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MATH 582

FINANCIAL DERIVATIVES

HOMEWORK #4

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$\boxed{7.1}$

While you can't make money on the stock itself, you can make money in the derivatives market for that stock. For example, you could <u>short a call</u> with some strike price $K > S(0)$, where $S(0)$ is the current price. The call will not be exercised at time T if $S(0) = S(T) < K$, and you will profit the price of the call. You could equivalently <u>long a put</u> by put-call-parity.

So, in short, your hunch can make you money.

$\boxed{7.2}$ <u>Selling a call</u>

<u>Maximum gain</u>: price of the call, C. (so $+C$)

<u>maximum loss</u>: theoretically, infinite, since the stock price $S(t)$ could grow without bound.
(so $-\infty$)

$\boxed{7.3}$ <u>Buying a call</u>

maximum gain: infinite ($+\infty$)

maximum loss: the price of the call ($-C$)

$\boxed{7.4}$ It depends on how much in the money the call actually is. If $S(t) = K + \epsilon$ for some small $\epsilon > 0$, then the call price will not move dollar-for-dollar. If $S(t)$ is sufficiently large, so that the probability of finishing in the money at expiration is $\cong 1$, then the call will increase dollar-for-dollar with stock. This is formalized in BSM with $\Delta := \frac{\partial C}{\partial S}$, which is close to 1 as $S(t)$ goes deeply into the money. So, for the most part, $\boxed{\text{AGREE}}$

$\boxed{7.5}$ $K = 80$ $S(\text{expiration}) = 84$

$C = 2$

(a)

$$\left.\begin{array}{l|l} t=0 & \text{Buy option: } -200 = -200 \\ t=T & \text{Exercise call: } -80 \cdot 100 = -8000 \\ t=T & \text{Sell stock: } +84 \cdot 100 = 8400 \end{array}\right\} +200.$$

You turned 200 into 400, so the rate of return is $\boxed{100\%}$

$$\left.\begin{array}{l|l} t=0 & \text{Buy stock: } -80 \cdot 100 = -8000 \\ t=T & \text{Sell stock: } 84 \cdot 100 = 8400 \end{array}\right\} +400$$

You turned 8000 into 8400.

So the rate of return is $\frac{400}{8000}$

$400/8000 = 1/20 = \boxed{5\%}$

The first case has a higher return rate as $100\% > 5\%$; however, since the second case risks more, it makes more money. So they do not have equivalent gain/loss.

See reverse for (b) →

(b) You can sell the option for $(84-80)\cdot 100 = 400$, so you still make 200. The same as the first case.

(c) It doesn't matter since both cases yield you the same payoffs. The only difference is in buying an option vs. buying stock in part (a). In that case it depends on your risk tolerance and what kind of bet you want to make.

$\boxed{7.6}$

(a) If you had the following position:

- long 100 shares at \$40/share
- Short 1 American call at C

Your gain is simply the option premium, $\boxed{C,}$

Since you already paid \$40 for stock and are calling it at \$40, breaking even on stock

(you are actually happy the stock went up, as your position's $\Delta > 0$)

(b) If you only shorted the call at C,

your P&L is the option premium $\boxed{C - \$600.}$

Since you take a naked \$600 drop on the exercise

7.7.1 *Conceptual Exercises*

7.1. If you believe that the market price of a stock will stay at approximately the same price for a period of time, can you still make money from the stock if your hunch is correct? Explain your answer. Yes, buy put/sell call

7.2. What is the possible maximum gain or loss if you sell a call? gain: price of call, loss: infinite

7.3. What is the possible maximum gain or loss if you buy a call? gain: inf, loss: price of call

7.4. If a call is in the money sufficiently close to the expiration date, then the call price will rise dollar for dollar with the stock price. Agree or disagree? Explain. depends on how far it is in the money, if $\mathbb{P}$(in the money) ≈ 1, then yes

7.5. An at-the-money American call with a strike price of \$80 is being sold for \$200. Assume that the stock goes up to \$84 per share on the day of expiration.

a) If you bought the option, what is your return rate from exercising the call and liquidating your stock position? If you did not buy the option, but had bought 100 shares of the stock in in the market at \$80 per share and then sold them on the option's expiration date at \$84 per share, what would be your return rate? Do the two scenarios have equivalent gain/loss?

b) If you do not exercise the option, what is your approximate return rate from selling the call right before expiration?

c) Which would you then prefer? Exercise the call or sell the call?

