MATH 311 Homework 4.3

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Problem 13

$y'' - y' + 9y = 3\sin 3t \qquad n^2 - n + 9 = 0 => (n-4)(n+5) = 0 => n = 4, -5$

$y_c = c_1 e^{4x} + c_2 e^{-5x}$
$y_p = A\sin 3t + B\cos 3t \quad y_p' = 3A\cos 3t - 3B\sin 3t \quad y_p'' = -9A\sin 3t - 9B\cos 3t$
$-9A\sin 3t - 9B\cos 3t - 3A\cos 3t + 3B\sin 3t + 9A\sin 3t + 9B\cos 3t = 3\sin 3t =$
$-A\cos 3t + B\sin 3t = \sin 3t => A = 0 \quad B = 1 \therefore$
$y_p = \cos 3t$

$y = c_1 e^{4t} + c_2 e^{-5t} + \cos 3t$

Problem 14

$2z'' + z = 9e^{2t} \qquad 2n^2 + 1 = 0 => n = \sqrt{2}i => \alpha = 0 \quad \beta = \sqrt{2}$

$z_c = c_1 \sin\left(\sqrt{2}t\right) + c_2 \cos\left(\sqrt{2}t\right)$
$z_p = Ae^{2t} \quad y_p' = 2Ae^{2t} \quad y_p'' = 4Ae^{2t}$
$8Ae^{2t} + Ae^{2t} = 9e^{2t} => A = 1$

$z = c_1 \sin\left(\sqrt{2}t\right) + c_2 \cos\left(\sqrt{2}t\right) + e^{2t}$

Problem 15

$y'' - 5y' + 6y = xe^x \qquad n^2 - 5n + 6 = 0 => (n-6)(n+1) = 0 => n = 6, -1$

$y_c = c_1 e^{6x} + c_2 e^{-x}$
$y_p = Axe^x + Be^x \quad y_p' = Axe^x + Ae^x + Be^x \quad y_p'' = Axe^x + 2Ae^x + Be^x$
$Axe^x + 2Ae^x + Be^x - 5Axe^x - 5Ae^x - 5Be^x + 6Axe^x + 6Be^x = xe^x =$
$2Ax - 3A + 2B = x => 2A = 1 \quad -3A + 2B = 0 => A = \frac{1}{2} \quad B = \frac{3}{4}$

$y = c_1 e^{6x} + c_2 e^{-x} + \frac{1}{2}xe^x + \frac{3}{4}e^x$

Problem 16

$\theta'' - \theta = t\sin t \qquad n^2 - 1 = 0 => n = -1, 1$

$\theta_c = c_1 e^{-t} + c_2 e^t$
$\theta_p = At\sin t + B\sin t + Ct\cos t + D\cos t$
$\theta_p' = At\cos t + A\sin t + B\cos t - Ct\sin t + C\cos t - D\sin t$
$\theta_p'' = -At\sin t + 2A\cos t - Ct\cos t - 2C\sin t - B\sin t - D\cos t$

$t\sin t = -2At\sin t + 2A\cos t - 2B\sin t - 2Ct\cos t - 2C\sin t - 2D\cos t$
$t\sin t \quad 1 = -2A => A = -\frac{1}{2}$
$t\cos t \quad 0 = -2C => C = 0$
$\sin t \quad 0 = -2C - 2B => B = 0$
$\cos t \quad 0 = -2A - 2D => D = \frac{1}{2}$

$\theta = c_1 e^{-t} + c_2 e^t - \frac{1}{2}t\sin t + \frac{1}{2}\cos t$

Problem 17

$y'' + 4y = 8\sin 2t \qquad n^2 + 4 = 0 => n = \dfrac{0 \pm \sqrt{0 - (4)(4)(1)}}{2} = 2i => \alpha = 0 \quad \beta = 2$

$y_c = c_1 \sin 2t + c_2 \cos 2t$
$y_p = At\sin 2t + Bt\cos 2t$
$y_p' = 2At\cos 2t + A\sin 2t - 2Bt\sin 2t + B\cos 2t$
$y_p'' = -4At\sin 2t + 4A\cos 2t - 4Bt\cos 2t - 4B\sin 2t$
$8\sin 2t = 4At\sin 2t + 4Bt\cos 2t - 4At\sin 2t - 4Bt\cos 2t + 4A\cos 2t - 4B\sin 2t$
$8\sin 2t = 4A\cos 2t - 4B\sin 2t$

