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# Christian Drappi
# Math 582 with Prof. Arlie Petters
# Homework #3, due Monday, Feb. 17, 2014




# Answers:

# Problem (a): P(S(2/10/2014) > S(2/7/2014)) = 0.5649

# Problem (b): P(1190 < S(2/14/2014) < 1200) = 0.0919

# Problem (c): P(1073.99 < S(3/7/2014) < 1435.08) = 0.9545





# I programmed this assignment in R.
# Below is all of the code

#### Load and prepare data
goog <- read.csv("Dropbox/workspace/math582/hw3/goog.csv")

# number of days in a year
t.day <- 1/252
# how many days of data will we analyze
num.days <- 121

# get the adjusted closing prices
adj.close <- rev(goog$Adj.Close[1:num.days])

# compute the log returns of these prices
log.returns <- diff(log(adj.close))

# get mu_(m) and sigma of the security
mu.m <- 1/t.day * mean(log.returns)
sigma <- sqrt(1/t.day * var(log.returns))
```

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#### Problem (a):
# Determine the probability that Google’s
# closing price on February 10, 2014
# is above the aforementioned current price.
# Compare your answer with the
# actual closing price.

#### Answer (a)

# the amount of trading days until 2/10/2014
time.1 <- 1

# Google's closing price will be above the
# current price in three days
# iff S(1/252) / S(0) > 1
# iff ln(S(1/252)-S(0)) > 0.
# The probability of this happening is
# one minus the value of the normal cdf at 0
# with mean = mu_(m) * t and sdev = sigma * sqrt(t)
# (since the normal cdf gives the probability of
# finishing below the current price)

prob.1 <- 1 - pnorm(0, mu.m * time.1 * t.day, sigma * sqrt(time.
1*t.day))
p.1 <- round(prob.1, digits = 4)
# 0.6131594
print(paste("Problem (a): ", "P(S(2/10/2014) > S(2/7/2014)) = ", p.1,
sep=""))
```

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#### Problem (b):
# Determine the probability that Google’s
# closing price on Valentine’s Day,
# February 14, 2014, will be between
# $1,190 and $1,200?

#### Answer (b)

# the amount of trading days until 2/14/2014
time.2 <- 5

# get the current price
feb.7.price <- adj.close[121]
# set the price bounds for the
# probability calculation
bottom.price <- 1190
top.price <- 1200

# compute the size of log returns that
# correspond to the prices
bottom.logr <- log(bottom.price/feb.7.price)
top.logr <- log(top.price/feb.7.price)

bottom.q <- pnorm(bottom.logr, mu.m * time.2*t.day, sigma * sqrt(time.
2*t.day))
top.q <- pnorm(top.logr, mu.m * time.2*t.day, sigma * sqrt(time.
2*t.day))
prob.2 <- top.q - bottom.q
p.2 <- round(prob.2, digits = 4)
print(paste("Problem (b): ", "P(1190 < S(2/14/2014) < 1200) = ", p.2,
sep=""))
```

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#### Problem (c):
# Find a 95.45% confidence interval for
# Google's closing price on March 7, 2014.


#### Answer (c)

# the amount of trading days until 3/7/2014
time.3 <- 20

conf <- 0.9545
bottom.conf <- (1-conf)/2
top.conf <- 1-bottom.conf

bottom.interval <- feb.7.price*exp(qnorm(bottom.conf, mu.m * time.
3*t.day, sigma * sqrt(time.3*t.day)))
top.interval <- feb.7.price*exp(qnorm(top.conf, mu.m * time.3*t.day,
sigma * sqrt(time.3*t.day)))

b.i <- round(bottom.interval, digits = 2)
t.i <- round(top.interval, digits = 2)

print(paste("Problem (c): ", "P(", b.i, " < S(3/7/2014) < ", t.i, ") =
", conf, sep=""))
```

