

TOC-04

Kleen & Positive
Closures

Theory of Computation

Kleen Closure:

$$\overline{\Sigma^*} = \{\epsilon, a, b, aa, ab, ba, bb, aaa, aab, \dots\}$$

$$\boxed{\Sigma = \{a, b\}}$$

$$\Sigma^1 = \{a, b\}$$

$$\Sigma^2 = \{aa, ab, ba, bb\}$$

$$\Sigma^3 = \{aaa, aab, aba, \dots, bbb\}$$

$$\Sigma^\circ = \{\epsilon\}$$

TOC-04

Kleen & Positive
Closures

Theory of Computation

Kleen Closure:

$$\Sigma = \{0, 1\} \quad \Sigma^* = \{\epsilon, 0, 1, 00, 01, 10, 11, 000, 001, \dots\}$$

$$\Sigma = \{a, b\} \quad \Sigma^* = \{\epsilon, a, b, aa, ab, bb, ba, aaa, aab, \dots\}$$

$$\Sigma = \{a\} \quad \Sigma^* = \{\epsilon, a, aa, aaa, aaaa, \dots\}$$

Theory of Computation
Positive Closure: (Σ^+)

$$\Sigma = \{a, b\}$$

$$\Sigma^+ = \{a, b, aa, ab, ba, bb, aaa, \dots\}$$

$$\Sigma = \{0, 1\}$$

$$\Sigma^+ = \{0, 1, 00, 01, 10, 11, 000, 001, \dots\}$$

Theory of Computation
Kleen & Positive Closure:

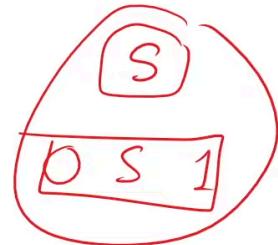
$$\Sigma^+ = \Sigma^* - \{\epsilon\}$$

$$\Sigma^* = \{\epsilon\} + \Sigma^+$$

$O_1^{n,n}$

$\underline{O^* 1^*} = \{011, 00011,$

$\underbrace{00000}_{S} \underbrace{11111}_{S}$



$S \rightarrow OS1$
$S \rightarrow \epsilon$

I

$$\begin{aligned} & \xrightarrow{S} 0[\underline{S}1] \\ & \Rightarrow 00\underline{S}11 \\ & \Rightarrow \overbrace{\underline{000}S111}^{\longrightarrow} \\ & \quad \overbrace{000111}^{\longleftarrow} \end{aligned}$$


$O_1^{n,n}$

$\underline{O^* 1^{2n}}$

$000 \sqcup \sqcup \sqcup$

$S \rightarrow OS1$
$S \rightarrow \epsilon$

$$\begin{aligned} & S \rightarrow OS11 \\ & S \rightarrow \epsilon \end{aligned}$$

$O_1^{3n, 5n}$

$$\begin{aligned} & S \rightarrow 000S11111 \\ & S \rightarrow \epsilon \end{aligned}$$

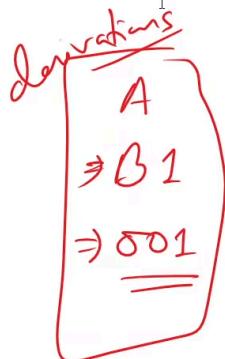
Context-Free Grammar

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$$A \rightarrow OA1 \quad | \quad B1$$

$$B \rightarrow 00$$

$$\Sigma = \{0, 1\}$$



$$\begin{aligned}
 A &\Rightarrow \underline{\underline{OA1}} \\
 &\Rightarrow 00A11 \\
 &\Rightarrow 00B111 \\
 &\Rightarrow \underline{\underline{0000111}}
 \end{aligned}$$

