# Mobile Application Development
# Junior infants crypto maths

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# Sign your app

## Learning objectives

An overview of:

- Mathematics underlying encryption.
- Extremely simple explanations.
- Real-world encryption uses huge numbers:
- Example: 600 digits; 2000 bits.
- We work with smallest possible quantities for learning purpose.
- We provide examples of:
  - Prime numbers.
  - Generators.
  - Modular arithmetic.
  - Symmetric key encryption.
  - Public key encryption.
  - Hashing

# Number Theory

## The briefest of introductions

*Number Theory*

# Crypto maths

## Prime number

- Natural numbers: whole numbers: $0, 1, 2, 3, \ldots$
- Prime: natural number divisible only by itself and one.
- Examples of primes: $2, 3, 5, 7, 11, 13$
- 4 is not prime because it is divisible by 2.
- Zero and one are not considered primes.

# Crypto maths

## Prime number

- There is an infinite number of primes.
- Primes still being discovered.
- Structure of pattern of primes still unsolved.
- In real-world cryptography huge prime numbers are used.
- Typically 600 digits, approximately 2000 bits.
- We will work with very small primes.

# Crypto maths

## Prime number

- All natural numbers are either prime or composite numbers.
- A number not a prime number is a composite.
- Prime: 7 because factors are itself and one only.
- Composite: 8 because factors are $1, 2, 4, 8$ and so not prime.

# Crypto maths

## Euclid's discoveries (300 BC)

- Realized all numbers prime or composite.
- Any number repeatedly divisible until set primes arrived at.
- $15 = 3 + 3 + 3$
- $25 = 5 + 5 + 5 + 5 + 5$
- $49 = 7 + 7 + 7 + 7 + 7 + 7 + 7$

# Crypto maths

## Euclid Fundamental Theorem of Arithmetic

Also called *Unique Factorization Theorem* or

*Unique Prime Factorization Theorem*

- Every integer greater than 1 either prime or product of primes
- Example: 30 = 2 x 15 (The prime 2 added 15 times)

# Crypto maths

## Euclid Fundamental Theorem of Arithmetic

- $30 = 2 \times 15$ (The prime 2 added 15 times)
- $30 = 3 \times 10$ (The prime 3 added 10 times)
- $30 = 5 \times 6$ (The prime 5 added 6 times)
- 2, 3 and 5 are the prime factors of 30.

# Crypto maths

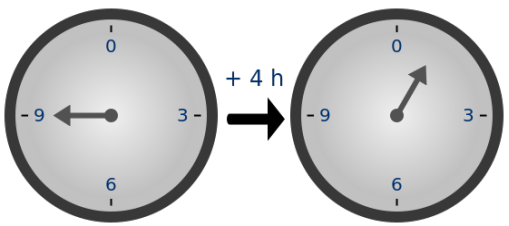
## Euclid Fundamental Theorem of Arithmetic)

- 2 x 3 x 5 is prime factorization of 30.
- Every number has one & only one prime factorization.
- Unique: no two numbers have same factorization.
- Analogy: each number different lock with unique key.
- The unique key: the prime factors.
- No two locks share same key.
- No two numbers share prime same factorization.

# Crypto maths

## Modular arithmetic

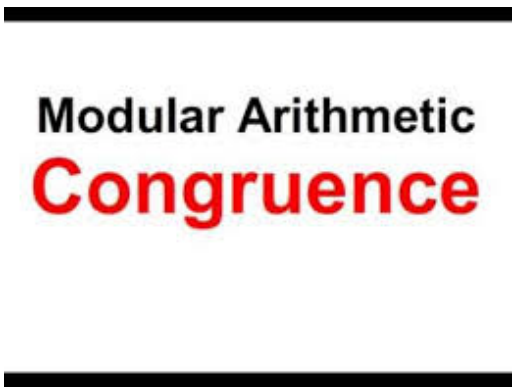
- Also referred to as *clock* arithmetic.
- Number wraps around when modulus reached.
- In case of 12-hour clock the modulus is 12
- Valid range numbers is 0 to 11.
- Example modular addition:
  - $9 + 2 \bmod 12 = 11$
  - $9 + 3 \bmod 12 = 0$
  - $9 + 4 \bmod 12 = 1$



# Crypto maths

## Modular arithmetic

- Java uses % operator for modular arithmetic.
- Example where modulus is 12:
  - 15 % 12 is 3
  - 3 is the remainder when 15 divided by 12.
  - Also expressed as 15 mod 12
- So 15 mod 12 is congruent to 3.
- Which may be expressed as 15 mod 12 $\equiv$ 3.



