# Final Paper: A brief introduction to Random Systems


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## Abstract

In this final paper, I will take a brief introduction to random systems, especially the random walk. I shall introduce this model in 1D,2D and 3D by using python tools. Also, I will summarize the diffusion model. Last but not least, I will simply give some other interesting models.


**[key words]** Random walks, Diffusion, Simulation


## 1 Background

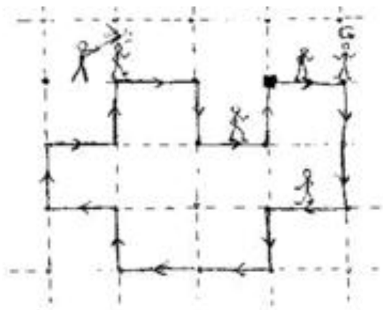
**Introduction**

A stochastic process (or random process) is a probability model used to describe phenomena that evolve over time or space. A stochastic or random process can be defined as a collection of random variables that is indexed by some mathematical set, meaning that each random variable of the stochastic process is uniquely associated with an element in the set. The set used to index the random variables is called the index set. Historically, the index set was some subset of the real line, such as the natural numbers, giving the index set the interpretation of time. Each random variable in the collection takes values from the same mathematical space known as the state space. This state space can be, for example, the integers, the real line or **n** n-dimensional Euclidean space. An increment is the amount that a stochastic process changes between two index values, often interpreted as two points in time. A stochastic process can have many outcomes, due to its randomness, and a single outcome of a stochastic process is called, among other names, a sample function or realization.

**Defination**

A stochastic process is defined as a collection of random variables defined on a common probability space ($\Omega$ ,$\mathcal{F}$,$P$), where $\Omega$ is a sample space, $\mathcal{F}$ is a $\sigma$-algebra, and $P$ is a probability measure, and the random variables, indexed by some set $T$, all take values in the same mathematical space $S$, which must be measurable with respect to some $\sigma$-algebra $\Sigma$.

# 2 Random walk




First, I shall definite random walk. A random walk is a mathematical object, known as a stochastic or random process, that describes a path that consists of a succession of random steps on some mathematical space such as the integers.
In theory, setting each step as s_i, then, we have

$$x_n = \sum_{i=1}^{n} s_i$$

$$x_n^2 = \sum_{i=1}^{n} (\sum_{j=1}^{n} s_i s_j)$$

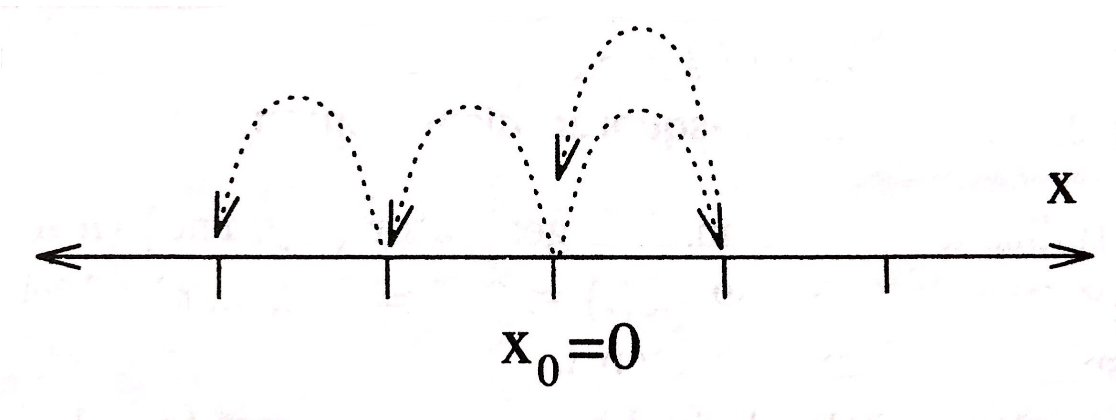
Thus, we will find

$$< x_n^2 >= \sum_{i=1}^{n} s_i^2 = n$$

This hints that $E(|S_n|)$ , the expected translation distance after n steps, should be of the order of $\sqrt{n}$. In fact,

