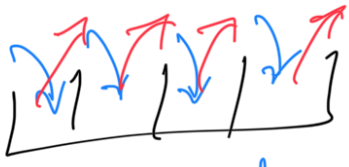
Pushdown Automata

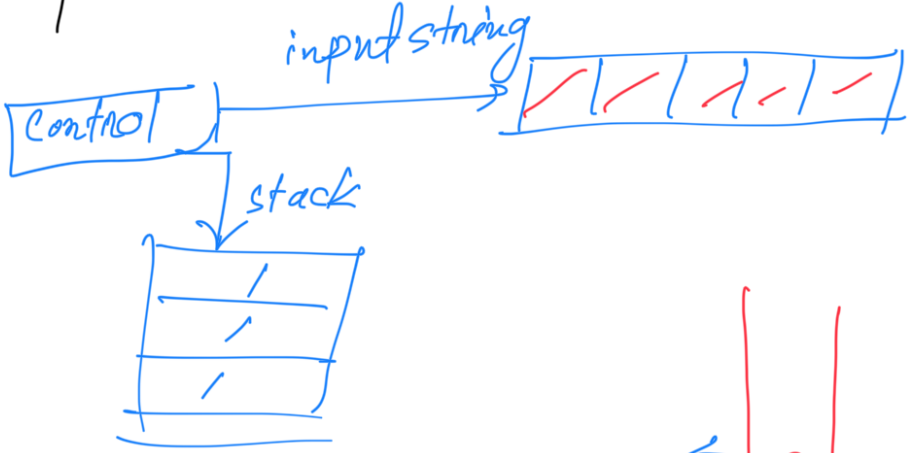
* This is like NFA but has additional component → Stack.

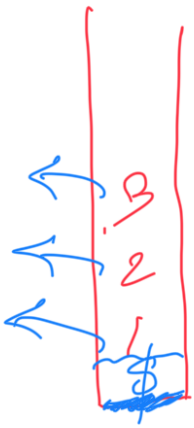


→ provides additional memory beyond the finite memory of automata.

→ Allows to recognize some non-regular language.

* Pushdown automata is equivalent in power to CFG.





Indication of the bottom of the stack ⇒ $

Write symbol on the stack ⇒ push

Read symbol from the stack → pop

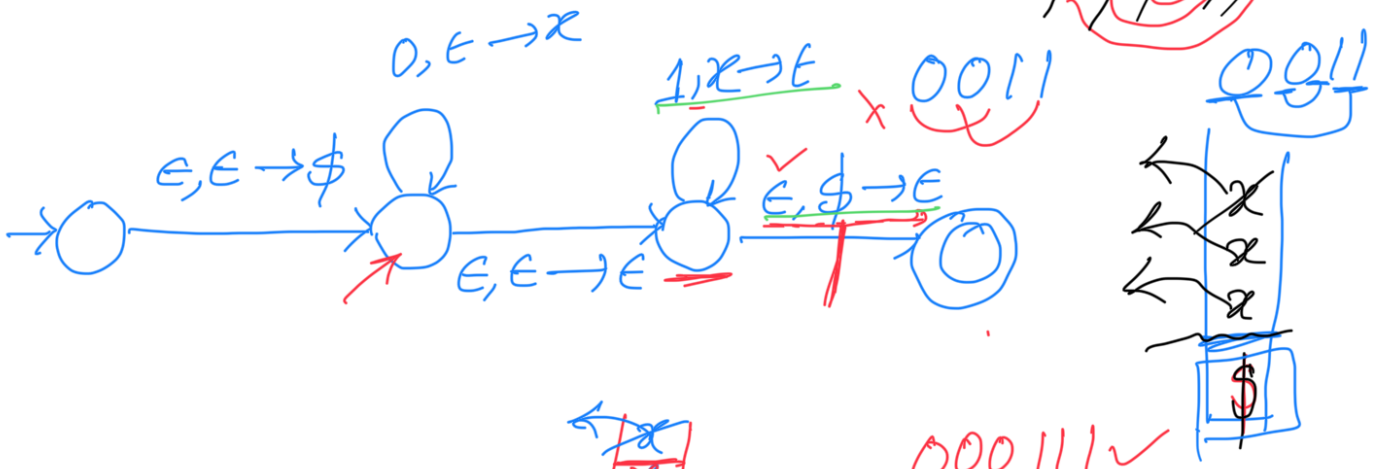
0, 1

→ NFA / DPA

Format input symbol, pop symbol → push symbol.

