

Response to critiques for article EJS2102-039

Third Revision

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April 21, 2022

In the following we provide responses to specific points, numbering comments as **ReviewerID.CommentNumber**. Reviewer comments are italicized and our response is in non-italic font.

1 Associate Editor

The authors have revised the paper after two rounds of refereeing. It is much better readable now, but some further improvements will be desirable.

(AE.1) *One issue is that the paper is exceptionally long, even for an electronic journal.*

Reply: We have further condensed the Simulation and Appendix (proofs) sections, which trims the entire manuscript by 9 pages; see also responses to (AE.4) and (AE.5) for more details.

(AE.2) *I can understand the authors' keenness in publishing distributional results analogous to those of the lasso, in both fixed p (less interesting) and growing p settings. Since consistent estimation and selection are easily possible by the basic lasso, results on coverage are of more interest. For instance, it will be helpful to point out the implications of the conditional distributional convergence for coverage and size of some appropriate confidence region constructed on the basis of the distributional results.*

Reply: We do not frame the distributional results in terms of coverage, however, we recognize the importance of this perspective for uncertainty quantification and we do report coverage probabilities and average CI widths in our numerical studies (see Table 2). We also add a comment after Theorem 3.4 that the results imply convergence of CI coverage to the nominal CI coverage.

(AE.3) *The analysis of the two step-procedure is nice, but I wonder if the basic consistency is sufficiently interesting — why not get a rate of convergence?*

Reply: Thank you for this suggestion. We have added a result about the convergence rate of $P\left(\widehat{S}_n^w \neq S_0 | \mathcal{F}_n\right)$ as a lemma at the end of the Appendix section in this latest draft. It follows readily using elements of the proof of Theorem 3.4. We refer to the result in the main paper after Theorem 3.4.

(AE.4) *The simulation section is a bit tiring, and must be selective. I think the focus should be only on coverage (and size) of confidence regions obtained from the random distributions, and probabilities of different models. There are several items in this section not very appealing. For instance, I do not understand the purpose of studying the sampling distribution of the variation distance between the random weighting distribution and a posterior distribution.*

Reply: The current draft has a streamlined Simulation section that puts more emphasis on the numerical results pertaining to conditional (on data) probabilities of selecting variables (Fig 1) as well as empirical coverage and average width of coverage CI's (Table 2). We have also completely removed Simulation part II (previously Section 4.2, now removed). This section focused on a narrow point about tuning parameter selection, and dropping it does not materially diminish the reported findings.

(AE.5) *I did not check the entire proof section thoroughly, but I feel that this part may be a little too elaborate. Some compactification appears to be possible. For instance, something like Lemma A.1 is quite well known, as a consequence of the result of convergence of random series of independent random variables and Kronecker's lemma, so I believe a citation of the source textbook will do the job. Condensation of the proofs without sacrificing clarity will be appreciated.*

Reply: Thank you for this suggestion; the proof of Lemma A.1 is now removed and relevant reference is cited accordingly. We have also trimmed many other parts of the proofs, which has shortened the Appendix section by 5 pages.

(AE.6) *Other points: Please introduce all notations (like μ_W , σ_W) even though guesswork is possible.*

Reply: The notations (μ_W, σ_W) are already defined in Section 2 (the paragraphs for Other Notation) in the preceding first- and second-revision drafts.