# Hashing

## What are hashes & how are they generated?

*Hash & Hash Algorithms*

# Hashing

## What are hashes & how are they generated?

- Cryptographic hash function:
  - Input: message variable length.
  - Output: fixed-size alphanumeric string.
  - Complexity: algorithmic complexity high.

# Hashing

## What are hashes & how are they generated?

- Cryptographic hash function:
  - Example: SHA-1
  - Output: fixed-size alphanumeric string.

# Hashing

## Trivial example hash function - definitely not cryptographic standard

```
// Input: any-length string
// Output: 3-digit integer
static int modulus = 1000;
public static int simpleHashAlgorithm(String s) {
  int hash = 0;
  char[] chars = s.toCharArray();
  for (char ch : chars) {
    hash += ch;
  }
  return padding(hash % modulus); // padding: ensure always minimum 3 digits
}
```

```
Input: "ICTSkills-2015"
Output: 950
```

```
Input: "ICTSkills-2016"
Output: 960
```

# SHA-1 hashing examples

## Observe differences between inputs and outputs

ICTSkills-2015

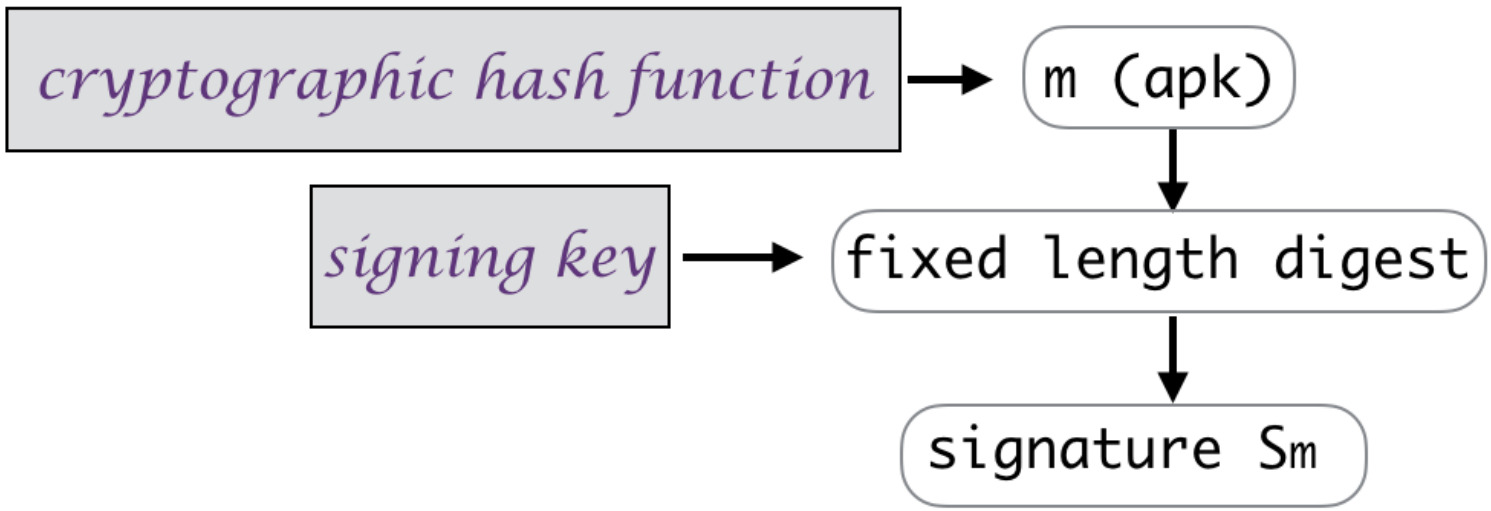
c83007996185ec1269ae9d1e78ef12d51ac0b078