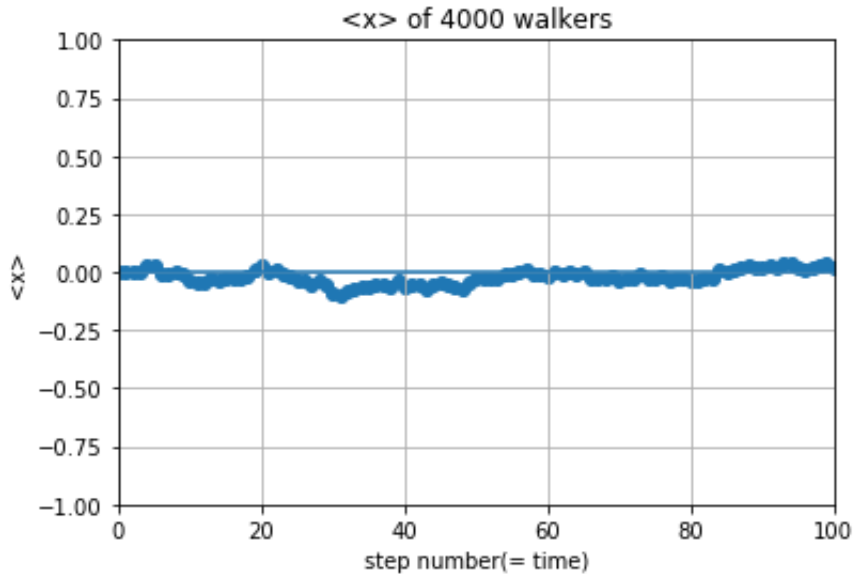
$$\lim_{n\to\infty} \frac{E(|S_n|)}{\sqrt{n}} = \sqrt{\frac{2}{\pi}}.$$

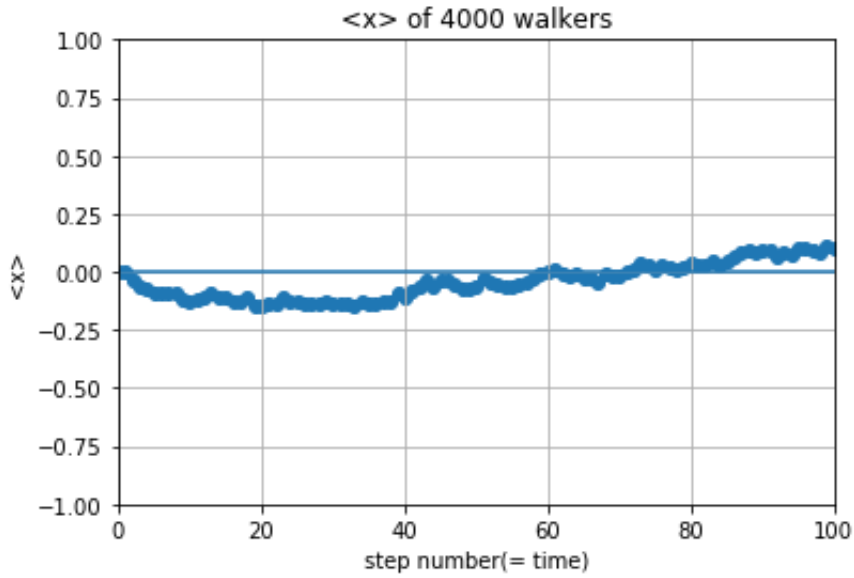
we can see a simple model of random walk.The simplist situation involves a walker that is able to take steps of length unity along a line. If we sketch of a random walk in one dimension. The walker began at x=$x_0$=0, and each step is indicated schematically by a dotted arrow. Here the first step happened to be to the right, while the next three steps were to the left, just like the picture showing below.

