MATH 311 Homework 4.1

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April 08, 2022

Problem 1

$y = c_1 + c_2 \cos x + c_3 \sin x; \qquad y(\pi) = 0, \quad y'(\pi) = 2, \quad y''(\pi) = 1$

$1 = -c_2 \cos \pi - c_3 \sin \pi => c_2 = 1$
$2 = -c_2 \sin \pi + c_3 \cos \pi => c_3 = -2$
$0 = c_1 + c_2 \cos \pi + c_3 \sin \pi => 0 = c_1 + 1 \cos \pi - 2 \sin \pi => 0 = c_1 - 1 => c_1 = 1$

$y = 1 + \cos x - 2 \sin x$

Problem 2

$y^{(4)} + y'' = 0$

$$\text{W=}\begin{vmatrix} 1 & x & \sin x & \cos x \\ 0 & 1 & \cos x & -\sin x \\ 0 & 0 & -\sin x & -\cos x \\ 0 & 0 & -\cos x & \sin x \end{vmatrix} = \begin{vmatrix} 1 & \cos x & -\sin x \\ 0 & -\sin x & -\cos x \\ 0 & -\cos x & \sin x \end{vmatrix} = \begin{vmatrix} \text{-sin}\, x & -\cos x \\ -\cos x & \sin x \end{vmatrix} =$$

$$-\sin^2 x - \cos^2 x = -1 \neq 0 \therefore$$
the set is linearly independent.

$1, \quad 0 + 0 = 0 => 0 = 0 \checkmark$
$x, \quad 0 + 0 = 0 => 0 = 0 \checkmark$
$\sin x, \quad \sin x - \sin x = 0 => 0 = 0 \checkmark$
$\cos x, \quad \cos x - \cos x = 0 => 0 = 0 \checkmark$
Therefore the functions form a Fundamental set of solutions for the DE.
General Solution: $y = c_1 + c_2 x + c_3 \sin x + c_4 \cos x$

Problem 3

$y'' + 9y = 0 \qquad y_1 = \sin 3x$

$y_2 = u \cdot \sin 3x \qquad y_2' = u' \sin 3x + 3u \cos 3x \qquad y_2'' = u'' \sin 3x + 6u' \cos 3x - 9u \sin 3x$

$u'' \sin 3x + 6u' \cos 3x - 9u \sin 3x + 9u \sin 3x = 0 \qquad w = u'$

$\int \frac{dw}{w} = \int \frac{6 \sin 3x}{\cos 3x} dx => \ln w = \ln u^{-2} => w = \frac{1}{\sin^2 3x} => \int w = \int \csc^2 3x$

$u = -\frac{1}{3} \cot 3x \therefore$

$y_2 = -\frac{1}{3} \cot 3x \cdot \sin 3x = \cos 3x$
General Solution: $y = c_1 \sin 3x + c_2 \cos 3x$

Problem 4

$x^2 y'' + 2xy' - 6y = 0, \qquad y_1 = x^2$
$y_2 = ux^2 \qquad y_2' = u'x^2 + 2ux \qquad y_2'' = u''x^2 + 4u'x + 2u$

$u''x^4 + 4u'x^3 + 2ux^2 + 2u'x^3 + 4ux^2 - 6ux^2 = 0 => u''x^4 + 6u'x^3 = 0 \quad w = u'$

$\frac{dw}{dx} + \frac{6w}{x} = 0 \; (I(x) = x^6) => \int (x^6 w)' = \int 0 => w = x^{-6} => u = x^{-5}$

$y_2 = x^{-5} x^2 = x^{-3}$
General Solution: $y = c_1 x^2 + c_2 x^{-3}$