marginally not Dirichlet distributed

Joint density for (ϕ, ψ):

$p(\phi, \psi) = \sum_{\pi \in \Pi} \omega_\pi \, p_\pi(\phi, \psi)$

for a fixed ϕ, marginal of ϕ, $f(\phi) = \int_\Omega p(\phi, \psi) d\psi$

We first look at integral for one component $p_\pi(\phi, \psi)$ of $p(\phi, \psi)$.

$$\int_\Omega p_\pi(\phi, \psi) d\psi = \int_{A(\phi)} p_\pi(\phi, \psi) d\psi + \int_{A(\phi)^c} p_\pi(\phi, \psi) d\psi$$

where $A(\phi) = \{\psi \textbf{ such that } \Phi_\pi = \Psi_\pi, \textbf{ given } \phi\}$

so $\int_{A(\phi)^c} p_\pi(\phi, \psi) d\psi = 0$

let $f(, \alpha)$ be the pdf of Dirichle(α) and $b(i)$ be the block contains index i for a given partition π

$$\begin{aligned}
\int_{A(\phi)} p_\pi(\phi, \psi) d\psi &= \int_{A(\phi)} \prod_{b \in \pi} \left[f(\tilde{\phi}, \alpha_b^1) f(\tilde{\psi}, \alpha_b^2) \right] f(\Phi_\pi, \beta_\pi) d\psi \\
&= \int_{A(\phi)} \prod_{b \in \pi} \left[f(\tilde{\phi}, \alpha_b^1) f(\tilde{\psi}, \alpha_b^2) \right] f(\Phi_\pi, \beta_\pi) d\psi_1 d\psi_2 ... d\psi_K \\
&= \int_{A(\phi)} \prod_{b \in \pi} \left[f(\tilde{\phi}, \alpha_b^1) f(\tilde{\psi}, \alpha_b^2) \right] f(\Phi_\pi, \beta_\pi) d\tilde{\psi}_1 \Phi_{b(1)} d\tilde{\psi}_2 \Phi_{b(2)} ... d\tilde{\psi}_K \Phi_{b(K)} \\
&= \int_{A(\phi)} \prod_{b \in \pi} \left[\Phi_b f(\tilde{\phi}, \alpha_b^1) f(\tilde{\psi}, \alpha_b^2) \right] f(\Phi_\pi, \beta_\pi) d\tilde{\psi}_1 d\tilde{\psi}_2 ... d\tilde{\psi}_K \\
&= \prod_{b \in \pi} \left[\Phi_b f(\tilde{\phi}, \alpha_b^1) \right] f(\Phi_\pi, \beta_\pi)
\end{aligned}$$

So ϕ is not marginally Dirichlet distributed