NORTHWESTERN UNIVERSITY | SCHOOL OF CONTINUING STUDIES

Handout: Problem Set #4 Solutions

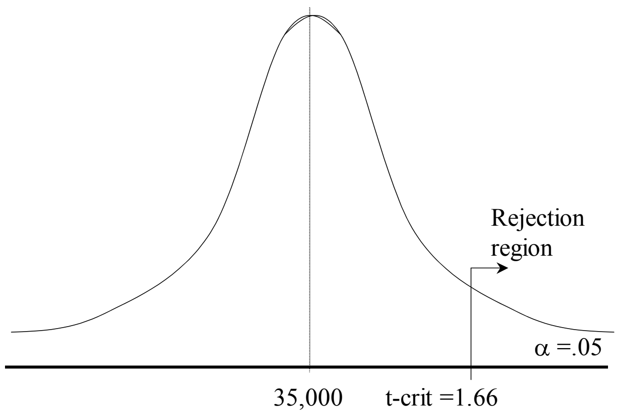
PREDICT 401: Introduction to Statistical Analysis

These problem sets are meant to allow you to practice and check the accuracy of your work. Please do not review the solutions until you have finalized your work. Although these problem sets are not submitted and graded, treat them as if they were. It is to your great benefit to work on and even struggle with the problem sets. Looking at the solutions before finalizing your work will, quite simply, make for a less meaningful learning experience.

1. You are doing an analysis of nurses' salaries in Illinois and want to know whether the average salary of Illinois' nurses is more than $35,000. You select a random sample of 100 Illinois nurses and find an average salary of $38,300 with a standard deviation of $3,000. Test your hypothesis at the .05 level. Be explicit in your work and go through the steps required to test a hypothesis. Drawing pictures would help.

H_o: $\mu \leq$ $35,000
H_a: $\mu >$ $35,000
Use t distribution because we don't know population standard deviation.
We set our critical value: $t_{.05}$ = 1.66 (rounding to nearest value given in table).

$t = \bar{x} - \mu \Big/ \frac{s}{\sqrt{n}} =$ **(38,300-35,000) / (3000/√100) = 11.**

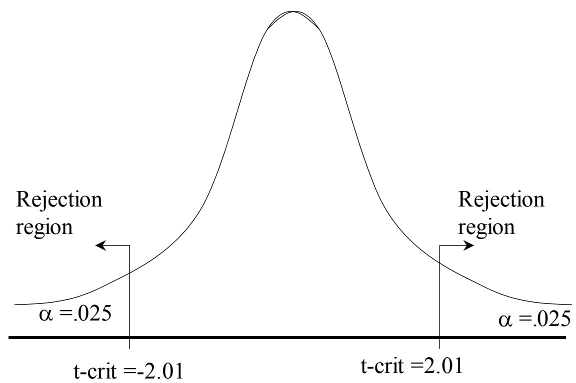


Since 11 falls to the right of 1.66, then we can reject the null in favor of the alternative hypothesis, and we are 95% confident that the true mean of Illinois nurses is not less than or equal to $35,000.

2. The Department of Social Services wishes to evaluate the effect of a new system for handling reported cases of child abuse. Under the old system, it took an average of 15 days for a caseworker to be assigned and to visit the home after someone called in a case of abuse. In a random sample of 50 reported cases of abuse under the new system, the average response time was 13.5 days with a sample standard deviation of 7.0 days. Can we reject the null hypothesis at the .05 level that the new system has not changed response times? Be explicit in your work and go through the steps required to test a hypothesis. Drawing pictures would help.

H_o: μ= 15 days
H_a: $\mu \neq$ 15 days

Use t distribution because we don't know population standard deviation.
We set critical values (b/c it's non-directional): $t_{.025, 49}$ = 2.01 (extrapolating from table).



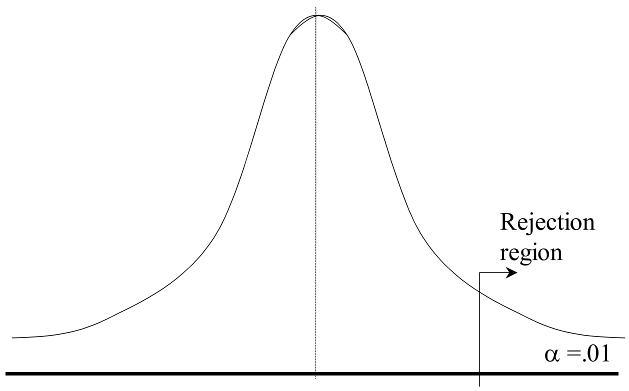
$t = \bar{x} - \mu \Big/ \frac{s}{\sqrt{n}}$ = **(13.5-15) / (7.0/√50) = -1.52**

Since -1.52 falls in the non-rejection region, then we cannot reject the null. We are not 95% confident that the response times have not stayed the same. In other words, we have no real evidence that the new system results in different response times.

3. You are the director of research at a large high school. The superintendent asks that you quickly let him know whether the students in the school are outperforming a state mandated cutoff point for standardized test scores.

   a. It would take too long to compile data on all students in your school, so you randomly select 100 students' scores. Their mean is 3 points higher than the cutoff. The standard deviation of their scores is 8 points. You want to be 99% sure of what you tell the superintendent. What do you tell him? Again, show all your work.