1

① $L = \{ w \in \{0,1\}^* : w = 0^n 1^n, n \geq 0 \}$

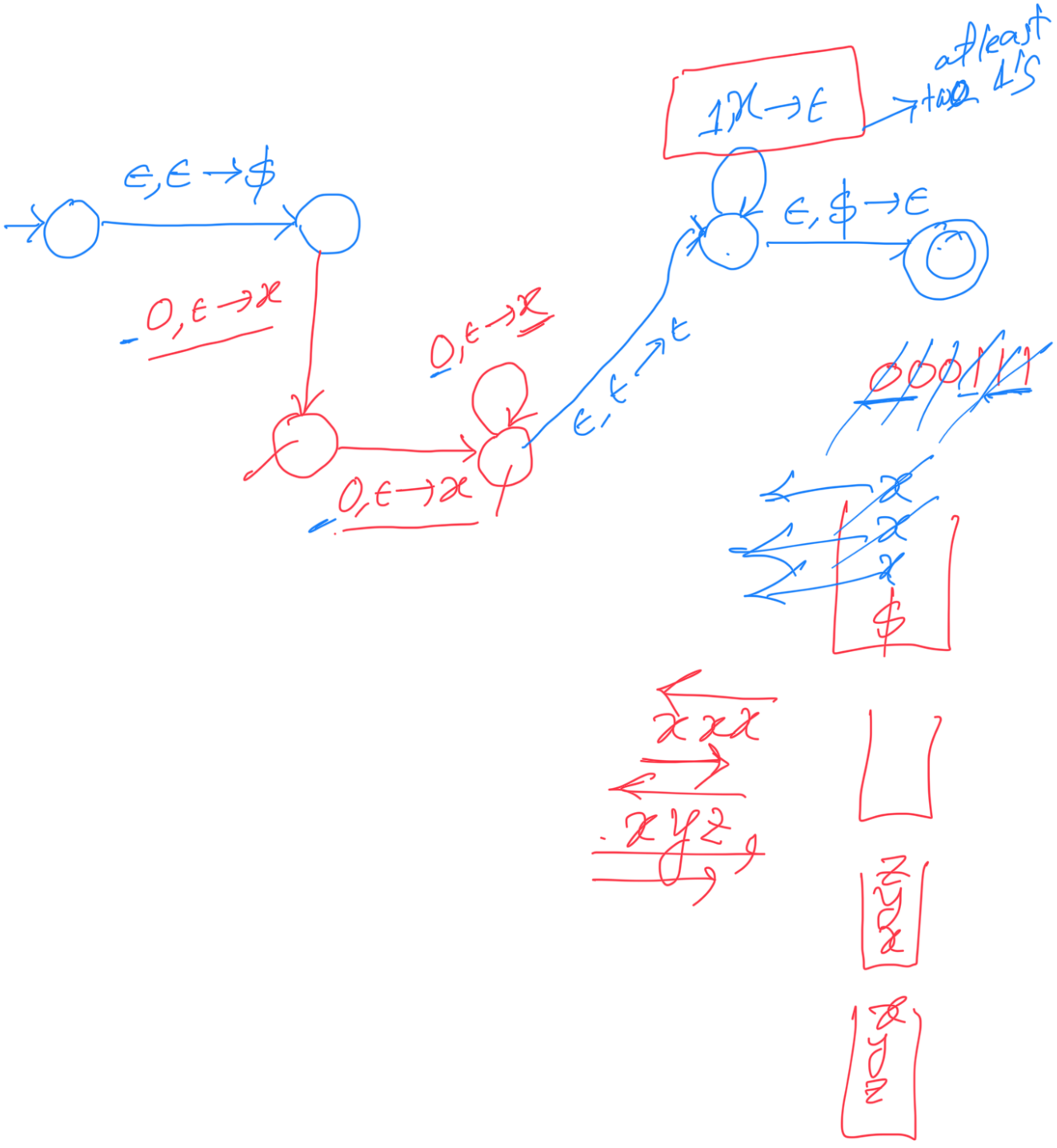


000111 ✓

→ 001 ✗

(11) $L = \{\omega \in \{0,1\}^* : \omega = 0^n 1^n, n \geq 2\}$

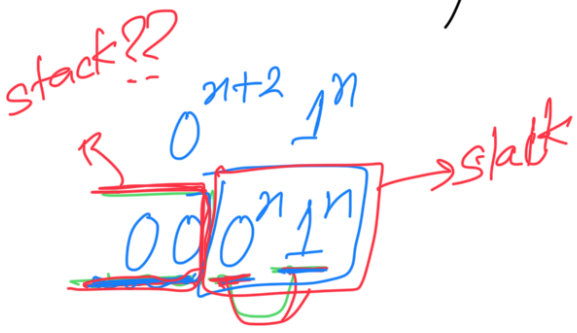
