Digital Communication Lecture-1, Prof. Dr. Habibullah Jamal

Under Graduate, Spring 2008

Course Books

Text: Digital Communications: Fundamentals and Applications, By "Bernard Sklar", Prentice Hall, 2^{nd} ed, 2001.

Probability and Random Signals for Electrical Engineers, Neon Garcia

References:

Digital Communications, Fourth Edition, J.G. Proakis, McGraw Hill, 2000.

Course Outline

- Review of Probability
- Signal and Spectra (Chapter 1)
- Formatting and Base band Modulation (Chapter 2)
- Base band Demodulation/Detection (Chapter 3)
- Channel Coding (Chapter 6, 7 and 8)
- Band pass Modulation and Demod./Detect. (Chapter 4)
- Spread Spectrum Techniques (Chapter 12)
- Synchronization (Chapter 10)
- Source Coding (Chapter 13)
- Fading Channels (Chapter 15)

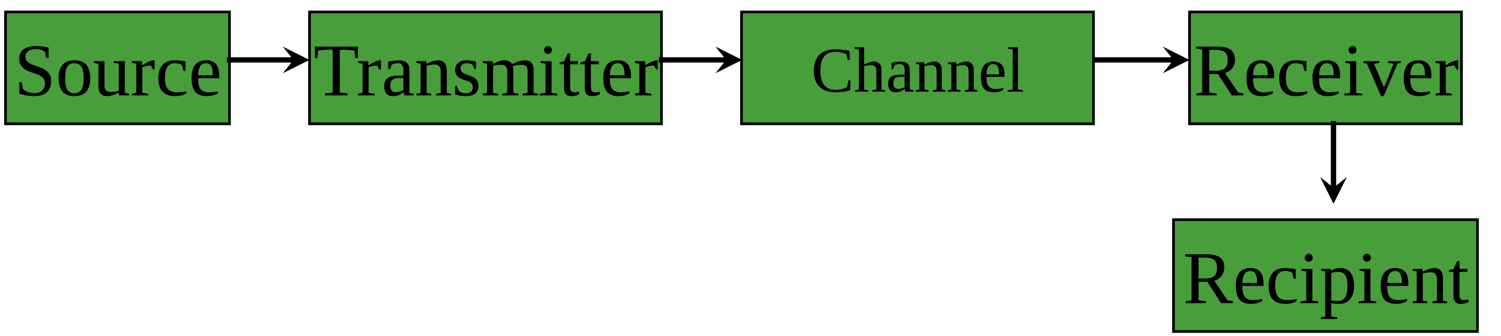
Today's Goal

- Review of Basic Probability
- Digital Communication Basic

Communication

- Main purpose of communication is to transfer information from a source to a recipient via a channel or medium.

- Basic block diagram of a communication system:



Brief Description

- **Source:** analog or digital
- **Transmitter:** transducer, amplifier, modulator, oscillator, power amp., antenna
- **Channel:** e.g. cable, optical fibre, free space
- **Receiver:** antenna, amplifier, demodulator, oscillator, power amplifier, transducer
- **Recipient:** e.g. person, (loud) speaker, computer

- **Types of information**
  Voice, data, video, music, email etc.

- **Types of communication systems**
  Public Switched Telephone Network (voice,fax,modem)
  Satellite systems
  Radio,TV broadcasting
  Cellular phones
  Computer networks (LANs, WANs, WLANs)

Information Representation

- Communication system converts information into electrical electromagnetic/optical signals appropriate for the transmission medium.
- Analog systems convert analog message into signals that can propagate through the channel.
- Digital systems convert bits(digits, symbols) into signals
  - Computers naturally generate information as characters/bits
  - Most information can be converted into bits
  - Analog signals converted to bits by sampling and quantizing (A/D conversion)

Why digital?

- Digital techniques need to distinguish between discrete symbols allowing regeneration versus amplification

- Good processing techniques are available for digital signals, such as medium.
  - Data compression (or source coding)
  - Error Correction (or channel coding)(A/D conversion)
  - Equalization
  - Security

- Easy to mix signals and data using digital techniques

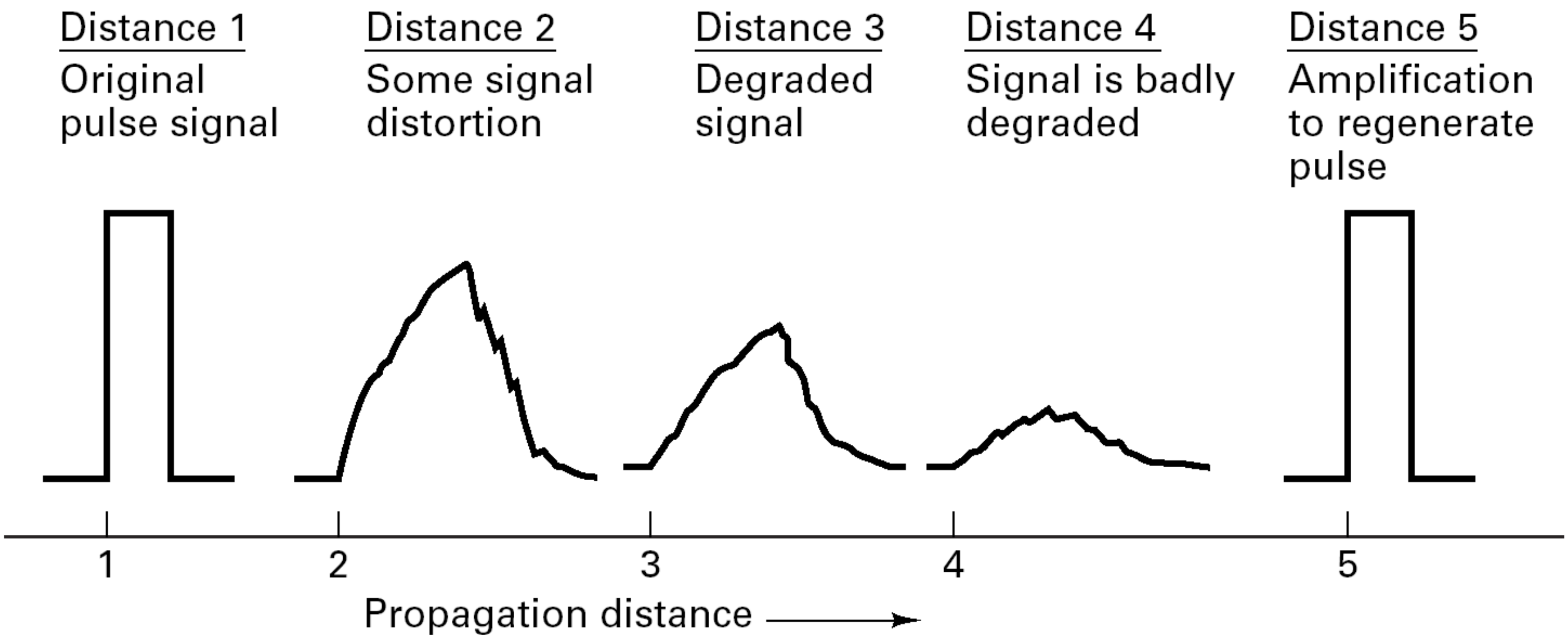


Figure 1.1 Pulse degradation and regeneration.