4 tuple $\langle V, T, P, S \rangle$

\downarrow
 non-variables
 terminals

\downarrow
 start symbol

Context-Free Grammar

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$$S \rightarrow OS1$$

$$V = \{S, A\}$$

$$S \rightarrow A$$

$$T = \{0, 1, \#\}$$

(S)

$$A \rightarrow \#$$

S

$$\Rightarrow OS1$$

$$\Rightarrow 0OS11$$

$$\Rightarrow 00A11$$

$$\Rightarrow 00\#11$$

$0^n \# 1^n$

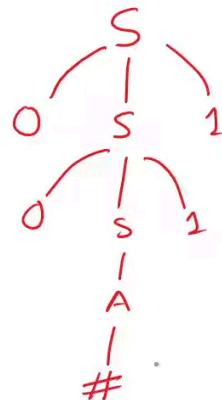
Context-Free Grammar

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$$S \rightarrow OS1$$

$$S \rightarrow A$$

$$A \rightarrow \#$$



$$V = \{S, A\}$$

$$T = \{0, 1, \#\}$$

S

$$\Rightarrow OS1$$

$$\Rightarrow O(OS1)1$$

$$\Rightarrow OO(A)11$$

$$\Rightarrow OO\#11$$

Context-Free Grammar

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$$S \rightarrow AOB$$

$$A \rightarrow OA | \epsilon$$

$$B \rightarrow 1B | \epsilon$$

~~rightmost~~

leftmost derivation

$$\Rightarrow AOB$$

$$\Rightarrow OAOB$$

$$\Rightarrow OOAOB$$

$$\Rightarrow OOB$$

$$\Rightarrow OOOB$$

$$\Rightarrow OOO1B$$

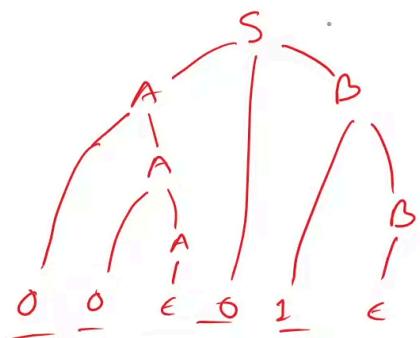
$$\Rightarrow OOO1$$

$$0^+ 1^n$$

Context-Free Grammar

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$$\begin{aligned} S &\rightarrow AOB \\ A &\rightarrow OA | \epsilon \\ B &\rightarrow LB | \epsilon \end{aligned}$$



~~rightmost~~

leftmost derivation

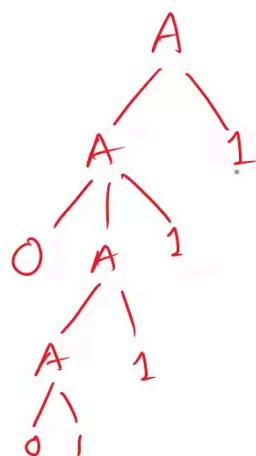
$$\begin{aligned} S &\Rightarrow AOB \\ &\Rightarrow OA\epsilon B \\ &\Rightarrow OOA\epsilon B \\ &\Rightarrow OOOB \\ &\Rightarrow OOO1B \\ &\Rightarrow OOO1 \end{aligned}$$

Context-Free Grammar

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$$A \rightarrow A1 | OA1 | \textcircled{O}1$$

"001111"



