of each outcome?

Outcome 1: None get funded:

$\overline{F}$ and $\overline{F}$ and $\overline{F}$ = .6*.6*.6 = .216

Outcome 2: Exactly one gets funded:

a) F and $\overline{F}$ and $\overline{F}$ = .4*.6*.6 = .144
b) $\overline{F}$ and F and $\overline{F}$ = .6*.4*.6 = .144
c) $\overline{F}$ and $\overline{F}$ and F = .6*.6*.4 = .144

P(a or b or c) = (.144+.144+.144) = .432

Outcome 3: Exactly two get funded:

a) F and F and $\overline{F}$ = .4*.4*.6 = .096
b) $\overline{F}$ and F and F = .6*.4*.4 = .096
c) F and $\overline{F}$ and F = .4*.6*.4 = .096

P(a or b or c) = (.096+.096+.096) = .288

Outcome 4: All three get funded:

F and F and F = .4*.4*.4 = .064

b. How many grant proposals would you expect to get funded under Strategy 1? What percentage of your time would you expect to get funded?

Expected value = E(X) = $\sum_{i=1}^{n}(x_i * p(x_i))$

X (# proposals funded)	p(X)	X * p(X)
0	.216	0
1	.432	.432
2	.288	.576
3	.064	.192
Sum	1.0	1.2

Percentage of time: same thing, but instead of number of proposals, use time funded.

X (time funded)	p(X)	X * p(X)
0	.216	0
.333	.432	.143
.667	.288	.193
1.0	.064	.064
Sum	1.0	.40

So, you'd expect 1.2 proposals funded, accounting for 40% of your time. (Note, 1.2 proposals * 1/3 time also equals 40% of time funded).

c. Which strategy would you choose? Discuss your answer. Provide evidence and an argument.

Answers can (and probably should) vary here. For me, I'd like to have the greater assurance that I get at least one grant, even if that means, on average, that I'll have less of my time funded (the expected value of the time funded for strategy 2 = .60). So, I'll take Strategy 1, where, on average, I'll have 1.2 grants funded, as opposed to Strategy 2, where I'll have only .6 funded on average.

2. Imagine that Chicago has had problems lately retaining science teachers. You want to run a study looking at why these teachers leave. You look at eight new Chicago science teachers. The average annual departure rate for new science teachers over the first 10 years of teaching is 0.3 (assume this means that any science teacher has a probability of leaving of 0.3 in any given year). Assume that whether any of the eight teachers leave is independent of whether any other teacher leaves.

a. What is the expected value of the number of teachers in your sample who will leave this year?

E(X) = n*p = 8*.3 = 2.4

b. What is the probability that exactly two teachers will leave in a year?

$\frac{n!}{(n-s)!*s!} * (\pi^s(1-\pi)^{n-s})$ **= [8! / (6!2!)] * 3^2(1-.3)^6 = 0.297**

You could do this by going to a binomial table for n=8, s=2, and π=0.3. This gives an answer of 0.296 (different than above probably due to rounding).

c. What is the probability that two or fewer teachers will leave in a year?

You could go to a binomial table and find P(0), P(1), and P(2), for n=8, s=0, 1, and 2, and π=0.3. These give 0.058, 0.198, and 0.296, respectively. Summing them gives a total probability of 0, 1, or 2 teachers leaving = 0.552.

Alternatively, you could realize that P(2 or fewer) = 1 – P(3 or more). Then, you could go to a cumulative binomial table to see that P(3 or more) for n=8 and π=0.3 is 0.448. Thus, 1-P(3 or more) = 1-0.448 = 0.552.

d. What is the probability that four or more teachers will leave in a year?

0.194, according to a cumulative binomial table for n=8, s=4+, and π=0.3.

e. What is the probability that at least one of the teachers will leave in a year?

P(at least one) = 1 – P(none).
P(none) = P(don't leave) for all seven teachers = 0.7^8 = .058.
1-.058 = .942

f. What is the probability that at least one of your current teachers will remain after seven years?

For each teacher:
P(stays 7 years) = P(stay yr1) * P(stay yr2) * . . . * P(stay yr7) = $(1-.3)^7$ = .0823
So, there are 8 teachers, each with a 0.0823 chance of staying for the 7 years.
Thus, each teacher has a 1-0.0823 = 0.9177 chance of leaving sometime within 7 years.

P(at least one stays for 7 years) = 1-P(none stay 7 years)
P(none stays 7 years) = P(teacher1 leaves within 7 years) * P(teacher2 leaves within 7 years) . . . * P(teacher7 leaves within 7 years)
So, P(none stays 7 years) = P(all leave within 7 years) = $(.9177)^8$.
Then , P(at least one stays for 7 years) = 1-$(.9177)^8$ = .497.