Homework 1

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Chapter 1.1

1. Second order, Linear ODE Dependent=x Independent=t
2. Second order, Linear ODE Dependent=y Independent=x
3. First order, Non-Linear ODE Dependent=y Independent=x
4. Second order, Linear PDE Dependent=u Independent=x, y
5. First order, Non-Linear ODE Dependent=y Independent=x
6. First order, Non-Linear ODE Dependent=x Independent=t
7. First order, Non-Linear ODE Dependent=p Independent=t
8. Second order, Non-Linear ODE Dependent=y Independent=x
9. Second order, Linear, ODE Dependent=y Independent=x
10. Fourth order, Linear ODE Dependent=y Independent=x
11. Second order, Linear PDE Dependent=N Independent=r
12. Second order, Non-Linear ODE Dependent=y Independent=x

Chapter1.2

Problem 3:

$y = \sin x + x^2$ $\qquad$ $\dfrac{d^2y}{dx^2} + y = x^2 + 2$

$y' = \cos x + 2x$ $\qquad$ $y'' = -\sin x + 2$

$-\sin x + 2 + \sin x + x^2 = x^2 + 2 => x^2 + 2 = x^2 + 2$

Therefore y is a solution to $\dfrac{d^2y}{dx^2} + y = x^2 + 2$.

Problem 4:

$x = 2\cos t - 3\sin t$ $\qquad$ $x'' + x = 0$

$x' = -2\sin t - 3\cos t$ $\qquad$ $x'' = -2\cos t + 3\sin t$

$2\cos t - 3\sin t - 2\cos t + 3\sin t = 0 => 0 = 0$

Therefore x is a solution to $x'' + x = 0$ is a solution.

Problem 5:

$\theta = 2e^{3t} - e^{2t}$ $\qquad$ $\dfrac{d^2\theta}{dt^2} - \theta\dfrac{d\theta}{dt} + 3\theta = -2e^{2t}$

$\theta' = 6e^{3t} - 2e^{2t}$ $\qquad$ $\theta'' = 18e^{3t} - 4e^{2t}$

$18e^{3t} - 4e^{2t} - (2e^{3t} - e^{2t})(6e^{3t} - 2e^{2t}) + 6e^{3t} - 3e^{2t} = -2e^{2t}$

$24e^{3t} - 7e^{2t} - 12e^{6t} + 10e^{5t} - 2e^{4t} \neq -2e^{2t}$

Therefore θ is not a solution to $\dfrac{d^2\theta}{dt^2} - \theta\dfrac{d\theta}{dt} + 3\theta = -2e^{2t}$

Problem 6:

$x = \cos 2t$ $\qquad$ $\dfrac{dx}{dt} + tx = \sin 2t$

$x' = -2\sin 2t$

$-2\sin 2t + t\cos 2t = \sin 2t => t\cos 2t \neq 3\sin 2t$

Therefore x is not a solution to $\dfrac{dx}{dt} + tx = \sin 2t$.

Problem 9:

$x^2 + y^2 = 4$ $\qquad$ $\dfrac{dy}{dx} = \dfrac{x}{y}$

$2x + 2yy' = 0 => 2yy' = -2x => y' = -\dfrac{x}{y}$

$-\dfrac{x}{y} \neq \dfrac{x}{y}$

Therefore $x^2 + y^2 = 4$ is not a solution to $\dfrac{dy}{dx} = \dfrac{x}{y}$.

Problem 10:

$y - \ln y = x^2 + 1$ $\qquad$ $\dfrac{dy}{dx} = \dfrac{2xy}{y-1}$

$y' - \dfrac{y'}{y} = 2x => y'\left(1 - \dfrac{1}{y}\right) = 2x => y' = \dfrac{2xy}{y-1}$

Therefore $y - \ln y = x^2 + 1$ is a solution to $\dfrac{dy}{dx} = \dfrac{2xy}{y-1}$.

Problem 22:

$\phi(x) = c_1 e^x + c_2 e^{-2x}$ $\qquad$ $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 2y = 0$

$y = c_1 e^x + c_2 e^{-2x}$
$y' = c_1 e^x - 2c_2 e^{-2x}$
$y'' = c_1 e^x + 4c_2 e^{-2x}$

$c_1 e^x + 4c_2 e^{-2x} + c_1 e^x - 2c_2 e^{-2x} - 2(c_1 e^x + c_2 e^{-2x}) = 0$
All values cancel making $0 = 0$ therefore ϕ is a solution.

Find c_1 and c_2 such that the initial conditions are met.

a) $y(0) = 2$, $y'(0) = 1$

After plugging in 0 into x need to find constant values such that,

$c_1 + c_2 = 2$
$c_1 - 2c_2 = 1$

$c_1 + 4c_2 = 3$ (get this one by plugging initial conditions into the ODE)

Using a bit of linear algebra and row reductions I came to the following matrix

$$\begin{bmatrix} 1 & 0 & \frac{5}{3} \\ \\ 0 & 1 & \frac{1}{3} \\ \\ 0 & 0 & 0 \end{bmatrix}$$

This tells us that $c_1 = \dfrac{5}{3}$ and $c_2 = \dfrac{1}{3}$

b) $y(1) = 1$, $y'(1) = 0$

Using the same idea as letter (a) we need to find constants such that,

$ec_1 + \dfrac{c_2}{e^2} = 1$
$ec_1 - \dfrac{2c_2}{e^2} = 0$
$ec_1 + \dfrac{4c_2}{e^2} = 2$

Use linear algebra again and come to a reduced matrix

$$\begin{bmatrix} 1 & 0 & \frac{2}{3e} \\ \\ 0 & 1 & \frac{e^2}{3} \\ \\ 0 & 0 & 0 \end{bmatrix}$$

The matrix indicates that $c_1 = \dfrac{2}{3e}$ and $c_2 = \dfrac{e^2}{3}$