(1)

(a) $\Delta(t) = N(d_1(t))$

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From the BSM formula,

$$C(S,t) = N(d_1)S - N(d_2)Ke^{-r(T-t)}$$

where $d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln(S/K) + (r + \frac{\sigma^2}{2})(T-t)\right]$

and $d_2 = d_1 - \sigma\sqrt{T-t}$

By def^n, $\Delta := \frac{\partial C}{\partial S}.$

Taking partial derivatives,

$$\frac{\partial C}{\partial S} = \frac{\partial}{\partial S}\left[N(d_1)S\right] - \frac{\partial}{\partial S}\left[N(d_2)Ke^{-r(T-t)}\right]$$

$$= N(d_1)\frac{\partial}{\partial S}[S] = N(d_1)$$

$$\therefore \Delta = N(d_1)$$

(see reverse for (b) and (c))

Recall $C(S,t) = N(d_1) S - N(d_2) K e^{-r(T-t)}$

with $d_1 = \frac{1}{\sigma\sqrt{T-t}} \left[\ln\left(S/K\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t) \right]$

(b) First suppose $S > K$ and that

$\tau \equiv T-t \to 0.$

Then $d_1 = \frac{1}{\sigma\sqrt{\tau}} \left[\ln(S/K) + (r + \frac{\sigma^2}{2}) \tau \right]$

$$= \frac{1}{\sigma\sqrt{\tau}} \ln(S/K) + \frac{(r + \frac{\sigma^2}{2})}{\sigma} \sqrt{\tau}$$

as $\tau \to 0^+$, $d_1 \to \frac{\ln(S/K)/\sigma}{\sqrt{\tau}} + 0$

$S > K \Rightarrow \ln(S/K) > 0$

$\sigma > 0 \Rightarrow \ln(S/K)\,\sigma > 0$

Hence $d_1 \to +\infty$.

$\therefore N(d_1) \to 1$ as $d_1 \to +\infty$

(c) This is an identical argument except

$S < K \Rightarrow \ln(S/K)\,\sigma < 0$

$\Rightarrow d_1 \to -\infty$

$\therefore N(d_1) \to 0$ as $d_1 \to -\infty$. $\square$

2. A firm sells 1,000 European Calls (in round lots) on a non dividend-paying stock with $S = \$60$, $K = \$60$, $\sigma = 0.15$, $\tau = 140/365$ and $r = 0.03$:

a. $$C = N(d_1)S - N(d_2)Ke^{-r\tau}$$

where $$d_1 = \frac{1}{\sigma\sqrt{\tau}}\left[\ln(S/K) + \left(r + \frac{\sigma^2}{2}\right)\tau\right]$$

and $$d_2 = d_1 - \sigma\sqrt{\tau}$$

$$d_1 = \frac{1}{0.15\sqrt{140/365}}\left[1 + \left(0.03 + \frac{0.0225}{2}\right)\frac{140}{365}\right]$$

$$\hat{=}\ 0.1703$$

$$d_2 = d_1 - \sigma\sqrt{\tau} \hat{=} 0.1703 - 0.15\sqrt{140/365}$$

$$\hat{=}\ 0.0774$$

$$\therefore N(d_1) \hat{=} 0.5676$$

and $$N(d_2) \hat{=} 0.5309$$

Hence $$C = 0.5676 \cdot 60 - 0.5309 \cdot 60 \cdot e^{-0.03 \cdot \frac{140}{365}}$$

$$\Rightarrow \boxed{C = \$2.57}$$

See reverse for (b) and (c)

(b) Fill in days 1, 139 and 140, Was the call exercised?

t	S(t)	Δ(t)	Funds borrowed to add shares (or received from selling shares) at t	Amount owed at t (with interest)
0	60.00	0.568	3,405,710.33	3,405,710.33
1	60.12	0.576	49478.76	3,455,469
⋮	⋮	⋮	⋮	⋮
138	56.12	0.00	0	257,300
139	55.86	0.00	0	257,321.10
140	55.64	0.00	0	257,342.30

$\boxed{\text{No}}$ the call would not have been exercised since K > S at expiration, so the firm would not have been assigned on their short calls

$\boxed{\text{c}}$ $\boxed{\text{Yes}}$ the firm made a profit of:

$$\$310,000 - \$257,342.30 = \boxed{\$52,657.70}$$

on day 140, or \$52,055.25 on day 0.