Regular Expression

* $0+1$
* $(0+1)^*$ / $\underline{(0^*1^*)^*}$ * $0+1$
* 0^*1^*
* $(0+1)^*1$
* $0^*1^* + (ab)^*$

7.

$$\frac{10}{0ab1}$$

* Construct Regular Expressions that generates the following languages.

① $L = \{ w \in \{0,1\}^* : w \text{ contains "101" as a substring} \}$

$$\underline{(0+1)}^*101\,\underline{(0+1)}^*$$

② $L = \{ w \in \{0,1\}^* : w \text{ starts with "101"} \}$

$$101(0+1)^*$$

③ $L = \{w \in \{0,1\}^*: w \text{ ends with "101"}\}$

$$(0+1)^* 101$$

④ $L = \{w \in \{0,1\}^*: w \text{ contains "00" or "11"}\}$

$$(0+1)^* 00 (0+1)^* + (0+1)^* 11 (0+1)^*$$

or,

$$(0+1)^* (00+11)(0+1)^*$$

⑤ $L = \{w \in \{0,1\}^*: w \text{ contains at least two 1's}\}$

$$(0+1)^* 1 (0+1)^* 1 (0+1)^*$$

⑥ $L = \{w \in \{0,1\}^*: w \text{ contains exactly two 1's}\}$

$$0^* 1 0^* 1 0^*$$

⑦ $L = \{w \in \{0,1\}^*: w \text{ contains at most two 1's}\}$

$$0^* + 0^* 1 0^* + 0^* 1 0^* 1 0^*$$

or, $0^* (\varepsilon+1) 0^* (\varepsilon+1) 0^*$

⑧ $L = \{w \in \{0,1\}^*: \text{length of } w \text{ is even/multiple of 2}\}$

0, 2, 4, 6.

$$((0+1)(0+1))^* \longrightarrow$$

9) $L = \{w \in \{0,1\}^* : \text{length of } w \text{ is odd}\}$

$1, 3, 5, 7, 9, \ldots$

$$\underline{(0+1)}\underbrace{((0+1)(0+1))^*}_{\epsilon,\ 2,\ 4} \rightarrow 1, 3, 5,$$

10) $L = \{w \in \{0,1\}^* : \text{length of } w \text{ is multiple of } 3\}$

$$((0+1)(0+1)(0+1))^*$$

11) $L = \{w \in \{0,1\}^* : \text{length of } w \text{ is } \boxed{\text{not}} \text{ multiple of } 3\}$

$$((0+1)(0+1)(0+1))^*(0+1)\{\epsilon + 0 + 1\}$$

$$= ((0+1)(0+1)(0+1))^*(0+1+00+01+10+11)$$

12) $L = \{w \in \{0,1\}^* : \text{Number of 1's in } w \text{ is multiple of } 3\}$

$$0^* + (0^*10^*10^*10^*)^*$$

or, $0^*(0^*10^*10^*10^*)^*$

13) $L = \{w \in \{0,1\}^* : w \text{ starts and ends with different symbol}\}$

$$0(0+1)^*1 + 1(0+1)^*0$$

(14) $L = \{w \in \{0,1\}^*$: w starts and ends with same symbol$\}$

$$0(0+1)^*0 + 1(0+1)^*1 + 0 + 1$$

(15) $L = \{w \in \{0,1\}^*$: w doesn't end with 01$\}$

$$(0+1)^*(00+11+10) + 0 + 1 + \epsilon$$

(16) $L = \{w \in \{0,1\}^*$: w doesn't contain 00$\}$

$$(1^*(01)^*)^* + (1^*(01)^*)^*0$$

$$\Rightarrow (1^*(01)^*)^*(0+\epsilon)$$

$$(1+01)^* + (1+01)^*0$$

$$\Rightarrow (1+01)^*(\epsilon+0)$$

(17) $L = \{w \in \{0,1\}^*$: w doesn't contain 10$\}$

$$0^*1^*$$

(18) $L = \{w \in \{0,1\}^*$: w doesn't contain 01$\}$

$$1^*0^*$$

(10) $L = \{w \in \{0,1\}^* : w \text{ contains } 0 \text{ in every 3rd position}\}$

$$\checkmark \quad \left((0+1)(0+1)0\right)^*(0+1)(0+1) \; +$$

$$\checkmark \quad \left((0+1)(0+1)0\right)^* \; +$$

$$\checkmark \quad \left((0+1)(0+1)0\right)^*(0+1)$$

Or,

$$\left((0+1)(0+1)0\right)^*\left(\underline{(0+1)(0+1)} + \epsilon + \underline{(0+1)}\right)$$

$$\Rightarrow \left((0+1)(0+1)0\right)^*\left((0+1)(0+1+\epsilon) + \epsilon\right)$$

$$\Rightarrow \left((0+1)(0+1)0\right)^* \underline{(0+1+\epsilon)}\,\underline{(0+1+\epsilon)}$$