ϵ ✗
01 ✗
0011 ✓
000111 ✓

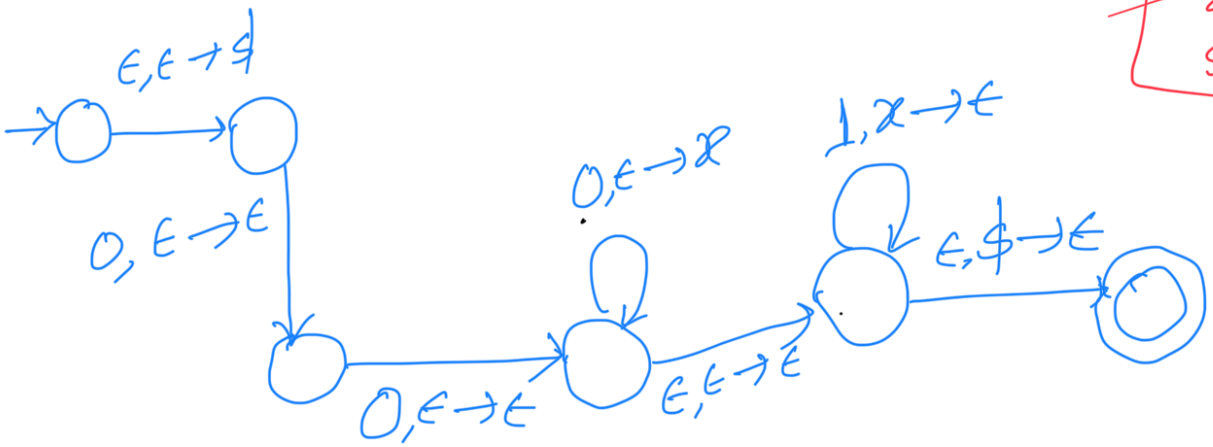


$\{0^{n+2}1^n : n \geq 0\}$

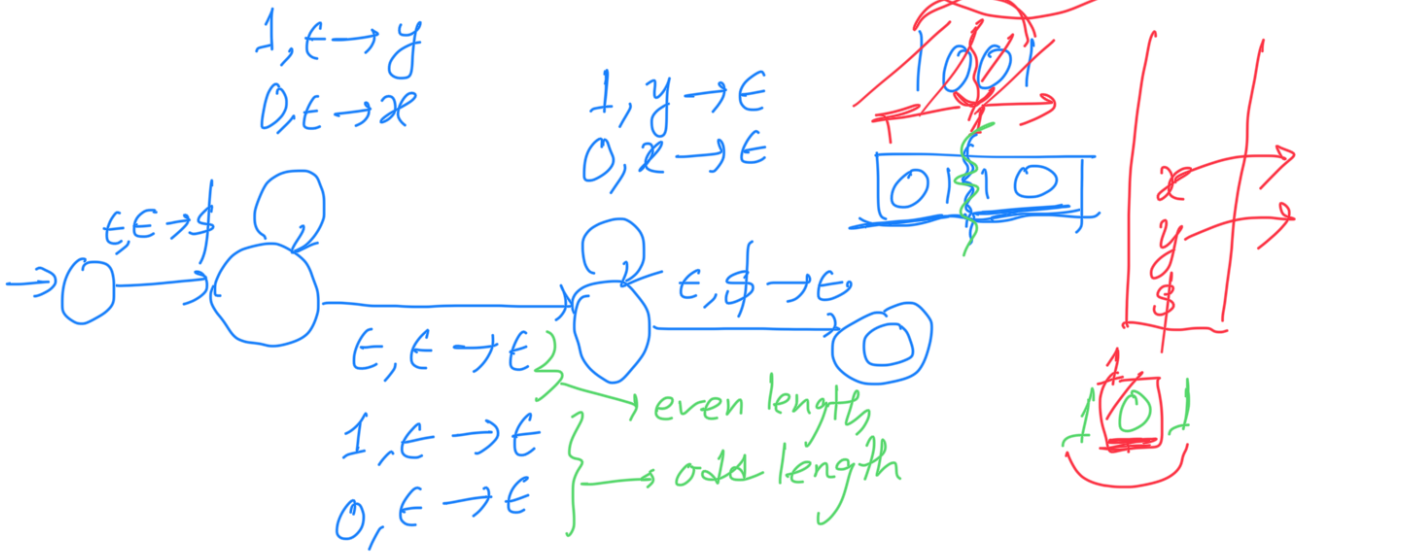
(III) $L = \{ w \in \{0,1\} : w = 0^{n+2} 1^n \}$

