CSE 330

Assignment - 05

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Ans. to. the. Q. on Part A

We have Known that,

$$Ax = b$$

where, A is the design matrix

b is the observation vector

x contains the unknown coefficient $[a, b]$.

$$\therefore \begin{bmatrix} 1 & 2 \\ 1 & 4 \\ 1 & 6 \\ 1 & 8 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} 15 \\ 22 \\ 28 \\ 35 \end{bmatrix}.$$

Ans. to. the. Q. on Part B

① If $x = 1$,

$$\therefore P_1 = U_1$$

$$= \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\therefore a_1 = \frac{1}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

if $u=2$

$\therefore P_2 = U_2 - (U_2^T a_1) \cdot q_1$

$$= \begin{pmatrix} 2 \\ 4 \\ 6 \\ 8 \end{pmatrix} - \left[(2 . 4 \ 6 \ 8) \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix} \right] \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 4 \\ 6 \\ 8 \end{pmatrix} - (1+2+3+4) \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 4 \\ 6 \\ 8 \end{pmatrix} - \begin{pmatrix} 5 \\ 5 \\ 5 \\ 5 \end{pmatrix}$$

$$= \begin{pmatrix} -3 \\ -1 \\ 1 \\ 3 \end{pmatrix}$$

$\therefore q_2 = \dfrac{1}{\sqrt{20}} \begin{pmatrix} -3 \\ -1 \\ 1 \\ 3 \end{pmatrix}$

$$= \begin{pmatrix} -\frac{3}{\sqrt{20}} \\ -\frac{1}{\sqrt{20}} \\ \frac{1}{\sqrt{20}} \\ \frac{3}{\sqrt{20}} \end{pmatrix}$$

$\therefore Q = (q_1 \ q_2)$

$$= \begin{pmatrix} 1/2 & -3/\sqrt{20} \\ 1/2 & -1/\sqrt{20} \\ 1/2 & 1/\sqrt{20} \\ 1/2 & 3/\sqrt{20} \end{pmatrix}$$

⑪ Now,

$$A = QR$$

$$\Rightarrow R = Q^T A.$$

$$\therefore R = \begin{bmatrix} u_1^T a_1 & u_2^T a_1 \\ 0 & u_2^T q_2 \end{bmatrix}$$

$$= \begin{bmatrix} (1\ 1\ 1\ 1)\begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix} & (2\ 4\ 6\ 8)\begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix} \\ 0 & (2,4\ 6\ 8)\begin{pmatrix} -3/\sqrt{20} \\ -1/\sqrt{20} \\ 1/\sqrt{20} \\ 3/\sqrt{20} \end{pmatrix} \end{bmatrix}$$

$$\therefore R = \begin{bmatrix} 2 & 10 \\ 0 & \sqrt{20} \end{bmatrix}$$

(iii)

given that,

$$A = QR$$

$$\therefore \Rightarrow A = \begin{pmatrix} 1/2 & -3/\sqrt{20} \\ 1/2 & -1/\sqrt{20} \\ 1/2 & 1/\sqrt{20} \\ 1/2 & 3/\sqrt{20} \end{pmatrix} \begin{pmatrix} 2 & 10 \\ 0 & \sqrt{20} \end{pmatrix}$$

$$= \begin{pmatrix} 1+0 & 5-3 \\ 1+0 & 5-1 \\ 1+0 & 5+1 \\ 1+0 & 5+3 \end{pmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 1 & 4 \\ 1 & 6 \\ 1 & 8 \end{bmatrix}$$

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Ans. to. the. Q on Part C

① given that,

$$Rx = Q^T b$$

$$= \begin{bmatrix} 1/2 & 1/2 & 1/2 & 1/2 \\ \dfrac{-3}{\sqrt{20}} & \dfrac{-1}{\sqrt{20}} & \dfrac{1}{\sqrt{20}} & \dfrac{3}{\sqrt{20}} \end{bmatrix} \begin{bmatrix} 15 \\ 22 \\ 28 \\ 35 \end{bmatrix}$$

$$\therefore Rx = \begin{bmatrix} 50 \\ 60/\sqrt{20} \end{bmatrix}$$

$$\therefore x_o = \begin{bmatrix} 50 \\ \dfrac{60}{\sqrt{20}} \end{bmatrix} R^{-1}$$

$$= \dfrac{1}{2\sqrt{20}} \begin{bmatrix} 20 & -10 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 50 \\ \dfrac{60}{\sqrt{20}} \end{bmatrix}$$

$$= \begin{bmatrix} 25 - \dfrac{330}{20} \\ \cdot \dfrac{60}{20} \end{bmatrix}$$

$$= \begin{bmatrix} 8.5 \\ 3.3 \end{bmatrix}_2$$

(11) The final best fit linear equation would be:

$$y = 8.5 + 3.3x.$$

Ans. to. the. Q. on Part D

If an athlete trains for 10 hours in a week, the performance score would be.

$$y = 8.5 + 3.3 \times 10$$
$$= 41.5$$