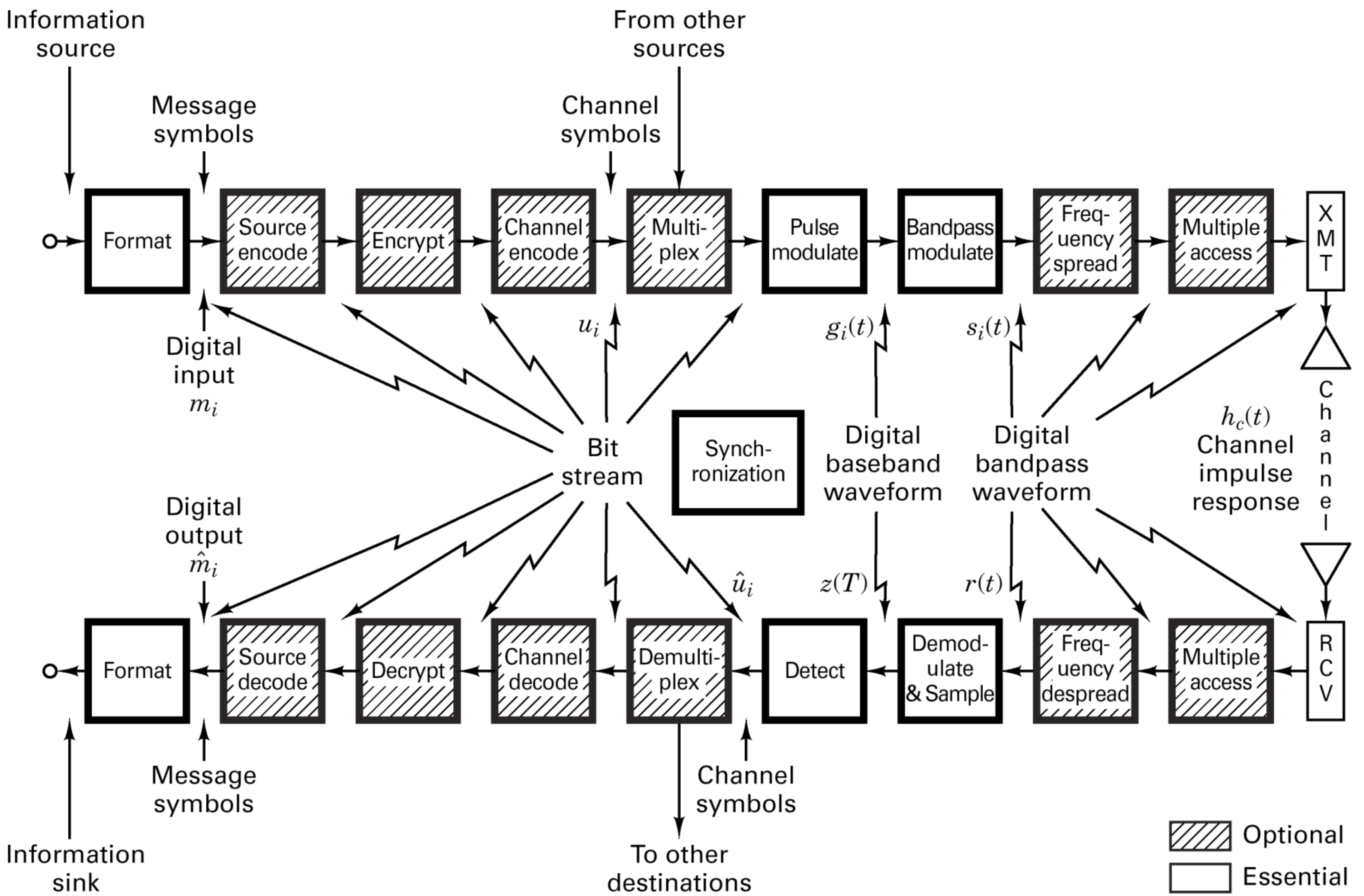


Figure 1.2 Block diagram of a typical digital communication system.

- Basic Digital Communication Transformations
  - Formatting/Source Coding
  - Transforms source info into digital symbols (digitization)
  - Selects compatible waveforms (matching function)
  - Introduces redundancy which facilitates accurate decoding despite errors
- *It is essential for reliable communication*
  - Modulation/Demodulation
  - Modulation is the process of modifying the info signal to facilitate transmission
  - Demodulation reverses the process of modulation. It involves the detection and retrieval of the info signal
    - Types
    - Coherent: Requires a reference info for detection
    - Noncoherent: Does not require reference phase information

Basic Digital Communication Transformations

- Coding/Decoding

  Translating info bits to transmitter data symbols

  Techniques used to enhance info signal so that they are less vulnerable to channel impairment (e.g. noise, fading, jamming, interference)

  - Two Categories
    - *Waveform Coding*
  - Produces new waveforms with better performance
    - *Structured Sequences*

- Involves the use of redundant bits to determine the occurrence of error (and sometimes correct it)
  - Multiplexing/Multiple Access Is synonymous with resource sharing with other users
  - Frequency Division Multiplexing/Multiple Access (FDM/FDMA

Formatting

- Character coding
- Sampling
- Quantization
- Pulse code modulation (PCM)

Source Coding

- Predictive coding
- Block coding
- Variable length coding
- Synthesis/analysis coding
- Lossless compression
- Lossy compression

Baseband Signaling

- PCM waveforms (line codes)
  - Nonreturn-to-zero (NRZ)
  - Return-to-zero (RZ)
  - Phase encoded
  - Multilevel binary
- *M*-ary pulse modulation
  - PAM, PPM, PDM

Equalization

- Maximum-likelihood sequence estimation (MLSE)
- Equalization with filters
  - Transversal or decision feedback
  - Preset or Adaptive
  - Symbol spaced or fractionally spaced

Bandpass Signaling

Coherent

- Phase shift keying (PSK)
- Frequency shift keying (FSK)
- Amplitude shift keying (ASK)
- Continuous phase modulation (CPM)
- Hybrids

Noncoherent

- Differential phase shift keying (DPSK)
- Frequency shift keying (FSK)
- Amplitude shift keying (ASK)
- Continuous phase modulation (CPM)
- Hybrids

Channel Coding

Waveform

- *M*-ary signaling
- Antipodal
- Orthogonal
- Trellis-coded modulation

Structured Sequences

- Block
- Convolutional
- Turbo

Synchronization

- Frequency synchronization
- Phase synchronization
- Symbol synchronization
- Frame synchronization
- Network synchronization

Multiplexing/Multiple Access

- Frequency division (FDM/FDMA)
- Time division (TDM/TDMA)
- Code division (CDM/CDMA)
- Space division (SDMA)
- Polarization division (PDMA)

Spreading

- Direct sequencing (DS)
- Frequency hopping (FH)
- Time hopping (TH)
- Hybrids

Encryption

- Block
- Data stream

Figure 1.3 Basic digital communication transformations.

Performance Metrics

- Analog Communication Systems
  - Metric is fidelity: want $\hat{m}(t) \approx m(t)$
  - SNR typically used as performance metric
- Digital Communication Systems
  - Metrics are data rate (R bps) and probability of bit error
    $$\left(P_b = p(\hat{b} \neq b) \right)$$
  - Symbols already known at the receiver
  - Without noise/distortion/sync. problem, we will never make bit errors

Main Points

- Transmitters modulate analog messages or bits in case of a DCS for transmission over a channel.
- Receivers recreate signals or bits from received signal (mitigate channel effects)
- Performance metric for analog systems is fidelity, for digital it is the bit rate and error probability.

Why Digital Communications?

  - Easy to regenerate the distorted signal
  - Regenerative repeaters along the transmission path can detect a digital signal and retransmit a new, clean (noise free) signal
  - These repeaters prevent accumulation of noise along the path
- This is not possible with analog communication systems
  - Two-state signal representation
- The input to a digital system is in the form of a sequence of bits (binary or M_ary)
  - Immunity to distortion and interference
  - Digital communication is rugged in the sense that it is more immune to channel noise and distortion

Why Digital Communications?

  - Hardware is more flexible
  - Digital hardware implementation is flexible and permits the use of microprocessors, mini-processors, digital switching and VLSI
- Shorter design and production cycle
  - Low cost
- The use of LSI and VLSI in the design of components and systems have resulted in lower cost
  - Easier and more efficient to multiplex several digital signals
  - Digital multiplexing techniques – Time & Code Division Multiple Access - are easier to implement than analog techniques such as Frequency Division Multiple Access

Why Digital Communications?

  - Can combine different signal types – data, voice, text, etc.
  - Data communication in computers is digital in nature whereas voice communication between people is analog in nature
  - The two types of communication are difficult to combine over the same medium in the analog domain.
- Using digital techniques, it is possible to combine both format for transmission through a common medium
- Encryption and privacy techniques are easier to implement
  - Better overall performance
  - Digital communication is inherently more efficient than analog in realizing the exchange of SNR for bandwidth
  - Digital signals can be coded to yield extremely low rates and high fidelity as well as privacy

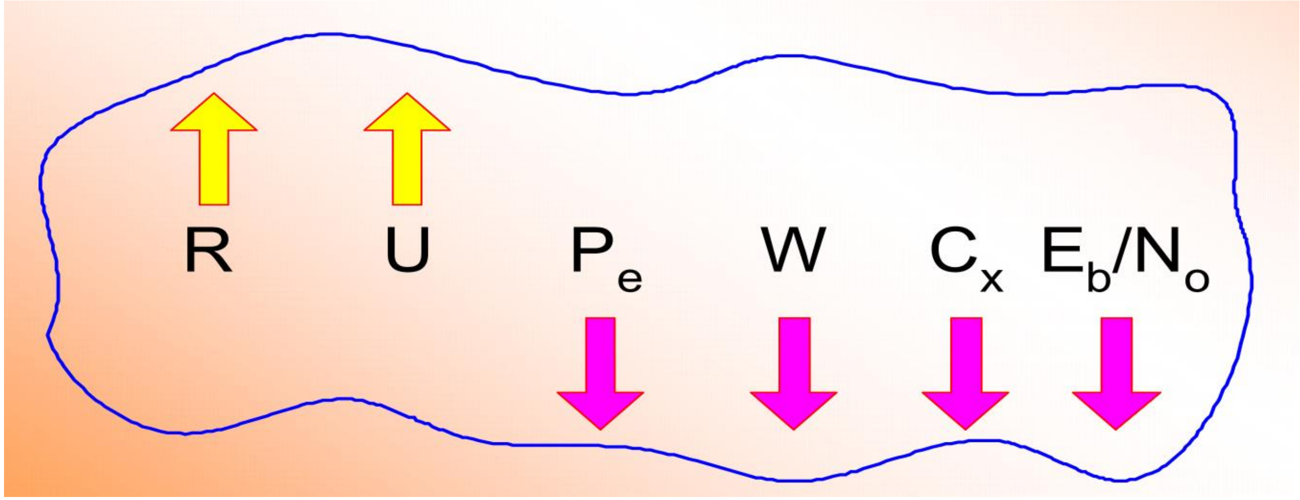
Why Digital Communications?

