MATH 311 Homework 4.2, 4.3

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Section 4.2

Problem 5

$y'' + 8y' + 16y = 0$
$n^2 + 8n + 16 = 0 => (n+4)^2 => n = -4$
$y = c_1e^{-4x} + c_2xe^{-4x}$

Problem 6

$y'' - 5y' + 6y = 0$
$n^2 - 5n + 6 = 0 => (n-3)(n-2) => n = 3, 2$
$y = c_1e^{2x} + c_2e^{3x}$

Problem 7

$6y'' + y' - 2y = 0$
$6n^2 + n - 2 = 0 => (3n-2)(2n+1) = 0 => n = -\frac{1}{2}, \frac{2}{3}$
$y = c_1e^{-\frac{1}{2}x} + c_2e^{\frac{2}{3}x}$

Problem 8

$z'' + z' - z = 0$
$n^2 + n - 1 = 0 => n = \frac{-1+\sqrt{5}}{2}, \frac{-1-\sqrt{5}}{2}$
$z = c_1e^{\frac{-1+\sqrt{5}}{2}} + c_2e^{\frac{-1-\sqrt{5}}{2}}$

Problem 9

$4y'' - 4y' + y = 0$
$4n^2 - 4n + 1 = 0 => (2n-1)^2 = 0 => n = \frac{1}{2}$
$y = c_1e^{\frac{1}{2}x} + c_2xe^{\frac{1}{2}x}$

Problem 10

$y'' - y' - 11y = 0$
$n^2 - n - 11 = 0 => n =, \frac{1+\sqrt{45}}{2}, \frac{1-\sqrt{45}}{2}$
$y = c_1 e^{\frac{1+\sqrt{45}}{2}} + c_2 e^{\frac{1-\sqrt{45}}{2}}$

Problem 14

$y'' + y' = 0 \qquad y(0) = 2, \quad y'(0) = 1$
$n^2 + n = 0 => n(n+1) = 0 => n = 0, -1$
$y = c_1 + c_2 e^{-x}$
$1 = -c_2 e^0 => c_2 = -1$
$2 = c_1 + c_2 => c_1 - 1 = 2 => c_1 = 3$
$y = 3 - e^{-x}$

Section 4.3

Problem 11

$z'' + 10z' + 25z = 0$
$n^2 + 10n + 25 = 0 => (n+5)^2 => n = -5$
$z = c_1 e^{-5x} + c_2 x e^{-5x}$

Problem 12

$u'' + 7u = 0$
$n^2 + 7 = 0 => n = \frac{0 \pm \sqrt{7}i}{2} => \alpha = 0, \quad \beta = \frac{\sqrt{7}}{2}$
$u = c_1 \sin \frac{\sqrt{7}}{2}x + c_2 \cos \frac{\sqrt{7}}{2}x$

Problem 13

$y'' - 2y' + 26y = 0$
$n^2 - 2n + 26 = 0 => n = \frac{2 \pm 10i}{2} => \alpha = 1, \quad \beta = 5$
$y = c_1 e^x \sin 5x + c_2 e^x \cos 5x$

Problem 14

$y'' + 2y' + 5y = 0$
$n^2 + 2n + 5 = 0 => n = \frac{-2 \pm 4i}{2} => \alpha = -1, \quad \beta = 2$
$y = c_1 e^{-x} \sin 2x + c_2 e^{-x} \cos 2x$

Problem 15

$y'' - 3y' - 11y = 0$
$n^2 - 3n - 11 = 0 => n = \frac{3 \pm \sqrt{53}}{2}$
$y = c_1 e^{\frac{3+\sqrt{53}}{2}x} + c_2 e^{\frac{3-\sqrt{53}}{2}x}$

Problem 16

$y'' + 10y' + 41y = 0$
$n^2 + 10y + 41 = 0 => n = \frac{-10 \pm 8i}{2} => \alpha = -5, \quad \beta = 4$
$y = c_1 e^{-5x} \sin 4x + c_2 e^{-5x} \cos 4x$

Problem 22

$y'' + 2y' + 17y = 0 \qquad y(0) = 1, \quad y'(0) = -1$
$n^2 + 2n + 17 = 0 => n = \frac{-2 \pm 8i}{2} => \alpha = -1, \quad \beta = 4$
$y = c_1 e^{-x} \sin 4x + c_2 e^{-x} \cos 4x$

$1 = c_1 \sin 0 + c_2 \cos 0 => c_2 = 1$
$-1 = 4c_1 \cos 0 - 4c_2 \sin 0 => c_1 = -\frac{1}{4}$
$y = -\frac{1}{4} e^{-x} \sin 4x + e^{-x} \cos 4x$