

①

a)

$$\Delta(t) = N(d_1(t))$$

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From the BSM formula,

$$C(S, t) = N(d_1) S - N(d_2) K e^{-r(T-t)}$$

$$\text{where } d_1 = \frac{1}{\sigma\sqrt{T-t}} \left[\ln(S/K) + \left(r + \frac{\sigma^2}{2}\right)(T-t) \right]$$

$$\text{and } d_2 = d_1 - \sigma\sqrt{T-t}$$

$$\text{By def, } \Delta := \frac{\partial C}{\partial S}.$$

Taking partial derivatives,

$$\frac{\partial C}{\partial S} = \frac{\partial}{\partial S} [N(d_1) S] - \frac{\partial}{\partial S} [N(d_2) K e^{-r(T-t)}]$$

$$= N(d_1) \frac{\partial}{\partial S} [S] = N(d_1)$$

$$\therefore \Delta = N(d_1)$$

(see reverse
for (b) and (c))

Recall $C(S, t) = N(d_1)S - N(d_2)Ke^{-r(T-t)}$

with $d_1 = \frac{1}{\sigma\sqrt{T-t}} \left[\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t) \right]$

⑥ First suppose $S > K$ and that $\tau \equiv T-t \rightarrow 0$.

Then $d_1 = \frac{1}{\sigma\sqrt{\tau}} \left[\ln(S/K) + \left(r + \frac{\sigma^2}{2}\right)\tau \right]$

$$= \frac{1}{\sigma\sqrt{\tau}} \ln(S/K) + \frac{\left(r + \frac{\sigma^2}{2}\right)\sqrt{\tau}}{\sigma}$$

as $\tau \rightarrow 0^+$, $d_1 \rightarrow \frac{\ln(S/K)/\sigma}{\sqrt{\tau}} + 0$

$S > K \Rightarrow \ln(S/K) > 0$

$\sigma > 0 \Rightarrow \ln(S/K)\sigma > 0$

Hence $d_1 \rightarrow +\infty$.

$\therefore N(d_1) \rightarrow 1$ as $d_1 \rightarrow +\infty$

⑦ This is an identical argument except

$S < K \Rightarrow \ln(S/K)\sigma < 0$

$\Rightarrow d_1 \rightarrow -\infty$

$\therefore N(d_1) \rightarrow 0$ as $d_1 \rightarrow -\infty$. \square

2 A firm sells 1,000 European Calls (in round lots) on a non dividend-paying stock with $S = \$60$, $K = \$60$, $\sigma = 0.15$, $T = 140/365$ and $r = 0.03$.

a $C = N(d_1)S - N(d_2)Ke^{-rT}$

where $d_1 = \frac{1}{\sigma\sqrt{T}} \left[\ln(S/K) + \left(r + \frac{\sigma^2}{2}\right)T \right]$

and $d_2 = d_1 - \sigma\sqrt{T}$

$$d_1 = \frac{1}{0.15\sqrt{140/365}} \left[1 + \left(0.03 + \frac{0.0225}{2}\right) \frac{140}{365} \right]$$

$$\approx 0.1703$$

$$d_2 = d_1 - \sigma\sqrt{T} \approx 0.1703 - 0.15\sqrt{140/365}$$

$$\approx 0.0774$$

$$\therefore N(d_1) \approx 0.5676$$

and $N(d_2) \approx 0.5309$

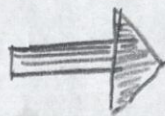
Hence $C = 0.5676 \cdot 60 - 0.5309 \cdot 60 \cdot e^{-0.03 \cdot \frac{140}{365}}$

$$\Rightarrow \boxed{C = \$2.57}$$

See reverse

for (b) and

(c)



(b) Fill in days 1, 139 and 140, Was the call exercised?

t	$S(t)$	$\Delta(t)$	Funds borrowed to add shares (or received from selling shares) at t	Amount owed at t (with interest)
0	60.00	0.568	3,405,710.33	3,405,710.33
1	60.12	0.576	49478.76	3,455,469
\vdots	\vdots	\vdots	\vdots	\vdots
138	56.12	0.00	0	257,300
139	55.86	0.00	0	257,321.10
140	55.64	0.00	0	257,342.30

No the call would not have been exercised since $K > S$ at expiration, so the firm would not have been assigned on their short calls

(c) Yes the firm made a profit of:

$$\$310,000 - \$257,342.30 = \underline{\$52,657.70}$$

on day 140, or \$52,055.25 on day 0.