Disadvantages

- Requires reliable “synchronization”
- Requires A/D conversions at high rate
- Requires larger bandwidth
- Nongraceful degradation
- Performance Criteria
- Probability of error or Bit Error Rate

Goals in Communication System Design

- To maximize transmission rate, ***R***
- To maximize system utilization, ***U***
- To minimize bit error rate, P_e
- To minimize required systems bandwidth, ***W***
- To minimize system complexity, C_x
- To minimize required power, E_b/N_o



Comparative Analysis of Analog and Digital Communication

Digital Signal Nomenclature

- **Information Source**
  - Discrete output values e.g. Keyboard
  - Analog signal source e.g. output of a microphone
- **Character**
  - Member of an alphanumeric/symbol (A to Z, 0 to 9)
  - Characters can be mapped into a sequence of binary digits using one of the standardized codes such as
    - **ASCII: American Standard Code for Information Interchange**
    - **EBCDIC: Extended Binary Coded Decimal Interchange Code**

Digital Signal Nomenclature

- **Digital Message**
  - Messages constructed from a finite number of symbols; e.g., printed language consists of 26 letters, 10 numbers, “space” and several punctuation marks. Hence a text is a digital message constructed from about 50 symbols
  - Morse-coded telegraph message is a digital message constructed from two symbols “**Mark**” and “**Space**”
- **M - ary**
  - A digital message constructed with *M* symbols
- **Digital Waveform**
  - Current or voltage waveform that represents a digital symbol
- **Bit Rate**
  - Actual rate at which information is transmitted per second

Digital Signal Nomenclature

- **Baud Rate**
  - Refers to the rate at which the signaling elements are transmitted, i.e. number of signaling elements per second.
- **Bit Error Rate**
  - The probability that one of the bits is in error or simply the probability of error

1.2 Classification Of Signals

1. Deterministic and Random Signals

- A signal is *deterministic* means that there is no uncertainty with respect to its value at any time.
- Deterministic waveforms are modeled by explicit mathematical expressions, example:

$$x(t) = 5\mathrm{Cos}(10t)$$

- A signal is *random* means that there is some degree of uncertainty before the signal actually occurs.
- Random waveforms/ Random processes when examined over a long period may exhibit certain regularities that can be described in terms of probabilities and statistical averages.

2. Periodic and Non-periodic Signals

- A signal x(t) is called *periodic in time* if there exists a constant $T_0 > 0$ such that

$$x(t) = x(t + T_0) \qquad \text{for} \qquad -\infty < t < \infty \qquad (1.2)$$

t denotes time

T_0 is the *period* of $x(t)$.

3. Analog and Discrete Signals

- An *analog signal* $x(t)$ is a continuous function of time; that is, $x(t)$ is uniquely defined for all t

- A *discrete signal* $x(kT)$ is one that exists only at discrete times; it is characterized by a sequence of numbers defined for each time, kT, where

  k is an integer

  T is a fixed time interval.

4. Energy and Power Signals

- The performance of a communication system depends on the received signal *energy;* higher energy signals are detected more reliably (with fewer errors) than are lower energy signals

- $x(t)$ is classified as an *energy signal* if, and only if, it has nonzero but finite energy ($0 < E_x < \infty$) for all time, where:

$$E_x = \lim_{T\to\infty} \int_{-T/2}^{T/2} x^2(t)\,dt \qquad = \int_{-\infty}^{\infty} x^2(t)\,dt \qquad (1.7)$$

- An energy signal has finite energy but *zero average power.*

- Signals that are both deterministic and non-periodic are classified as energy signals

4. Energy and Power Signals

- *Power* is the *rate* at which energy is delivered.

- A signal is defined as a power signal if, and only if, it has finite but nonzero power ($0 < P_x < \infty$) for all time, where

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt \qquad (1.8)$$

- Power signal has finite average power but *infinite energy.*

- As a general rule, periodic signals and random signals are classified as power signals

5. The Unit Impulse Function

- *Dirac delta function* $\delta(t)$ or impulse function is an abstraction—an infinitely large amplitude pulse, with zero pulse width, and unity weight (area under the pulse), concentrated at the point where its argument is zero.

$$\int_{-\infty}^{\infty} \delta(\mathrm{t})\,\mathrm{dt} = 1 \qquad (1.9)$$

$$\delta(\mathrm{t}) = 0 \quad \text{for} \quad \mathrm{t} \neq 0 \qquad (1.10)$$

$$\delta(\mathrm{t}) \text{ is bounded at } \mathrm{t} = 0 \qquad (1.11)$$

- Sifting or Sampling Property

$$\int_{-\infty}^{\infty} x(t)\,\delta(\mathrm{t}-\mathrm{t}_0)\mathrm{dt} = \mathrm{x}(\mathrm{t}_0) \qquad (1.12)$$

1.3 Spectral Density

- The *spectral density* of a signal characterizes the distribution of the signal's energy or power in the frequency domain.

- This concept is particularly important when considering filtering in communication systems while evaluating the signal and noise at the filter output.

- The energy spectral density (ESD) or the power spectral density (PSD) is used in the evaluation.

1. Energy Spectral Density (ESD)

- Energy spectral density describes the signal energy per unit bandwidth measured in joules/hertz.
- Represented as $\psi_x(f)$, the squared magnitude spectrum

$$\psi_x(f) = |X(f)|^2 \qquad (1.14)$$

- According to Parseval's theorem, the energy of x(t):

$$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-\infty}^{\infty} |X(f)|^2\,df \qquad (1.13)$$

- Therefore:

$$E_x = \int_{-\infty}^{\infty} \psi_x(f)\,df \qquad (1.15)$$

- The Energy spectral density is symmetrical in frequency about origin and total energy of the signal x(t) can be expressed as:

$$E_x = 2\int_{0}^{\infty} \psi_x(f)\,df \qquad (1.16)$$

2. Power Spectral Density (PSD)

- The *power spectral density* (PSD) function $G_x(f)$ of the periodic signal $x(t)$ is a real, even, and nonnegative function of frequency that gives the distribution of the power of $x(t)$ in the frequency domain.
- PSD is represented as:

$$G_x(f) = \sum_{n=-\infty}^{\infty} |C_n|^2 \delta(f - nf_0) \qquad (1.18)$$

- Whereas the average power of a periodic signal x(t) is represented as:

$$P_x = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x^2(t)\,dt = \sum_{n=-\infty}^{\infty} |C_n|^2 \qquad (1.17)$$

- Using PSD, the average normalized power of a real-valued signal is represented as:

$$P_x = \int_{-\infty}^{\infty} G_x(f)\,df = 2\int_{0}^{\infty} G_x(f)\,df \qquad (1.19)$$

1.4 Autocorrelation

1. Autocorrelation of an Energy Signal

- Correlation is a matching process; *autocorrelation* refers to the matching of a signal with a delayed version of itself.
- Autocorrelation function of a real-valued energy signal *x*(*t*) is defined as:

$$R_x(\tau) = \int_{-\infty}^{\infty} x(t)\, x(t+\tau)\, dt \qquad \text{for} \qquad -\infty < \tau < \infty \tag{1.21}$$

- The autocorrelation function $R_x(\tau)$ provides a measure of how closely the signal matches a copy of itself as the copy is shifted τ units in time.
- $R_x(\tau)$ is not a function of time; it is only a function of the time difference τ between the waveform and its shifted copy.

1. Autocorrelation of an Energy Signal

- The autocorrelation function of a real-valued *energy* signal has the following properties:

$$R_x(\tau) = R_x(-\tau)$$

symmetrical in about zero

$$R_x(\tau) \le R_x(0) \text{ for all } \tau$$

maximum value occurs at the origin

$$R_x(\tau) \leftrightarrow \psi_x(f)$$

autocorrelation and ESD form a Fourier transform pair, as designated by the double-headed arrows

value at the origin is equal to the energy of the signal

$$R_x(0) = \int_{-\infty}^{\infty} x^2(t)\,dt$$

2. Autocorrelation of a Power Signal

- Autocorrelation function of a real-valued power signal *x*(*t*) is defined as:

$$R_x(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, x(t+\tau)\, dt \quad \text{for} \quad -\infty < \tau < \infty \qquad (1.22)$$

- When the power signal *x*(*t*) is periodic with period T_0, the autocorrelation function can be expressed as

$$R_x(\tau) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t)\, x(t+\tau)\, dt \quad \text{for} \quad -\infty < \tau < \infty \qquad (1.23)$$

2. Autocorrelation of a Power Signal

- The autocorrelation function of a real-valued *periodic* signal has the following properties similar to those of an energy signal:

$$R_x(\tau) = R_x(-\tau)$$

symmetrical in about zero

$$R_x(\tau) \le R_x(0) \text{ for all } \tau$$

maximum value occurs at the origin

$$R_x(\tau) \leftrightarrow G_x(f)$$

autocorrelation and PSD form a Fourier transform pair

$$R_x(0) = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x^2(t)\,dt$$

value at the origin is equal to the average power of the signal

1.5 Random Signals

1. Random Variables

- All useful message signals appear random; that is, the receiver does not know, a priori, which of the possible waveform have been sent.