ICTSkills-2016

33f87c1b7e03bc33b34e62313a638123260ca0b0

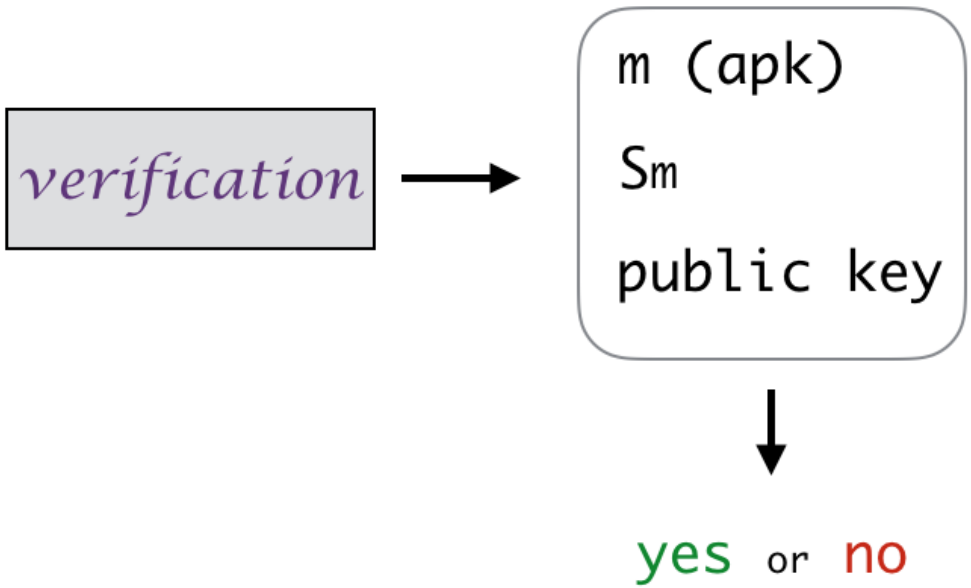
# Sign your app

## Signing



# Sign your app

## Verifying



# Symmetric Key Encryption

## Example using one-time pad

*Symmetric Key Encryption*

# One Time Pad

## Key same length as plaintext

Exclusive OR denoted by $\oplus$.

- m denotes plaintext or message text
- k denotes key
- c denotes the cipher text or encrypted message
- $c = m \oplus k$

| a | b | a $\oplus$ b |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

| m | 0 | 1 | 1 | 0 | 1 | 1 |
|---|---|---|---|---|---|---|
| k | 1 | 0 | 1 | 1 | 0 | 0 |
| c | 1 | 1 | 0 | 1 | 1 | 1 |

# One Time Pad

## Key same length as plaintext

Observe from table:

- $c = m \oplus k$
- $m = c \oplus k$

| m | 0 | 1 | 1 | 0 | 1 | 1 |
|---|---|---|---|---|---|---|
| k | 1 | 0 | 1 | 1 | 0 | 0 |
| c | 1 | 1 | 0 | 1 | 1 | 1 |
| $c \oplus k$ | 0 | 1 | 1 | 0 | 1 | 1 |

