Math 311
Mechanical Lung Project

In medicine, mechanical ventilation is a procedure that assists or replaces spontaneous breathing for critically ill patients, using a medical device called a ventilator. Some people attribute the first mechanical ventilation to Andreas Vesalius in 1555. Negative pressure ventilators (iron lungs) came into use in the 1940s-1950s in response to *poliomyelitis* (polio) epidemics. Philip Drinker and Louis Shaw are credited with its invention. Modern ventilators use positive pressure to inflate the lungs of the patient. In the ICU (intensive care unit), common indications for the initiation of mechanical ventilation are acute respiratory failure, acute exacerbation of chronic obstructive pulmonary disease, coma, and neuromuscular disorders. The goals of mechanical ventilation are to provide oxygen to the lungs and to remove carbon dioxide.

When a person is placed on a respirator, their lungs are filled using mechanical ventilation. There are two primary properties to describe the way a person's lungs inflate and deflate, compliance (C) and resistance (R). Pulmonary compliance (or lung compliance) is a measure of the lung's ability to stretch and expand. Airway resistance is the resistance of the respiratory tract to airflow during inspiration and expiration. At the completion of the breath, there will be some lost pressure at the airway, P_{ex}, related to the resistance, compliance and applied pressure.

In Pressure Controlled Ventilation, the ventilator applies a constant pressure at the airway, P_{app} to inflate the lungs (inspiration). The ventilator then removes that applied pressure and the lungs deflate on their own (expiration). We use the following assumptions:

We model the inhalation and exhalation of the lungs using differential equations and the following assumptions:

1. We fix the length of a breath (t_{tot}) which includes time of inhalation (t_i) and exhalation (t_{ex}).
2. During inspiration the ventilator applies a constant pressure (P_{app})to the patient's airway, and during expiration this pressure is zero, relative to atmospheric pressure.
3. We assume the lungs are modeled using a single compartment.
4. At the airway there is pressure balance: $P_r + P_e + P_{ex} = P_{aw}$
5. We denote the volume of the lungs as $V(t)$ where V_T is the tidal volume or maximum volume of the lung during a breath.
6. The resistive pressure P_r is proportional to the flow or $P_r = R\frac{dV}{dt}$ where R is the resistance of the lung.

7. The elastic pressure P_e is proportional to the flow or $P_e = \frac{1}{C}V$ where C is the compliance of the lung.
8. P_{ex} is a residual pressure that remains in the lung at the completion of the breath.
9. P_{aw} denotes the pressure applied at the airway, P_{app} during inspiration and zero during expiration.

Consider the following set of parameters

$$\begin{aligned} C &= \textit{varies}* \text{ L/cm water pressure} \\ R &= \textit{varies}* \text{ cm water pressure/(L/sec)} \\ P_{app} &= 10 \text{ cm water pressure} \end{aligned}$$

*see MyClasses for the assigned parameters.

First we calculate the length of the inspiration, expiration and the total breath. The length of the inspiration is based on the resistance and compliance of the lungs. We first find a time constant, t_s, by multiplying the resistance and compliance. From empirical data, studies have found that in five time steps the lungs will inflate to 98% of the total volume. For our calculations, we will use this value as the time to fully inflate the lungs. Thus we find the time for inspiration, t_i by using five times the time constant.

Then we consider the ratio of inspiration to expiration. For this example, we will use a $1 : 5.6$ ratio of inspiration time to expiration time. The total breath time is then the sum of inspiration and expiration.

The modeling of the lungs is done by considering all the pressures that are being applied at the airway. There is pressure at the airway, pressure from the volume of air in the lungs wanting to escape, pressure from the flow of air into and out of the lungs, and a small amount of pressure that escapes into the atmosphere, P_{ex}, which is related to the resistance, compliance and applied pressure.

1. Find the time for inspiration, t_i, expiration, t_e, and total breath time t_{tot}.
2. Use the assumptions above to create two differential equations (one for inspiration and one for expiration) for the volume in the lungs with respect to time. Also include three conditions on the volume in the lungs.
3. Solve the two differential equations using the three conditions. You will be able to determine the function for $V_i(t)$, $V_e(t)$, and P_{ex}.

4. In addition to the volume in the lungs, practitioners also monitor the flow rate of air into the lungs, $F = \dfrac{dV}{dt}$, and the pressure in the lungs, $P = \dfrac{1}{C}V(t)$. Determine the piecewise function for F and P considering

$$V(t) = \begin{cases} V_i(t) & 0 \leq t \leq t_i \\ V_e(t) & t_i < t \leq t_{tot} \end{cases}.$$

5. Create one page containing a graph of each of the piecewise functions: volume, flow, and pressure for one full breath.

6. Describe why it make sense that the graph of flow is negative during expiration, both from a lung mechanics point of view and a calculus point of view.