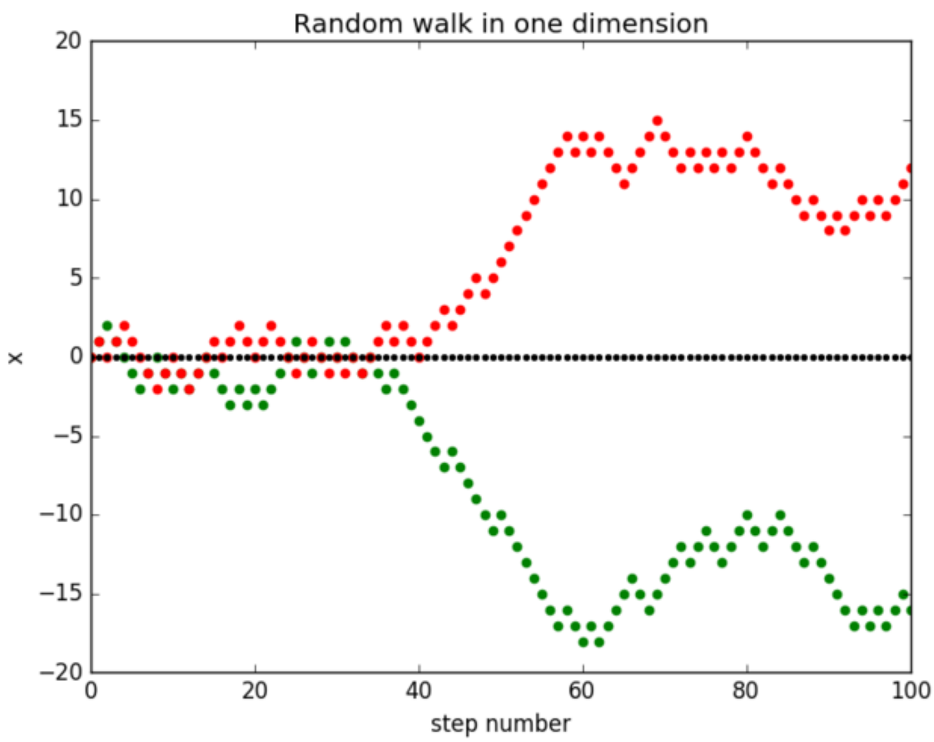



## 2.1 Random walks in one dimension

1. First of all, I shall stimulate the random walk for one dimension. I will take the step to be 4000 steps without the lossing of generality specific codes



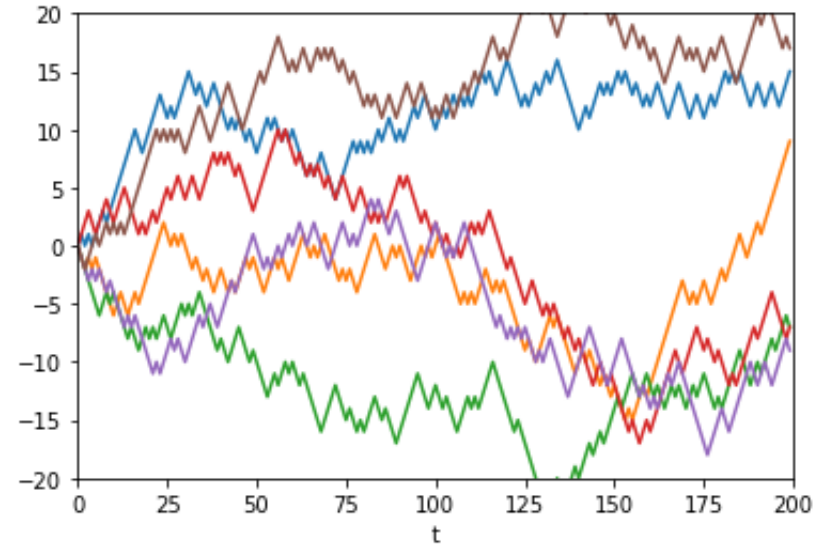


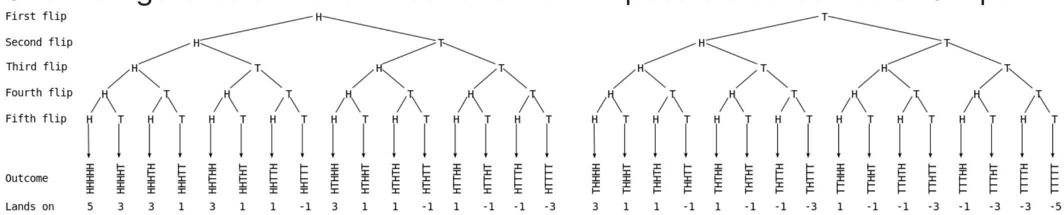

codes

As the figures showing above, the mean displacement of the walker will fluctuate near 0, which is bacause the possibility of the walker to go left or right is same according to the theory.

