MATH 311 Homework 2.6

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Problem 17

$\frac{dy}{dx} = \sqrt{x+y} - 1$

$u = x + y \qquad \frac{du}{dx} - 1 = \frac{dy}{dx}$

$\frac{du}{dx} = u^{\frac{1}{2}} => \int u^{-\frac{1}{2}} du = \int dx$

$2\sqrt{u} = x + C => 2\sqrt{x+y} = x + C$

Problem 18

$\frac{dy}{dx} = (x+y+2)^2$

$u = x + y + 2 \qquad \frac{du}{dx} - 1 = \frac{dy}{dx}$

$\frac{du}{dx} = u^2 + 1 => \int dx = \int \frac{du}{u^2+1}$

$x + C = \tan^{-1}(u) => \tan^{-1}(x+y+2) = x + C$

$y = \tan(x+C) - x - 2$

Problem 19

$\frac{dy}{dx} = (x-y+5)^2$

$u = x - y + 5 \qquad 1 - \frac{du}{dx} = \frac{dy}{dx}$

$-\frac{du}{dx} = u^2 - 1 => \int dx = \int \frac{du}{1-u^2}$

$$\ln\left(\sqrt{1-u}\right)+\ln\left(\sqrt{1+u}\right)=x+C$$

Problem 20

$$\frac{dy}{dx}=\sin\left(x-y\right)$$

$$u=x-y \qquad \frac{du}{dx}=1-\frac{dy}{dx}$$

$$1-\frac{du}{dx}=\sin u => \int dx=\int\frac{du}{1-\sin u}$$

$$x=-\frac{2}{\frac{\sin u}{\cos u+1}-1}$$

$$x=-\frac{2}{\frac{\sin\left(x-y\right)}{\cos\left(x-y\right)+1}-1}$$

Problem 21

$$\frac{dy}{dx}+\frac{y}{x}=x^2y^2$$

$$u=y^{-1} \qquad \frac{dy}{dx}=u^{-2}\frac{du}{dx}$$

$$-u^{-2}\frac{du}{dx}-\frac{u^{-1}}{x}=x^2u^{-2} => \frac{du}{dx}-\frac{u}{x}=-x^2 => I(x)=x^{-1}$$

$$x^{-1}\frac{du}{dx}-\frac{u}{x^2}=-x => \int(x^{-1}u)'=\int -x => x^{-1}u=C-\frac{x^2}{2} => x^{-1}y^{-1}=C-\frac{x^2}{2}$$

$$y=\left(Cx-\frac{x^3}{2}\right)^{-1}$$

Problem 22

$$\frac{dy}{dx}-y=e^{2x}y^3 => y^{-3}\frac{dy}{dx}-y^{-2}=e^{2x}$$

$$u=\frac{1}{y^2} => y=\frac{1}{\sqrt{u}} \qquad y^{-3}\frac{dy}{dx}=-\frac{1}{2}\frac{du}{dx}$$

$$\frac{du}{dx}+2u=-2e^{2x} => I(x)=e^{2x}$$

$$e^{2x}\frac{du}{dx}+2e^{2x}u=-2e^{4x} => \int(e^{2x}u)'=\int -2e^{4x}$$

$$e^{2x}u=-\tfrac{1}{2}e^{4x}+C => e^{2x}y^{-2}=-\tfrac{1}{2}e^{4x}+C$$

$y^2 = -\frac{1}{2}e^{2x} + Ce^{-2x}$

Problem 23

$$\frac{dy}{dx} = \frac{2y}{x} - x^2y^2$$

$$u = y^{-1} \qquad \frac{dy}{dx} = -u^{-2}\frac{du}{dx}$$

$$-u^{-2}\frac{du}{dx} = \frac{2u^{-1}}{x} - x^2u^{-2} => -u^{-2}\frac{du}{dx} - \frac{2u^{-1}}{x} = -x^2u^{-2} => \frac{du}{dx} + \frac{2u}{x} = x^2 => I(x) = x^2$$

$$x^2\frac{du}{dx} + 2xu = x^4 => \int (x^2u)' = \int x^4$$

$$x^2u = \tfrac{1}{5}x^5 + C => x^2y^{-1} = \tfrac{1}{5}x^5 + C$$

$$y = \tfrac{1}{5}x^3 + Cx^{-2}$$

Problem 24

$$\frac{dy}{dx} + \frac{y}{x-2} = 5(x-2)y^{\frac{1}{2}}$$

$$u = y^{\frac{1}{2}} \qquad \frac{dy}{dx} = 2u\frac{du}{dx}$$

$$2u\frac{du}{dx} + \frac{u^2}{x-2} = 5(x-2)u => \frac{du}{dx} + \frac{u}{2(x-2)} = \tfrac{5}{2}(x-2) => I(x) = e^{x^2-4x}$$

$$e^{x^2-4x}\frac{du}{dx} + e^{x^2-4x}\frac{u}{2x-4} = e^{x^2-4x}\tfrac{5}{2}(x-2) => \int (e^{x^2-4x}u)' = \tfrac{5}{2}\int e^{x^2-4x}(x-2)$$

$$e^{x^2-2x}u = \tfrac{5}{4}e^{x^2-4x} + C$$

Problem 25

$$\frac{dx}{dt} + tx^3 + \frac{x}{t} = 0 => \frac{dx}{dt} + \frac{x}{t} = -tx^3 => -x^{-3}\frac{dx}{dt} - x^{-2}t = t$$

$$u = x^{-2} \qquad -x^{-3}\frac{dx}{dt} = \tfrac{1}{2}\frac{du}{dt}$$

$$\frac{du}{dt} - 2ut^{-1} = -2t => I(x) = t^{-2}$$

$$t^{-2}\frac{du}{dt} - 2ut^{-3} = -2t^{-1} => \int (ut^{-2})' = \int -2t^{-1} => ut^{-2} = \ln(t^{-2}) + C$$

$y^{-2}t^{-2} = \ln(t^{-2}) + C$

Problem 26

$\dfrac{dy}{dx} + y = e^x y^{-2} => y^2 \dfrac{dy}{dx} + y^3 = e^x$

$u = y^3 \qquad 3y^2 \dfrac{dy}{dx} = \dfrac{du}{dx}$

$\dfrac{du}{dx} + u = e^x => I(x) = e^x$

$e^x \dfrac{du}{dx} + ue^x = e^{2x} => \int (ue^x)' = \int e^{2x}$

$ue^x = \frac{1}{2}e^{2x} + C$

$y^3 = \dfrac{e^{2x} + C}{2e^x}$

Problem 27

$\dfrac{dr}{d\theta} = \dfrac{r^2 + 2r\theta}{\theta^2} => r^{-2}\dfrac{dr}{d\theta} - 2\theta^{-1}r^{-1} = 0$

$u = r^{-1} \qquad -\dfrac{1}{r^2}\dfrac{dr}{d\theta} = \dfrac{du}{d\theta}$

$\dfrac{du}{d\theta} + 2u\theta^{-1} = 0 => I(x) = \theta^2$

$\theta^2 \dfrac{du}{d\theta} + 2u\theta = 0 => \int (u\theta^2)' = \int d\theta$

$u\theta^2 = \theta + C => u = \dfrac{\theta + C}{\theta^2}$

$r = \dfrac{\theta^2}{\theta + C}$