BIMCT Tiebreaker Problems

BIMCT Team

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Tiebreaker Round

1. What value x is such that if Bob has an infinite number of coins with values 5 and x, 71 would be the largest value he could not make?

2. In a convex decagon, all diagonals are drawn. Given that no three diagonals intersect at one point, find the number of different triangles with vertices at the intersection points of the diagonals or vertices of the decagon.

3. How many ways are there to place two pieces on an 8 by 8 chessboard, such that they are not in the same row or column? Two configurations are considered the same if a rotation of the board can make the pieces on the same positions.

4. Points A and B are such that $AB = d$. Circle α has radius 240 and is centered at A, and circle β has radius 240 and is centered at B. Point P is an intersection between α and β, and point C is on α such that $\angle CAP = 60^\circ$ and CB is maximized. then, D is the intersection between BC and β, and E is on ray AC such that $AE = BC$. Then, O is the circumcenter of CDE. Over all $0 < d < 480$, find the minimum length of OP.

5. Let S contain all positive integers x and y such that

$$((x+2y)^4 + (x-2y)^4)((y+2x)^4 + (y-2x)^4) = (x-y)^9.$$

Assume that we have polynomials $P_x(x)$ and $P_y(x)$ such that the set containing $(P_x(i), P_y(i))$ for all positive integers i is equivalent to S. Find $P_x(0)$.