Week 3 - Lecture 1

The binomial distribution, one of the more useful discrete distributions, is based on the idea of a bernoulli trial. A bernoulli expt. is where each trial results in 2 and only 2 possible outcomes. That is, if X is a bernoulli random variable, then

$$X = \begin{cases} 1 & \text{with prob } p \\ 0 & \text{with prob } 1-p \end{cases}, \quad 0 \le p \le 1.$$

1 = success
0 = failure

$$EX = 1 \times p + 0 \times (1-p) = p,$$

$$VX = (1-p)^2 \times p + (0-p)^2 \times (1-p) = p(1-p).$$

Examples ⇒ Tossing a coin,
Roulette,
Voting.

If n identical Bernoulli trials are performed, define the events

$$A_i = \{X=1 \text{ on the } i^{th} \text{ trial}\},$$
$$i=1,\dots,n.$$

Let $A_1, \dots, A_n$ be a collection of independent events. It is easy to derive the distribution of the total number of successes in n trials.

Let Y = # number of successes in n trials.

It can be shown that

$$P(Y=y|n,p) = \binom{n}{y} p^y (1-p)^{n-y}$$
$$y=0,1,2,\dots,n$$

Y is called a binomial (n,p) random variable.

It can be shown that
$EY = n \times p$ and $VY = n \times p \times (1-p)$

Ex. Suppose we are interested in finding the prob. of obtaining at least one 6 in 4 rolls of a fair die.

Here $n=4$ and $p=\frac{1}{6}$.

$$P\{Y \geq 1\} = P(Y=1) + P(Y=2) + P(Y=3) + P(Y=4)$$

$$= 1 - P(\cancel{X}=0)$$

$$= 1 - \binom{4}{0}\left(\frac{1}{6}\right)^{0}\left(\frac{5}{6}\right)^{4-0}$$

$$= 1 - (1)(1)\left(\frac{5}{6}\right)^{4}$$

$$= 1 - \left(\frac{5}{6}\right)^{4}$$

$$= 51.8\%$$

Excel Functions $\Rightarrow$ "binomdist"