- Let a *random variable* $X(A)$ represent the functional relationship between a random event A and a real number.

- The *(cumulative) distribution function* $F_X(x)$ of the random variable X is given by

$$F_X(x) = P(X \leq x) \qquad (1.24)$$

- Another useful function relating to the random variable X is the *probability density function* (pdf)

$$P_X(x) = \frac{dF_X(x)}{dx} \qquad (1.25)$$

1.1 Ensemble Averages

$$m_X = E\{X\} = \int_{-\infty}^{\infty} x\, p_X(x)dx$$

$$E\{X^2\} = \int_{-\infty}^{\infty} x^2 p_X(x)dx$$

$$\mathrm{var}(X) = E\{(X - m_X)^2\}$$

$$= \int_{-\infty}^{\infty} (x - m_X)^2 p_X(x)dx$$

$$\mathrm{var}(X) = E\{X^2\} - E\{X\}^2$$

- *The first moment of a probability distribution of a random variable X is called mean value* m_X, or *expected value* of a random variable X
- *The second moment of a probability distribution is the mean-square value of X*
- *Central moments* are the moments of the difference between X and m_X and the second central moment is the *variance* of X
- Variance is equal to the difference between the mean-square value and the square of the mean

Example Problem
(Hwei P. Hsu page 177)

7.1. Consider a random process $X(t)$ given by

$$X(t) = A\cos(\omega t + \Theta) \tag{7.74}$$

where A and ω are constants and Θ is a uniform random variable over $[-\pi, \pi]$. Show that $X(t)$ is WSS.

From Eq. (*6.105*) (Prob. 6.23), we have

$$f_\Theta(\theta) = \begin{cases} \dfrac{1}{2\pi} & -\pi \leqslant \theta \leqslant \pi \\ 0 & \text{otherwise} \end{cases}$$

Thus,

$$\mu_X(t) = E[X(t)] = \int_{-\infty}^{\infty} A\cos(\omega t + \theta) f_\Theta(\theta)\, d\theta$$

$$= \frac{A}{2\pi}\int_{-\pi}^{\pi} \cos(\omega t + \theta)\, d\theta = 0 \tag{7.75}$$

2. Random Processes

- A random process *X*(*A, t*) can be viewed as a function of two variables: *an event A* and *time.*

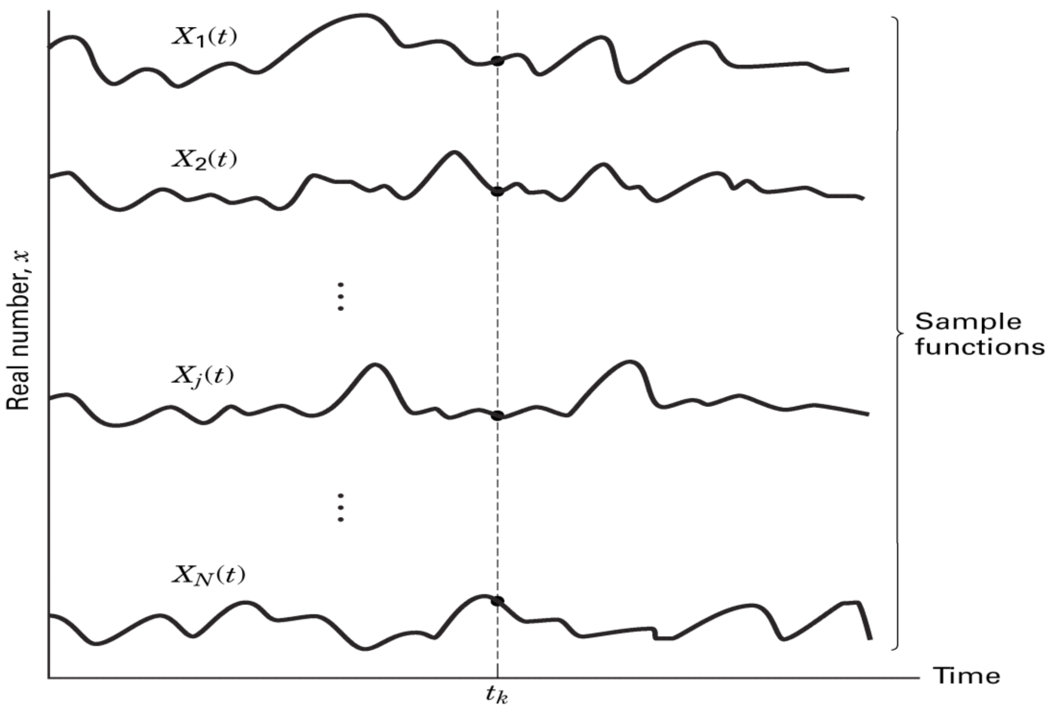


Figure 1.5 Random noise process.

1.5.2.1 Statistical Averages of a Random Process

- A random process whose distribution functions are continuous can be described statistically with a probability density function (pdf).

- A partial description consisting of the mean and autocorrelation function are often adequate for the needs of communication systems.

- Mean of the random process $X(t)$:

$$E\{X(t_k)\} = \int_{-\infty}^{\infty} x p_{X_k}(x)\, dx = m_X(t_k) \qquad (1.30)$$

- Autocorrelation function of the random process $X(t)$

$$R_X(t_1, t_2) = E\{X(t_1) X(t_2)\} \qquad (1.31)$$

1.5.5. Noise in Communication Systems

- The term *noise* refers to *unwanted* electrical signals that are always present in electrical systems; e.g spark-plug ignition noise, switching transients, and other radiating electromagnetic signals.

- Can describe thermal noise as a zero-mean *Gaussian* random process.

- A Gaussian process $n(t)$ is a random function whose amplitude at any arbitrary time t is statistically characterized by the Gaussian probability density function

$$p(n) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{n}{\sigma}\right)^2\right] \tag{1.40}$$

Noise in Communication Systems

- The *normalized* or *standardized Gaussian density function* of a zero-mean process is obtained by assuming unit variance.

$$p(n) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{n}{\sigma}\right)^2\right]$$

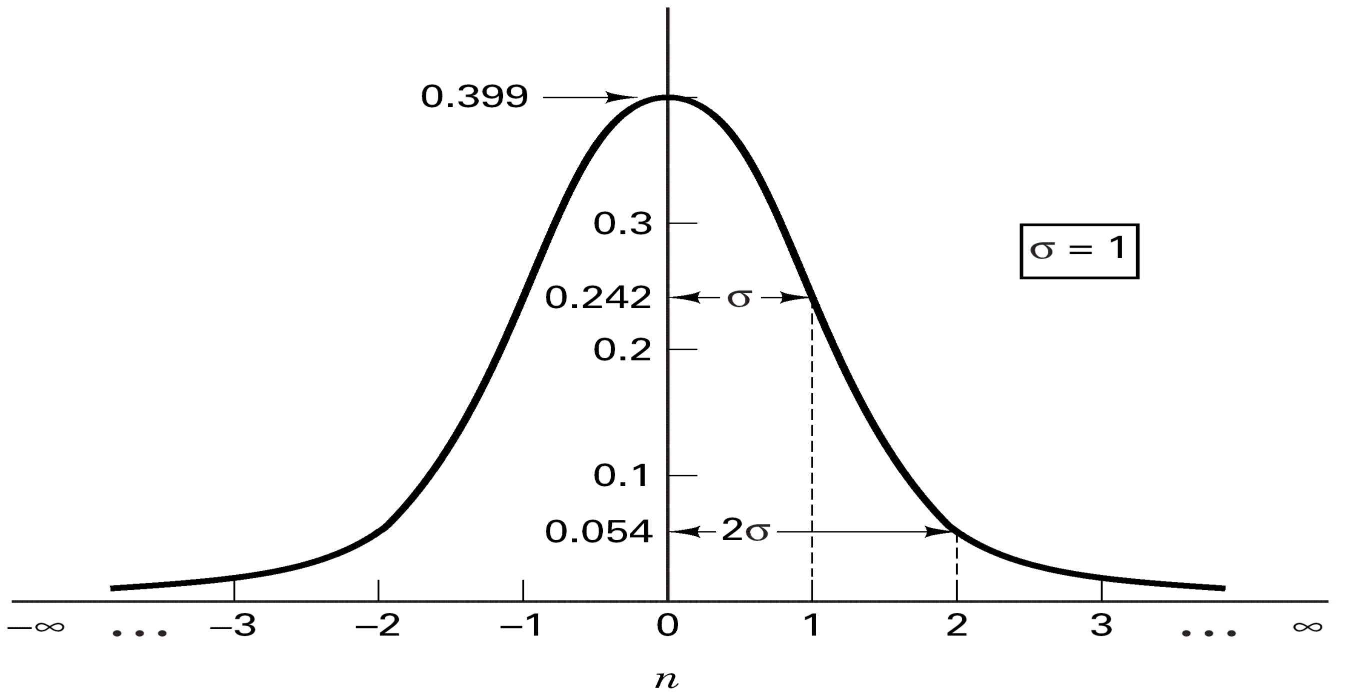


Figure 1.7 Normalized ($\sigma = 1$) Gaussian probability density function.