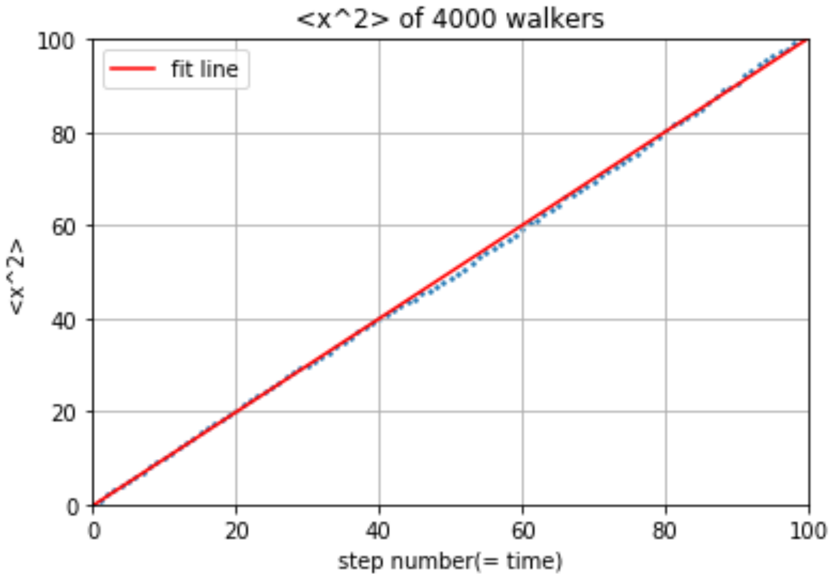
Six times stimulation of a 1D random walk. codes



From the wikipedia, we can know that This walk can be illustrated as follows. A marker is placed at zero on the number line and a fair coin is flipped. If it lands on heads, the marker is moved one unit to the right. If it lands on tails, the marker is moved one unit to the left. After five flips, the marker could now be on 1, −1, 3, −3, 5, or −5. With five flips, three heads and two tails, in any order, will land on 1. There are 10 ways of landing on 1 (by flipping three heads and two tails), 10 ways of landing on −1 (by flipping three tails and two heads), 5 ways of landing on 3 (by flipping four heads and one tail), 5 ways of landing on −3 (by flipping four tails and one head), 1 way of landing on 5 (by flipping five heads), and 1 way of landing on −5 (by flipping five tails). See the figure below for an illustration of the possible outcomes of 5 flips.



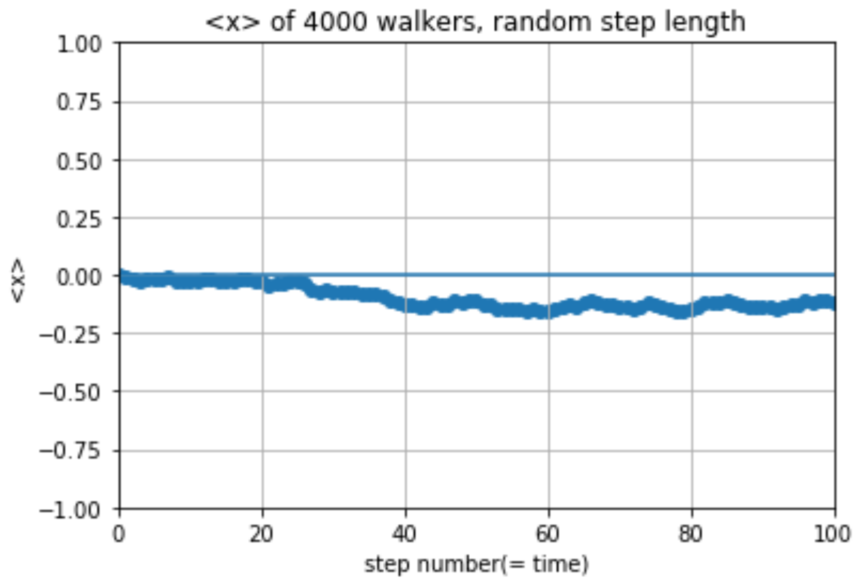

What's more, we can plot the mean value of the square of the displacement.

