MATH 311 Homework 3.2 and 3.3

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Section 3.2

Problem 14

$\frac{dP}{dt} = kP => \int \frac{dP}{P} = \int k\, dt$

$\ln P = kt + C => P = Ae^{kt}$

$P(0) = 1500 => 1500 = A$

$P(26) = 6000 => 6000 = 1500e^{26k} => 4 = e^{26k} => \ln 4 = 26k => k = \frac{\ln 4}{26}$

$P(40) = 1500e^{\frac{\ln 4}{26}(40)} \approx 12657.19$

Problem 15

$p_0 = P(0) = 1500 \quad p_a = P(13) = 4100 \quad p_b = P(26) = 6000 \quad p_1 = P(40)$

$p_1 = \frac{(4100)(6000) - 2(1500)(6000) + (1500)(4100)}{(4100)^2 - (1500)(6000)}(4100) \approx 6693.34$

Problem 21

If V = volume $\quad S_a$ = surface area $\quad t$ = time $\quad A$ = diameter

$\frac{dA}{dt} = -k\frac{dV}{dS_a} \qquad \frac{dV}{dS_a} = \frac{\frac{4}{3}\pi\left(\frac{A}{2}\right)^3}{4\pi\left(\frac{A}{2}\right)^2} = \frac{A}{6}$

$\frac{dA}{dt} = -\frac{Ak}{6} => \int \frac{dA}{A} = -\int \frac{kdt}{6} => A = Ce^{-\frac{kt}{6}}$

$A(0) = 4 \qquad C = 4$

$A(.5) = 3 \qquad 3 = 4e^{-\frac{k}{12}} => k = -12\ln\frac{3}{4}$

$A(t) = 2 \qquad 2 = 4e^{12 \ln \frac{3}{4} t} => \ln .5 = 12 \ln \frac{3}{4} t => t = \dfrac{\ln .5}{12 \ln \frac{3}{4}} => t \approx$ hrs

$A(t) \approx 0$ I honestly don't know how to answer this one

Problem 23

$\dfrac{dA}{dt} = -kA => \ln A = -kt + C => A = Ce^{-kt}$

$A(0) = 50 \qquad 50 = C$

$A(3) = 10 \qquad 10 = 50e^{-3k} => \frac{1}{5} = e^{-3k} => \ln\left(\frac{1}{5}\right) = -3k => k = -\frac{\ln\left(\frac{1}{5}\right)}{3}$

$A(4) = 50e^{\frac{4 \ln\left(\frac{1}{5}\right)}{3}} \approx 5.85 \quad 11.7\%$ remaining

Problem 24

Using equation in Problem 23:

$A(0) = 300 \qquad C = 300$

$A(5) = 200 \qquad 200 = 300e^{-5k} => \frac{2}{3} = e^{-5k} => k = -\ln\left(\frac{2}{3}\right)\frac{1}{5}$

$A(?) = 10 \qquad 10 = 300e^{\ln\left(\frac{2}{3}\right)\frac{1}{5}t} => \frac{1}{30} = e^{\ln\left(\frac{2}{3}\right)\frac{1}{5}t} => \ln\frac{1}{30} = \ln\left(\frac{2}{3}\right)\frac{1}{5}t => t = \dfrac{5 \ln \frac{1}{30}}{\ln\left(\frac{2}{3}\right)}$

$t \approx 41.94$ yrs

Section 3.3

Problem 1

$T(t) = 21 + Ae^{kt}$

$T(0) = 95 \qquad 95 = 21 + A(1) => A = 74$

$T(5) = 80 \qquad 80 = 21 + 74e^{5k} => 59 = 74e^{5t} => \frac{59}{74} = e^{5k} => \ln\left(\frac{59}{74}\right) = 5k => k = \dfrac{\ln\left(\frac{59}{74}\right)}{5}$

$T(?) = 50 \qquad 50 = 21 + 74e^{\frac{\ln\left(\frac{59}{74}\right)}{5}t} => \frac{29}{74} = e^{\frac{\ln\left(\frac{59}{74}\right)}{5}t} => \ln\frac{29}{74} = \frac{\ln\left(\frac{59}{74}\right)}{5}t => t = \dfrac{5 \ln\left(\frac{29}{74}\right)}{\ln\left(\frac{59}{74}\right)}$

$t \approx 20.68$ min

Problem 2

$T(t) = T_m + Ae^{kt}$

$T(0) = 35 \qquad 35 = 70 + A => A = -35$

$T(3) = 40 \qquad 40 = 70 - 35e^{3k} => \ln\left(\frac{6}{7}\right) = 3k => k = \frac{\ln\left(\frac{6}{7}\right)}{3}$

$T(20) = 70 - 35e^{\frac{20\ln\left(\frac{6}{7}\right)}{3}} \approx 57.48°F$

Problem 3

$T(t) = T_m + Ae^{kt}$

$T(0) = 70 \qquad 70 = 32 + A => A = 38$

$T(15) = 60 \qquad 60 = 32 + 38e^{15k} => 28 = 38e^{15k} => k = \frac{\ln\frac{14}{19}}{15}$

$T(?) = 56 \qquad 56 = 32 + 38e^{\frac{\ln\frac{14}{19}}{15}t} => \ln\frac{12}{19} = \frac{\ln\frac{14}{19}}{15}t => t = \frac{15\ln\frac{12}{19}}{\ln\frac{14}{19}}$

$t \approx 22.57$ min