MATH 311 Homework 2.4

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Problem 1

$(x^2y + x^4\cos x)dx - x^3dy = 0 => x^2y + x^4\cos x = x^3\frac{dy}{dx} => x^{-1}y + x\cos x = \frac{dy}{dx}$

$M = (x^2y + x^4\cos x)dx \quad N = -x^3dy$

$M_y = x^2$

$N_x = -4x^4$

Linear ✓
Separable
Exact

Problem 2

$(x^{\frac{10}{3}} - 2y)dx + x\,dy = 0 => x^{\frac{10}{3}} - 2y = x\frac{dy}{dx}$

$M = x^{\frac{10}{3}} - 2y\,dx \quad N = x\,dy$

$M_y = -2$

$N_x = 1$

Linear ✓
Separable
Exact

Problem 3

$\sqrt{-2y-y^2}dx + (3+2x-x^2)dy = 0 => (-2y-y^2)^{-\frac{1}{2}}dy = (3+2x-x^2)^{-1}dx$

$M = \sqrt{-2y-y^2}\,dx \quad N = 3+2x-x^2\,dy$

$M_y = -1 - y(-2y-y^2)^{\frac{3}{2}}$

$N_x = 2-2x$

Linear
Separable ✓
Exact

Problem 4

$(ye^{xy}+2x)dx + (xe^{xy}-2y)dy = 0$

$M = ye^{xy}+2x\,dx \quad N = xe^{xy}-2y\,dy$

$M_y = e^{xy}$

$N_x = e^{xy}$

Linear
Separable
Exact ✓

Problem 5

$xy\,dx + dy = 0 => xy + \dfrac{dy}{dx} = 0$

$xy\,dx + dy = 0 => x\,dx = y^{-1}\,dy$

$M = xy\,dx \quad N = dy$

$M_y = x$

$N_x = 0$

Linear ✓
Separable ✓

Exact

Problem 6

$y^2\,dx + (2xy + \cos y)\,dy = 0 => (2xy + \cos y)\,dy = -y^2\,dx$

$M = y^2\,dx \quad N = (2xy + \cos y)\,dy$

$M_y = 2y$

$N_x = 2y$

Linear ✓
Separable
Exact ✓

Problem 9

$(2xy + 3)dx + (x^2 - 1)dy = 0$

$M = 2xy + 3\,dx \quad N = x^2 - 1\,dy$

$M_y = 2x \qquad N_x = 2x$

$\int 2xy + 3\,dx => x^2y + 3x \qquad \int x^2 - 1\,dy => x^2y - y$

$x^2y + 3x - y = C$

Problem 10

$(2x + y)\,dx + (x - 2y)\,dy = 0$

$M = 2x + y\,dx \quad N = x - 2y\,dy$

$M_y = 1 \qquad N_x = 1$

$\int 2x + y\,dx => x^2 + xy \qquad \int x - 2y\,dy => xy - y^2$

$xy + x^2 - y^2 = C$

Problem 11

$(e^x \sin y - 3x^2)dx + \left(e^x \cos y + \frac{y^{-\frac{2}{3}}}{3}\right) dy = 0$

$M = e^x \sin y - 3x^2\, dx \quad N = e^x \cos y + \frac{y^{-\frac{2}{3}}}{3}\, dy$

$M_y = e^x \cos y \quad N_x = e^x \cos y$

$\int e^x \sin y - 3x^2\, dx => e^x \cos y - x^3 \qquad \int e^x \sin y + \frac{y^{-\frac{2}{3}}}{3}\, dy => e^x \sin y + \sqrt[3]{y}$

$e^x \sin y - x^3 + \sqrt[3]{y} = C$

Problem 12

$\cos x \cos y + 2x\, dx - \sin x \sin y + 2y\, dy = 0$

$M = \cos x \cos y + 2x\, dx \quad N = -\sin x \sin y + 2y\, dy$

$M_y = -\cos x \sin y \quad N_x = -\cos x \sin y$

$\int \cos x \cos y + 2x\, dx => \sin x \cos y + x^2 \qquad -\int \sin x \sin y + 2y\, dy => \sin x \cos y y^2$

$\sin x \cos y + x^2 + y^2 = C$

Problem 21

$(x^{-1} + 2y^2 x)dx + (2yx^2 - \cos y)dy = 0 \qquad y(1) = \pi$

$M = x^{-1} + 2y^2 x\, dx \quad N = 2yx^2 - \cos y\, dy$

$M_y = 4xy \quad N_x = 4xy$

$\int x^{-1} + 2y^2 x\, dx => \ln|x| + y^2 x^2 \qquad \int 2yx^2 - \cos y\, dy => y^2 x^2 - \sin y$

$y^2 x^2 + \ln|x| - \sin y = C$

Find C:

$\pi^2 + \ln|1| - \sin \pi = C => C = \pi^2$

Final Solution: $y^2 x^2 + \ln|x| - \sin y = \pi^2$

Problem 22

$(ye^{xy} - y^{-1})dx + (xe^{xy} + xy^{-2})dy = 0 \qquad y(1) = 1$

$M = ye^{xy} - y^{-1}\,dx \quad N = xe^{xy} + xy^{-2}\,dy$

$M_y = yxe^{xy} + y^{-2} \quad N_x = yxe^{xy} + y^{-2}$

$\int ye^{xy} - y^{-1}\,dx => e^{xy} - xy^{-1} \qquad \int xe^{xy} + xy^{-2}\,dy => e^{xy} - xy^{-1}$

$e^{xy} - xy^{-1} = C$

Find C:

$e - 1 = C$

Final Solution: $e^{xy} - xy^{-1} = e - 1$