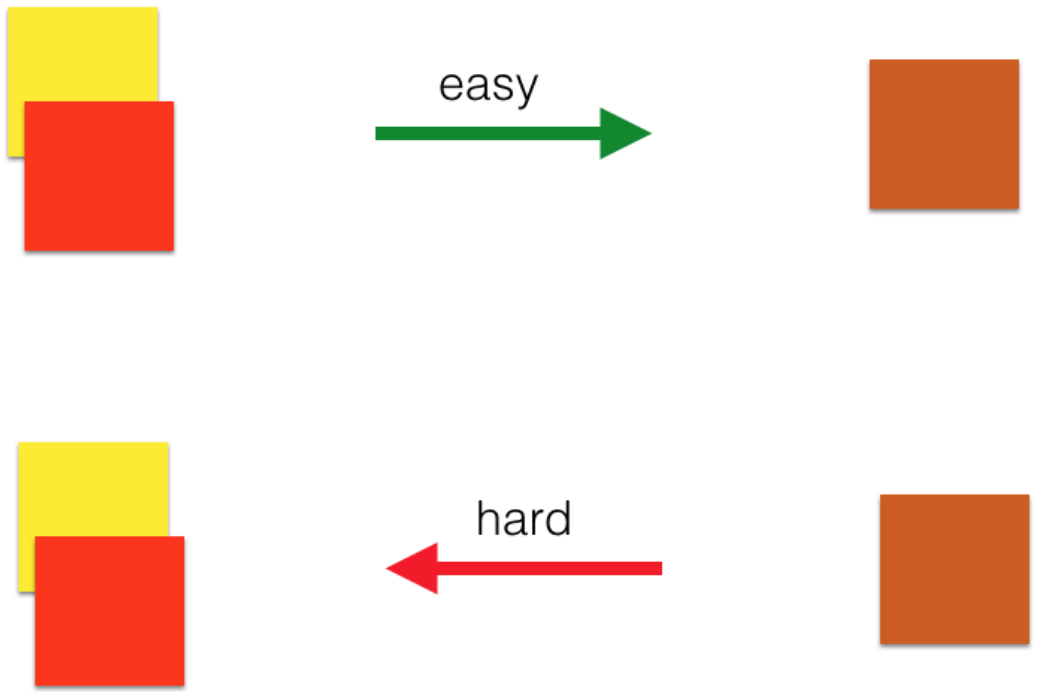
# Key Exchange

## Discovered independently by Diffie & Hellman (Stanford) & Christopher Cocks (GCHQ)

*Diffie-Hellman*

# Key Exchange

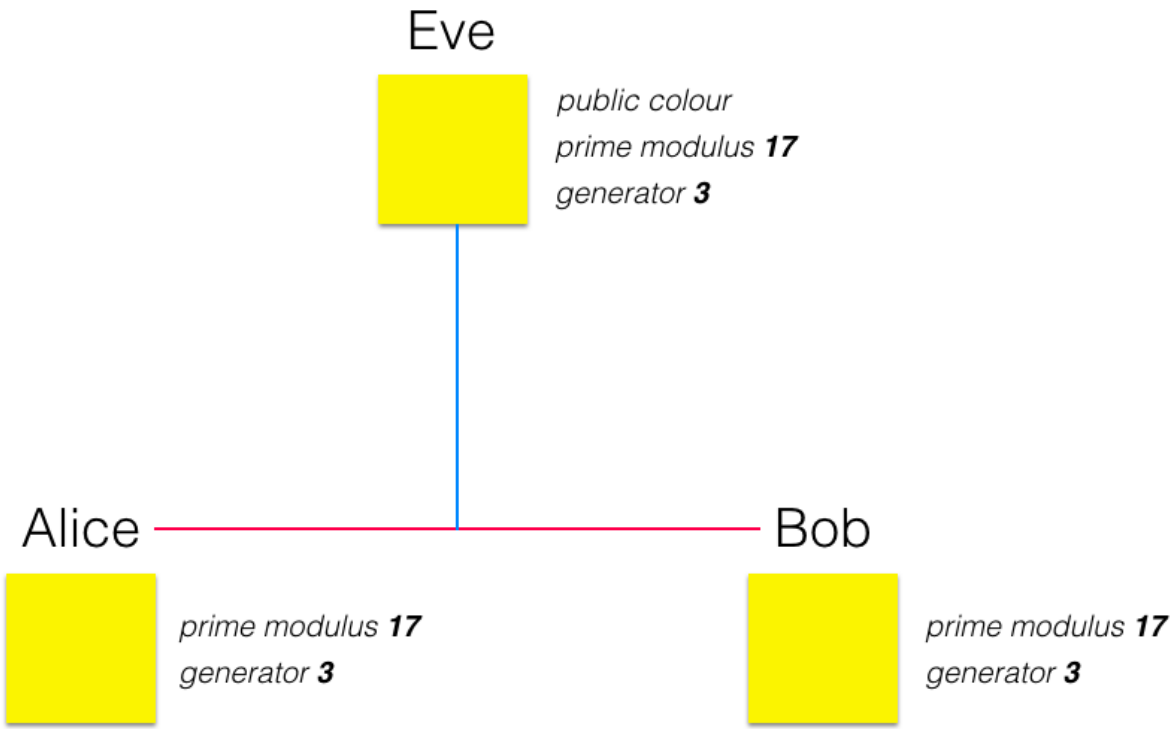
## Uses One-Way Function




One-Way function