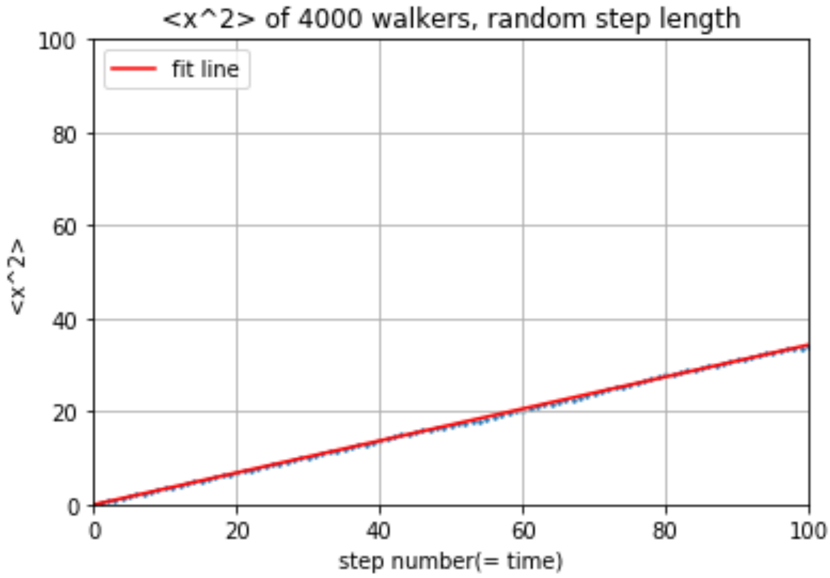



codes

As the figure showing above, given the chance that we make infinite walks every for every step,those scattering points is just the straight line.

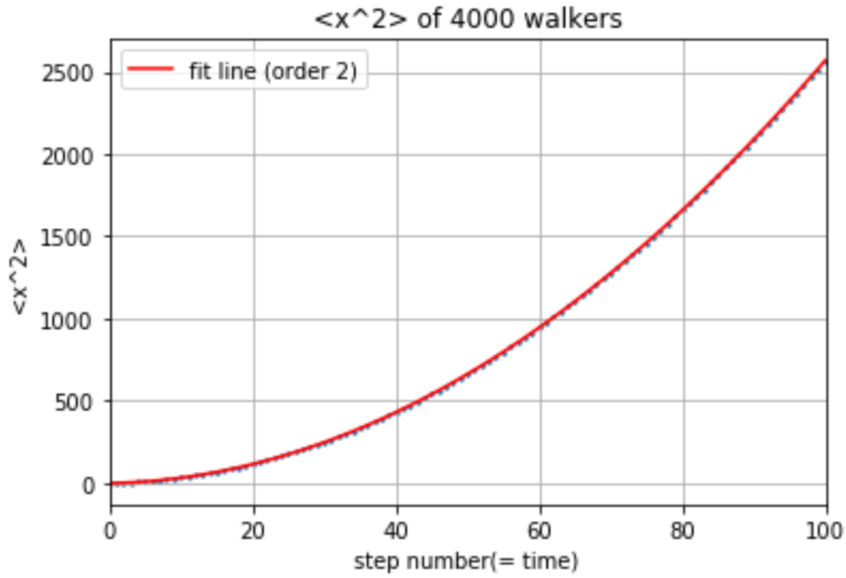
We can we generalize the situation.codes



