Assignment 02: Polynomial Interpolation

Course Code: CSE330	Course Name: Numerical Methods	
Faculty: SADF	Submission Date: 17/07/25	Total Marks: 15
Student ID: 21301289	Section: 08	Marks Achieved:
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Instructions: Answer all questions. Show all your work and reasoning clearly.		

Question 1: Vandermonde matrix and Lagrange Polynomial

Consider the following data-table:

Time (sec) t	Velocity (m/s) $v(t)$
2	10
4	20
6	25

a) **[4+1 marks]** Find an interpolating polynomial of velocity that passes through the above data points using the **Vandermonde Matrix Method**. From the polynomial approximate the acceleration by differentiating $v(t)$. Now, compute an approximate value of the **acceleration** at time $t = 7$ seconds.

b) **[4 marks]** Find an interpolating polynomial of velocity that passes through the above data points using the **Lagrange method**.

c) **[1 mark]** If a new data point is added to the above scenario, which method would be more suitable for constructing a new interpolating polynomial? Also, what will be the **degree** of the new polynomial?

Question 2: Newton's divided-difference method

(a) **[2 marks]** Consider the nodes $\left[-\frac{\pi}{2}, 0, \frac{\pi}{2}\right]$. Find an interpolating polynomial of appropriate degree using **Newton's divided-difference method** for the function -

$$f(x) = x\sin(x)$$

(b) **[1 marks]** Use the interpolating polynomial to find an approximate value at $x = \frac{\pi}{4}$, and compute the **percentage relative error** at $x = \frac{\pi}{4}$.

$$\text{Percentage Relative Error} = \left(\frac{|f(x) - P_n(x)|}{|f(x)|}\right) \times 100\%$$

(c) **[2 marks]** Add a new node $x = \pi$ to the above set of nodes, and find the interpolating polynomial of appropriate degree.

Assignment-02 (CSE330)

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Ans. to the Q. no. 1

a) We assume a quadratic interpolating polynomial

$$V(t) = a_0 + a_1 t + a_2 t^2$$

We use the vandermonde matrix formed from the time values

$$\begin{bmatrix} 1 & 2 & 4 \\ 1 & 4 & 16 \\ 1 & 6 & 36 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \\ 25 \end{bmatrix}$$

from solving this system, we get,

$$a_0 = 5, \quad a_1 = 5, \quad a_2 = 0$$

So, the interpolating polynomial ~~is~~, is,

$$V(t) = 5 + 5t. \text{ Ans}$$

Now, Acceleration at $t = 7$ seconds

$$a(t) = \frac{dv}{dt} = 5$$

∴ The acceleration at $t = 7$ is: $5\ ms^{-2}$. Ans.

b)

Using Lagrange basis polynomial:

$$V(t) = V_0 L_0(t) + V_1 L_1(t) + V_2 L_2(t)$$

where, $V_0 = 10, \ t_0 = 2$
$V_1 = 20, \ t_1 = 4$
$V_2 = 25, \ t_2 = 6$

Lagrange basis polynomials;

$$L_0(t) = \frac{(t-4)(t-6)}{(2-4)(2-6)} = \frac{(t-4)(t-6)}{8}$$

$$L_1(t) = \frac{(t-2)(t-6)}{(4-2)(4-6)} = -\frac{(t-2)(t-6)}{4}$$

$$L_2(t) = \frac{(t-2)(t-4)}{(6-2)(6-4)} = \frac{(t-2)(t-4)}{8}$$

$$\therefore V(t) = 10 \cdot \frac{(t-4)(t-6)}{8} - 20 \cdot \frac{(t-2)(t-6)}{4} + 25 \cdot \frac{(t-2)(t-4)}{8}$$

c) If a new data point is added, the Largange method becomes less ~~than~~ efficient because the entire polynomial must be recalculated from scratch.

The Vandermonde method also requires reconstructing and solving a larger system.

However, Newton's divided difference method is more suitable for updating interpolation when new data points are added.