H_o: $\mu \leq$ cutoff
H_a: $\mu >$ cutoff
Use t distribution because we don't know population standard deviation.
We set critical value: $t_{.01}$ = 2.36 (estimating from table)



To calculate, you can choose anything as your cutoff "dummy" value. The easiest is probably zero.

$t = \bar{x} - \mu \Big/ \frac{s}{\sqrt{n}} =$ **(3-0) / (8/√100) = 3.75.**

Since 3.75 falls in the rejection region, we can reject the null that the scores are less than or equal to the cutoff. We are 99% sure that the true mean population is higher than the cutoff.

   b. What would your answer be if you had been able to randomly select only 25 students (with the same mean and standard deviation as the 100 in part (a))? Make sure to show evidence. Explain why your answer differs/does not differ from your answer in part (a) for a non-statistician audience.

Here, all of the calculations and concepts remain the same as in part (a), except that *n* is now 25 instead of 100. Therefore, we merely need to find the new t-crit, which is 2.49 (t at 0.01 with 49 degrees of freedom) and calculate a new value of t.

$t = \bar{x} - \mu \Big/ \frac{s}{\sqrt{n}}$ **= (3-0) / (8/√25) = 1.875.**

Since the value of t is less than t-critical for a 1-tailed test, it falls within the non-rejection region. Thus, I would have told the superintendent that—at 99% confidence—we can't reject the null hypothesis that the student population's achievement scores are less than or equal to the cutoff. In other words, we don't have sufficient evidence that the school's students score higher than the cutoff.

For the non-statistician audience: Given the values of the sample's mean and the spread of their scores (standard deviation), only sampling 25 students is not enough to provide sufficient evidence that the students' scores are higher than the cutoff. The more students we sample, the more sure we can be of the population's true mean. With this particular mean and standard deviation, a sample of 25 students simply does not yield the required evidence, while sampling 100 does. (*You could say something also about 25 students being enough only with a higher sample mean and/or a lower sample standard deviation.)*

4. You are the parent of a high school boy who wants to be admitted into an Ivy League college. However, to do so, you think that he will need to improve his SAT scores. You like one particular course, called SAT-up. But, before spending the $1,000 fee for the course, you want evidence that it actually works. You ask the facilitator for SAT-up for the average gain for students in the course. (Here, "gain" = practice SAT score after course minus practice SAT score before the course.) The facilitator says that they don't keep such extensive data but that they do have gain data for a group of randomly selected students. They give you the data for 100 randomly selected students:

Gain score	% of students
- 100	10
- 50	5
0	15
50	20
100	30
150	15
200	5

a. What is the mean gain of this 100-person sample?

μ = (.1)(-100)+(.05)(-50)+(.15)(0)+(.2)(50)+(.3)(100)+(.15)(150)+(.05)(200) = 60. The mean gain is 60 points.

b. What is the standard deviation of gain of this 100-person sample?

$\sigma^2 = \Sigma[(x-\mu)^2 \cdot p(x)] = (-100-60)^2 (.1) + (-50-60)^2 (.05) + (0-60)^2 (.15) + (50-60)^2 (.2) + (100-60)^2 (.3) + (150-60)^2 (.15) + (200-60)^2 (.05) = 6400.$

σ = 80.

c. What is the 90% confidence interval of the gain for students who take the class?

$$\bar{x} - (t_{\alpha/2,df} * \frac{s}{\sqrt{n}}) \le \mu \le \bar{x} + (t_{\alpha/2,df} * \frac{s}{\sqrt{n}})$$

$t_{(0.05,\ 99)} = 1.66$

60-(1.66(80/10)) ≤ μ ≤ 60 + (1.66(80/10))

(60-13.28) ≤ μ ≤ (60+13.28)

46.72 ≤ μ ≤ 73.28

We are 90% confident that the true mean gain for students who take the class is between 46.72 and 73.28 points.

d. What is the 95% confidence interval of the gain for students who take the class?

$$\bar{x} - (t_{\alpha/2,df} * \frac{s}{\sqrt{n}}) \le \mu \le \bar{x} + (t_{\alpha/2,df} * \frac{s}{\sqrt{n}})$$

$z_{.025,99} = 1.98$

60-(1.98(80/10)) ≤ μ ≤ 60 + (1.98(80/10))

(60-15.84) ≤ μ ≤ (60+15.84)

$44.16 \le \mu \le 75.84$

We are 95% confident that the true mean gain for students who take the class is between 44.16 and 75.84 points.

e. For kids who take the program, do you think that their post-program scores are greater than their pre-program scores? Why/why not?

Yes, I do. We are quite sure that taking the class results in an increase in test scores. Even our lowest bound at the 95% confidence level is an increase of 44.16 points. Thus, I am quite confident that the true gain of students in the class is a positive one.

You are discussing SAT prep courses with another parent who says, "You know, you can get practice tests for free. And research has shown that the average student gains 50 points just by taking practice tests."

f. Assuming that the conclusion of this research that the parent mentions is correct, do you think the program is effective? Would you enroll your child? Explain your answers.

I am still relatively confident that the program is related to a rise in test scores. However, it isn't statistically different than just taking practice tests. (The 50-point gain falls within both our 90% and 95% confidence intervals.) Thus, we can't conclude that the program is related to gains that are any different than those from simply taking practice tests. Since we can't tell whether the program is any more effective than simply taking tests, then I feel that the program isn't very effective. I would probably not enroll my child and simply have him take practice tests for free.