MATH 311 Homework 1.4

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For all problems:
$$n = 4 \quad h = \frac{1-0}{4} = 0.25$$
$$y_{i+1} = hf(x_i, y_i) + y_i$$

Problem 1

$$\frac{dy}{dx} = -\frac{x}{y} \qquad y(0) = 4 \quad y(1) = ?$$

$y(0.25) = 0.25(0) + 4 = 4.25$
$y(0.5) = 0.25(-\frac{0.25}{4.25}) + 4.25 \approx 4.24$
$y(0.75) = 0.25(-\frac{0.5}{4.24}) + 4.24 \approx 4.21$
$y(1) = 0.25(-\frac{0.75}{4.21}) + 4.21 \approx 4.17$

Problem 2

$$\frac{dy}{dx} = y(2 - y) \qquad y(0) = 3 \quad y(1) = ?$$

$y(0.25) = 0.25(-3) + 3 = 2.25$
$y(0.5) = 0.25(-0.5625) + 2.25 \approx 2.1094$
$y(0.75) = 0.25(-0.23076836) + 2.1094 \approx 2.0517$
$y(1) = 0.25(-0.10607289) + 2.0517 \approx 2.0252$

Problem 3

$$\frac{dy}{dx} = x + y \qquad y(0) = 1 \quad y(1) = ?$$

$y(0.25) = 0.25(1) + 1 = 1.25$
$y(0.5) = 0.25(1.5) + 1.25 = 1.625$
$y(0.75) = 0.25(2.125) + 1.625 \approx 2.1563$

$y(1) = 0.25(2.90625) + 2.1563 \approx 2.8829$

Problem 4

$$\frac{dy}{dx} = \frac{x}{y} \qquad y(0) = -1 \quad y(1) = ?$$

$y(0.25) = 0.25(0) - 1 = -1$
$y(0.5) = 0.25(-0.25) - 1 = -1.0625$
$y(0.75) = 0.25(-0.470588235) - 1.0625 \approx -1.1801$
$y(1) = 0.25(-0.635539361) - 1.1801 \approx -1.3390$