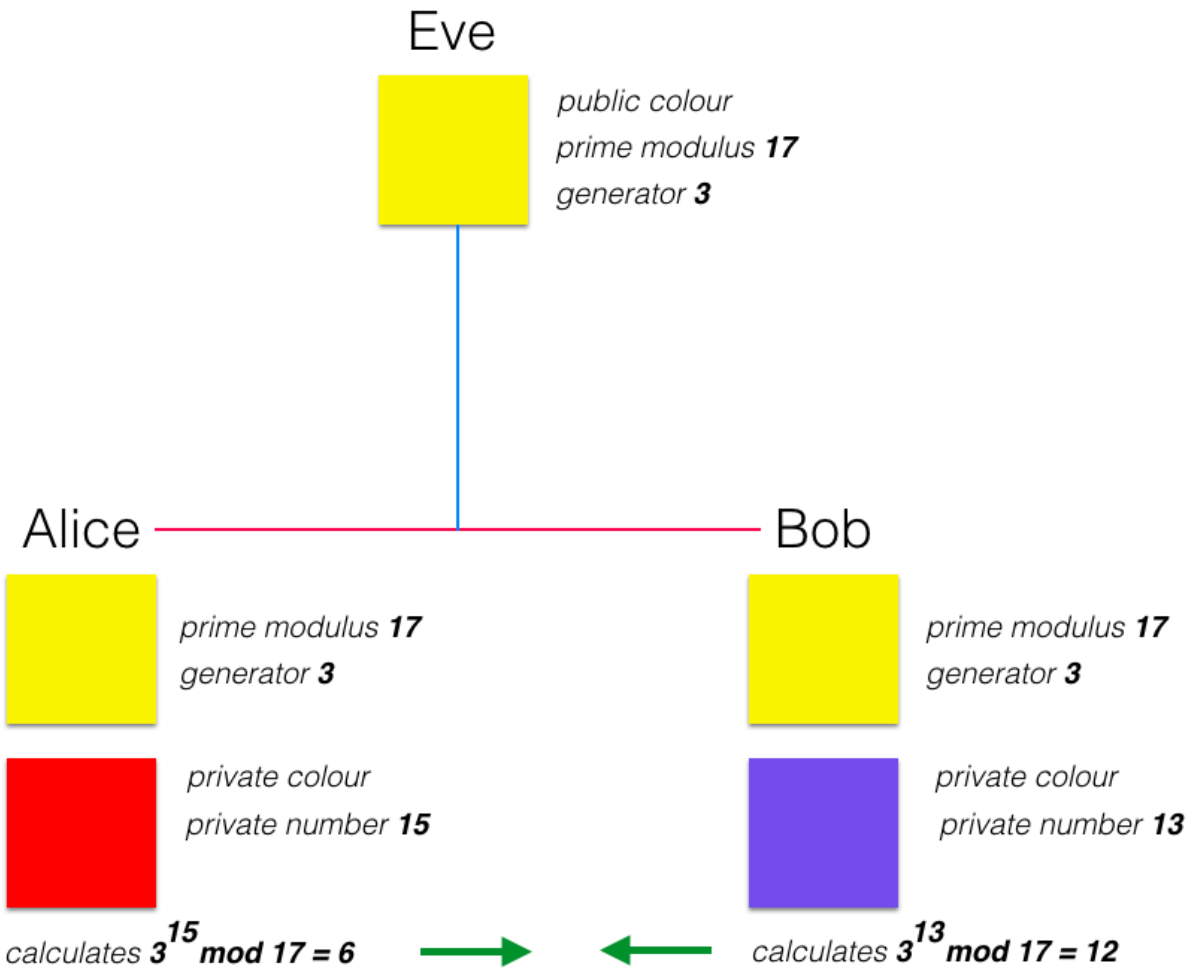
# Key Exchange

## One-Way Function - underlying mathematical theory



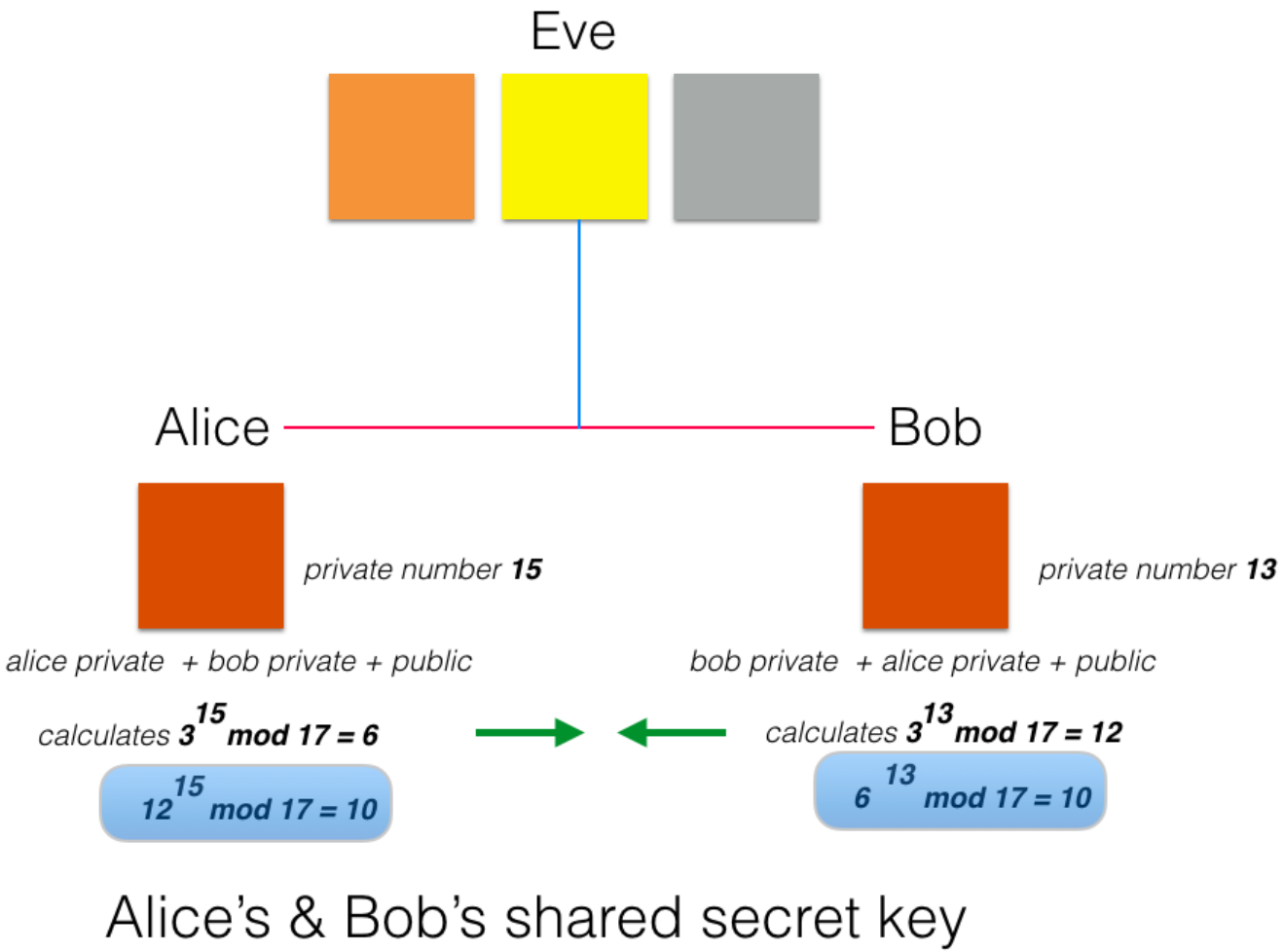
# Key Exchange

## One-Way Function - underlying mathematical theory



# Key Exchange

## One-Way