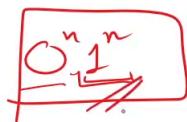


Context-Free Grammar

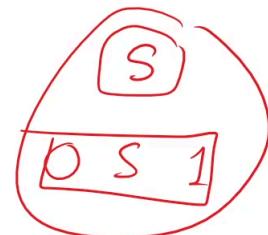
Saturday, November 26, 2022 9:49 PM

$$\begin{array}{|c|} \hline S \rightarrow 0S1 \\ S \rightarrow \epsilon \\ \hline \end{array}$$



$$\frac{0^* 1^*}{S} = \{0^{11}, 000^{11}, \dots\}$$

$$\begin{array}{c} S \\ \Rightarrow 0[S]1 \\ \Rightarrow 00[\square]11 \\ \Rightarrow \underline{\underline{000S111}} \\ \underline{\underline{000111}} \end{array}$$



Context-Free Grammar

Saturday, November 26, 2022 9:49 PM

$$\begin{array}{|c|} \hline S \rightarrow 0S1 \\ S \rightarrow \epsilon \\ \hline \end{array}$$



$$0^{n+2n}$$

$$\underline{\underline{000}} \quad \underline{\underline{111111}}$$

$$\begin{array}{c} S \rightarrow 0S11 \\ S \rightarrow \epsilon \end{array}$$

$$0^{3n} 1^{5n}$$

$$\begin{array}{c} S \rightarrow 000S11111 \\ S \rightarrow \epsilon \end{array}$$

On CFG | Derivation of String in a Grammar

Saturday, November 26, 2022 9:49 PM

$$A \rightarrow OA1 \mid B1$$

$$B \rightarrow 00$$

$$\begin{array}{l} A \\ \Rightarrow B1 \\ \Rightarrow 001 \\ \hline \end{array}$$

$$0^{n+1} 1^n$$

$$\begin{array}{l} A \\ \Rightarrow OA1 \\ \Rightarrow OOA11 \\ \Rightarrow OOB111 \\ \Rightarrow \underline{OOO111} \end{array}$$

Context-Free Grammar

Saturday, November 26, 2022 9:49 PM

$$A \rightarrow OA1 \mid B1$$

$$B \rightarrow 00$$

$$\begin{array}{l} A \\ \Rightarrow B1 \\ \Rightarrow 001 \\ \hline \end{array}$$

$$\Sigma = \{0, 1\}$$

$$\begin{array}{l} A \\ \Rightarrow OA1 \\ \Rightarrow OOA11 \\ \Rightarrow OOB111 \\ \Rightarrow \underline{OOO111} \end{array}$$

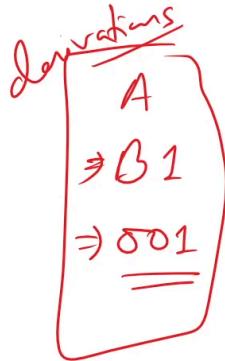
4 tuple $\langle V, T, P, S \rangle$
non/
variables
terminal
start symbol

$$A \rightarrow OA1 \mid B1$$

$$B \rightarrow 00$$

I

$$\Sigma = \{0, 1\}$$



$$\begin{aligned} A &\Rightarrow \underline{\underline{OA1}} \\ &\Rightarrow 0 OA1 1 \\ &\Rightarrow 00 B1 11 \\ &\Rightarrow \underline{\underline{0000111}} \end{aligned}$$

4 tuple $\langle V, T, P, S \rangle$

\downarrow

non-variables terminals

start symbol

$$S \rightarrow OS1$$

$$V = \{S, A\}$$

$$S \rightarrow A$$

$$T = \{0, 1, \#\}$$

S

$$A \rightarrow \#$$

S

$$\Rightarrow OS1$$

$$\Rightarrow 0 OS1 1$$

$$\Rightarrow 00 A11$$

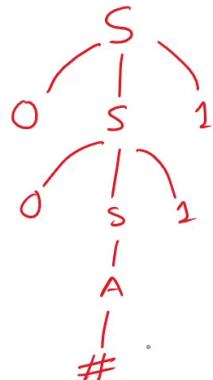
$$\Rightarrow 00 \# 11$$

$$0^n \# 1^n$$

$$S \rightarrow OS1$$

$$S \rightarrow A$$

$$A \rightarrow \#$$



$$V = \{S, A\}$$

$$T = \{0, 1, \#\}$$

S

$$\begin{aligned}
 &\Rightarrow OS1 \\
 &\Rightarrow O(OS1)1 \\
 &\Rightarrow OOA11 \\
 &\Rightarrow OO\#11
 \end{aligned}$$

