Regular Expression

* $0+1$
* $(0+1)^*$ / $(0^*1^*)^*$
* 0^*1^*
* $(0+1)^*1$
* $0^*1^* + (ab)^*$

* $0+1$

7.

10

0ab1

* Construct Regular Expressions that generates the following languages.

① $L = \{w \in \{0,1\}^* : w \text{ contains "101" as a substring}\}$

$$(0+1)^*101(0+1)^*$$

② $L = \{w \in \{0,1\}^* : w \text{ starts with "101"}\}$

$$101(0+1)^*$$

③ $L = \{w \in \{0,1\}^* : w \text{ ends with "101"}\}$

$$(0+1)^* 101$$

④ $L = \{w \in \{0,1\}^* : w \text{ contains "00" or "11"}\}$

$$(0+1)^* 00 (0+1)^* + (0+1)^* 11 (0+1)^*$$

or,

$$(0+1)^* (00+11) (0+1)^*$$

⑤ $L = \{w \in \{0,1\}^* : w \text{ contains "00" and "11"}\}$

$$(0+1)^* 00 (0+1)^* 11 (0+1)^*$$

$$+$$

$$(0+1)^* 11 (0+1)^* 00 (0+1)^*$$

⑥ $L = \{w \in \{0,1\}^* : w \text{ contains at least two 1's}\}$

$$(0+1)^* 1 (0+1)^* 1 (0+1)^*$$

⑦ $L = \{w \in \{0,1\}^* : w \text{ contains exactly two 1's}\}$

$$0^* 1 0^* 1 0^*$$

⑧ $L = \{w \in \{0,1\}^* : w \text{ contains at most two 1's}\}$

$0^* + 0^*10^* + 0^*10^*10^*$

or, $0^*(\epsilon+1)0^*(\epsilon+1)0^*$

(9) $L = \{w \in \{0,1\}^* : \text{length of } w \text{ is even / multiple of 2}\}$

0, 2, 4, 6.

$((0+1)(0+1))^* \longrightarrow$

(10) $L = \{w \in \{0,1\}^* : \text{length of } w \text{ is odd}\}$

1, 3, 5, 7, 9, ...

$(0+1)((0+1)(0+1))^*$

$\epsilon, 2, 4 \longrightarrow 1, 3, 5,$

(11) $L = \{w \in \{0,1\}^* : \text{length of } w \text{ is multiple of 3}\}$

$((0+1)(0+1)(0+1))^*$

(12) $L = \{w \in \{0,1\}^* : \text{length of } w \text{ is } \boxed{\text{not}} \text{ multiple of 3}\}$

$((0+1)(0+1)(0+1))^*(0+1)\{\epsilon + 0 + 1\}$

$= ((0+1)(0+1)(0+1))^*(0+1+00+01+10+11)$

(13) $L = \{w \in \{0,1\}^* : \text{Number of 1's in } w \text{ is multiple of 3}\}$

$0^* + (0^*10^*10^*10^*)^*$

or, $0^*(0^*10^*10^*10^*)^*$

(14) $L = \{w \in \{0,1\}^* :$ w starts and ends with different symbol$\}$

$$0(0+1)^*1 + 1(0+1)^*0$$

(15) $L = \{w \in \{0,1\}^* :$ w starts and ends with same symbol$\}$

$$0(0+1)^*0 + 1(0+1)^*1 + 0 + 1$$

(16) $L = \{w \in \{0,1\}^* :$ w doesn't end with 01$\}$

$$(0+1)^*(00+11+10) + 0 + 1 + \epsilon$$

(17) $L = \{w \in \{0,1\}^* :$ w doesn't contain 00$\}$

$$(1^*(01)^*)^* + (1^*(01)^*)^*0$$

$$\Rightarrow (1^*(01)^*)^*(0+\epsilon)$$

$$(1+01)^* + (1+01)^*0$$

$$\Rightarrow (1+01)^*(\epsilon+0)$$

(18) $L = \{w \in \{0,1\}^* :$ w doesn't contain 10$\}$

$0^* 1^*$

(19) $L = \{w \in \{0,1\}^* : w$ doesn't contain 01 $\}$

$1^* 0^*$

(20) $L = \{w \in \{0,1\}^* : w$ contains 0 in every 3rd position$\}$

✓ $((0+1)(0+1)0)^* (0+1)(0+1) +$

✓ $((0+1)(0+1)0)^* +$

✓ $((0+1)(0+1)0)^* (0+1)$

Or,

$$((0+1)(0+1)0)^* \left(\underline{(0+1)(0+1)} + \epsilon + \underline{(0+1)} \right)$$

$(\quad \quad \quad \quad \epsilon) + \epsilon$

$$\Rightarrow \left((0+1)(0+1)0\right)^* \left((0+1)(0+1+\epsilon) \ldots\right)$$

$$\Rightarrow \left((0+1)(0+1)0\right)^* \underline{(0+1+\epsilon)}\,\underline{(0+1+\epsilon)}$$