MATH 311 The Tank Problem

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Problem 1

$\frac{dA}{dt} = 8(0.05) - \frac{8A}{100} => \frac{dA}{dt} + \frac{2A}{25} = 0.4 => I(t) = e^{\int \frac{2dt}{25}} = e^{\frac{2}{25}t}$

$e^{\frac{2}{25}t}\frac{dA}{dt} + \frac{2}{25}e^{\frac{2}{25}t}A = 0.4e^{\frac{2}{25}t} => \int (e^{\frac{2}{25}t}A)' = \int 0.4e^{\frac{2}{25}t} => e^{\frac{2}{25}t}A = 0.4\frac{25}{2}e^{\frac{2}{25}t} + C$

$A = 5 + Ce^{-\frac{2}{25}t}$
$A(0) = 0.5 \qquad 0.5 = 5 + C => C = -4.5$

$A(t) = 5 - 4.5e^{-\frac{2}{25}t}$

$A(t) = 2 \qquad 2 = 5 - 4.5e^{-\frac{2}{25}t} => \ln\frac{2}{3} = -\frac{2}{25}t => t = -\frac{25\ln\frac{2}{3}}{2}$

$t \approx 5.07$ min

Problem 2

$\frac{dA}{dt} = 6(0.05) - \frac{6A}{50} => \frac{dA}{dt} + \frac{3A}{25} = 0.3 => I(t) = e^{\int \frac{3dt}{25}} = e^{\frac{3}{25}t}$

$e^{\frac{3}{25}t}\frac{dA}{dt} + \frac{3}{25}e^{\frac{3}{25}t}A = 0.3e^{\frac{3}{25}t} => \int (e^{\frac{3}{25}t}A)' = \int 0.3e^{\frac{3}{25}t} => e^{\frac{3}{25}t}A = 0.3\frac{25}{3}e^{\frac{3}{25}t} + C$

$A = 2.5 + Ce^{-\frac{3}{25}t}$
$A(0) = 0.5 \qquad 0.5 = 2.5 + C => C = -2$

$A(t) = 2.5 - 2e^{-\frac{3}{25}t}$

$A(t) = 1.5 \qquad 1.5 = 2.5 - 2e^{-\frac{3}{25}t} => 0.5 = e^{-\frac{3}{25}t} => \ln 0.5 = -\frac{3}{25}t => t = -\frac{25\ln 0.5}{3}$

$t \approx 5.78$ min

Problem 4

$\frac{dA}{dt} = 0.8 - \frac{3A}{100+t} => \frac{dA}{dt} + \frac{3A}{100+t} = 0.8 => I(t) = (100+t)^3$

$\frac{dA}{dt}(100+t)^3 + 3A(100+t)^2 = 0.8(100+t)^3 => \int (A(100+t)^3)' = \int 0.8(100+t)^3 =>$

$A(100+t)^3 = 0.2(100+t)^4 + C$

$A = 0.2(100+t) + C(100+t)^{-3}$

$A(0) = 0 \qquad 0 = 20 + 100^{-3}C => C = -2 \cdot 10^7$

$A(t) = 0.2(100+t) - \frac{2 \cdot 10^7}{(100+t)^3}$

$A(?) = 0.1(100+t) \qquad \frac{0.2(100+t)^4 - 2 \cdot 10^7}{(100+t)^4} = 0.1 => \frac{2 \cdot 10^7}{(100+t)^4} = 0.1$

$(100+t)^4 = \frac{20000000}{0.1} => t = \sqrt[4]{\frac{20000000}{0.1}} - 100 => t \approx 18.92 \text{ min}$

Problem 5

$\frac{dA}{dt} = 5(0.001) - \frac{A}{2000} => \frac{dA}{dt} + \frac{A}{2000} = 0.005 => I(t) = e^{\int 0.0005} = e^{0.0005t}$

$e^{0.0005t}\frac{dA}{dt} + 0.0005e^{0.0005t}A = 0.005e^{0.0005t} => \int (Ae^{0.0005t})' = \int 0.005e^{0.0005t}$

$Ae^{0.0005t} = 10e^{0.0005t} + C => A = 10 + Ce^{-0.0005t}$

$A(0) = 0.01(10000) = 100 \qquad 100 = 10 + C => C = 90$

$A(t) = 10 + 90e^{-0.0005t}$

$A(60) = 10 + 90e^{-0.0005(60)} \approx 97.34$ then $\frac{97.43}{10000} \approx 0.0097\%$

$A(?) = 0.002(10000) = 20 \qquad 20 = 10 + 90e^{-0.0005t} => \frac{1}{9} = e^{-0.0005t} => \ln\frac{1}{9} = -0.0005t$

$t = -2000 \ln\frac{1}{9} \approx 4394.45$ min or 73.24 hrs