$$S \rightarrow AOB$$

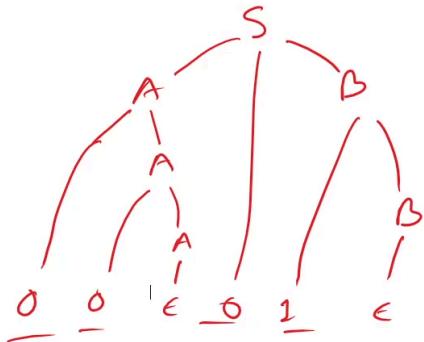
$$A \rightarrow OA | \epsilon$$

$$B \rightarrow LB | \epsilon$$

~~rightmost~~ leftmost derivation

$$\begin{aligned}
 &\Rightarrow AOB \\
 &\Rightarrow OAOB \\
 &\Rightarrow OOAOB \\
 &\Rightarrow OOOB \\
 &\Rightarrow OOO1B \\
 &\Rightarrow OOO1
 \end{aligned}
 \quad \quad \quad 0^+ 1^*$$

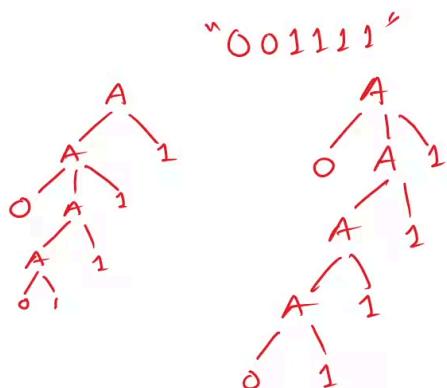
$$\begin{aligned} S &\rightarrow AOB \\ A &\rightarrow OA \mid \epsilon \\ B &\rightarrow LB \mid \epsilon \end{aligned}$$



~~rightmost~~ leftmost derivation

$$\begin{aligned} I &\quad S \\ &\Rightarrow AOB \\ &\Rightarrow OA\epsilon B \\ &\Rightarrow OOA\epsilon B \\ &\Rightarrow OOOB \\ &\Rightarrow OOO1B \\ &\Rightarrow OOO1 \end{aligned}$$

Antiquity $A \rightarrow A1 \mid OA1 \mid 01$



$$\begin{aligned} A & \\ \Rightarrow A1 & \\ \Rightarrow OA1 & \\ \Rightarrow OA11 & \\ \Rightarrow O\cancel{A}111 & \\ \Rightarrow "001111" & \end{aligned} \qquad \begin{aligned} A & \\ \Rightarrow OA1 & \\ \Rightarrow OA11 & \\ \Rightarrow O\cancel{A}111 & \\ \Rightarrow "001111" & \end{aligned}$$

CFG | Regular Operations

Saturday, November 26, 2022 9:49 PM

OR , Concatenation, Kleene Closure.

$0^n 1^n$

$S \rightarrow 0S1 \mid \epsilon$

$1^n 0^n$

$S \rightarrow 1S0 \mid \epsilon$

$ny_1 \mid ny_2$

$S \rightarrow S_a \mid S_b$
$S_a \rightarrow 0S_a1 \mid \epsilon$
$S_b \rightarrow 1S_b0 \mid \epsilon$

Context-Free Grammar

Saturday, November 26, 2022

9:49 PM

OR , Concatenation, Kleene Closure.

$0^n 1^n$

$S \rightarrow 0S1 \mid \epsilon$

$1^n 0^n$

$S \rightarrow 1S0 \mid \epsilon$

0000111 1100

$ny_1 \mid ny_2$

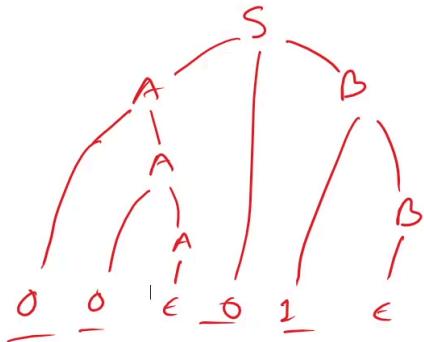
$S \rightarrow S_a S_b$

$S_a \rightarrow 0S_a1 \mid \epsilon$

$S_b \rightarrow 1S_b0 \mid \epsilon$

$\Rightarrow \begin{matrix} S \\ \downarrow \\ S_a S_b \\ \downarrow \quad \downarrow \\ 0000111 1100 \end{matrix}$

