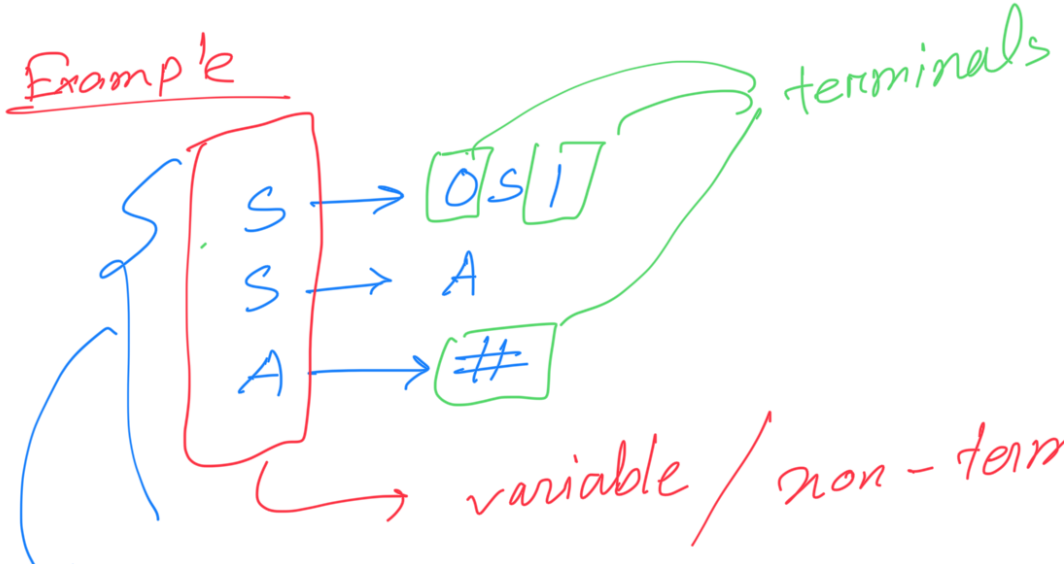
Context Free Grammar

Finite Automata (DFA, NFA)
Regular Expression
→ methods of Describing Languages

→ Regular Languages

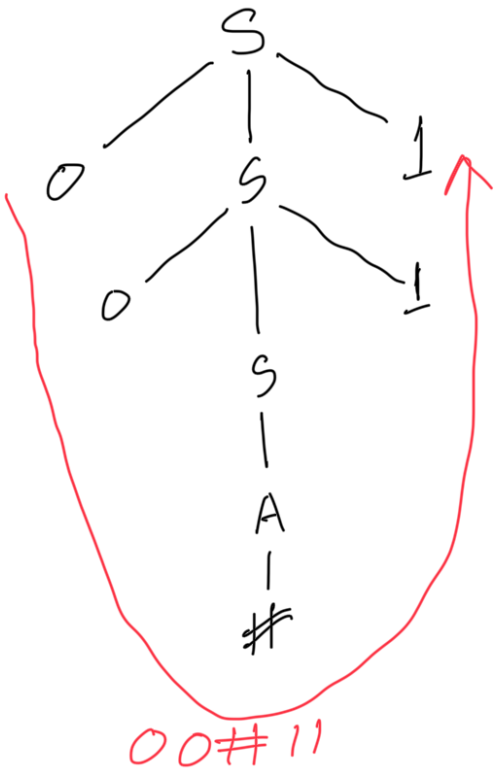
CFG
↳ Associated Languages (CFL)
↳ RL + additional languages