00
0001
000011





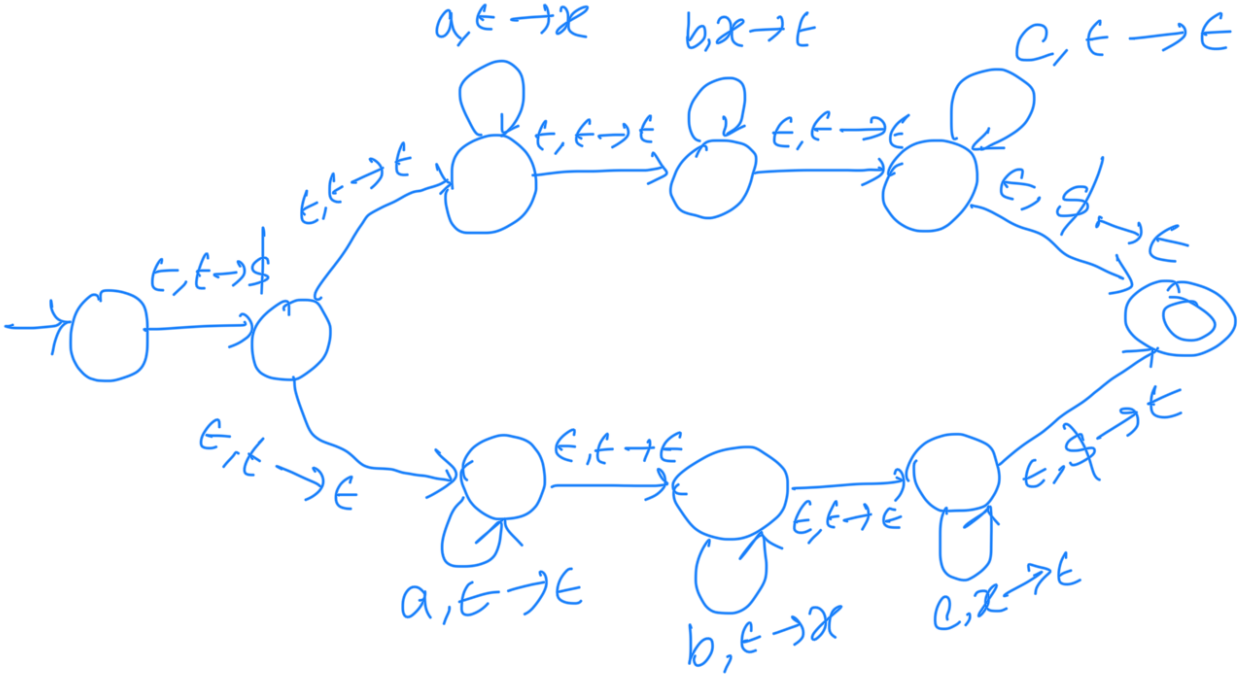
(IV) $L = \{ w \in \{0,1\}^* : w \text{ is a palindrome} \}$



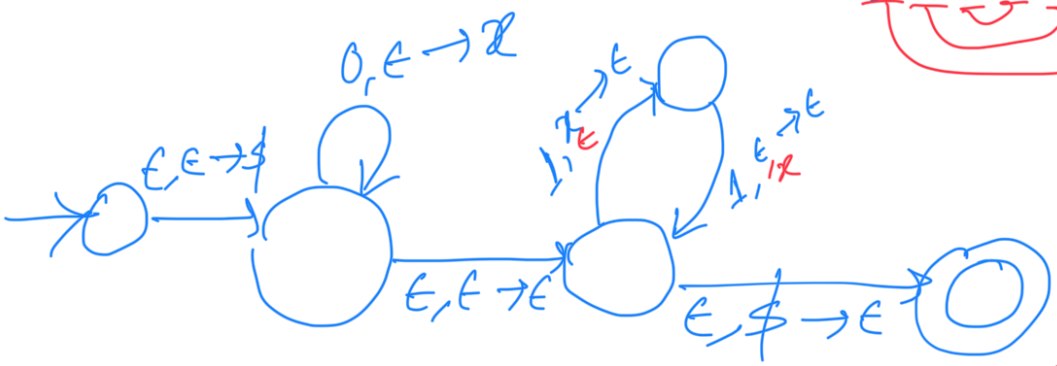
(V) $L = \{ w \in \{a,b,c\}^* : w = a^i b^j c^k, \text{ where } i = j \text{ or } j = k \}$

