remains at rest at a position with a value of +5, whereas observer B claims the position of P continuously changes with time, even passing him and moving behind him! Again, both observers are correct, with the difference in their measurements arising from their different frames of reference.

We explore this phenomenon further by considering two observers watching a man walking on a moving beltway at an airport in Figure 4.20. The woman standing on the moving beltway sees the man moving at a normal walking speed. The woman observing from the stationary floor sees the man moving with a higher speed because the beltway speed combines with his walking speed. Both observers look at the same man and arrive at different values for his speed. Both are correct; the difference in their measurements results from the relative velocity of their frames of reference.

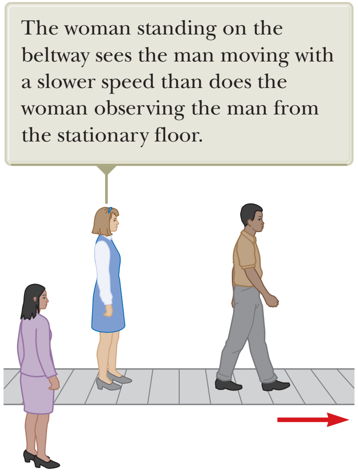

Figure 4.20 Two observers measure the speed of a man walking on a moving beltway.

In a more general situation, consider a particle located at point P in Figure 4.21. Imagine that the motion of this particle is being described by two observers, observer A in a reference frame S_A fixed relative to the Earth and a second observer B in a reference frame S_B moving to the right relative to S_A (and therefore relative to the Earth) with a constant velocity $\vec{\mathbf{v}}_{BA}$. In this discussion of relative velocity, we use a double-subscript notation; the first subscript represents what is being observed, and the second represents who is doing the observing. Therefore, the notation $\vec{\mathbf{v}}_{BA}$ means the velocity of observer B (and the attached frame S_B) as measured by observer A. With this notation, observer B measures A to be moving to the left with a velocity $\vec{\mathbf{v}}_{AB} = -\vec{\mathbf{v}}_{BA}$. For purposes of this discussion, let us place each observer at her or his respective origin.

We define the time $t = 0$ as the instant at which the origins of the two reference frames coincide in space. Therefore, at time t, the origins of the reference frames will be separated by a distance $v_{BA}t$. We label the position P of the particle relative to observer A with the position vector $\vec{\mathbf{r}}_{PA}$ and that relative to observer B with the position vector $\vec{\mathbf{r}}_{PB}$, both at time t. From Figure 4.21, we see that the vectors $\vec{\mathbf{r}}_{PA}$ and $\vec{\mathbf{r}}_{PB}$ are related to each other through the expression

$$\vec{\mathbf{r}}_{PA} = \vec{\mathbf{r}}_{PB} + \vec{\mathbf{v}}_{BA}t \tag{4.29}$$

By differentiating Equation 4.29 with respect to time, noting that $\vec{\mathbf{v}}_{BA}$ is constant, we obtain

$$\frac{d\vec{\mathbf{r}}_{PA}}{dt} = \frac{d\vec{\mathbf{r}}_{PB}}{dt} + \vec{\mathbf{v}}_{BA}$$

$$\vec{\mathbf{u}}_{PA} = \vec{\mathbf{u}}_{PB} + \vec{\mathbf{v}}_{BA} \tag{4.30}$$

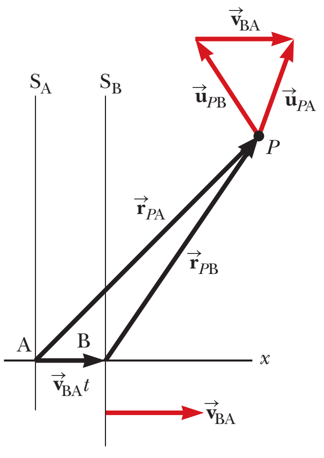

Figure 4.21 A particle located at P is described by two observers, one in the fixed frame of reference S_A and the other in the frame S_B, which moves to the right with a constant velocity $\vec{\mathbf{v}}_{BA}$. The vector $\vec{\mathbf{r}}_{PA}$ is the particle's position vector relative to S_A, and $\vec{\mathbf{r}}_{PB}$ is its position vector relative to S_B. The red vectors at the top of the figure show a vector addition for the velocities of the particle at time t, representing Equation 4.30.

where $\vec{\mathbf{u}}_{PA}$ is the velocity of the particle at P measured by observer A and $\vec{\mathbf{u}}_{PB}$ is its velocity measured by B. (We use the symbol $\vec{\mathbf{u}}$ for particle velocity rather than $\vec{\mathbf{v}}$, which we have already used for the relative velocity of two reference frames.) Equation 4.30 is demonstrated by the red vectors at the top of Figure 4.21. Vector $\vec{\mathbf{u}}_{PB}$ is the velocity of the particle at time t as seen by observer B. When you add the relative velocity $\vec{\mathbf{v}}_{BA}$ of the frames, the sum is the velocity of the particle as measured by observer A.

Equations 4.29 and 4.30 are known as **Galilean transformation equations.** They relate the position and velocity of a particle as measured by observers in relative motion.

Although observers in two frames measure different velocities for the particle, they measure the *same acceleration* when $\vec{\mathbf{v}}_{BA}$ is constant. We can verify that by taking the time derivative of Equation 4.30:

$$\frac{d\vec{\mathbf{u}}_{PA}}{dt} = \frac{d\vec{\mathbf{u}}_{PB}}{dt} + \frac{d\vec{\mathbf{v}}_{BA}}{dt}$$

Because $\vec{\mathbf{v}}_{BA}$ is constant, $d\vec{\mathbf{v}}_{BA}/dt = 0$. Therefore, we conclude that $\vec{\mathbf{a}}_{PA} = \vec{\mathbf{a}}_{PB}$ because $\vec{\mathbf{a}}_{PA} = d\vec{\mathbf{u}}_{PA}/dt$ and $\vec{\mathbf{a}}_{PB} = d\vec{\mathbf{u}}_{PB}/dt$. That is, the acceleration of the particle measured by an observer in one frame of reference is the same as that measured by any other observer moving with constant velocity relative to the first frame.