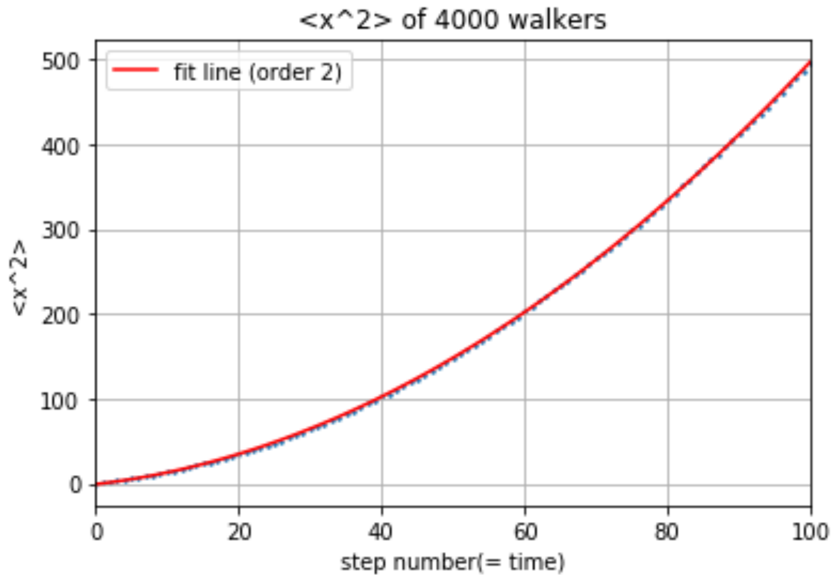


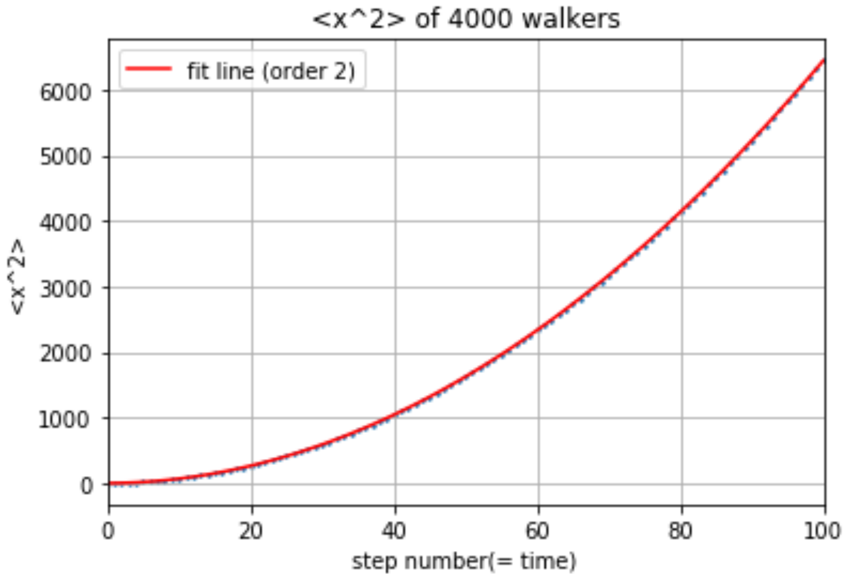

From the figures, we can easily find that the average value of x fluctuates around zero, the average value of $x^2$ is approximately linear with the number of steps. It's also a diffusion process.

2. Now, we change the probability about going to left and going to right. Going to right with the probability 3/4 and goint to left with the probability 1/4. And we set the step as 1. We can study some values of this random process. codes




As the figures showing above, the average value of x increases linearly with the number of steps, and the average of squares has a second order relationship with the number of steps.