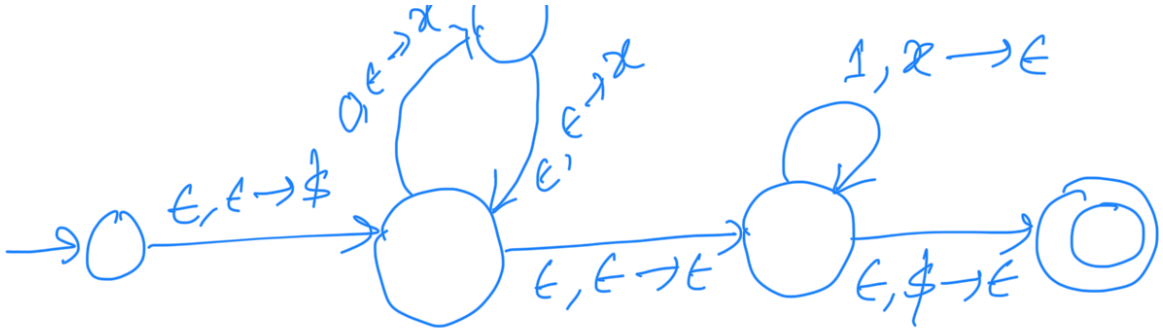
$a^i b^j c^k$ / $a^i b^j c^d$

① ②



(vi) $L = \{ \omega \in \{0,1\}^* : \omega = 0^n 1^{2n},\ n \geq 0 \}$





(VII) $L = \{ w \in \{0,1\}^* : w = 0^{3n} 1^{2n}, n \geq 0 \}$

ε

000 11

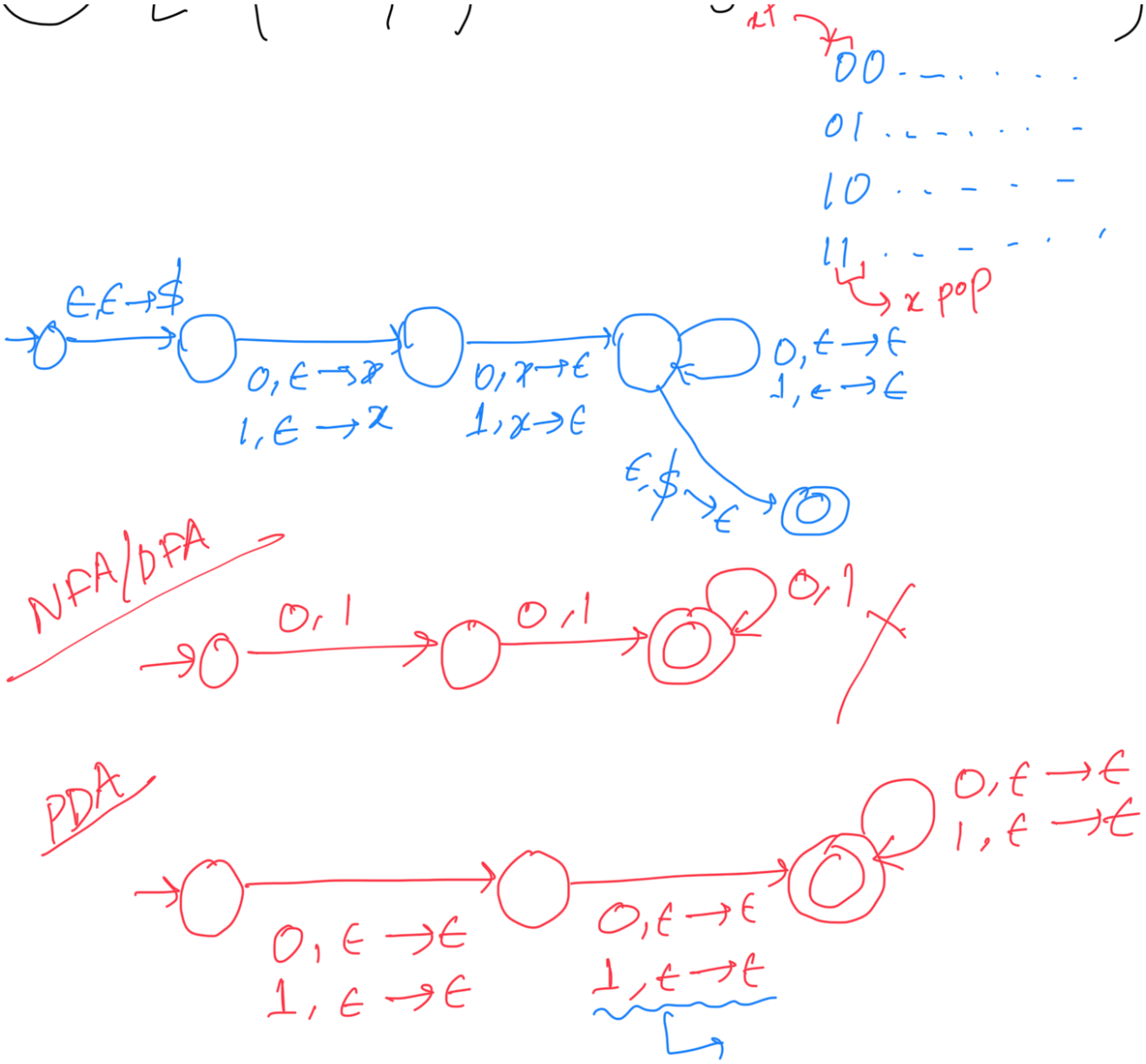
x 000 000 1 1 1

x x

000 1 1

(VIII) $L = \{ w \in \{0,1\}^* :$ The length of w is at least two $\}$

push



(IX) $L = \{ w_1 \# w_2 :$ The number of '00' in w_1 is the same as the number of '11' in $w_2 \}$