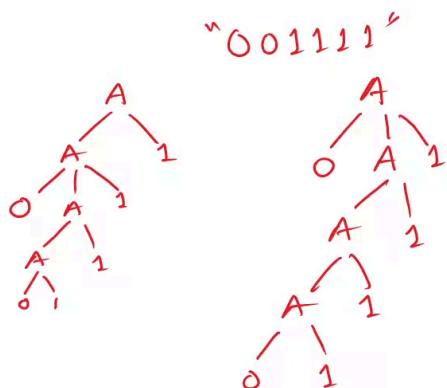
$$\begin{aligned} S &\rightarrow AOB \\ A &\rightarrow OA \mid \epsilon \\ B &\rightarrow LB \mid \epsilon \end{aligned}$$



~~rightmost~~ leftmost derivation

$$\begin{aligned} I & \quad S \\ & \Rightarrow AOB \\ & \Rightarrow OA\epsilon B \\ & \Rightarrow OOA\epsilon B \\ & \Rightarrow OOOB \\ & \Rightarrow OOO1B \\ & \Rightarrow OOO1 \end{aligned}$$

Antiquity $A \rightarrow A1 \mid OA1 \mid O1$



$$\begin{aligned} A & \\ \Rightarrow A1 & \\ \Rightarrow OA1 & \\ \Rightarrow OA11 & \\ \Rightarrow O\underline{A}111 & \\ \Rightarrow "001111" & \end{aligned}$$

$$\begin{aligned} A & \\ \Rightarrow OA1 & \\ \Rightarrow OA11 & \\ \Rightarrow O\underline{A}111 & \\ \Rightarrow "001111" & \end{aligned}$$

CFG | Regular Operations

Saturday, November 26, 2022 9:49 PM

OR , Concatenation, Kleene Closure.

$0^n 1^n$

$S \rightarrow 0S1 \mid \epsilon$

$1^n 0^n$

$S \rightarrow 1S0 \mid \epsilon$

$ny_1 \mid ny_2$

$S \rightarrow S_a \mid S_b$
$S_a \rightarrow 0S_a1 \mid \epsilon$
$S_b \rightarrow 1S_b0 \mid \epsilon$

Context-Free Grammar

Saturday, November 26, 2022

9:49 PM

OR , Concatenation, Kleene Closure.