1.5.5.1 White Noise

- The primary spectral characteristic of thermal noise is that its power spectral density is *the same* for all frequencies of interest in most communication systems
- Power spectral density $G_n(f)$

$$G_n(f) = \frac{N_0}{2} \quad watts/hertz \tag{1.42}$$

- Autocorrelation function of white noise is

$$R_n(\tau) = \mathfrak{I}^{-1}\{G_n(f)\} = \frac{N_0}{2}\delta(\tau) \tag{1.43}$$

- The average power P_n of white noise is *infinite*

$$p(n) = \int_{-\infty}^{\infty} \frac{N_0}{2} df = \infty \tag{1.44}$$

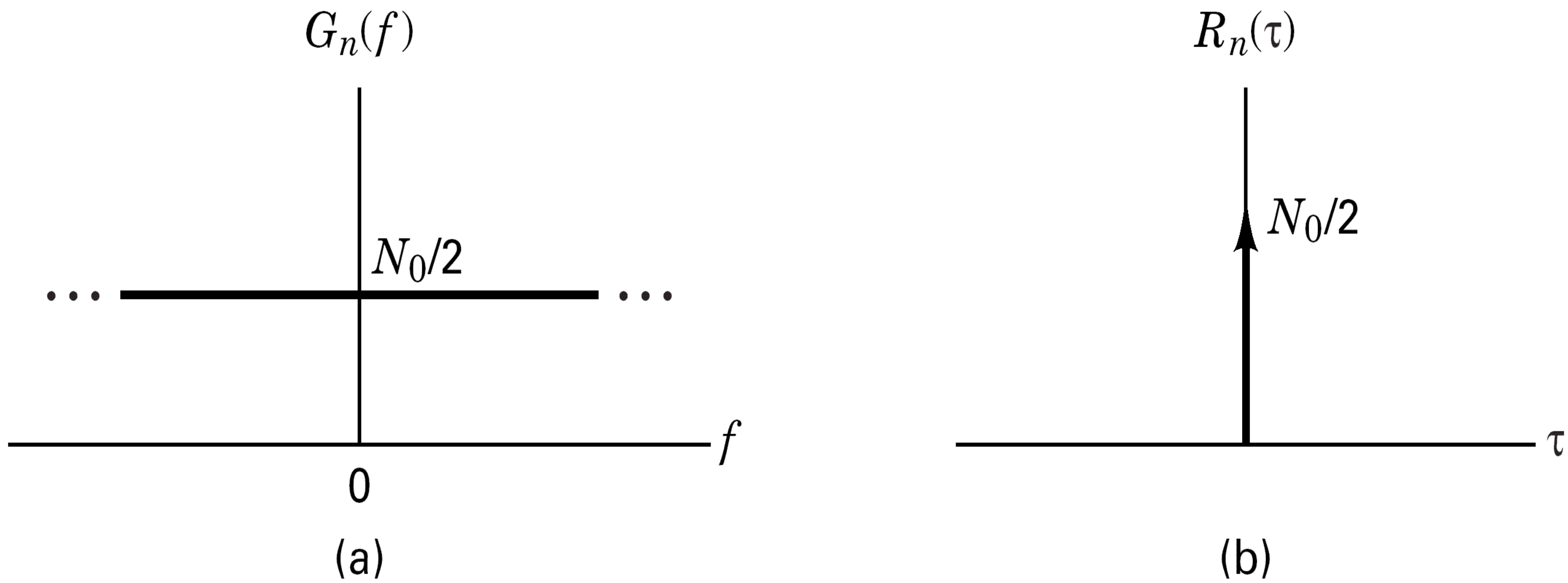


Figure 1.8 (a) Power spectral density of white noise. (b) Autocorrelation function of white noise.

- The effect on the detection process of a channel with *additive white Gaussian noise* (AWGN) is that the noise affects each transmitted symbol *independently.*

- Such a channel is called a *memoryless channel.*

- The term “additive” means that the noise is simply superimposed or added to the signal

1.6 Signal Transmission through Linear Systems

- A system can be characterized equally well in the time domain or the frequency domain, techniques will be developed in both domains

- The system is assumed to be linear and time invariant.

- It is also assumed that there is no stored energy in the system at the time the input is applied

1.6.1. Impulse Response

- The linear time invariant system or network is characterized in the time domain by an impulse response *h* (*t*),to an input unit impulse δ(*t*)

$$y(t) = h(t) \ \ when\, x(t) = \delta(t) \qquad (1.45)$$

- The response of the network to an arbitrary input signal *x* (*t*)is found by the convolution of *x* (*t*)with *h* (*t*)

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \qquad (1.46)$$

- The system is assumed to be *causal,*which means that there can be *no* output prior to the time, *t* =0,when the input is applied.
- The *convolution integral* can be expressed as:

$$y(t) = \int_{0}^{\infty} x(\tau)h(t-\tau)d\tau \qquad (1.47a)$$

1.6.2. Frequency Transfer Function

- The frequency-domain output signal $Y(f)$ is obtained by taking the Fourier transform

$$Y(f)=X(f)H(f) \tag{1.48}$$

- *Frequency transfer function* or the *frequency response* is defined as:

$$H(f)=\frac{Y(f)}{X(f)} \tag{1.49}$$

$$H(f)=|H(f)|e^{j\theta(f)} \tag{1.50}$$

- The phase response is defined as:

$$\theta(f)=\tan^{-1}\frac{\mathrm{Im}\{H(f)\}}{\mathrm{Re}\{H(f)\}} \tag{1.51}$$

1.6.2.1. Random Processes and Linear Systems

- If a random process forms the input to a time-invariant linear system,the output will also be a random process.

- The input power spectral density $G_X(f)$and the output power spectral density $G_Y(f)$are related as:

$$G_Y(f) = G_X(f)\,|H(f)|^2 \tag{1.53}$$

1.6.3. Distortionless Transmission

What is the required behavior of an *ideal* transmission line?

- The output signal from an ideal transmission line may have some time delay and different amplitude than the input
- It must have no distortion—it must have the same shape as the input.
- For ideal distortionless transmission:

Output signal in time domain
$$y(t) = Kx(t - t_0) \tag{1.54}$$

Output signal in frequency domain
$$Y(f) = KX(f)e^{-j2\pi f t_0} \tag{1.55}$$

System Transfer Function
$$H(f) = Ke^{-j2\pi f t_0} \tag{1.56}$$

What is the required behavior of an *ideal* transmission line?

- The overall system response must have a constant magnitude response
- The phase shift must be linear with frequency
- All of the signal's frequency components must also arrive with identical time delay in order to add up correctly
- Time delay t_0 is related to the phase shift θ and the radian frequency $\omega = 2\pi f$ by:

  t_0 (seconds) = θ (radians) / $2\pi f$ *(radians/seconds)* (1.57a)

- Another characteristic often used to measure delay distortion of a signal is called *envelope delay* or *group delay:*

(1.57b)

$$\tau(f) = -\frac{1}{2\pi}\frac{d\theta(f)}{df}$$

1.6.3.1. Ideal Filters

- For the ideal low-pass filter transfer function with bandwidth $W_f = f_u$ hertz can be written as:

$$H(f) = |H(f)| e^{-j\theta(f)}$$

(1.58)

Where

$$|H(f)| = \begin{cases} 1 & for\,|f| < f_u \\ 0 & for\,|f| \geq f_u \end{cases}$$

(1.59)

$$e^{-j\theta(f)} = e^{-j2\pi f t_0}$$

(1.60)

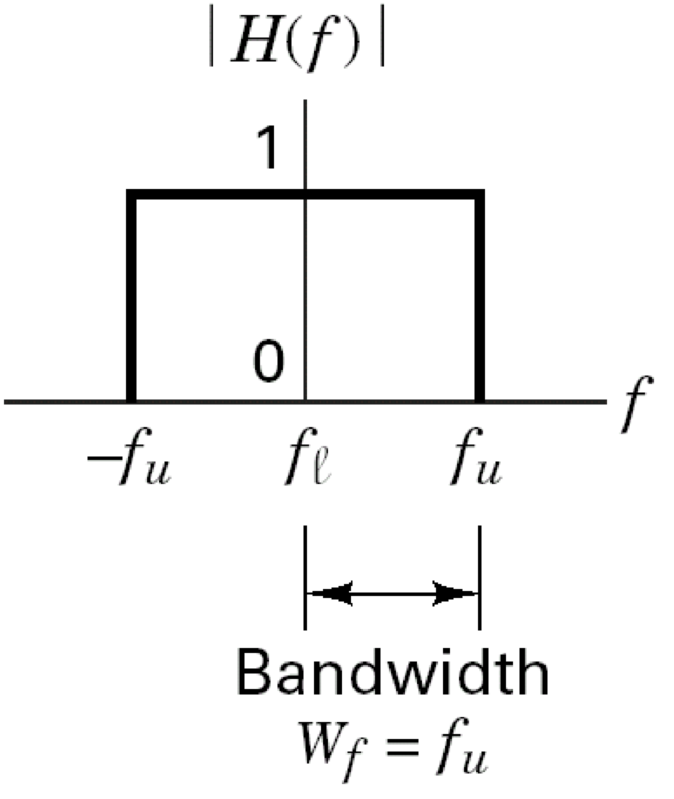


Figure1.11 (b) Ideal low-pass filter

Ideal Filters

- The impulse response of the ideal low-pass filter:

$$
\begin{aligned}
h(t) &= \mathfrak{I}^{-1}\{H(f)\} \\
&= \int_{-\infty}^{\infty} H(f)e^{j2\pi ft}\,df \\
&= \int_{-f_u}^{f_u} e^{-j2\pi ft_0}e^{j2\pi ft}\,df \\
&= \int_{-f_u}^{f_u} e^{j2\pi f(t-t_0)}\,df \\
&= 2f_u \frac{\sin 2\pi f_u(t-t_0)}{2\pi f_u(t-t_0)} \\
&= 2f_u \operatorname{sin}nc\, 2f_u(t-t_0)
\end{aligned}
$$