$$A$$

$$\Rightarrow A1$$

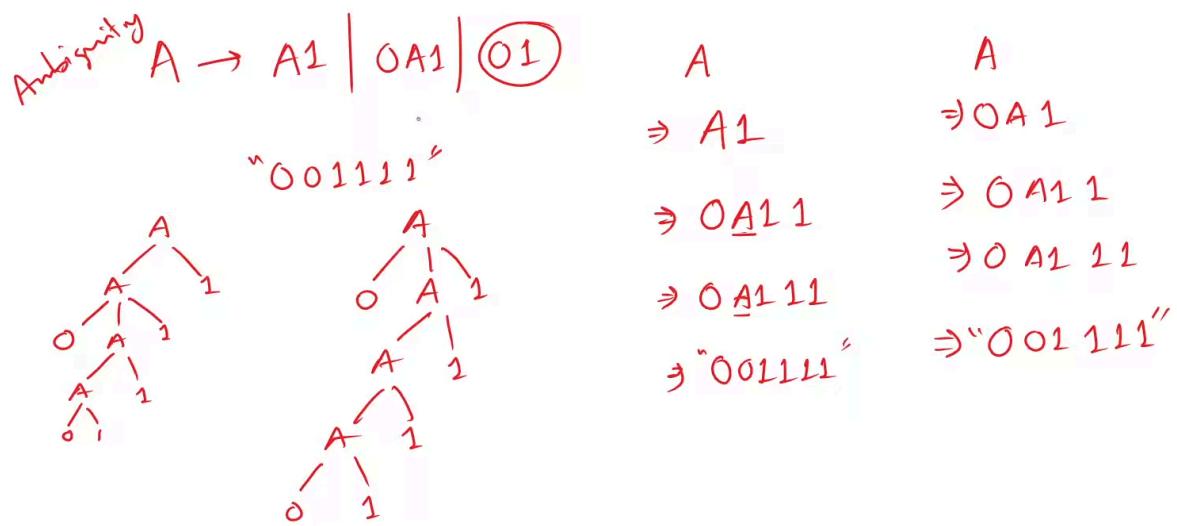
$$\Rightarrow OA\underline{A}11$$

$$\Rightarrow O\underline{A}111$$

$$\Rightarrow "001111"$$

Context-Free Grammar

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CFG | Regular Operations

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OR , Concatenation, Kleene Closure.

$\textcircled{O^n}1^n$

$S \rightarrow OS1 \mid \epsilon$

$1^n O^n$

$S \rightarrow 1SO \mid \epsilon$

$n_1 \mid n_2$

I

$S \rightarrow S_a \mid S_b$

$S_a \rightarrow OS_a1 \mid \epsilon$

$S_b \rightarrow 1S_b0 \mid \epsilon$

on CFG & Regular Operations

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OR, Concatenation, Kleene Closure.

$0^n 1^n$ $\underline{000111} \underline{1100}$ $w_1 | w_2$

$S \rightarrow 0S1 | \epsilon$

$S \rightarrow 1S0 | \epsilon$

$S \rightarrow S_a S_b$

$S_a \rightarrow 0S_a 1 | \epsilon$

$S_b \rightarrow 1S_b 0 | \epsilon$

$\Rightarrow \begin{matrix} S \\ S_a S_b \\ \hline \end{matrix} \downarrow \downarrow$

$\boxed{000111 1100}$

Context-Free Grammar

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OR, Concatenation, Kleene Closure.