7.6. You sell an American call on 1 round lot of a stock at \$40 per share. A month later, the market value of that stock is \$46 per share. If the buyer exercises the option, you will be obligated to deliver 100 shares at \$6 below current market value.

a) If you own those shares, what is your gain/loss from settling the position?

b) If you had naked short sold the American call, what is your gain/loss from settling the position?

7.7. Fill in the blanks. Why does an American option's value change when the underlying stock price moves? The reason is that an option sets the stock's value at __expiration__. For example, when you buy a call, you take the position, "I am willing to pay for the __right__ to buy this stock at a fixed price at some point in the future." If the stock's price __increases__ higher than the identified __strike__ price during the term of the contract, then the value of the call __rises__ because the potential for a __profit/exercise__. If the price is __less__, then value of the __call falls__ due to a __decreased__ [decreasing] potential for a positive __exercise/trade__

The same argument applies for a put buyer, but in __reverse__. If the stock's price __falls lower__ than the __strike price__, then the put becomes __exercisable.__.

7.8

You can take the following actions:

(a) sell the call.

(b) long a put at the same strike price and/or short stock.

(c) if the stock pays an enormous dividend before next month AND the call is deep in the money, see if it's profitable to exercise early.

(d) hold your position and hope for a rally!

Consequences

(a) You lost \$200, but that was a sunk cost anyway. By selling, you no longer have a position.

(b) Hedge the position by shorting stock or longing puts. This would make your portfolio less sensitive to changes in price.

- For Δ-neutrality, short 100Δ shares or:
  to fully neutralize, short a put and long 100 shares

(c) you lose optionality but profit dividends.

(d) you get paid off $\max\{0, S-K\}$ at expiration, which is unlikely to be over \$300!

$\boxed{7.14}$ Establish the following bounds for European options:

a) $S(t_0) > C_E(t_0) \geq \max\{S(t_0)e^{-q\tau} - Ke^{r_f T}, 0\}$

Clearly $S(t_0) > C_E(t_0)$ since buying an option that costs as much or more than the stock price would make no sense (as long as the strike $K > 0$). Formally, if $C_E(t_0) \geq S(t_0)$ then you could sell a call and use that money to buy stock. So your portfolio started at 0 and at expiration t_f equals:

$e^{r(t_f - t_0)}[C_E(t_0) - S(t_0)] + \min\{S(t), K\} > 0$ since if you are assigned you get paid K for the stock you already own. If not, you can sell the stock for $S(t)$ in the market. So this is a clear arbitrage, and so $S(t_0) > C_E(t_0)$

To see $C_E(t_0) \geq \max\{S(t_0)e^{-q\tau} - Ke^{r_f\tau}, 0\}$, consider the following two portfolios:

(1) A call $C_E(t_0)$ plus $Ke^{-r_f\tau}$

(2) Stock worth $S(t_0)e^{-q\tau}$.

At time t_f, portfolio 2 is worth $S(t_f)$ and portfolio 1 is worth $\max\{S(t_f), K\}$.

(continued) →

At time t_f, portfolio 1 is worth as much or more than portfolio 2, so at time t_0, this must also be true. Hence,

$$C_E(t_0) + Ke^{-r_f \tau} \geq S(t_0)e^{-q\tau}$$

$$\Rightarrow C_E(t_0) \geq S(t_0)e^{-q\tau} - Ke^{-r_f \tau}$$

Since $C_E(t_0)$ can never be worth negative money, $C_E(t_0) \geq 0$

$$\Rightarrow C_E(t_0) \geq \max\{S(t_0)e^{-q\tau} - Ke^{-r_f\tau}, 0\}$$

(b) Goal: show $Ke^{-r_f\tau} > P_E(t_0) \geq \max\{Ke^{-r_fT} - S(t_0)e^{-q\tau}, 0\}$

Clearly $Ke^{-r_f\tau} > P_E(t_0)$ since the max value of a put at t_f is K, and $Ke^{-r_f\tau}$ is guaranteed to be worth K at t_f.