10# ✓

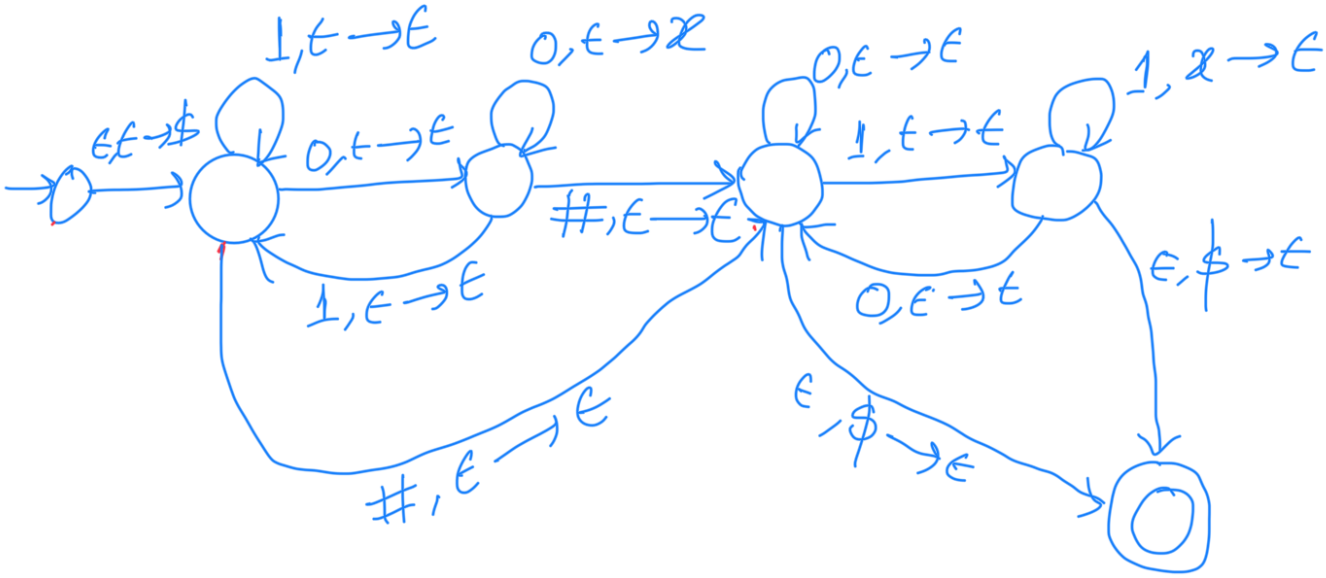
#10 ✓

111#000 ✓

001 # 001011

000 # 11011

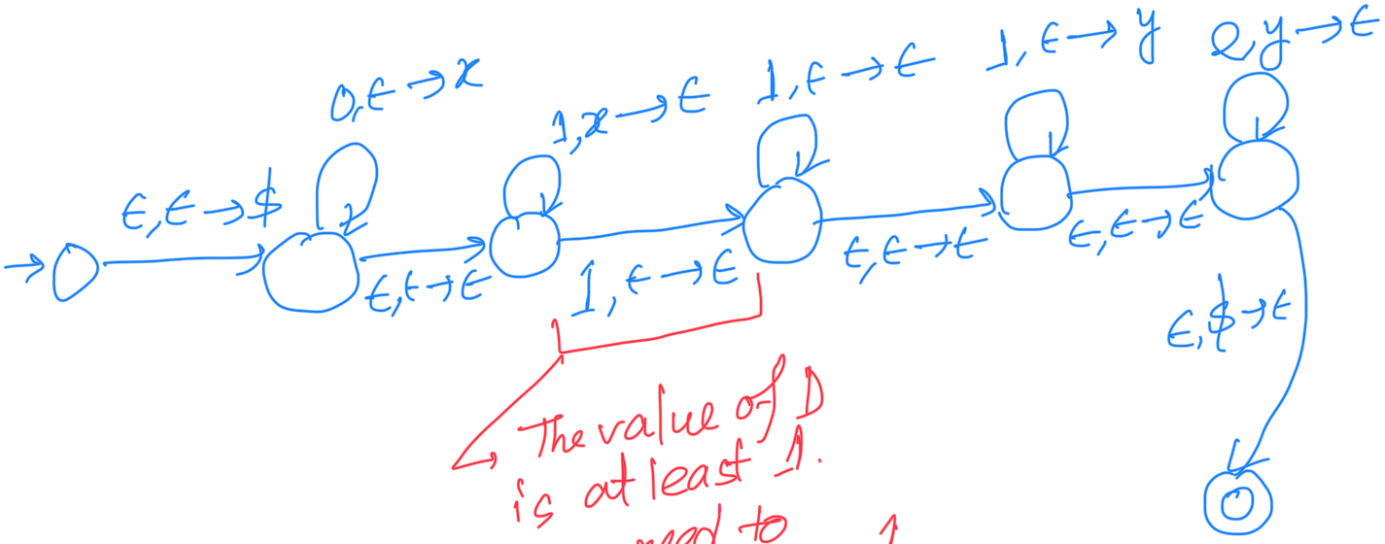
011 # 000



(X) $L = \{ w \in \{0,1,2\}^* : w = 0^i 1^j 2^k,\ j > i+k,\ i,k \geq 0 \}$