$0^n 1^n$ $\overbrace{000000}^A$ $0^* = \{\epsilon, 0, 00, 000, \dots\}$

$S \rightarrow 0S1 | \epsilon$

$S \rightarrow 1S0 | \epsilon$

$(01001)^* = \{\epsilon, \underline{01001}, \underline{0100101001}, \dots\}$

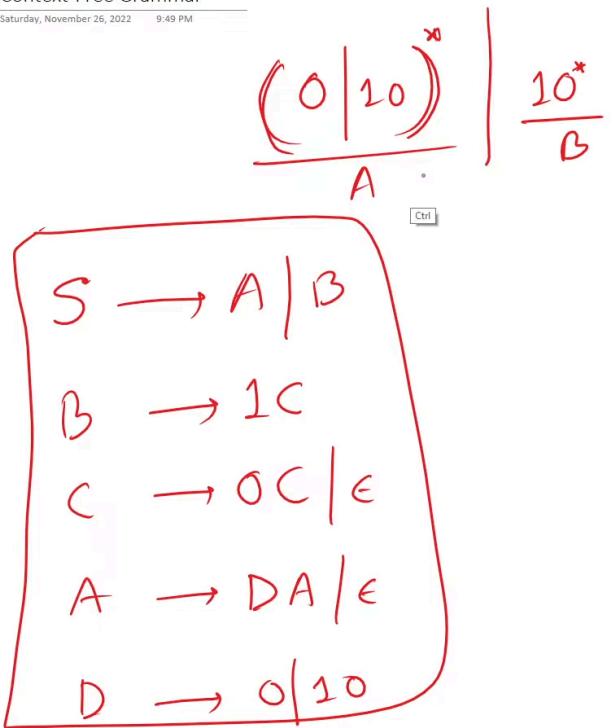
$A \rightarrow 0A | \epsilon$

$B \rightarrow CB | \epsilon$

$C \rightarrow 0\underline{1001}$

Context-Free Grammar

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Context-Free Grammar

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$$0^n 1 2^n$$

$$S \rightarrow 0S2 \mid 1$$

$$\frac{0^n 1^n 2}{S}$$

$$A \rightarrow S2$$

$$S \rightarrow 0S1 | \epsilon$$

$$0^m 1^n 2^k$$

$$k = m+n$$

$$0^m 1^n 2^{\underline{k}} 2^m$$

$$S \rightarrow 0S2 \mid X$$

$$X \rightarrow 1X2 \mid \epsilon$$



$$S \Rightarrow 0S2$$

$$\Rightarrow 00S22$$

$$\Rightarrow \underline{00} \underline{X} \underline{22}$$

$$\Rightarrow 0011X2222$$

$$\Rightarrow \underline{00111} \underline{2222}$$

Activate Windows
Go to Settings to activate Windows.

Context-Free Grammar

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$$0^n 1 2^n$$

$$S \rightarrow 0S2 \mid 1$$

$$\frac{0^n 1^n 2}{S}$$

$$A \rightarrow S2$$

$$S \rightarrow 0S1 \mid \epsilon$$

$$0^m 1^{n+k} 2^k \quad n = m+k$$

$$0^m 1^{m+k} 2^k$$

$$\underline{\underline{0}} \underline{\underline{1}} \underline{\underline{1}} \underline{\underline{k}} \underline{\underline{2}} \underline{\underline{k}}$$

$$S \rightarrow A B$$

$$A \rightarrow 0A1 \mid \epsilon$$

$$B \rightarrow 1B2 \mid \epsilon$$

Context-Free Grammar

Saturday, November 26, 2022 9:49 PM

$$0^m 1^m \quad m \geq 2$$

$$S \rightarrow \underline{\underline{0}} \underline{\underline{0}} \underline{\underline{A}} \underline{\underline{1}} \underline{\underline{1}}$$

$$A \rightarrow 0A1 \mid \epsilon$$

$$0^m 1^n \quad m > n$$

$$S \rightarrow 0S1 \mid A$$

$$A \rightarrow 0A \mid 0$$

$$\begin{array}{l} S \\ \Rightarrow 0S1 \\ \Rightarrow 00S11 \\ \Rightarrow 0000111 \\ \downarrow \\ 0000 \\ 0^+ \end{array}$$

$\{w \mid w \text{ contains at least three } 1's\}$ $(0|1)^*$
 $\Sigma^* 1 \Sigma^* 1 \Sigma^* 1 \Sigma^*$

$S \rightarrow A 1 A 1 A 1 A$

$A \rightarrow 0A \mid 1A \mid \epsilon$

$S \rightarrow B 1 B 1 B 1 A$

$A \rightarrow 0A \mid 1A \mid \epsilon$

$B \rightarrow 0B \mid \epsilon$

$\{w \mid w \text{ starts and ends with same symbol}\}$ $\Sigma = \{0, 1\}$

$0 \Sigma^* 0 \mid 1 \Sigma^* 1$

$S \rightarrow 0AO \mid 1A1$ •

$A \rightarrow 0A \mid 1A \mid \epsilon$

$\{w \mid \text{the length of } w \text{ is odd}\}$

$\underbrace{\Sigma (\Sigma \Sigma)^*}_{(0|1)}$

$S \rightarrow AB$

$A \rightarrow 0|1^*$

$B \rightarrow AAB \mid \epsilon$

$\{w \mid \text{the length of } w \text{ is odd and its middle symbol is } 0\}$

$S \rightarrow OS0 \mid 1S1 \mid 0S1 \mid 1S0 \mid 0$

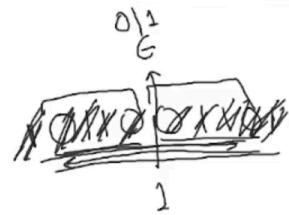
~~100
1100101~~

~~10000101~~

S
 $\Rightarrow \begin{cases} S1 \\ 0S01 \\ 101\Sigma 101 \\ 101 \ 100101 \end{cases}$

Activate Windows
Go to Settings to activate Windows.