6.3

CHRISTIAN DRAPPI

Given a probability p, determine security prices K_1 and K_2 that satisfy the following:

$$\mathbb{P}(K_1 \leq S(t) \leq K_2) = p \quad \text{with}$$

$$\mathbb{P}(S(t) < K_1) = \frac{1-p}{2} = \mathbb{P}(S(t) > K_2)$$

assuming that the security price $S(t)$ is modeled by geometric Brownian motion

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$$S(t) = S_0 \exp\{\mu_{(m)} t + \sigma \mathcal{B}(t)\}$$

$$\mathbb{P}(K_1 \leq S(t) \leq K_2) = p = \mathbb{P}\left(\frac{K_1}{S(0)} \leq \frac{S(t)}{S(0)} \leq \frac{K_2}{S(0)}\right)$$

$$= \int_{K_1/S(0)}^{K_2/S(0)} \frac{dx}{x\sqrt{2\pi\sigma^2 t}} \exp\left\{\frac{-1}{2} \frac{(\ln x - \mu_{(m)} t)^2}{\sigma^2 t}\right\}$$

$$\ln x = u \Rightarrow x = e^u \Rightarrow dx = e^u du$$

$$\Rightarrow p = \int_{\ln(K_1/S(0))}^{\ln(K_2/S(0))} \frac{du\, e^u}{e^u\sqrt{2\pi\sigma^2 t}} \exp\left\{\frac{-1}{2\sigma^2 t}(u - \mu_{(m)} t)^2\right\}$$

$$= \int_{\ln\left(\frac{K_1}{S(0)}\right)}^{\ln\left(\frac{K_2}{S(0)}\right)} \frac{du}{\sqrt{2\pi}\cdot\sigma\sqrt{t}} \exp\left\{\frac{-1}{2\sigma^2 t}(u - \mu_{(m)} t)^2\right\}$$

Let $z = \frac{u - \mu_{(m)} t}{\sigma\sqrt{t}}$

$\Rightarrow du = \sigma\sqrt{t}\, dz$

$$= \int_{a_1}^{a_2} dz\, e^{-\frac{1}{2}z^2} = \mathcal{N}(a_2) - \mathcal{N}(a_1)$$

where

$$a_1 = \frac{\ln\left(\frac{K_1}{S(0)}\right) - \mu_{(m)} t}{\sigma\sqrt{t}}$$

$$a_2 = \frac{\ln\left(\frac{K_2}{S(0)}\right) - \mu_{(m)} t}{\sigma\sqrt{t}}$$

(see reverse)

Recall $a_1 = \dfrac{\ln(K_1/S(0)) - \mu_{(m)} t}{\sigma\sqrt{t}}$

$a_2 = \dfrac{\ln(K_2/S(0)) - \mu_{(m)} t}{\sigma\sqrt{t}}$

Then $p = \mathcal{N}(a_2) - \mathcal{N}(a_1)$, where $\mathcal{N}(\cdot)$ is the standard normal cdf

In the limit $K_2 \to \infty$, $\mathcal{N}(a_2) \to 1$.

$\Rightarrow \mathbb{P}(K_1 \le S(t)) = 1 - \mathcal{N}(a_1)$

$\Rightarrow \mathbb{P}(S(t) \le K_1) = \mathcal{N}(a_1) = \frac{1-p}{2}$

$\Rightarrow a_1 = \mathcal{N}^{-1}\left(\frac{1-p}{2}\right)$

In the limit $K_1 \to 0$, $\mathcal{N}(a_1) \to 0$

$\Rightarrow \mathbb{P}(S(t) \le K_2) = \mathcal{N}(a_2)$

$\Rightarrow \mathbb{P}(S(t) \ge K_2) = 1 - \mathcal{N}(a_2) = \frac{1-p}{2}$

$\Rightarrow \mathcal{N}(a_2) = \frac{1+p}{2} \Rightarrow a_2 = \mathcal{N}^{-1}\left(\frac{1+p}{2}\right)$

By the formula at the top of the page,

$\ln\left(\frac{K_1}{S(0)}\right) = \mu_{(m)} t + \sigma\sqrt{t}\,\mathcal{N}^{-1}\left(\frac{1-p}{2}\right)$

$\ln\left(\frac{K_2}{S(0)}\right) = \mu_{(m)} t + \sigma\sqrt{t}\,\mathcal{N}^{-1}\left(\frac{1+p}{2}\right)$

So

$$\boxed{\begin{aligned} K_1 &= S(0)\exp\left\{\mu_{(m)} t + \sigma\sqrt{t}\,\mathcal{N}^{-1}\left(\frac{1-p}{2}\right)\right\} \\ K_2 &= S(0)\exp\left\{\mu_{(m)} t + \sigma\sqrt{t}\,\mathcal{N}^{-1}\left(\frac{1+p}{2}\right)\right\}\end{aligned}}$$