Construct the following portfolios:

(1) long a put $P_E(t_0)$ and stock $S(t_0)e^{-q\tau}$

(2) cash worth Ke^{-r_fT}

At expiration portfolio 2 is worth K, and portfolio 1 is worth more, since its min value happens at $S(t_f) = 0$ giving it a value of K from exercising the put (if $S(t_f) > K$ then portfolio 1 is already worth more). Then at t_0, (1) > (2) so $P_E + S(t_0)e^{-q\tau} \geq Ke^{-r_f\tau}$

Since $P_E(t_0) \geq 0$, we get $P_E \geq \max\{Ke^{-r_f\tau} - S(t_0)e^{-q\tau}, 0\}$ $\square$

7.15 These are nearly identical arguments as before.

For (a): note that portfolio (1) is strictly more valuable than (2) which is more valuable than (3) at expiration

(1) long stock $S(t_0)$

(2) long a call $C_E(t_0)$

(2) long stock $S(t_0)e^{-q\tau}$ and short $Ke^{-r_f\tau}$ in cash

Since $S(t_0)e^{q\tau} > \max\{S(t_0)-K, 0\} \geq S(t_0)-K \quad \forall K$

It follows that $S(t_0) > C_E(t_0) \geq S(t_0)e^{-q\tau} - Ke^{-r_f\tau}$

Also since $C_E(t_0) \geq 0$, we get what is desired:

$$S(t_0) > C_E(t_0) \geq \max\{S(t_0)e^{-q\tau} - Ke^{-r_f\tau}, 0\}$$

For (b): Construct 3 portfolios at t_0 for some τ.

(1) long K in cash

(2) long a put $P_E(t_0)$

(3) long $Ke^{-r_f\tau}$ cash and short $S(t_0)e^{-q\tau}$ stock.

At expiration τ, portfolio (1) > (2) $\geq$ 3

$Ke^{r\tau} > \max\{K - S(\tau), 0\} \geq K - S(\tau)$.

so at t_0, $K > P_A(t_0) \geq Ke^{-r_f\tau} - S(t_0)e^{-q\tau}$

Since $P_A(t_0) \geq 0$ always we finally get

$$K > P_A(t_0) \geq \max\{Ke^{-r_f\tau} - S(t_0)e^{-q\tau}, 0\} \quad \square$$

$\boxed{7.16}$ Consider two portfolios:

(1) an American call $C_A(t_0)$ plus $Ke^{-r_f(t_f-t_0)}$ in cash

(2) $S(t_0)$ in stock.

At expiration, (1) has k in cash., and at some earlier τ, it has $ke^{-r_f(t_f-\tau)}$. If the call is exercised at τ, the value of the portfolio is $S(\tau) - k + ke^{-r_f(t_f-\tau)} < S$ when $\tau < t_f$

So $(1) < (2)$ when the American call is exercised early.

On the other hand, if there is no early exercise, $(1) \geq (2)$ since $\max\{S(t_f), k\} \geq S(t_f)$.

Hence early exercising at any $0 < \tau < t_f$ is not optimal, as it takes portfolio (1) from being greater than (2) to less than (2) $\forall\ \tau \in [0, t_f)$

7.17 It is sometimes advantageous to exercise an American put early, even if there are no dividends. By Put-call parity,

$$C - P = S - K + PV(int - div)$$
$$= S - K + PV(int)$$
$$\Rightarrow PV(int) - C = (S - K) + P$$

If the interest one would gain by exercising the put and investing profit at the risk free rate is greater than the value of the current call price, then you should exercise early. This happens when puts are deep in-the-money and therefore the calls are deep out-of-the-money and therefore cheap.

By exercising a put, you are essentially selling stock plus your optionality, which is the same as shorting stock and longing a call at the same price. And this is profitable when

$PV(\text{interest}) > C$ by put-call parity.

$\boxed{7.18}$ Consider 3 portfolios:

(1) long $S(t_0)e^{-qT}$ stock and short K in cash

(2) long an American call $C_A(t_0)$ and short (a put) $P_A(t_0)$

(3) long $S(t_0)$ stock and short $Ke^{-r_f T}$ in cash.

As a rational trader, you will correctly exercise your call, so at worst it is worth $S(t_e)-K$ at any time t_e. This is because you have "locked in" the stock price at K by selling it at K or worse (lower) but covering your short at K or better (lower).

Portfolio (1) is worth $S(t_0)e^{-q(T-t_e)} - Ke^{r_f t_e}$

and Portfolio (3) is worth $S(t_0)e^{q t_e} - Ke^{-r_f t_e}$

So we then have:

$$S(t_e)e^{-q(T-t_e)} - Ke^{r_f t_e} \le S(t_e) - K$$

$$\le S(t_e)e^{q t_e} - Ke^{-r_f(t_e - T)}$$

So (1) $\le$ (2) $\le$ (3)

and hence at t_0,

$$S(t_0)e^{-qT} - K \le C_A(t_0) - P_A(t_0) \le S(t_0) - Ke^{-r_f T} \quad \square$$