Function - underlying mathematical theory

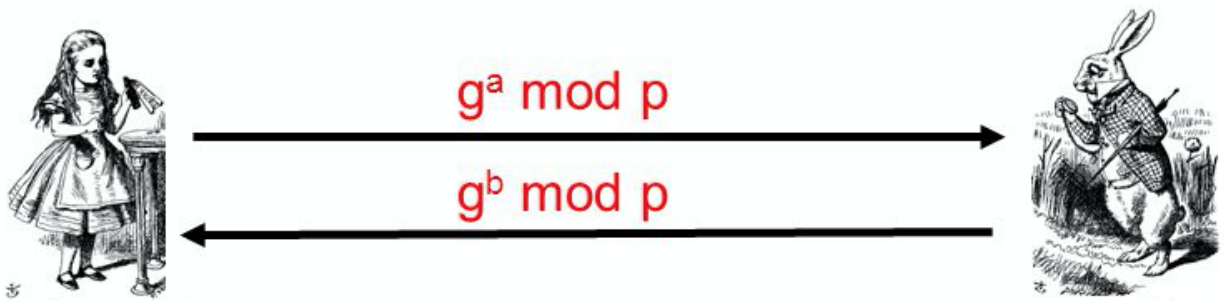


# Key Exchange

## Diffie-Hellman & Christopher Cocks

### Diffie-Hellman

- **Public:** g and p
- **Private:** Alice's exponent a, Bob's exponent b




