BIMCT Estimathon

BIMCT Team

October 2022

Estimathon

Rules

Estimathons are completed by individuals or teams, with teams up to six people.

In an estimathon, you are trying to answer each problem to the best of your ability. To answer, you must enter a pair of positive integers (a, b), such that $a < b$. Submissions that do not meet these requirements will not be considered, and you will receive 0 points for that problem. Your score will be calculated as follows: if the correct answer A satisfies $A \in [a, b)$, then you will receive $\lfloor \frac{101a}{b} \rfloor$ points for the problem. Otherwise, you will receive 0 points for that problem. (Thus, the maximum possible score you can receive for each problem is 100)

You must have your camera on at all times, and using Google or any calculators will result in an automatic disqualification.

---

1. 24 points are equally spaced around a circle of radius 20, and lines are drawn such that every pair of points is connected. What is the sum of the lengths of all of the lines? **Answer: 7323.4** *Problem Proposed by Adam Tang*

2. What is the area of a regular polygon with 1000 sides and side length 1? **Answer: 79577.210** *Problem Proposed by Larry Xing*

3. How many 1s are in the binary representation of 2022!? **Answer: 8647** *Problem Proposed by Larry Xing*

4. How many rational numbers $\frac{a}{b}$ exist such that a and b are coprime positive integers with $0 < \frac{a}{b} < 100$ and $a + b < 1000$? **Answer: 300797** *Problem Proposed by Ary Cheng*

5. Let $S_0(n)$ be the sum of the even digits of n, and $S_1(n)$ be the sum of the odd digits of n. Find

$$\sum_{n=0}^{2022} S_1(n) - S_0(n)$$

   Answer: 3967 *Problem Proposed by Larry Xing*

6. The Fibonacci sequence is defined as $F_0 = 0, F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for all $n \geq 2$. Let $T_n(x)$ return the number of times the integer n appears in the number x. For example, $T_2(2022) = 3$. Find

$$\sum_{n=1}^{9} \sum_{i=1}^{1000} (-1)^n \cdot T_n(F_i)$$

   Answer: 12914 *Problem Proposed by Derrick Liu*

7. A dimensionless particle is released from a corner at an angle of 45 degrees to one of the sides of a rectangular room with sidelengths of 1792 and 4051 meters. Whenever the ball hits a wall, it bounces of the wall such that the angle between the incoming path and the wall is equal to the angle between the departing path and the wall such that the departing path is not identical to the incoming path. What is the total distance the particle travels before hitting a corner for the first time? **Answer: 7259392** *Problem Proposed by Derrick Liu*

8. In an infinitely large room with an infinte amount of people, a person is randomly infected. Every minute, each infected person interacts with a non-infected person, and the probability of infecting a non-infected person is 99/100. After 60 minutes, what is the expected number of infected people? **Answer: 853,462,777,143,902,694** *Problem Proposed by Derrick Liu*

9. Estimate the value of $\log_{10} n$, where n is the smallest integer n for which the sum

$$\sum_{x=2}^{100} x^{\frac{1}{\log_x n}}$$

is less than 100. **Answer: 615.16** *Problem Proposed by Derrick Liu*

10. Define $\mathbb{P}$ as the set of primes. Let q be the highest degree of x in a polynomial with minimal degree with all real coefficients such that one of its roots is

$$\sum_{n=1, n\in\mathbb{P}}^{2022} \sqrt{n}$$

Estimate $\log_5 q$. **Answer: 131.787 Answer:** *Problem Proposed by Derrick Liu*

11. Estimate the number of mountains in the Apennines with a height above 2000 meters. **Answer: 249** *Problem Proposed by Derrick Liu*

12. Estimate the number of programming languages that have ever existed as of 9/10/2022. **Answer: 8,945** *Problem Proposed by Ary Cheng*

13. Estimate the total number of employees at NASA as of 9/10/2022. **Answer: 17,960** *Problem Proposed by Derrick Liu*

14. Estimate the total number of exoplanets discovered as of 9/10/2022 **Answer: 5157** *Problem Proposed by Ary Cheng*

15. Estimate the number of seconds it would take walk from Los Angeles to Beijing if you walk at a constant speed of 5 miles per hour. **Answer: 4498560** *Problem Proposed by Derrick Liu*

16. Estimate the number of years it would take 96.73009% of a 100 kilogram sample of pure Technetium-99 to decay. **Answer: 1041285** *Problem Proposed by Derrick Liu*

17. Estimate the number of hours it would take to watch all the episodes of *Guiding Light* in one sitting. **Answer: 14185** *Problem Proposed by Derrick Liu*

18. Estimate the total number of Yu-Gi-Oh cards in the Official Card Game as of February 2022. **Answer: 12,456** *Problem Proposed by Derrick Liu*

19. Estimate the number of Big Macs sold every minute. **Answer: 1020** *Problem Proposed by Derrick Liu*

20. If the number of distinguishable ways that two decks of 52 cards can be shuffled is N, estimate $\log_{10}(N)$. (The two decks have the same 52 cards, and a card from the first deck is indistinguishable from another card of the same number and suit from the second deck.) **Answer: 150.359** *Problem Proposed by Ary Cheng*