$0^i 1^i \boxed{1^D} 1^* 2^k$ — $D \geq 1$ too

$j = i + k + D$, $D \geq 1$ to ∞

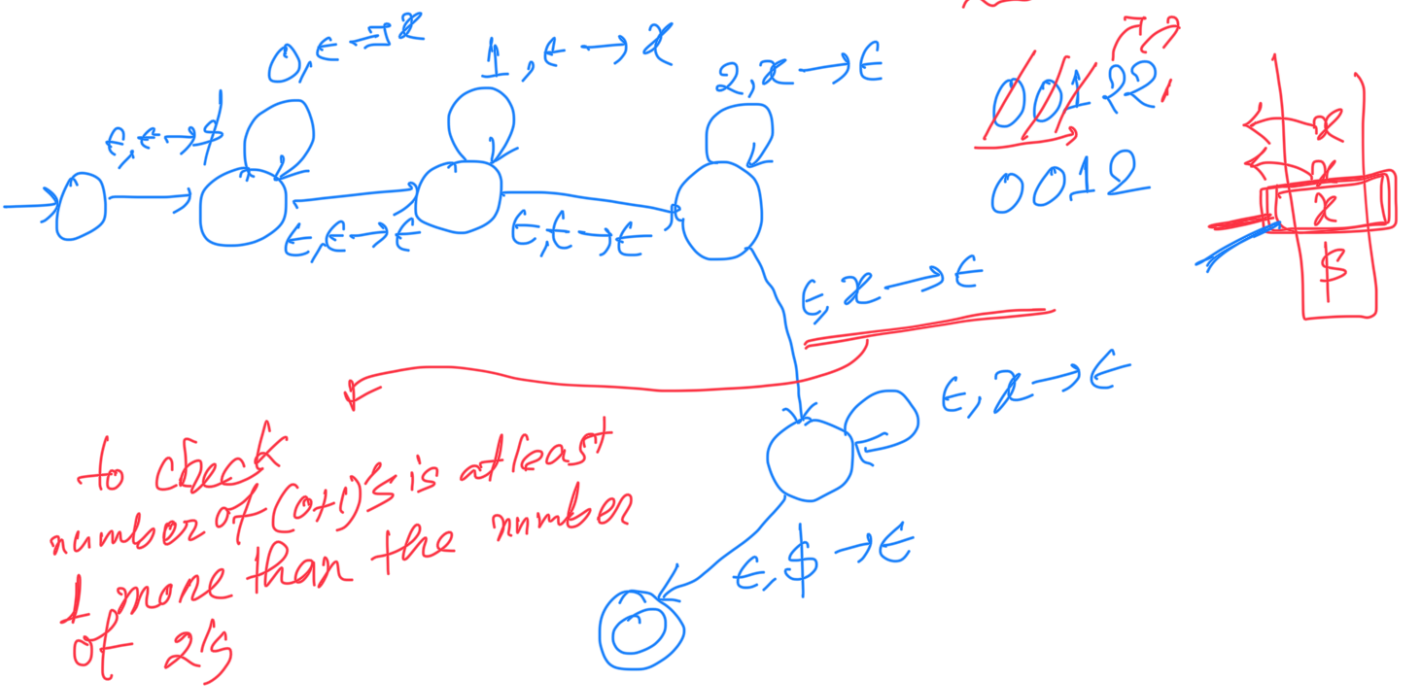


The value of D is at least 1.
So we need to read atleast one-1 without pushing/poping anything to/from the stack.