$0^n 1^n$

$S \rightarrow 0S1 \mid \epsilon$

$1^n 0^n$

$S \rightarrow 1S0 \mid \epsilon$

0000111 1100

$ny_1 \mid ny_2$

$S \rightarrow S_a S_b$

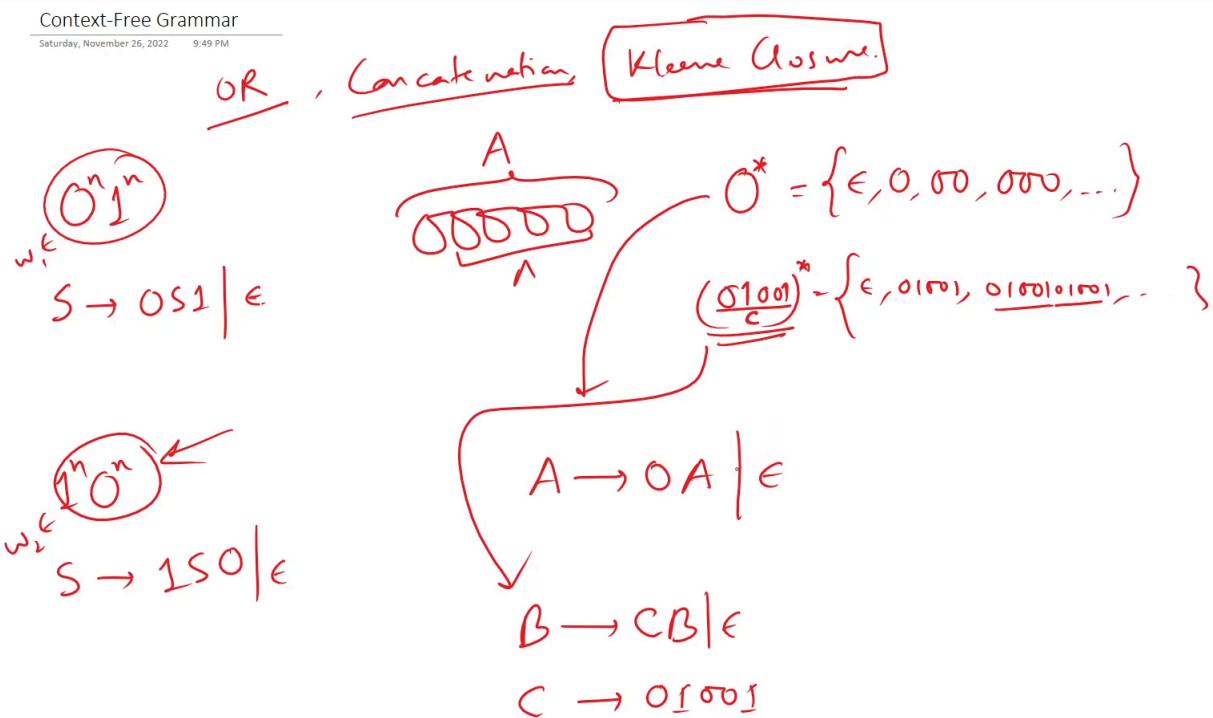
$S_a \rightarrow 0S_a1 \mid \epsilon$

$S_b \rightarrow 1S_b0 \mid \epsilon$

$\Rightarrow \begin{matrix} S \\ \downarrow \\ S_a S_b \\ \downarrow \quad \downarrow \\ 0000111 1100 \end{matrix}$

