

CSE 330

# Assignment - 04

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Sec: 08 [SADE]

Ans. to the Q. no.1

a) given that,

$$4x_1 + 2x_2 + x_3 = 11$$

$$x_1 + 3x_2 + 2x_3 = 10$$

$$2x_1 + x_2 + 5x_3 = 13$$

$$\therefore \text{Augmented Matrix} = \left[ \begin{array}{ccc|c} 4 & 2 & 1 & 11 \\ 1 & 3 & 2 & 10 \\ 2 & 1 & 5 & 13 \end{array} \right]$$

① ———  $a_{11} = 4$

② ———  $m_{21} = \frac{1}{4}$  ,  $m_{31} = \frac{2}{4} = \frac{1}{2}$

$$\therefore \left[ \begin{array}{ccc|c} 4 & 2 & 1 & 11 \\ 0 & 2.5 & 1.75 & 7.25 \\ 0 & 0 & 4.5 & 7.5 \end{array} \right] \quad \begin{array}{l} \therefore R_2 = R_2 - m_{21} R_1 \\ \quad \quad \quad R_2 - \frac{1}{4} R_1 \\ \therefore R_3 = R_3 - m_{31} R_1 \\ \quad \quad \quad R_3 - \frac{1}{2} R_1 \end{array}$$

b) from "a" we get.

$$\left[ \begin{array}{ccc|c} 4 & 2 & 1 & 11 \\ 0 & 2.5 & 1.75 & 7.25 \\ 0 & 0 & 4.5 & 7.5 \end{array} \right]$$

Here,  $a_{32} = 0$ .

$$\therefore m_{32} = \frac{a_{32}}{u_{22}} = \frac{0}{2.5} = 0$$

P.T.O

$$\therefore m_{21} = \frac{1}{4}, \quad m_{31} = \frac{1}{2} \quad [\text{from "a"}]$$

$$\therefore m_{32} = 0$$

$\therefore$  Final upper-triangular matrix U.

$$U = \begin{bmatrix} 4 & 2 & 1 \\ 0 & 2.5 & 1.75 \\ 0 & 0 & 4.5 \end{bmatrix}$$

c) given that

$$4x_1 + 2x_2 + x_3 = 11 \quad \text{--- (I)}$$

$$x_1 + 3x_2 + 2x_3 = 10 \quad \text{--- (II)}$$

$$2x_1 + x_2 + 5x_3 = 13 \quad \text{--- (III)}$$

~~from (III)  $\times 4$~~

And we get

$$\Rightarrow \begin{bmatrix} 4 & 2 & 1 \\ 0 & 2.5 & 1.75 \\ 0 & 0 & 4.5 \end{bmatrix} \begin{bmatrix} 11 \\ 10 \\ 7.5 \end{bmatrix} \quad \because \text{from "a"}$$

from the last equation:

$$4.5x_3 = 7.5$$

$$\Rightarrow x_3 = \frac{7.5}{4.5} = \frac{5}{3} \approx 1.67$$

P.T.O

(2)

and from second equation:

$$2.5x_2 + 1.75x_3 = 7.25$$

$$\Rightarrow 2.5x_2 + 1.75 \times \frac{5}{3} = 7.25$$

$$\Rightarrow 2.5x_2 + 2.9167 = 7.25$$

$$\Rightarrow 2.5x_2 = 4.3333$$

$$\Rightarrow x_2 = \frac{4.3333}{2.5} \approx 1.733$$

And from the first equation:

$$4x_1 + 2x_2 + x_3 = 11$$

$$\Rightarrow 4x_1 + 2(1.733) + 1.667 = 11$$

$$\Rightarrow 4x_1 + 3.466 + 1.667 = 11$$

$$\Rightarrow 4x_1 + 5.133 = 11$$

$$\Rightarrow 4x_1 = 5.867$$

$$\therefore x_1 \approx 1.467$$

$$\therefore x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1.467 \\ 1.733 \\ 1.667 \end{bmatrix}$$



Ans. to the Q. no. 2

a)

given that,

$$4x_1 + 2x_2 + x_3 = 11$$

$$x_1 + 3x_2 + 2x_3 = 10$$

$$2x_1 + x_2 + 5x_3 = 13$$

$$A = \begin{bmatrix} 4 & 2 & 1 \\ 1 & 3 & 2 \\ 2 & 1 & 5 \end{bmatrix}, \quad b = \begin{bmatrix} 11 \\ 10 \\ 13 \end{bmatrix}$$

When pivot  $a_{11} = 4$ ,  $u_{11} = 4$ ,  $u_{12} = 2$ ,  $u_{13} = 1$   
 $\therefore l_{21} = \frac{a_{21}}{u_{11}} = \frac{1}{4}$ ,  $l_{31} = \frac{a_{31}}{u_{11}} = \frac{1}{2}$

When pivot  $u_{22}$ ,  $u_{22} = a_{22} - l_{21}u_{12} = 3 - \frac{1}{4} \cdot 2 = 2.5$   
 $u_{23} = a_{23} - l_{21}u_{13} = 2 - \frac{1}{4} \cdot 1 = 1.75$   
 $\therefore l_{32} = \frac{a_{32} - l_{31}u_{12}}{u_{22}} = \frac{1 - \frac{1}{2} \cdot 2}{2.5} = 0$

Last pivot  $u_{33}$ ,  $u_{33} = a_{33} - l_{31}u_{13} - l_{32}u_{23}$   
 $= 5 - \frac{1}{2} \cdot 1 - 0 \cdot 1.75$   
 $= 4.5$

$$\therefore L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{4} & 1 & 0 \\ \frac{1}{2} & 0 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 4 & 2 & 1 \\ 0 & 2.5 & 1.75 \\ 0 & 0 & 4.5 \end{bmatrix}$$

b)

Forward substitution:  $Ly = b$ 

$$y_1 = b_1 = 11$$

$$y_2 = b_2 - l_{21}y_1 = 10 - \frac{1}{4} \times 11 = \frac{29}{4} = 7.25$$

$$\begin{aligned} y_3 &= b_3 - l_{31}y_1 - l_{32}y_2 = 13 - \frac{1}{2} \times 11 - 0 \times y_2 \\ &= \frac{15}{2} \\ &= 7.5 \end{aligned}$$

$$\therefore y = \begin{bmatrix} 11 \\ 7.25 \\ 7.5 \end{bmatrix}$$

c) Back substitution:  $Ux = y$ 

$$4.5x_3 = 7.5$$

$$\Rightarrow x_3 = 1.667$$

$$2.5x_2 + 1.75x_3 = 7.25$$

$$\Rightarrow x_2 = 1.793$$

$$4x_1 + 2x_2 + x_3 = 11$$

$$\Rightarrow x_1 = 1.467$$

$$\therefore x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1.467 \\ 1.793 \\ 1.667 \end{bmatrix}$$