Example



00#11

parse tree



Derivation

$S \Rightarrow 0S1 \Rightarrow 00S11 \Rightarrow 00A11$

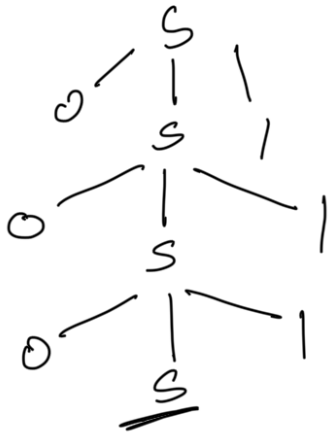
↓

00#11

Example 1

$L = \{ w \in \{0,1\}^* : w = 0^m 1^n, \text{ where } m = n, m \geq 0 \}$

ϵ 01 0011 000111



$S \longrightarrow 0S1$

$S \longrightarrow \epsilon$

$S \longrightarrow 0S1 \mid \epsilon$

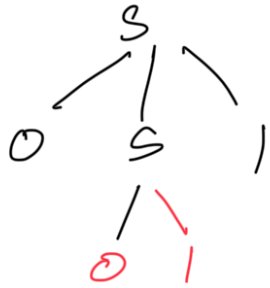
Example 2

$L = \{ w \in \{0,1\}^* : w = 0^m 1^n$, where $m = n$, $m \geq 1 \}$

01

0011

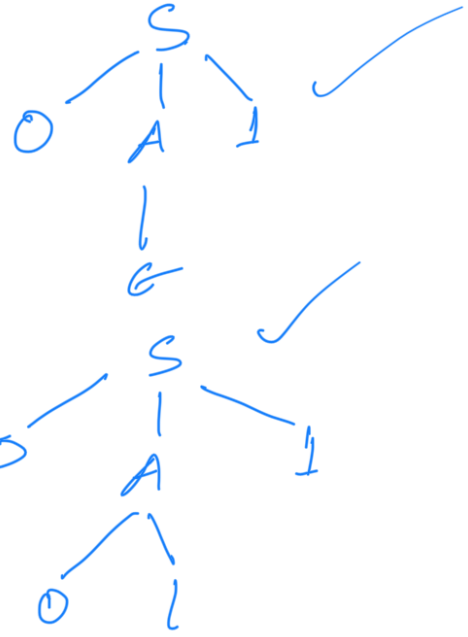
$S \rightarrow 0S1 \mid 01$

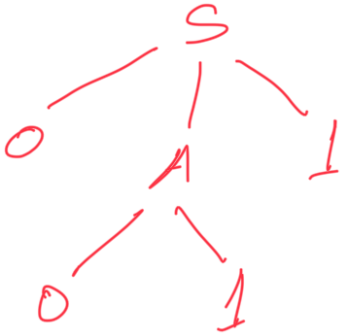


0011

$S \rightarrow 0A1$

$A \rightarrow 0A1 \mid \epsilon$

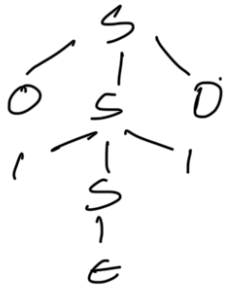




Example 3

$L = \{ w \in \{0,1\}^* : w \text{ is a valid palindrome} \}$

101 | 0110



$S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1 \mid \epsilon$

0 | 1 → odd length

ϵ → even length

Example 4

$L = \{ w \in \{0,1\}^* : w = 1^n, \text{ where } n \geq 0 \}$

$S \rightarrow 1S \mid \epsilon$

$S \rightarrow S1 \mid \epsilon$

$S \rightarrow 1S1 \mid 1 \mid \epsilon$

1S1 → pairwise

1 → odd lengths

ϵ → even length

Example 5

$i \; k$ where $i = k$

$L = \{ w \in \{0,1,2\}^* : w = 0^i 2^j 1^k ;$ where and $i, j, k \geq 0 \}$

00222 11

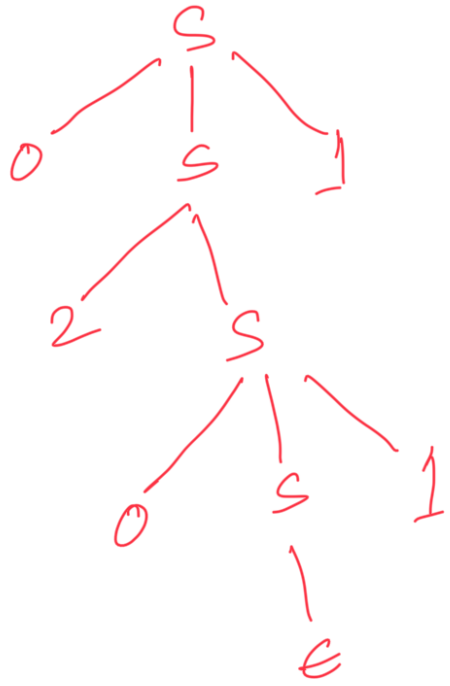
0 1 pair ✓

count of 2 ✓

$S \longrightarrow 0S1 \mid A$