Degree of the new polynomial will be;

$(n-1)$

currently : 3 data points $\longrightarrow$ degree 2

After adding 1 more point : 4 data points $\longrightarrow$ degree 3

Ans. to the Q. no.2

a) Given nodes:

$$x_0 = -\frac{\pi}{2}, \quad x_1 = 0, \quad x_2 = \frac{\pi}{2}$$

$$\therefore f(x) = x \sin(x)$$

$$\therefore f(x_0) = -\frac{\pi}{2} \cdot \sin\left(-\frac{\pi}{2}\right)$$

$$= -\frac{\pi}{2} \cdot (-1)$$

$$= \frac{\pi}{2}$$

$$\therefore f(x_1) = 0 \cdot \sin(0)$$

$$= 0$$

$$\therefore f(x_2) = \frac{\pi}{2} \cdot \sin\left(\frac{\pi}{2}\right)$$

$$= \frac{\pi}{2} \cdot 1$$

$$= \frac{\pi}{2}$$

~~let's~~ denote,

$$f|x_0| = f(x_0) = \frac{\pi}{2}$$

$$f|x_1| = 0$$

$$f|x_2| = \frac{\pi}{2}$$

first order differces.

$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0} = \frac{0 - \frac{\pi}{2}}{0 + \frac{\pi}{2}} = -1$$

$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} = \frac{\frac{\pi}{2} - 0}{\frac{\pi}{2} - 0} = 1$$

second order difference,

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} = \frac{1 - (-1)}{\pi} = \frac{2}{\pi}$$

Now, construct the Newton's Polynomial.

$$P_2(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$$

$$\therefore P_2(x) = \frac{\pi}{2} - 1 \cdot \left(x + \frac{\pi}{2}\right) + \frac{2}{\pi} \cdot \left(x + \frac{\pi}{2}\right) \cdot x$$

b)

Approximate value at $x = \frac{\pi}{4}$:

$$\therefore P_2\left(\frac{\pi}{4}\right) = \frac{\pi}{2} - \left(\frac{\pi}{4} + \frac{\pi}{2}\right) + \frac{2}{\pi} \cdot \left(\frac{\pi}{4} + \frac{\pi}{2}\right) \cdot \frac{\pi}{4}$$

$$= \frac{\pi}{2} - \frac{3\pi}{4} + \frac{2}{\pi} \cdot \frac{3\pi}{4} \cdot \frac{\pi}{4}$$

$$= -\frac{\pi}{4} + \frac{2}{\pi} \cdot \frac{3\pi^2}{16}$$

$$= -\frac{\pi}{4} + \frac{3\pi}{8}$$

$$= \frac{\pi}{8}$$

$$\therefore P_2\left(\frac{\pi}{4}\right) = \frac{\pi}{8}$$

Now compute the actual,

$$f\left(\frac{\pi}{4}\right) = \frac{\pi}{4} \cdot \sin\left(\frac{\pi}{4}\right)$$

$$= \frac{\pi}{4} \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{\pi\sqrt{2}}{8}.$$

$$\text{Percentage Relative error} = \left| \frac{f(x) - P_2(x)}{f(x)} \right| \times 100$$

$$= \left| \frac{\sqrt{2}-1}{\sqrt{2}} \right| \times 100$$

$$\approx 29.29\%$$

c)

Add new node, $x_3 = \pi$.

Compute, $f(\pi) = \pi \sin(\pi)$

$= \pi . 0$

$= 0$

Now, nodes are,

$x_0 = -\frac{\pi}{2}, \quad x_1 = 0, \quad x_2 = \frac{\pi}{2}, \quad x_3 = \pi$

$\therefore f(x_0) = \frac{\pi}{2}, \quad f(x_1) = 0, \quad f(x_2) = \frac{\pi}{2}, \quad f(x_3) = 0$

$$\therefore P_3(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$$
$$+ f[x_0, x_1, x_2, x_3](x - x_0)(x - x_1)(x - x_2)$$