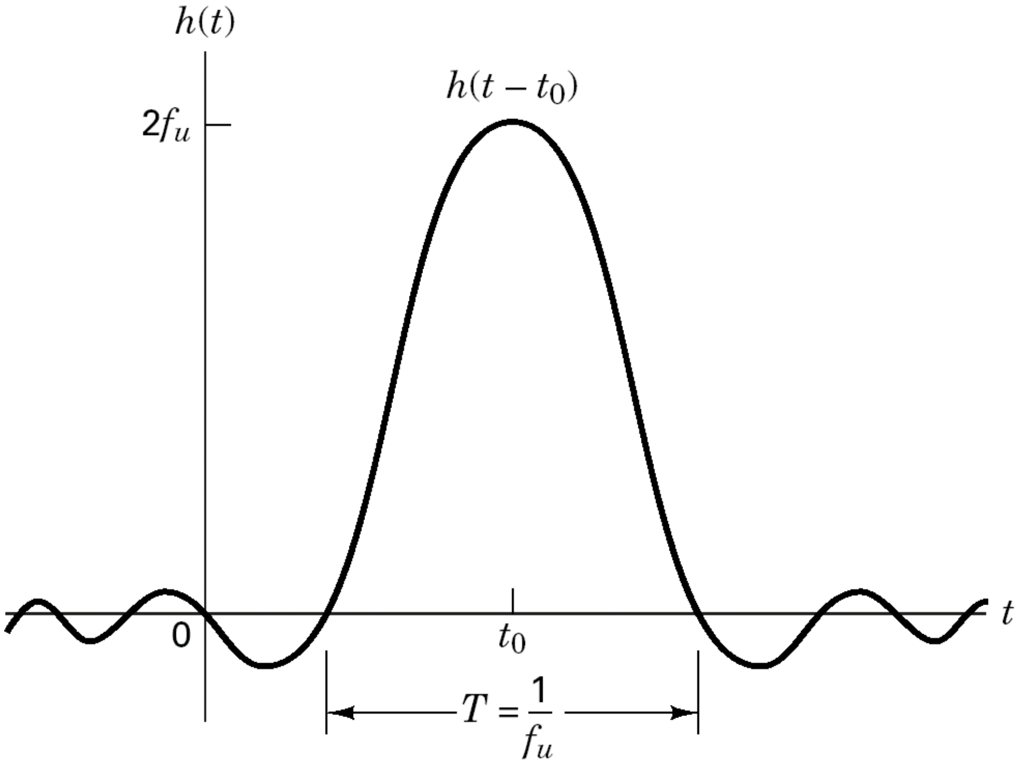


Figure 1.12 Impulse response of the ideal low-pass filter.

Ideal Filters

- For the ideal band-pass filter transfer function

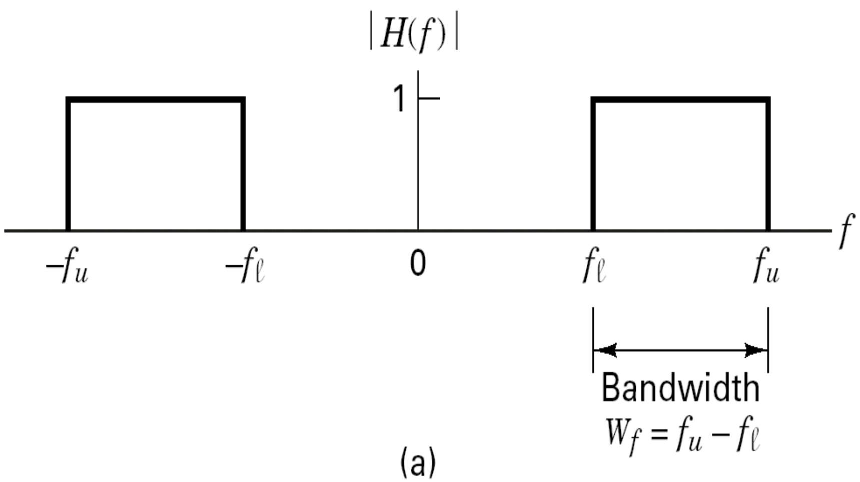


Figure1.11 (a) Ideal band-pass filter

- For the ideal high-pass filter transfer function

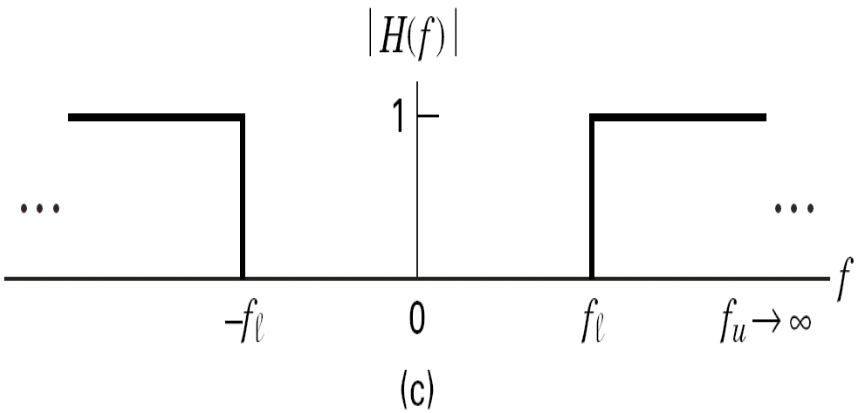


Figure1.11 (c) Ideal high-pass filter

1.6.3.2. Realizable Filters

- The simplest example of a realizable low-pass filter; an R*C filter*

$$H(f) = \frac{1}{1 + j2\pi f \Re C} = \frac{1}{\sqrt{1 + (2\pi f \Re C)^2}} e^{-j\theta(f)} \qquad 1.63)$$