$A \longrightarrow 2A \mid \epsilon$

$S \longrightarrow 0S1 \mid 2S \mid \epsilon$ ✗



0 2 0 1 1

00222211

Example 6

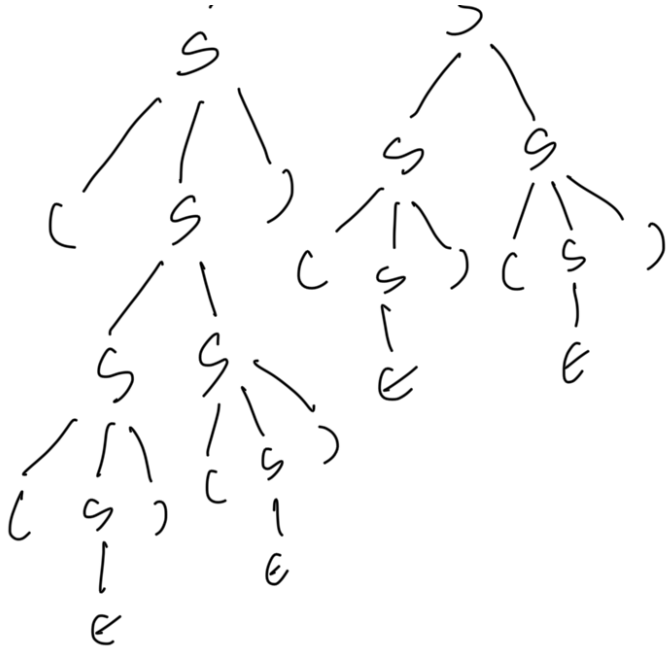
$L = \{ w \in \{(,)\}^* : w \text{ is a valid parentheses} \}$

$S \rightarrow (S) \mid SS \mid \epsilon$

(( )) ( ) ( )

( ) ( )) ( )

Draw parse tree for (( ) ( )) ( ) ( )



Example 7

$L = \{ w \in \{a,b,c\}^* : w = a^i b^j c^k$ where $i=j$ or $j=k$ and $i,j,k \geq 0 \}$

X Y

$S \rightarrow X \mid Y$ ✓

$i=j$: $a^i b^i c^k$, P Q

$X \rightarrow PQ$

$P \rightarrow aPb \mid \epsilon$

$Q \rightarrow cQ \mid \epsilon$

$j=k$: $a^i b^k c^k$, M N

$Y \rightarrow MN$

$M \rightarrow aM \mid \epsilon$

$N \rightarrow bNc \mid \epsilon$

Example 8

$L = \{ w \in \{a,b,c\}^* : w = a^i b^j c^k$ where $i+j=k$ and $i,j \geq 0 \}$

$w = a^i b^j c^k$

$= a^i b^{i+j} c^{i+j}$

$= a^i b^j c^i c^j$

$= a^i b^j c^j c^i$

$S \to aSc \mid X$

$X \to bXc \mid \epsilon$

$a^3 b^2 c^5$

aaa bb cc ccc

Example 9

$L = \{ w \in \{0,1\}^* : w = 0^i 1^j 0^k$ where $j = i+k$ and $i,k \geq 0 \}$

$w = 0^i 1^j 0^k$

$= 0^i 1^{i+k} 0^k$

$= 0^i 1^i 1^k 0^k$

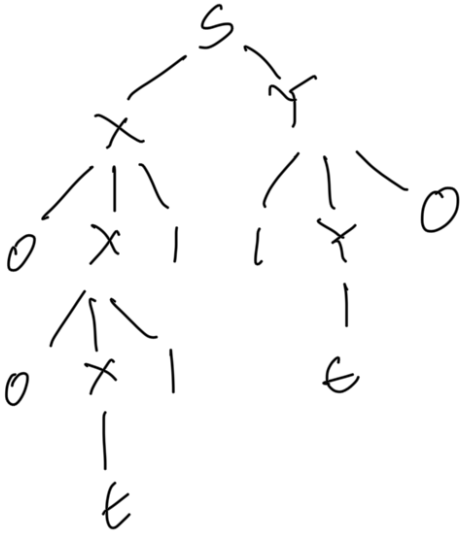
X Y

$S \to XY$

$X \to 0X1 \mid \epsilon$

$Y \to 1Y0 \mid \epsilon$

00 1 1 1 0



Example 10

$L = \{ w \in \{0,1\}^* : w = 0^m 1^n$, where $m > n$, $m \geq 1$ and $n \geq 0 \}$

$m = n + D$ ↳ min val 1.