$8\sin 2t \quad 8 = -4B => B = -2$
$0\cos 2t \quad 0 = 4A => A = 0$

$y = c_1 \sin 2t + c_2 \cos 2t - 2t\cos 2t$

Problem 18

$y'' - 2y' + y = 8e^t \qquad n^2 - 2n + 1 = 0 => (n-1)^2 = 0 => n = 1, 1$

$y_c = c_1 e^t + c_2 te^t$
$y_p = At^2 e^t \quad y_p' = At^2 e^t + 2Ate^t \quad y_p'' = At^2 e^t + 4Ate^t + 2Ae^t$
$At^2 e^t + 4Ate^t + 2Ae^t - 2At^2 e^t - 4Ate^t + At^2 e^t = 8e^t => 2A = 8 => A = 4$

$y = c_1 e^t + c_2 te^t + 4t^2 e^t$

Problem 27

$y'' + 9y = 4t^3 \sin 3t \qquad n^2 + 9 = 0 => n = 3i => \alpha = 0 \quad \beta = 3$

$y_c = c_1 \sin 3t + c_2 \cos 3t$
$y_p = At^4 \sin 3t + Bt^3 \sin 3t + Ct^2 \sin 3t + Dt \sin 3t + Et^4 \cos 3t + Ft^3 \cos 3t + Gt^2 \cos 3t + Ht \cos 3t$

Problem 28

$y'' - 6y' + 9y = 5t^6 e^{3t} \qquad n^2 - 6n + 9 = 0 => (n-3)^2 = 0 => n = 3, 3$

$y_c = c_1 e^{3t} + c_2 t e^{3t}$
$y_p = At^8 e^{3t} + Bt^7 e^{3t} + Ct^6 e^{3t} + Dt^5 e^{3t} + Et^4 e^{3t} + Ft^3 e^{3t} + Gt^2 e^{3t}$

Problem 29

$y'' + 3y' - 7y = t^4 e^t \qquad n^2 + 3n - 7 = 0 => n = \frac{-3 \pm \sqrt{9 - 4(-7)}}{2} = \frac{-3 \pm \sqrt{37}}{2}$

$y_c = c_1 e^{\frac{-3-\sqrt{37}}{2} t} + c_2 e^{\frac{-3+\sqrt{37}}{2} t}$
$y_p = At^4 e^t + Bt^3 e^t + Ct^2 e^t + Dte^t + Ee^t$

Problem 30

$y'' - 2y' + y = 7e^t \cos t \qquad n^2 - 2n + 1 = 0 => (n-1)^2 = 0 => n = 1, 1$

$y_c = c_1 e^t + c_2 t e^t$
$y_p = Ae^t \sin t + Be^t \cos t$

Problem 31

$y'' + 2y' + 2y = 8t^3 e^{-t} \sin t \qquad n^2 + 2n + 2 = 0 => n = \frac{-2 \pm 2i}{2} => \alpha = -1 \quad \beta = 1$

$y_c = c_1 e^{-t} \sin t + c_2 e^{-t} \cos t$
$y_p = e^{-t} \sin t (At^4 + Bt^3 + Ct^2 + Dt) + e^{-t} \cos t (Et^4 + Ft^3 + Gt^2 + Ht)$

Problem 32

$y'' - y' - 12y = 2t^6 e^{-3t} \qquad n^2 - n - 12 = 0 => (n-4)(n+3) = 0 => n = -3, 4$

$y_c = c_1 e^{-3t} + c_2 e^{4t}$
$y_p = e^{-3t}(At^7 + Bt^6 + Ct^5 + Dt^4 + Et^3 + Ft^2 + Gt)$

Problem 33

$y''' - y'' + y = \sin t \qquad n^3 - n^2 + 1 = 0 =>$

$y_c = c_1 \cdot ? + c_2 \cdot ? + c_3 \cdot ?$
$y_p = A\sin t + B\cos t \quad y_p' = A\cos t - B\sin t \quad y_p'' = -A\sin t - B\cos t \quad y_p''' = -A\cos t + B\sin t$
$2A\sin t + 2B\cos t - A\cos t + B\sin t = \sin t$
$2A + B = 1 \quad 2B - A = 0$
$B = 1 - 2A => 2 - 4A - A => A = \frac{2}{5}$
$\frac{4}{5} + B = 1 => B = \frac{1}{5}$

$y = c_1 \cdot ? + c_2 \cdot ? + c_3 \cdot ? + \frac{2}{5}\sin t + \frac{1}{5}\cos t$