Ambiguity

same string $\longleftarrow$ way 1 $\Big\}$ $\rightarrow$ different ways to generate a string from a CFG
way 2

different parse tree ✓
different derivation ✗

Derivation $\Big\langle$ leftmost derivation
right most derivation

$$
\boxed{
\begin{array}{l}
S \longrightarrow AB \\
A \longrightarrow a \\
B \longrightarrow b
\end{array}
}
\longrightarrow CFG
$$

string : ab

Leftmost derivation

$S \longrightarrow AB \longrightarrow aB \longrightarrow ab$

Rightmost derivation

$S \longrightarrow AB \longrightarrow Ab \longrightarrow ab$

S
/＼
A   B
| |
a   b

S
/＼
A   B
| |
a   b
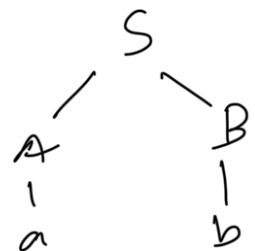
different parse trees $\Big\}$ $\rightarrow$ The CFG
different leftmost derivations

is ambiguous

Example 1

$$S \longrightarrow S+S \mid S*S \mid a \mid b$$

Given the context free grammar, answer the following questions.

a) Give a left most derivation fore the string $a + a * b$

b) sketch a parse tree corresponding to the derivation you gave in (a)

c) Give a rightmost derivation for the string $a + a * b$.

d) sketch a parse tree corresponding to the derivation you gave in (c)

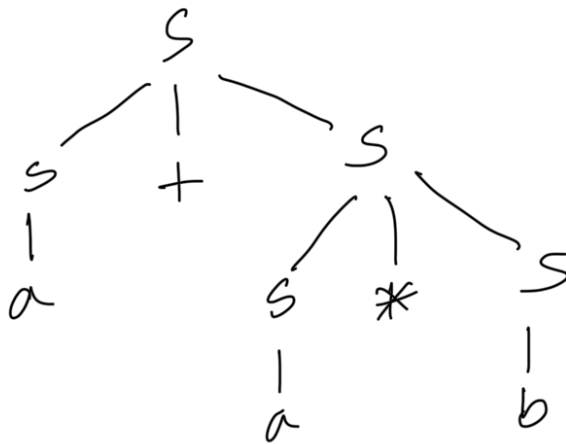e) Demonstrate one more parse tree (apart from the one you already found in

(b) & (d))

$S \rightarrow S \underline{+} S \mid S \underline{*} S) \ a \mid b$

a) leftmost derivation of $a \underset{1}{\underline{+}} a \underset{2}{\underline{*}} b$

$$S \longrightarrow S + S$$
$$\rightarrow a + S$$
$$\rightarrow a + S * S$$
$$\rightarrow a + a * S$$
$$\rightarrow a + a * b$$

b)

c) Rightmost derivation of $a \underset{1}{\underline{+}} a \underset{2}{\underline{*}} b$

$$S \longrightarrow S + S$$
$$\rightarrow S + S * S$$
$$\rightarrow S + S * b$$
$$\rightarrow S + a * b$$
$$\rightarrow a + a * b$$

d)

e)

$$a \underset{2}{+} a \underset{1}{*} b$$
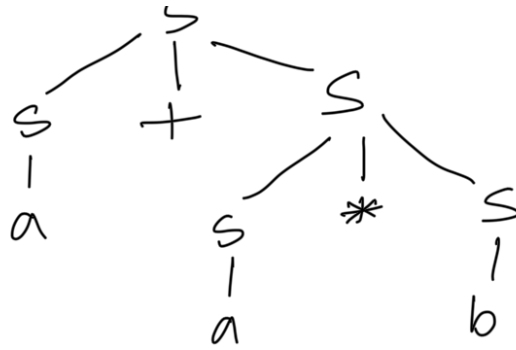
$$S \implies S*S$$
$$\implies S+S*S$$
$$\implies a+S*S$$
$$\implies a+a*S$$
$$\implies a+a*b$$

