MATH 311 Homework Chp 2.3

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Problem 1

$x^2\frac{dy}{dx} + \sin x - y = 0$ DE is not separable.

$x^2\frac{dy}{dx} + \sin x - y = 0$ DE is linear.

DE is solvable via the linear method.

Problem 2

$\frac{dx}{dt} + xt = e^x$ equation not separable.

$\frac{dx}{dt} + xt = e^x$ DE is non linear.

DE is solvable neither via the linear or separable methods.

Problem 3

$(t^2+1)\frac{dy}{dt} = yt - y => (t^2+1)\frac{dy}{dt} = (t-1)y => y^{-1}\,dy = \frac{t-1}{t^2-1}\,dt$ DE is separable

$(t^2+1)\frac{dy}{dt} - yt = -y$ DE is linear.

DE is solvable via the separable and linear methods.

Problem 4

$3t = e^t \frac{dy}{dt} + y \ln t$ DE not separable.

$3t = e^t \frac{dy}{dt} + y \ln t$ DE is linear.

DE is solvable via the linear method.

Problem 5

$x\frac{dx}{dt} + t^2 x = \sin t$ equation not separable.

$x\frac{dx}{dt} + t^2 x = \sin t$ The DE is non linear.

DE is solvable neither via the linear or separable methods.

Problem 6

$3r = \frac{dr}{d\theta} - \theta^3$ equation not separable.

$3r = \frac{dr}{d\theta} - \theta^3$ DE is linear.

DE is solvable via the linear method.

Problem 7

$\frac{dy}{dx} - y - e^{3x} = 0 => \frac{dy}{dx} - y = e^{3x}$

Find $I(x)$:

$I(x) = e^{-\int dx} = e^{-x}$

Solve:

$e^{-x}\frac{dy}{dx} - e^{-x}y = e^{3x}e^{-x} => \int (e^{-x}y)' = \int e^{2x} => e^{-x}y = \frac{e^{2x}}{2} + C =>$
$y = e^x \left(\frac{e^{2x}}{2} + C\right)$

Problem 8

$\frac{dy}{dx} = \frac{y}{x} + 2x + 1 => \frac{dy}{dx} - \frac{y}{x} = 2x + 1$

Find $I(x)$:

$I(x) = e^{-\int x^{-1}} = e^{-\ln x} = x^{-1}$

Solve:

$x^{-1}\frac{dy}{dx} - \frac{y}{x^2} = 2 + x^{-1} => \int \left(\frac{y}{x}\right)' = \int 2 + x^{-1} => \frac{y}{x} = 2x + \ln|x| + C$

$y = 2x^2 + x\ln|x| + Cx$

Problem 9

$\frac{dr}{d\theta} + r\tan\theta = \sec\theta$

Find $I(x)$:

$I(x) = e^{\int \tan\theta} = e^{-\ln\cos|\theta|} = \sec\theta$

Solve:

$\sec\theta\frac{dr}{d\theta} + r\sec\theta\tan\theta = \sec^2\theta => r\sec\theta = \tan\theta + C$

$r = \frac{\tan\theta + C}{\sec\theta}$

Problem 10

$x\frac{dy}{dx} + 2y = x^{-3} => \frac{dy}{dx} + \frac{2y}{x} = x^{-4}$

Find $I(x)$: $I(x) = e^{\int \frac{2}{x}} = e^{\ln(x^2)} = x^2$

Solve:

$x^2\frac{dy}{dx} + 2xy = x^{-2} = \int (x^2y)' = \int x^{-2}\,dx => x^2y = -x^{-1} + C$

$y = -\frac{x^{-1} + C}{x^2}$

Problem 11

$(t+y+1)dt - dy = 0 => t+y+1 = \frac{dy}{dt} => t+1 = \frac{dy}{dt} - y$

Find $I(x)$:

$I(x) = e^{-\int dt} = e^{-t}$ Solve:

$e^{-t}(t+1) = e^{-t}\frac{dy}{dt} - e^{-t}y => \int e^{-t}(t+1)\,dt = \int (ye^{-t})' => -(t+2)e^{-t} + C = ye^{-t}$

$y = Ce^{t} - t - 2$

Problem 12

$\frac{dy}{dx} = x^2e^{-4x} - 4y = \frac{dy}{dx} + 4y = x^2e^{-4x}$

Find $I(x)$:

$I(x) = e^{4\int dx} = e^{4x}$

Solve:

$e^{4x}\frac{dy}{dx} + 4e^{4x}y = x^2 => \int (e^{4x}y)' = \int x^2\,dx => e^{4x}y = \frac{x^3}{3} + C$

$y = e^{-4x}\left(\frac{x^3}{3} + C\right)$

Problem 13

$\frac{dx}{dy} + \frac{2x}{y} = 5y^2$

Find $I(x)$:

$I(x) = e^{\int \frac{2}{y}\,dy} = e^{\ln y^2} = y^2$

Solve:

$y^2\frac{dx}{dy} + 2xy = 5y^4 => \int (xy^2)' = \int 5y^4\,dy => xy^2 = y^5 + C$

$x = \frac{y^5 + C}{y^2}$

Problem 17

$\frac{dy}{dx} - \frac{y}{x} = xe^x \qquad y(1) = e - 1$

$I(x) = e^{\int -\frac{1}{x}\,dx} = e^{-\ln x} = x^{-1}$

Solve:

$x^{-1}\frac{dy}{dx} - \frac{y}{x^2} = e^x => \int \left(\frac{y}{x}\right)' = \int e^x\,dx => \frac{y}{x} = e^x + C => y = x(e^x + C)$

Find C:

$e - 1 = 1(e + C) => C = 1$

Final Solution:

$y = xe^x + x$

Problem 18

$\frac{dy}{dx} + 4y - e^{-x} = 0 \qquad y(0) = \frac{4}{3}$

$\frac{dy}{dx} + 4y = e^{-x}$

Find $I(x)$:

$I(x) = e^{\int 4dx} = e^{4x}$

Solve:

$e^{4x}\frac{dy}{dx} + 4e^{4x}y = e^{3x} => \int (e^{4x}y)' = \int e^{3x}\,dx => e^{4x}y = \frac{1}{3}e^{3x} + C => y = \frac{\frac{1}{3}e^{3x} + C}{e^{4x}}$

Find C:

$0 = \frac{\frac{1}{3}e^4 + C}{e^{\frac{16}{3}}} => C = -\frac{1}{3}e^4$

Final Solution:

$y = \frac{3e^{3x} - \frac{1}{3}e^4}{e^{4x}}$