Example 4.8 A Boat Crossing a River

A boat crossing a wide river moves with a speed of 10.0 km/h relative to the water. The water in the river has a uniform speed of 5.00 km/h due east relative to the Earth.

(A) If the boat heads due north, determine the velocity of the boat relative to an observer standing on either bank.

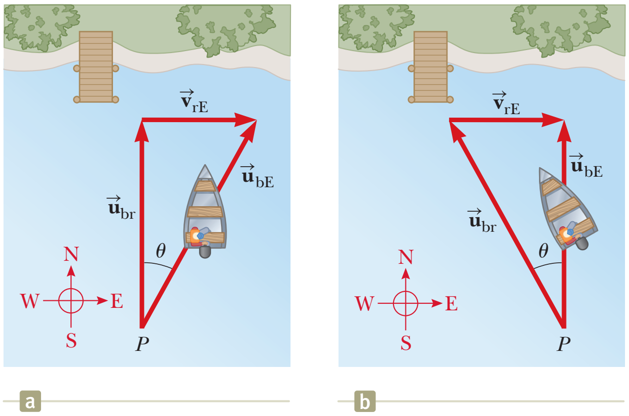

Figure 4.22 (Example 4.8) (a) A boat aims directly across a river and ends up downstream. (b) To move directly across the river, the boat must aim upstream.

SOLUTION

Conceptualize Imagine moving in a boat across a river while the current pushes you down the river. You will not be able to move directly across the river, but will end up downstream as suggested in Figure 4.22a. Imagine observer A on the shore, so that she is on the Earth, represented by letter E. Observer B is represented by letter r in the figure; this observer is on a cork floating in the river, at rest with respect to the water and carried along with the current. When the boat begins from point P and is aimed straight across the river, the velocities $\vec{u}_{br}$, the boat relative to the river, and $\vec{v}_{rE}$, the river relative to the Earth, add to give the velocity $\vec{v}_{bE}$, the velocity of the boat relative to observer A on the Earth. Compare the vector addition in Figure 4.22a to that in Figure 4.21. As the boat moves, it will follow along vector $\vec{v}_{bE}$, as suggested by its position after some time in Figure 4.22a.

Categorize Because of the combined velocities of you relative to the river and the river relative to the Earth, we can categorize this problem as one involving relative velocities.

Analyze We know $\vec{u}_{br}$, the velocity of the *boat* relative to the *river,* and $\vec{v}_{rE}$, the velocity of the *river* relative to the *Earth.* What we must find is $\vec{u}_{bE}$, the velocity of the *boat* relative to the *Earth.* The relationship between these three quantities is $\vec{u}_{bE} = \vec{u}_{br} + \vec{v}_{rE}$. The terms in the equation must be manipulated as vector quantities; the vectors are shown in Figure 4.22a. The quantity $\vec{u}_{br}$ is due north; $\vec{v}_{rE}$ is due east; and the vector sum of the two, $\vec{u}_{bE}$, is at an angle θ as defined in Figure 4.22a.

Find the speed u_{bE} of the boat relative to the Earth using the Pythagorean theorem:

$$u_{bE} = \sqrt{u_{br}^2 + v_{rE}^2} = \sqrt{(10.0\text{ km/h})^2 + (5.00\text{ km/h})^2}$$

$$= \boxed{11.2\text{ km/h}}$$

Find the direction of $\vec{u}_{bE}$:

$$\theta = \tan^{-1}\left(\frac{v_{rE}}{u_{br}}\right) = \tan^{-1}\left(\frac{5.00}{10.0}\right) = \boxed{26.6°}$$

Finalize The boat is moving at a speed of 11.2 km/h in the direction 26.6° east of north relative to the Earth. Notice that the speed of 11.2 km/h is faster than your boat speed of 10.0 km/h. The current velocity adds to yours to give you a higher speed. Notice in Figure 4.22a that your resultant velocity is at an angle to the direction straight across the river, so you will end up downstream, as we predicted.

(B) If the boat travels with the same speed of 10.0 km/h relative to the river and is to travel due north as shown in Figure 4.22b, what should its heading be?

SOLUTION

Conceptualize/Categorize This question is an extension of part (A), so we have already conceptualized and categorized the problem. In this case, however, we must aim the boat upstream so as to go straight across the river.

Analyze The analysis now involves the new triangle shown in Figure 4.22b. As in part (A), we know $\vec{v}_{rE}$ and the magnitude of the vector $\vec{u}_{br}$, and we want $\vec{u}_{bE}$ to be directed across the river. Notice the difference between the triangle in Figure 4.22a and the one in Figure 4.22b: the hypotenuse in Figure 4.22b is no longer $\vec{u}_{bE}$.

Use the Pythagorean theorem to find u_{bE}:

$$u_{bE} = \sqrt{u_{br}^2 - v_{rE}^2} = \sqrt{(10.0\text{ km/h})^2 - (5.00\text{ km/h})^2} = 8.66\text{ km/h}$$

Find the direction in which the boat is heading:

$$\theta = \tan^{-1}\left(\frac{v_{rE}}{u_{bE}}\right) = \tan^{-1}\left(\frac{5.00}{8.66}\right) = \boxed{30.0°}$$

continued