leftmost derivation
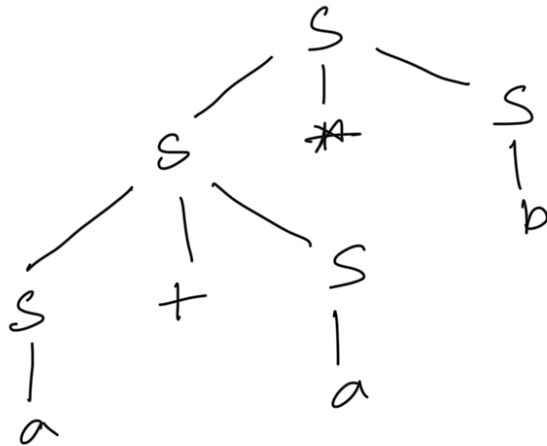
Example 2

$$S \rightarrow B1B$$
$$B \rightarrow 0B \mid 1B \mid \epsilon$$

... grammar, answer the

Given the context free grammar, solve the following questions.

a) Give a leftmost derivation fore the string (0101)

b) sketch a parse tree corresponding to the derivation you gave in (a)

c) Demonstrate one more parse tree (apart from the one you already found in (b)) for the string 0101

d) find a string w of length six such that w has exactly one parse tree in the grammar above.

$$S \rightarrow B1B$$
$$B \rightarrow 0B \mid 1B \mid \epsilon$$

a)   0101

leftmost derivation

$$S \rightarrow B \underline{1} B$$

$\rightarrow$ 0B$\underline{1}$B

$\rightarrow$ 0ϵ1B

$\rightarrow$ 01B

$\rightarrow$ 0$\underline{1}$0B

$\rightarrow$ 0$\underline{1}$01B

$\rightarrow$ 0$\underline{1}$01ϵ

$\rightarrow$ 0$\underline{1}$0$\underline{1}$

b)
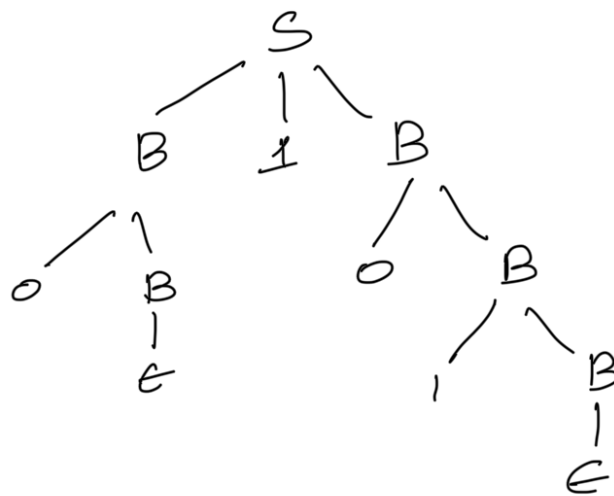
c)   0101

Another leftmost derivation

$S \longrightarrow$ B$\underline{1}$B

$\longrightarrow$ 0B$\underline{1}$B

$\longrightarrow$ 01B$\underline{1}$B

$\longrightarrow$ 010B$\underline{1}$B

$\longrightarrow$ 010$\epsilon$$\underline{1}$B

$\longrightarrow$ 0$\underline{1}$0$\underline{1}$B

$\longrightarrow$ 0101ϵ

$\longrightarrow$ 0$\underline{1}$0$\underline{1}$.

```
        S
     B  |  B
   0  B 1  |
      1 B   E
         0 B
             |
             E
```

d)

0 0 0 1 0 0 0 1 ̸X

1 0 0 0 0 0
0 1 0 0 0 0
0 0 1 0 0 0
0 0 0 1 0 0
0 0 0 0 1 0
0 0 0 0 0 1