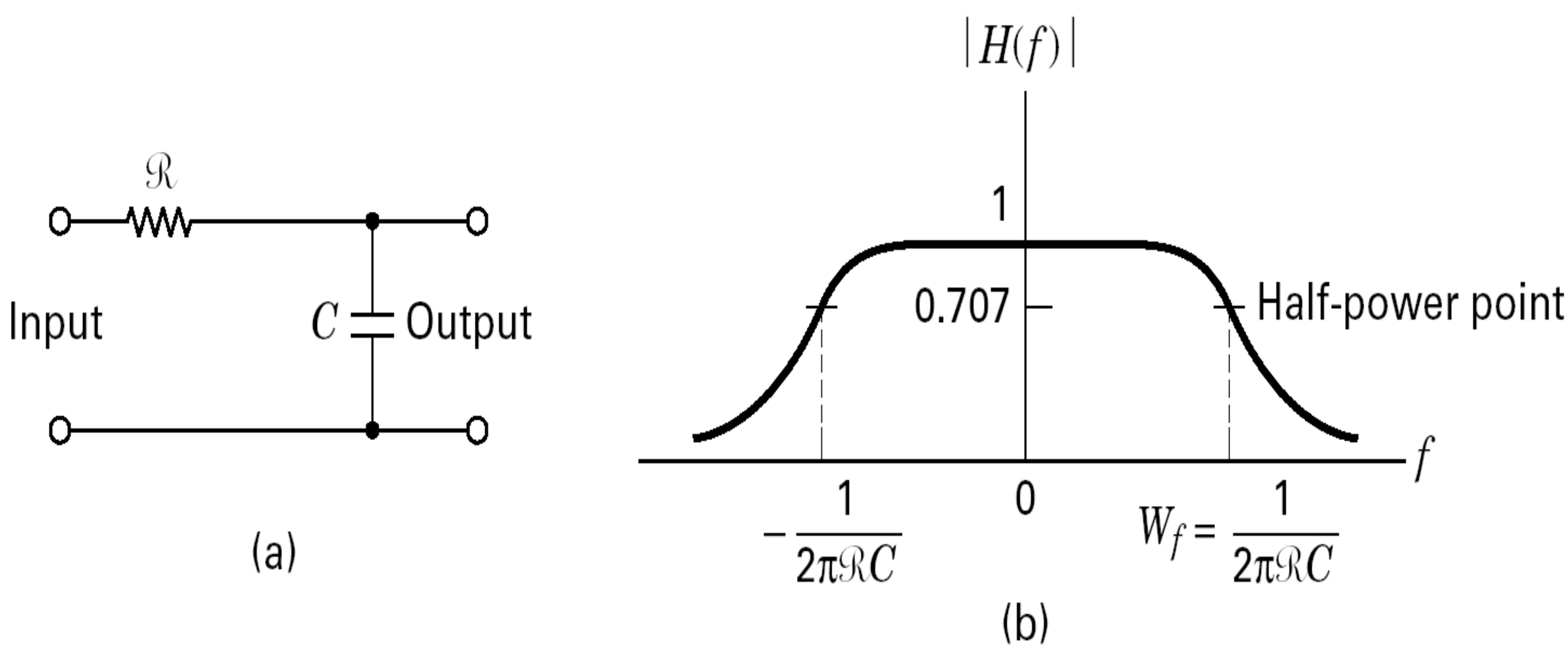


Figure 1.13

Realizable Filters

- Phase characteristic of RC filter

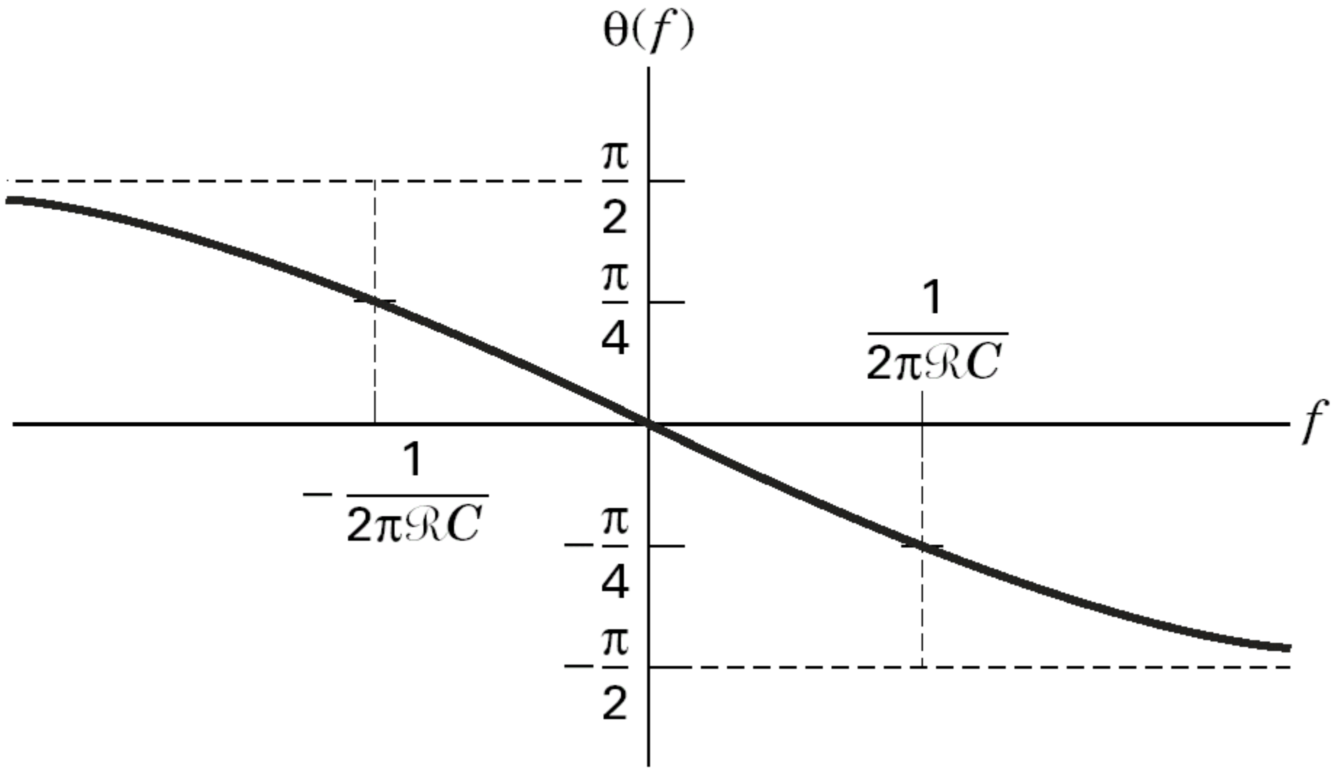


Figure 1.13 (c)

Realizable Filters

- There are several useful approximations to the ideal low-pass filter characteristic and one of these is the *Butterworth filter*

$$|H_n(f)| = \frac{1}{\sqrt{1+(f/f_u)^{2n}}} \qquad n \geq 1 \tag{1.65}$$

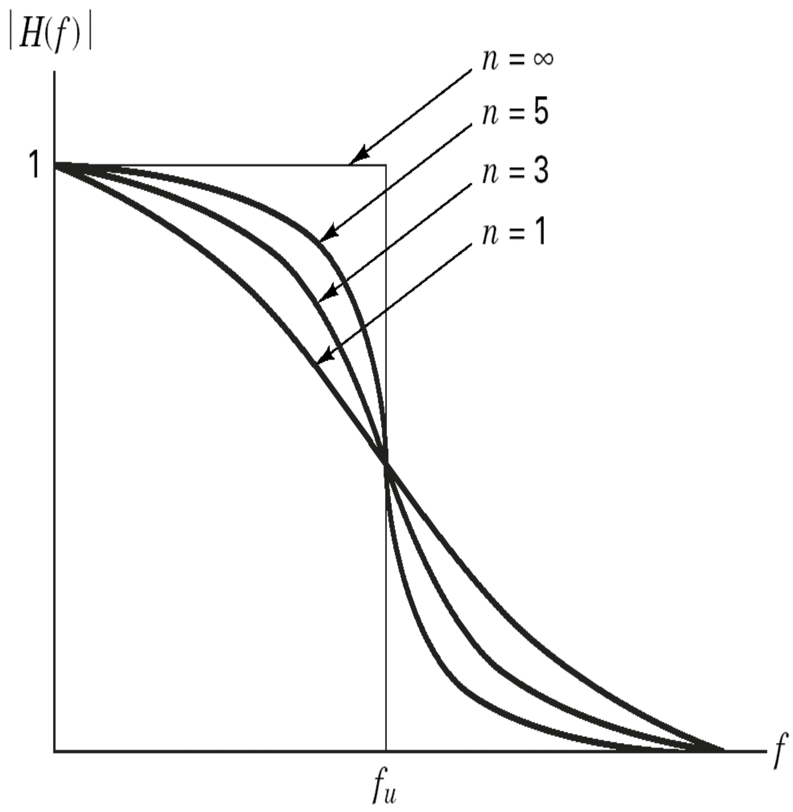


Figure 1.14 Butterworth filter magnitude response.

- Butterworth filters are popular because they are the best approximation to the ideal, in the sense of *maximal flatness* in the filter passband.

1.7. Bandwidth Of Digital Data

1.7.1 Baseband versus Bandpass

- An easy way to translate the spectrum of a low-pass or baseband signal x(t) to a higher frequency is to multiply or heterodyne the baseband signal with a carrier wave cos $2\pi f_c t$
- $x_c(t)$ is called a double-sideband (DSB) modulated signal

  $x_c(t) = x(t) \cos 2\pi f_c t$ (1.70)
- From the frequency shifting theorem

  $X_c(f) = 1/2\,[X(f-f_c) + X(f+f_c)]$ (1.71)
- Generally the carrier wave frequency is much higher than the bandwidth of the baseband signal