- Alice computes $(g^b)^a = g^{ba} = g^{ab} \bmod p$
- Bob computes $(g^a)^b = g^{ab} \bmod p$
- Use $K = g^{ab} \bmod p$ as symmetric key

# RSA Encryption

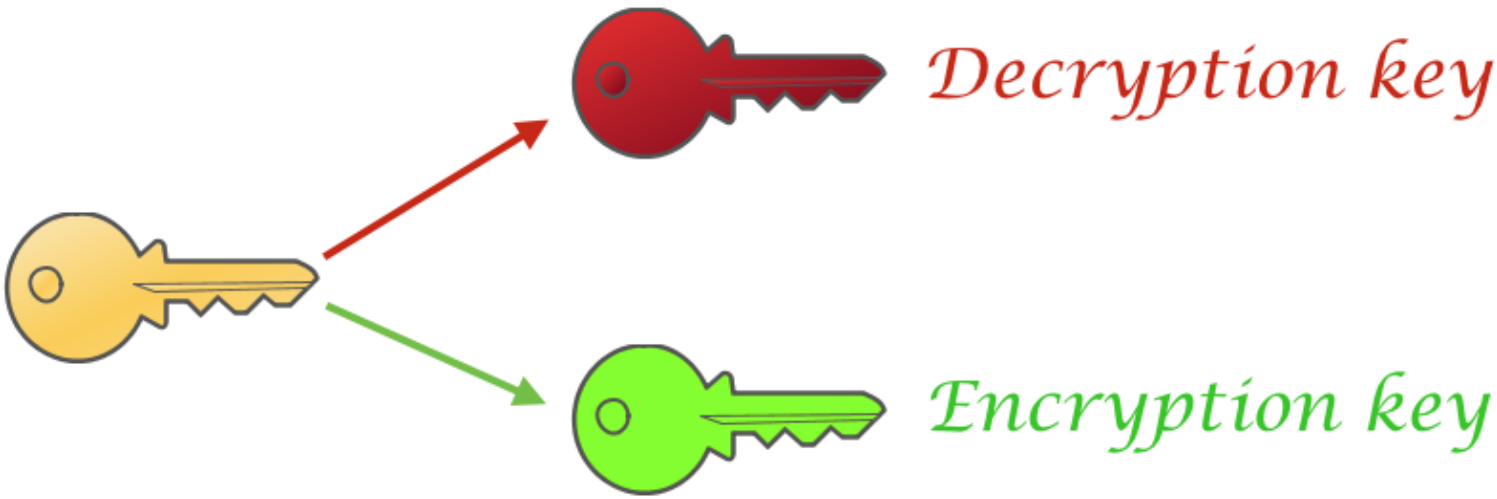
## Discovered by Rivest, Shamir, Adleman (RSA) & Christopher Cocks (GCHQ)

