CSE330

Assignment - 06

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Sec: 08 [SADF]

Ans. to. the. Q. no. part a

$$I_1(f) = \frac{b-a}{2}\left[f(a)+f(b)\right]$$

degree, $n=1$, so root node $=2$.

$$P_1(x) = l_0(x)\,f(x_0) + l_1(x)\,f(x_1)$$

$$= f(a)\cdot\frac{x-b}{a-b} + f(b)\cdot\frac{x-a}{b-a}$$

interval: $[a,b]$
$x_0 = a$
$x_1 = b$

Now, $$I_1(f) = \int_a^b P_1(x)\,dx$$

$$= f(a)\int_a^b \frac{x-b}{a-b}\,dx + f(b)\int_a^b \frac{x-a}{b-a}\,dx$$

$$= f(a)\cdot\frac{1}{a-b}\left[\frac{x^2}{2}-bx\right]_a^b + f(b)\cdot\frac{1}{b-a}\left[\frac{x^2}{2}-ax\right]_a^b$$

$$= f(a)\cdot\frac{1}{a-b}\left[\frac{b^2-a^2}{2} - b(b-a)\right]$$

$$+ f(b)\cdot\frac{1}{b-a}\left[\frac{b^2-a^2}{2} - a(b-a)\right]$$

P.t.o

$$= f(a) \cdot \frac{1}{a-b}\left(-\frac{(b-a)^2}{2}\right) + f(b) \cdot \frac{1}{b-a} \cdot \frac{(b-a)^2}{2}$$

$\because$ Here, $a-b = -(b-a)$

$\therefore$ Now, $$I_1(f) = f(a)\,\frac{\frac{-(b-a)^2}{2}}{-(a-b)} + f(b)\,\frac{\frac{(b-a)^2}{2}}{b-a}$$

$$= f(a)\,\frac{(b-a)}{2} + f(b)\,\frac{(b-a)}{2}$$

$$\therefore I_1(f) = \frac{b-a}{2}\left[f(a) + f(b)\right]$$

(shown).

Ans. to. the. Q. on Part b

$$h = \frac{b-a}{m}$$

$$\therefore x_i = a + ih \; ; \quad i = 0, 1 \ldots m$$

$$\int_{x_i}^{x_i+1} f(x) \cdot dx \approx \frac{h}{2}\left(f(x_i) + f(x_i+1)\right)$$

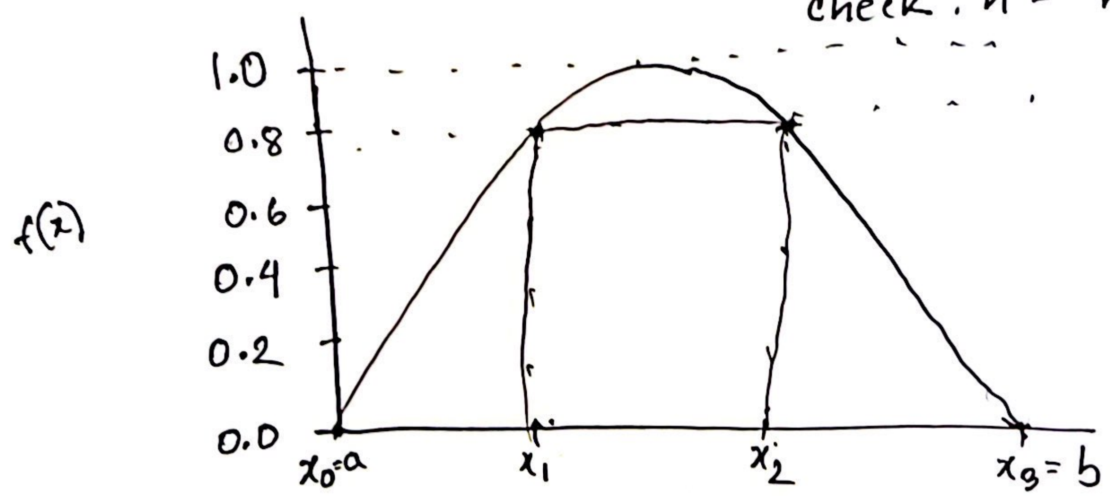
Now, summing area,

$i = 0, 1 \ldots m-1$

$$C_{1,m}(f) = \sum_{i=0}^{m-1} \frac{h}{2}\left(f(x_i) + f(x_i+1)\right)$$

$$= \frac{h}{2}\left(f(x_0) + 2\sum_{i=1}^{m-1} f(x_i) + f(x_m)\right)$$

check. $h = \frac{b-a}{m}$

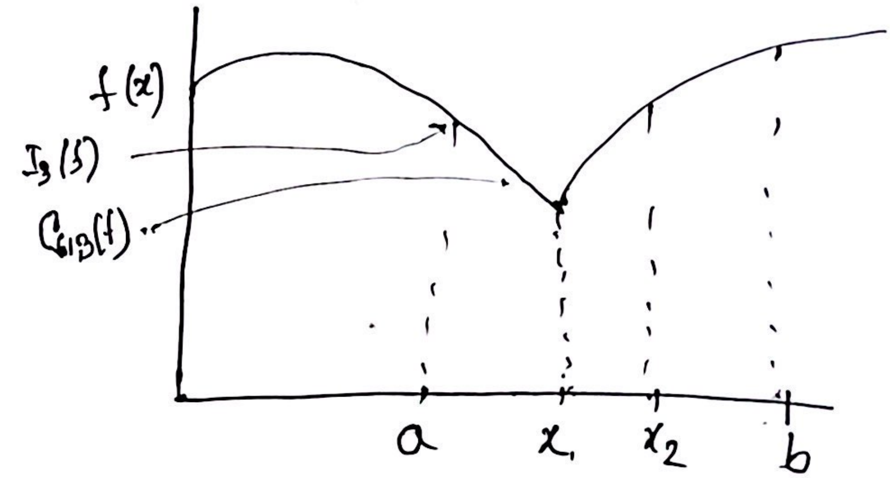


from x_0, x_1, x_2, x

composite slope and $C_{1,3}(f)$ shown as three smaller

Ans. to the. Q. on Prt. e

the gralphical Difference between $I_1(f)$ and $C_{1,3}(f)$ given bellow,



$I_2(f)$: One trapezoid across $[a, b]$

$C_{1,3}(f)$: three trapezoid (piecewise linear, dotted