$\{ w \mid w = w^R, \text{ i.e. } w \text{ is a palindrome} \}$



$$S \rightarrow 0S0 \mid 1S1 \in \{1\}^*$$



Activate Windows

Context Free Grammar

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$\hookrightarrow w \in \Sigma^* \quad \Sigma = \{0, 1\}$

#0's in w = #1's in w

$S \rightarrow 0S1 \mid 1S0 \mid SS \mid \epsilon$

Activate Win
Go to Settings to

\Rightarrow CFG | Equal Number of 0s and 1s

#0's in w = #1's in w

$$S \rightarrow_1 OS1 \mid 1SO \mid SS \mid \epsilon$$

001110

$$\begin{aligned} & S \\ & \Rightarrow SS \\ & \Rightarrow OS1 S \\ & \Rightarrow OOS11 S \\ & \Rightarrow OO11 S \\ & \Rightarrow OO11 1SO \\ & \Rightarrow \boxed{OO1110} \end{aligned}$$

OneDrive - OneDrive

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Palindrome $\epsilon|0|1$
 $\cancel{0}|1|0|1|1|0|\cancel{1}$

$$S \rightarrow OS0 \mid 1S1 \mid \epsilon \mid 0 \mid 1$$

$$O^{i \underline{j} \underline{k}} \quad \frac{i=j \text{ or } j \neq k}{A \quad B}$$

$$\begin{array}{l} A \rightarrow PQ \\ Q \rightarrow 2Q | \epsilon \\ P \rightarrow OP1 | \epsilon \end{array}$$

$$O^{i \underline{j} \underline{k}} \quad \frac{i=j \text{ or } j \neq k}{\begin{array}{l} A \quad B \\ w \quad x \end{array}} \quad \begin{array}{l} j > k \\ j \leq k \end{array}$$



$$S \rightarrow A | B$$

$$A \rightarrow PQ$$

$$Q \rightarrow 2Q | \epsilon$$

$$P \rightarrow OP1 | \epsilon$$

$$\begin{array}{l} B \rightarrow WX \\ w \rightarrow ow | \epsilon \end{array}$$

$$X \rightarrow Y | Z$$

$$Y \rightarrow 1Y2 | Y'$$

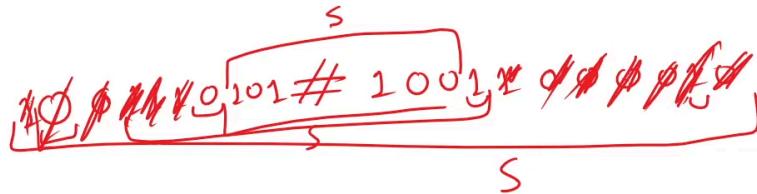
$$Y' \rightarrow 2Y' | \epsilon$$

$$Z \rightarrow 1Z2 | Z'$$

$$Z' \rightarrow 1Z' | \epsilon$$

(L) $\omega_1 \# \omega_2$

no. of 0's in ω_1 = no. of 1's in ω_2



I

$$S \rightarrow 1S \mid S0 \mid OS1 \mid \#$$



Question 1: [15 Points]

- a) Give a context-free grammar for the following language. (5 points)
 $A = \{w \in \{0, 1\}^*: w \text{ contains odd number of } 1s\}$

- b) Give a context-free grammar for the following language. (5 points)
 $W = \{w \in \{0, 1\}^*: w \text{ starts and ends with same characters}\}$

- c) Give a context-free grammar for the following language. (5 points)
 $L = \{w \in \{0, 1\}^*: w = \underbrace{u0^{2i}}_{B} v1^{3i} \text{ where } u \in B, v \in A \text{ and } i \geq 0\}$

$0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0 \ 1$

$w \ B \rightarrow 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0 \ 1$

$P \rightarrow QP \mid \epsilon$

$Q \rightarrow 0 \ 1$

$(0101)^* 010$

$A \rightarrow BCBC$

$C \rightarrow 0C \mid \epsilon$

$B \rightarrow DB \mid \epsilon$

$D \rightarrow C1C1$