$\mathcal{RSA}$

# RSA Encryption

## public-private key pair

- Ron **R**ivest, Adi **S**hamir & Leonard **A**dleman
- Key generator produces two components.
- The private (secret) key (SK) used to decrypt.
- The public key (PK) used to encrypt.
- Keys have inverse functionality.
  - Encrypt with PK => decrypt with SK.
  - Sign (encrypt) with SK => verify (decrypt) with PK.



# RSA Encryption

## Mathematical explanation

- Let modulus be 14.
- Alice uses key generator to output public-private key pair.
- Gives (somehow) public key to Bob.
  - Private key: (11, 14)
  - Public key: (5, 14)
  - There is a mathematical relationship between the 11 & 5
  - Brief explanation follows
  - More detailed explanations referenced materials

# RSA Encryption

## Mathematical explanation

- Let modulus Z be p * q where p, q very large primes
- p & q remain secret - trapdoor function
- Calculation Z very easy
- Derivation p, q given Z very hard
- Applying set rules using p, q (see ref material):
  - Choose number e.
  - (e, Z) the public key
  - Choose secret number d
  - (d, Z) the private key.

# RSA Encryption

## Plaintext m: Ciphertext c

- Encryption:

$$c = m^e \mod Z$$

- Decryption:

$$m = c^d \mod Z$$

# RSA Encryption

## Mathematical explanation

Bob encrypts plaintext 2 using public key (5, 14):

$$\begin{aligned} c &= 2^5 \, mod \, 14 \\ &= 4 \end{aligned}$$

Hint: Use Paul Trow's online modular arithmetic calculator:

`https://goo.gl/MhfqcO`

# RSA Encryption

## Mathematical explanation

Alice uses private key (11, 14) to decrypt $c = 4$:

$$\begin{aligned} m &= 4^{11} \, mode \, 14 \\ &= 2 \end{aligned}$$

# RSA Encryption

## Mathematical explanation

Alice uses private key (11, 14) to sign (encrypt) a message $m = 2$:

$$\begin{aligned} c &= 2^{11} \, mode \, 14 \\ &= 4 \end{aligned}$$

Bob verifies signed message 4 using public key (5, 14):

$$\begin{aligned} m &= 4^5 \, mod \, 14 \\ &= 2 \ (verified) \end{aligned}$$

# RSA Encryption

## The importance of **p** & **q**

The modulus **Z** is the product of **p** & **q**.

- Individual numbers **p** & **q** required in calculation of:
  - **e** the public key exponent (e, Z)
  - **d** the private key (d, Z)
- **Z** is public.
- Therefore: if Z factorizable then boom goes eCommerce and lots more besides.

# References

## Encryption & Digital Signing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# References

## Encryption & Digital Signing


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## Encryption & Digital Signing


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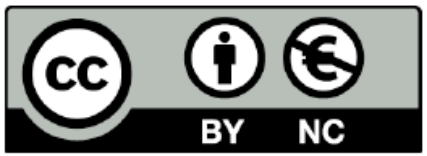




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