(XI) $L = \{ w \in \{0,1,2\}^*,\ w = 0^i 1^j 2^k,\ i+j > k \}$

$i+j = k+D \rightarrow (1 \to \infty)$

$k = i+j-D$

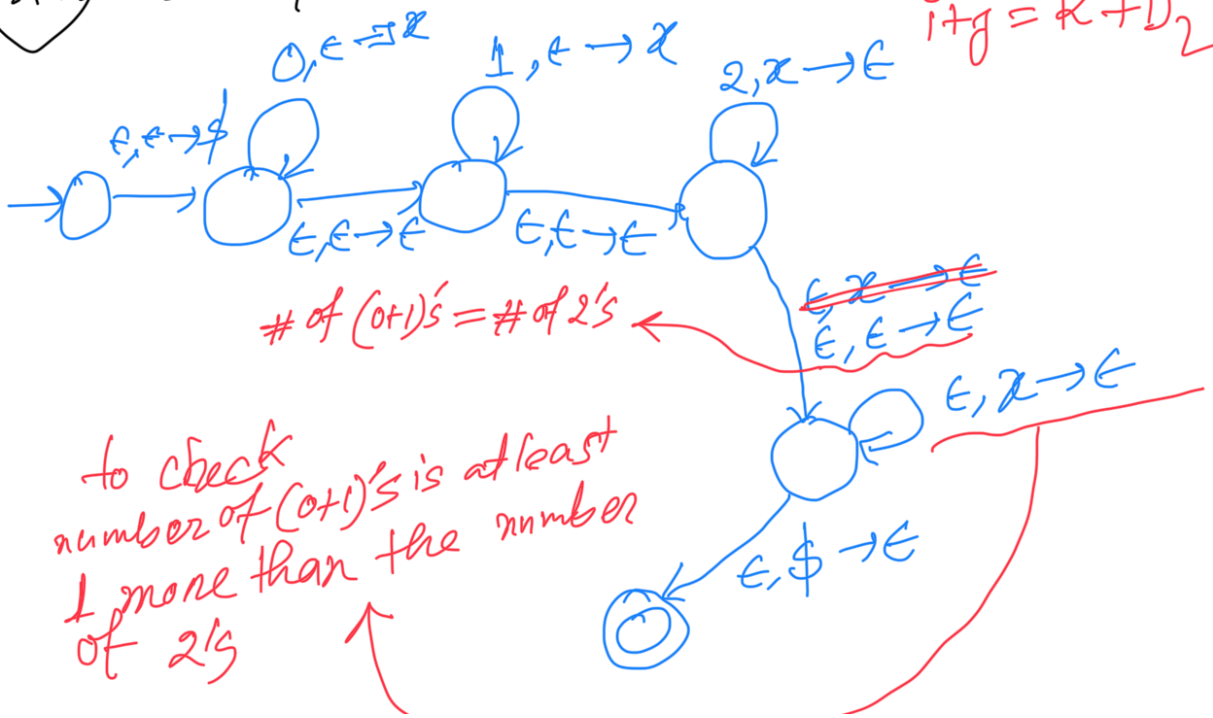


0012

to check number of (0+1)'s is atleast 1 more than the number of 2's

(XII) $L = \{ w \in \{0,1,2\}^*,\ w = 0^i 1^j 2^k,\ i+j \geq k \}$

$i+j = k+D \rightarrow (0 \to \infty)$



of (0+1)'s = # of 2's

to check number of (0+1)'s is atleast 1 more than the number of 2's

Practice

(1) $L = \{ w \in \{a,b\}^* : w = a^i b^j$, where $i > j, j \geq 0 \}$

(2) $L = \{ w \in \{0,1,2\}^* : w = 0^i 1^j 2^k$, where $i + j = k$, $i, j \geq 0 \}$

(3) $L = \{ w \in \{0,1\}^* :$ 0 and 1 alternates in $w \}$

(4) $L = \{ w \in \{a,b\}^* :$ the count of "a" in w is a multiple of 3 $\}$