Context-Free Grammar

Saturday, November 26, 2022 9:49 PM



CFG I: Regular Operations

Saturday, November 26, 2022 9:49 PM

$$\frac{(0|10)^\infty}{A} \mid \frac{10^*}{B}$$

$$S \rightarrow A | B$$

$$B \rightarrow 1C$$

$$C \rightarrow 0C | \epsilon$$

$$A \rightarrow DA | \epsilon$$

$$D \rightarrow 0|10$$

Context-Free Grammar

Saturday, November 26, 2022 9:49 PM

$0^n 1 2^n$

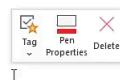
$$S \rightarrow 0S2 \mid 1$$

$\frac{0^n 1^n 2}{S}$

$$A \rightarrow S2$$

$$S \rightarrow 0S1 \mid \epsilon$$

A
 $\Rightarrow S2$
 $\Rightarrow 0001112$



CFG Examples

Syntax Tree Examples

Saturday, November 26, 2022 9:49 PM

$0^n 1 2^n$

$0^n 1^{n+1}$

$$S \rightarrow 0S2 \mid 1$$

$0^n 1^n$

$\frac{0^n 1^n 2}{S}$

$$A \rightarrow S2$$

$$S \rightarrow 0S1 \mid \epsilon$$

Context-Free Grammar Examples

Saturday, November 26, 2022 9:49 PM

$$0^n 1 2^n$$

$$S \rightarrow 0S2 \mid 1$$

$$0^n 1^{n+1}$$

$$0^n 1^n 1$$

$$\underbrace{0^n 1^n}_S 2$$

$$A \rightarrow S2$$

$$S \rightarrow 0S1 \mid \epsilon$$

Context-Free Grammar

Saturday, November 26, 2022 9:49 PM

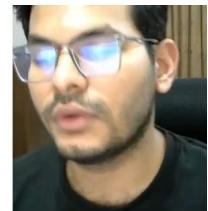
$$0^n 1 2^n$$

$$S \rightarrow 0S2 \mid 1$$

$$0^m 1^n 2^k$$

$$k = m+n$$

$$0^m 1^n 2^k 2^m$$



$$\underbrace{0^n 1^n}_S 2$$

$$S \rightarrow 0S2 \mid X$$

$$S \Rightarrow 0S2$$

$$\Rightarrow 00S22$$

$$\Rightarrow \underline{00} \underline{X} \underline{22}$$

$$\Rightarrow 001X222$$

$$\Rightarrow 00111X2222$$

$$\underline{\underline{+0011122222}}$$

$$A \rightarrow S2$$

$$S \rightarrow 0S1 \mid \epsilon$$

$$X \rightarrow 1X2 \mid \epsilon$$

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Context-Free Grammar

Saturday, November 26, 2022 9:49 PM

$$0^n 1^m 2^n$$

$$S \rightarrow 0S2 \mid 1$$

$$\begin{array}{c} 0^n 1^m 2^n \\ \hline S \end{array}$$

$$A \rightarrow S2$$

$$S \rightarrow 0S1 \mid \epsilon$$

$$0^m 1^n 2^k \quad n = m+k$$

$$0^m 1^{m+k} 2^k$$

$$\begin{array}{c} 0^m 1^m 1^k 2^k \\ \hline \hline \end{array}$$

$$S \rightarrow A\beta$$

$$A \rightarrow 0A1 \mid \epsilon$$

$$\beta \rightarrow 1\beta 2 \mid \epsilon$$

Context-Free Grammar

Saturday, November 26, 2022 9:49 PM

$$0^m 1^m \quad m \geq 2$$

$$S \rightarrow \underline{00} \underline{A} \underline{11}$$

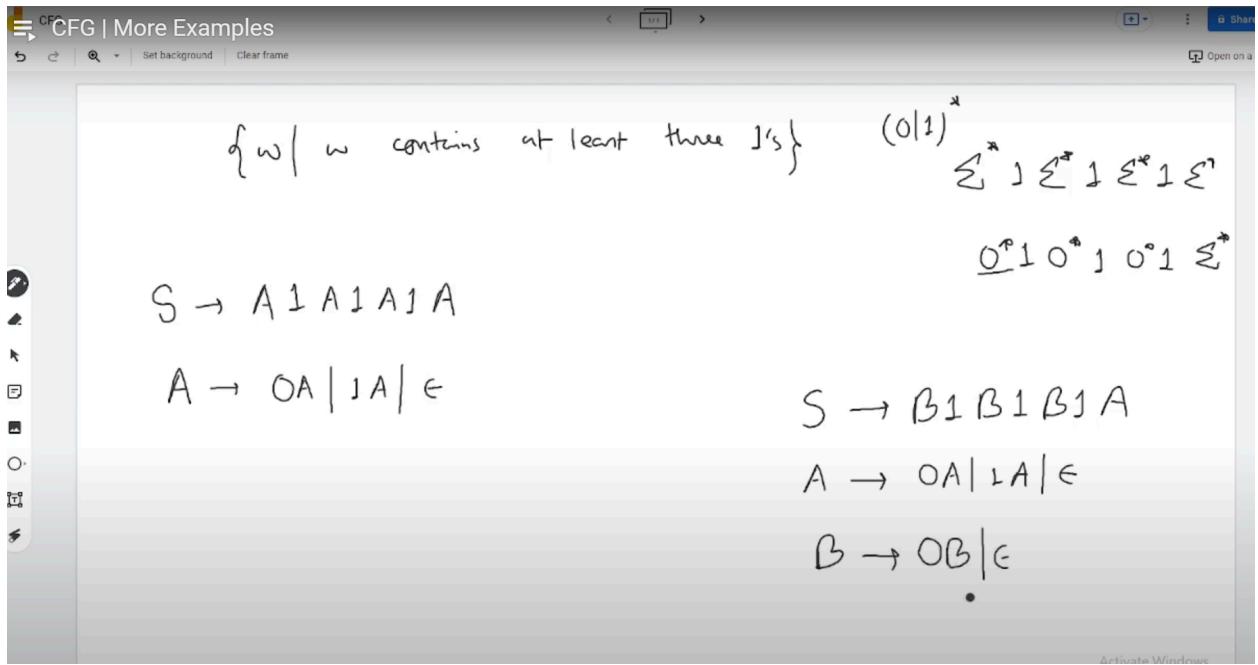
$$A \rightarrow 0A1 \mid \epsilon$$

$$0^m 1^n \quad m > n$$

$$S \rightarrow 0S1 \mid A$$

$$A \rightarrow 0A \mid 0$$

$$\begin{array}{l} S \\ \Rightarrow 0S1 \\ \Rightarrow 00S11 \\ \Rightarrow 000S111 \\ \downarrow \\ 0^+ \quad \underline{00}^+ \end{array}$$



$\{ w \mid w \text{ starts and ends with same symbol} \}$ $\Sigma = \{0, 1\}$

$0 \in^* 0 \mid 1 \in^* 1$

$S \rightarrow 0 A 0 \mid 1 A 1$

$A \rightarrow 0A \mid 1A \mid \epsilon$

$\{\omega \mid \text{the length of } \omega \text{ is odd}\}$

$\underbrace{\Sigma (\Sigma \Sigma)^*}_{(0|1)}$

$S \rightarrow AB$

$A \rightarrow 0|1$

$B \rightarrow AAB \mid \epsilon$

$\{\omega \mid \text{the length of } \omega \text{ is odd and its middle symbol is } 0\}$

