MATH 311 Homework 2.5

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*Note the use of lower case mu instead of Capital mu

Problem 7

$(2xy)dx + (y^2 - 3x^2)dy = 0$

$M = 2xy\,dx \qquad N = y^2 - 3x^2\,dy$

$M_y = 2x \qquad N_x = -6x \quad M_y \neq N_x$

$\mu = e^{\int \frac{N_x - M_y}{M}\,dy} = e^{\int \frac{-4}{y}\,dy} = y^{-4}$

$\hat{M} = 2xy^{-3}\,dx \qquad \hat{N} = y^{-2} - 3x^2y^{-4}\,dy$

$\hat{M}_y = -6xy^{-3} \qquad \hat{N}_x = -6xy^{-3} \quad M_y = N_x$

$\int 2xy^{-3}\,dx => x^2y^{-3} \qquad \int y^{-2} - 3x^2y^{-4}\,dy => -y^{-1} + x^2y^{-3}$

Solution: $x^2y^{-3} - y^{-1} = C$

Problem 8

$(3x^2 + y)dx + (x^2y - x)dy = 0$

$M = 3x^2 + y \qquad N = x^2y - x$

$M_y = 1 \qquad N_x = 2xy - 1 \quad M_y \neq N_x$

$\mu = e^{\int \frac{N_x - M_y}{M}\,dy} = e^{\int \frac{-2(xy-1)}{x(xy-1)}\,dx} = e^{\ln x^{-2}} = x^{-2}$

$\hat{M} = 3 + yx^{-2}\,dx \qquad \hat{N} = y - x^{-1}\,dy$

$\hat{M}_y = x^{-2} \qquad \hat{N}_x = x^{-2} \quad M_y = N_x$

$\int 3 + yx^{-2}\, dx => 3x - x^{-1}y \qquad \int y - x^{-1}\, dy => \frac{1}{2}y^2 - x^{-1}y$

$3x + \frac{1}{2}y^2 - x^{-1}y = C$

Problem 9

$(x^4 - x + y)dx - x\, dy = 0$

$M = x^4 - x + y\, dx \qquad N = -x\, dy$

$M_y = 1 \qquad N_x = -1 \quad M_y \neq N_x$

$\mu = e^{\int -\frac{2}{x}\, dx} = x^{-2}$

$\hat{M} = x^2 - x^{-1} + yx^{-2}\, dx \qquad \hat{N} = -x^{-1}\, dy$

$\hat{M}_y = x^{-2} \qquad \hat{N}_x = x^{-2} \quad M_y = N_x$

$\int x^2 - x^{-1} + yx^{-2}\, dx => \frac{1}{3}x^3 - \ln|x| - x^{-1}y \qquad \int -x^{-1}\, dy => -x^{-1}y$

Final Solution: $\frac{1}{3}x^3 - \ln|x| - yx^{-1} = C$

Problem 10

$(2y^2 + 2y + 4x^2)dx + (2xy + x)dy = 0$

$M = 2y^2 + 2y + 4x^2 \qquad N = 2xy + x$

$M_y = 4y + 2 \qquad N_x = 2y + 1 \quad M_y \neq N_x$

$\mu = e^{\int \frac{2y+1}{x(2y+1)}\, dx} = x$

$\hat{M} = 2y^2x + 2yx + 4x^3\, dx \qquad \hat{N} = 2x^2y + x^2\, dy$

$\hat{M}_y = 4xy + 2x \qquad \hat{N}_x = 4xy + 2x \quad M_y = N_x$

$\int 2y^2x + 2yx + 4x^3\, dx => x^2y^2 + x^2y + x^4 \qquad \int 2x^2y + x^2\, dy => x^2y^2 + x^2y$

Final Solution: $x^2y^2 + x^2y + x^4 = C$

Problem 11

$(y^2 + 2xy)dx - x^2dy = 0$

$M = y^2 + 2xy \qquad N = -x^2$

$M_y = 2y + 2x \qquad N_x = -2x \quad M_y \neq N_x$

$\mu = e^{\int \frac{-2(y+2x)}{y(y+2x)}\,dy} = y^{-2}$

$\hat{M} = 1 + 2xy^{-1}\,dx \qquad \hat{N} = -y^{-2}x^2\,dy$

$\hat{M}_y = -2xy^{-2} \qquad \hat{N}_x = -2xy^{-2} \quad M_y = N_x$

$\int 1 + 2xy^{-1}\,dx => x + x^2y^{-1} \qquad \int -y^{-2}x^2\,dy => x^2y^{-1}$

Final Solution: $x^2y^{-1} + x = C$

Problem 12

$(2xy^3 + 1)dx + (3x^2y^2 - y^{-1})dy = 0$

$M = 2xy^3 + 1 \qquad N = 3x^2y^2 - y^{-1}$

$M_y = 6xy^2 \qquad N_x = 6xy^2 \quad M_y = N_x$

$\int 2xy^3 + 1\,dx => x^2y^3 + x \qquad \int 3x^2y^2 - y^{-1}\,dy => x^2y^3 - \ln|y|$

Final Solution: $x^2y^3 + x - \ln|y| = C$