MATH 311 Homework 3.6 and 3.7

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April 4, 2022

Calculate

$$\frac{dy}{dx} = x^2 + 2y$$
$$y(1) = 3 \quad y(3) = ?$$

Euler's Method (Using GeoGebra)

$n = 2$
$y(2) = 10$
$y(3) = 34$

$n = 4$
$y(1.5) = 6.5$
$y(2) = 14.125$
$y(2.5) = 30.25$
$y(3) = 36.625$

Improved Euler's Method

$n = 2$
$\tilde{y}(2) = (1^2 + 2(3)) + 3 = 10$
$y(2) = \frac{1}{2}((1^2 + 2(3)) + (2^2 + 2(10))) + 3 = 18.5$
$\tilde{y}(3) = (2^2 + 2(18.5)) + 18.5 = 59.5$
$y(3) = \frac{1}{2}((2^2 + 2(18.5)) + (3^2 + 2(59.5))) + 18.5 = 103$

$n = 4$ (Using eMathHelp):
$\tilde{y}(1.5) = 6.5$
$y(1.5) = 8.5625$
$\tilde{y}(2) = 18.25$
$y(2) = 23.53125$
$\tilde{y}(2.5) = 49.0625$
$y(2.5) = 62.390625$

$\tilde{y}(3) = 127.90625$
$y(3) = 161.3515625$

Runge-Kutta 4 (Using eMathHelp)

$n = 4$
$y(1.5) = 9.388020833333333$
$y(2) = 27.933702256944442$
$y(2.5) = 79.8334644458911952$
$y(3) = 222.494278707621979$

$n = 10$
$y(1.2) = 4.7698933333333333$
$y(1.4) = 7.5274568817777777$
$y(1.6) = 11.778030345777304$
$y(1.8) = 18.275445801140863$
$y(2) = 28.14421968308853$
$y(2.2) = 43.061830641919262$
$y(2.4) = 65.5306294962390275$
$y(2.6) = 99.2834563738562981$
$y(2.8) = 149.888713321427234$
$y(3) = 225.652971285350385$