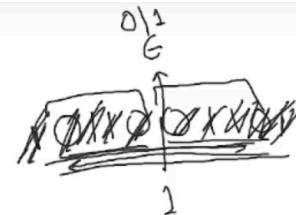
$S \rightarrow 0S0 \mid 1S1 \mid 0S1 \mid \underline{1S0} \mid 0$

~~100
011 0 011~~
~~100 0 10~~

S
 $\Rightarrow \{ S1$
 $\quad \quad \quad \overbrace{0S01}$
 $\quad \quad \quad 101 \underline{S101}$
 $\quad \quad \quad \overbrace{101 \quad \overbrace{0010}}$

Activate Windows
Go to Settings to activate V

$\{ w \mid w = w^R, \text{ i.e. } w \text{ is a palindrome} \}$



$$S \rightarrow 0S0 \mid 1S1 \mid \epsilon \mid 1 \mid 0$$



Context-Free Grammar

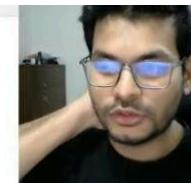
Saturday, November 26, 2022 5:49 PM

$$\hookrightarrow w \in \Sigma^* \quad \Sigma = \{0, 1\}$$

#0's in $w =$ #1's in w

$$S \rightarrow 0S1S \mid 1S0S \mid \epsilon$$

$$S \rightarrow S0S1 \mid S1S0 \mid \epsilon$$



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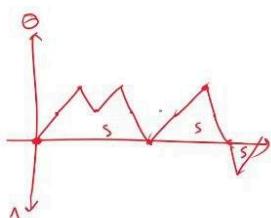
Context-Free Grammar

Saturday, November 26, 2022 5:49 PM

$$\hookrightarrow w \in \Sigma^* \quad \Sigma = \{0, 1\}$$

#0's in $w =$ #1's in w

$$\underline{\quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 1 \quad}, \underline{\quad 1 \quad 0 \quad 1 \quad 1 \quad 1 \quad 0 \quad}$$



Activate Windows
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$\#0's \text{ in } w = \#1's \text{ in } w$

$$S \rightarrow OS1 | 1S0 | SS | \epsilon$$

$$\begin{array}{l}
 \underline{001110} \\
 \downarrow S \\
 \Rightarrow SS \\
 \Rightarrow OS1 \downarrow S \\
 \Rightarrow OOS1 \downarrow S \\
 \Rightarrow O011 \downarrow S \\
 \Rightarrow O011 | 1S0 \\
 \Rightarrow \boxed{O01110}
 \end{array}$$

Activate Windows
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$\#0's \text{ in } w = \#1's \text{ in } w$

$$S \rightarrow OS1 | 1S0 | SS | \epsilon$$

$$S \rightarrow SOS1 | S1S0 | \epsilon$$

$$S \rightarrow OS1 | 1S0 | SS | \epsilon$$

$$S \rightarrow OAS | 1BS | \epsilon$$

$$A \rightarrow 1 | OAA$$

$$\beta \rightarrow O | 1BB$$

$$00011011$$

Activate Windows
Go to Settings to activate Windows.

$$S \rightarrow OS1 | 1S0 | SS | \epsilon$$

$$S \rightarrow SOS1 | S1S0 | \epsilon$$

$$S \rightarrow OS1 | 1S0 | SS | \epsilon$$

$$S \rightarrow OAS | 1BS | \epsilon$$

$$A \rightarrow 1 | OAA$$

$$\beta \rightarrow O | 1BB$$

$$00011011$$

Activate Windows
Go to Settings to activate Windows.

$$S \rightarrow OS1 | 1S0 | SS | \epsilon$$
$$S \rightarrow SOS1 | S1S0 | \epsilon$$
$$S \rightarrow OAS | 1BS | \epsilon$$
$$A \rightarrow 1 | OAA$$
$$B \rightarrow 0 | LB\beta$$

00011011

$S \Rightarrow OAS$
 $\Rightarrow 00AAS$
 $\Rightarrow 000AAAS$
 $\Rightarrow 0001AAS$
 $\Rightarrow 00011AS$
 $\Rightarrow 000110AAS$
 $\Rightarrow \underline{00011011}$