$0^m 1^n$

$0^{n+D} 1^n$

$0^n 0^D 1^n$

$0^D 0^n 1^n$

$X \quad Y$

$S \rightarrow 0S1 \mid X$

$X \rightarrow 0X \mid 0$

$0^n 1$

$S \rightarrow XY$

$X \rightarrow 0X \mid 0$

$Y \rightarrow 0Y1 \mid \epsilon$

Example 11

$L = \{ w \in \{0,1\}^* :$ the length of w is even $\}$

$S \rightarrow 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid \epsilon$

1010

1001

$S \rightarrow 00S \mid 01S \mid 10S \mid 11S \mid \epsilon$

1010

1001

Example 12

... with different

$L = \{w \in \{0,1\}^* : w$ starts and ends [illegible] symbol$\}$

$S \rightarrow 0X1 \mid 1X0$

$X \rightarrow 0X \mid 1X \mid \epsilon$

$0\underbrace{(0+1)^*}_{M}\underbrace{1}_{A} + \underbrace{1\underbrace{(0+1)^*}_{M}\underbrace{0}_{X}}_{B}$ (S)

$S \rightarrow A \mid B$

$A \rightarrow 0X1$

$B \rightarrow 1X0$

$X \rightarrow MX \mid \epsilon$

$M \rightarrow 0 \mid 1$

Convert Regular expression into CFG

$\Big(\underbrace{\underbrace{(ab+bc)}_{P}\underbrace{bb}_{Q}\underbrace{(\underbrace{ca+cc}_{W})^*}_{R} + \underbrace{aa}_{Y}}_{X}\Big)^*$ (T), (S)

$S \rightarrow TS \mid \epsilon$

$T \rightarrow X \mid Y$

$X \rightarrow PQR$

[illegible]

P → ab | bb

Q → bb

R → WR | ϵ

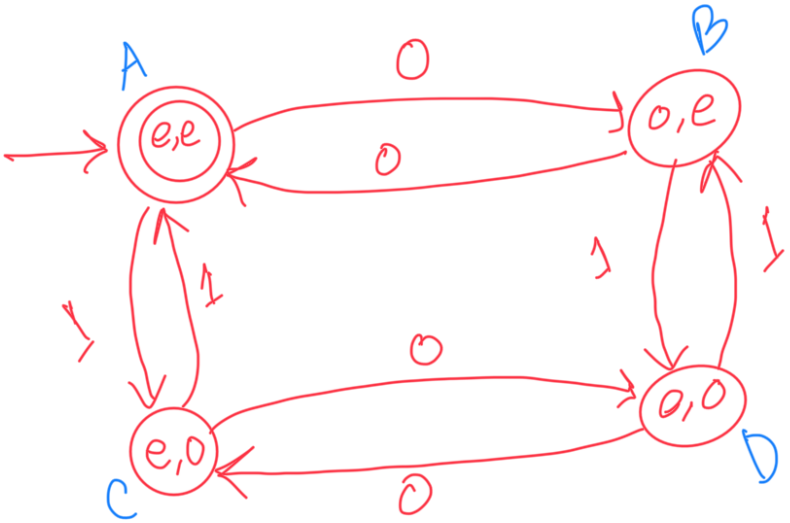
W → ca | cc

Y → aa

Convert DFA into CFG

$L = \{ w \in \{0,1\}^* : w$ contains even number of 0s and 1s $\}$

0	1
e	e
e	o
o	e
o	o



$R_i \xrightarrow{x} R_j$

if R_i is an accepting state

$\rightarrow \epsilon$

$R_i \rightarrow xR_j$ $R_i \rightarrow$

$S \rightarrow A$

$A \rightarrow 0B \mid 1C \mid \epsilon$

$B \rightarrow 0A \mid 1D$

$C \rightarrow 0D \mid 1A$

$D \rightarrow 0C \mid 1B$

Practice

1. $L = \{w \in \{0,1\}^* :$ the length of w is odd and the mid is $0\}$

2. $L = \{w \in \{0,1\}^* :$ w contains even numbers of 0s$\}$

3. $L = \{w \in \{0,1\}^* :$ w starts and ends with same symbol$\}$

4. $L = \{w \in \{0,1\}^* :$ w contains exactly three 1s$\}$

5. $L = \{w \in \{0,1\}^* :$ w starts with $101\}$

6. $L = \{w \in \{a,b\}^* :$ w ends with "ba"$\}$

6.