  $f_c >> f_m$ and therefore $W_{DSB} = 2f_m$

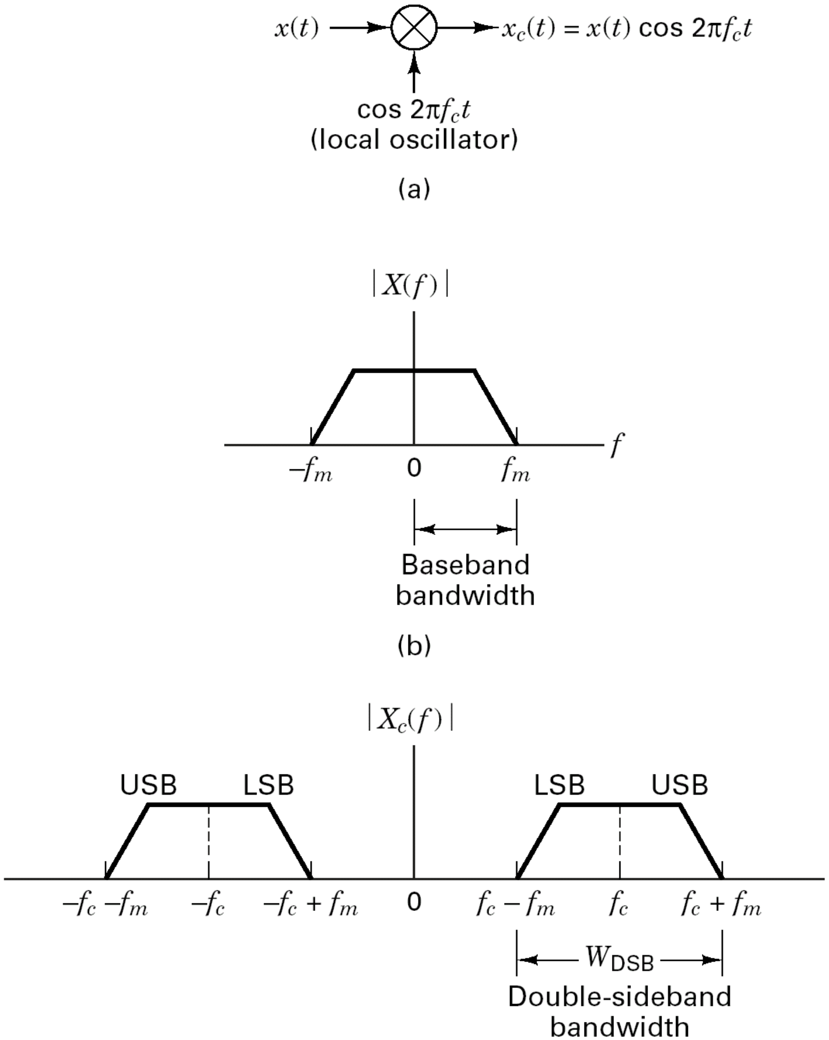


Figure 1.18 Comparison of baseband and double-sideband spectra. (a) Heterodyning. (b) Baseband spectrum. (c) Double-sideband spectrum.

1.7.2 Bandwidth Dilemma

- Theorems of communication and information theory are based on the assumption of *strictly bandlimited* channels

- The mathematical description of a real signal does not permit the signal to be strictly duration limited and strictly bandlimited.

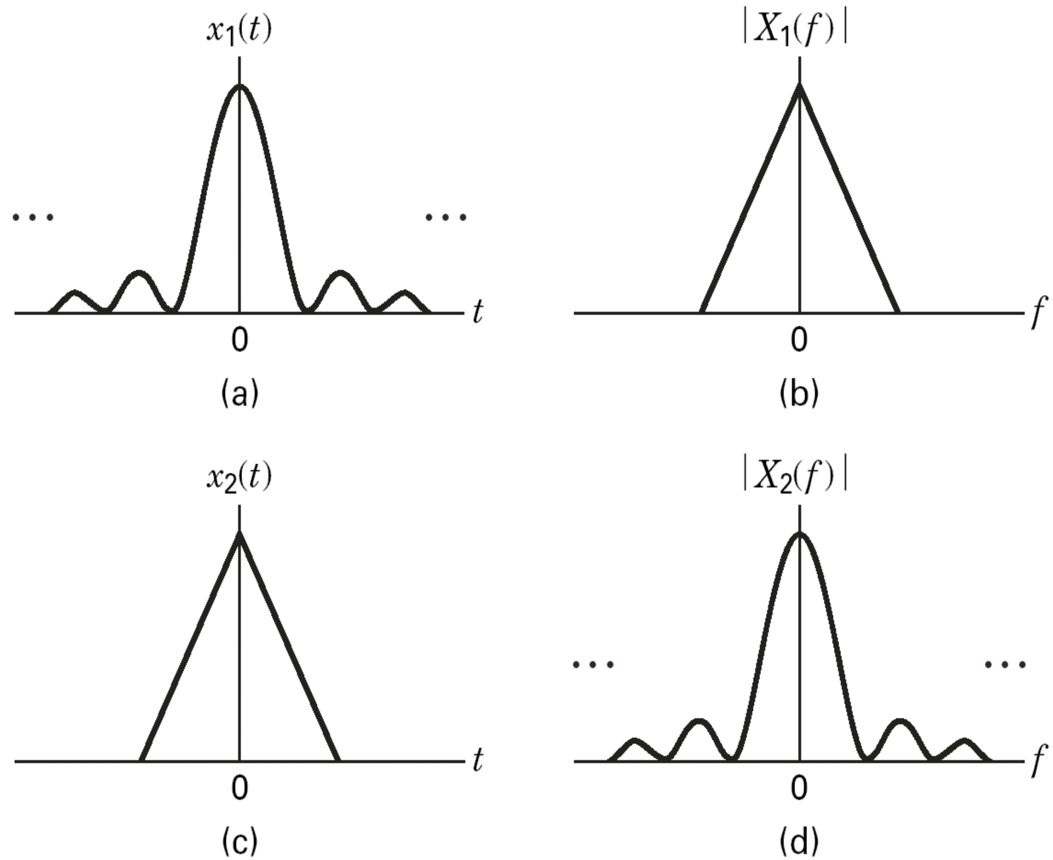


Figure 1.19 (a) Strictly bandlimited signal in the time domain. (b) In the frequency domain. (c) Strictly time limited signal in the time domain. (d) In the frequency domain.

1.7.2 Bandwidth Dilemma

- All bandwidth criteria have in common the attempt to specify a measure of the width, *W*, of a nonnegative real-valued spectral density defined for all frequencies $f < \infty$

- The single-sided power spectral density for a single heterodyned pulse $x_c(t)$ takes the analytical form:

$$G_x(f) = T\left[\frac{\sin \pi(f - f_c)T}{\pi(f - f_c)T}\right]^2 \qquad (1.73)$$

Different Bandwidth Criteria

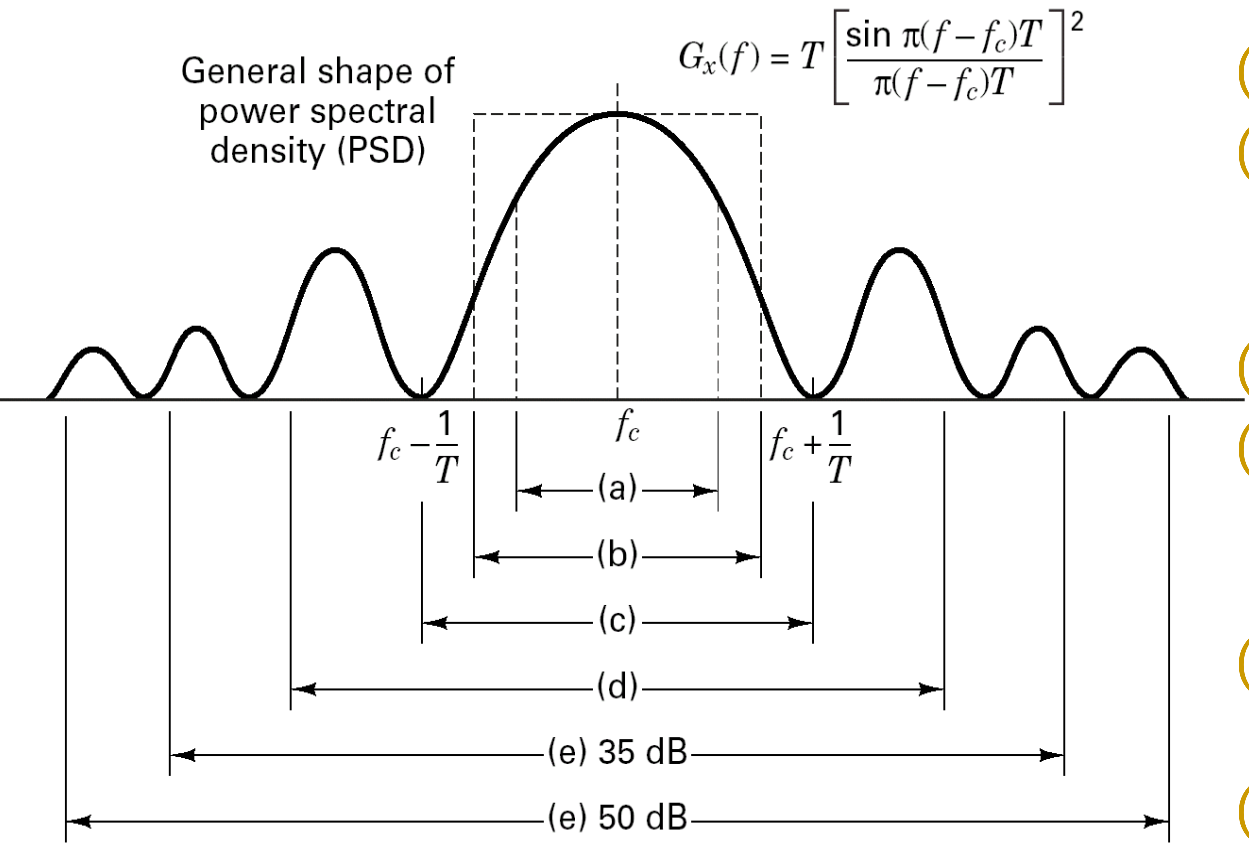


Figure 1.20 Bandwidth of digital data. (a) Half-power. (b) Noise equivalent. (c) Null to null. (d) 99% of power. (e) Bounded PSD (defines attenuation outside bandwidth) at 35 and 50 dB.

(a) Half-power bandwidth.

(b) Equivalent rectangular or noise equivalent bandwidth.

(c) Null-to-null bandwidth.

(d) Fractional power containment bandwidth.

(e) Bounded power spectral density.

(f) Absolute bandwidth.