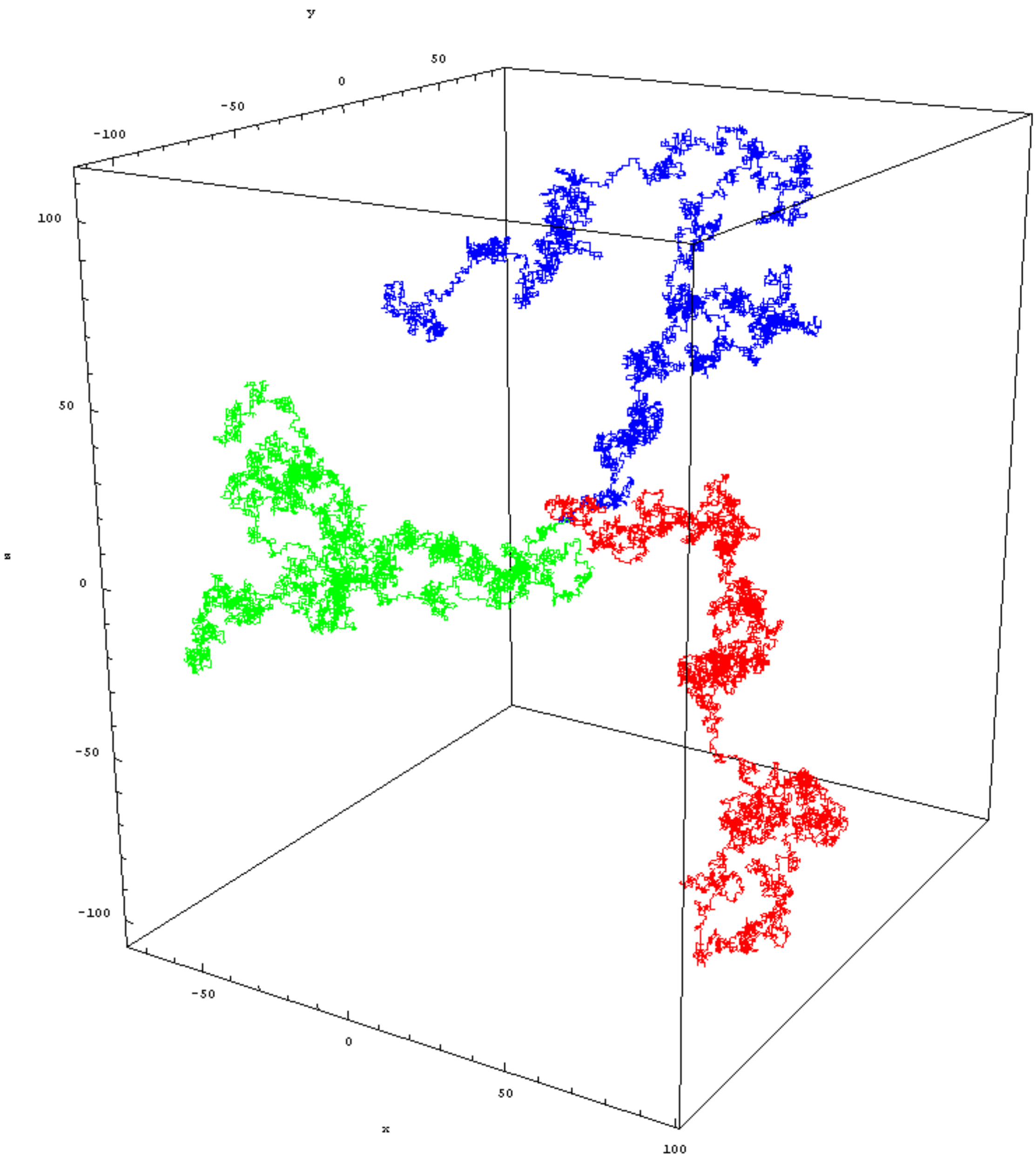

From these two figures, we can also find that even we changed the probability, the relationship of x^2 and the step would never change.

## 2.2 Random walks in 2D

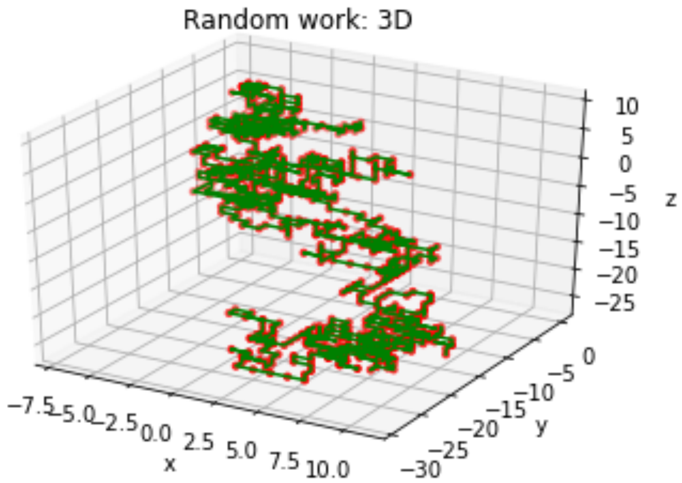
We can also show the process in 2D codes

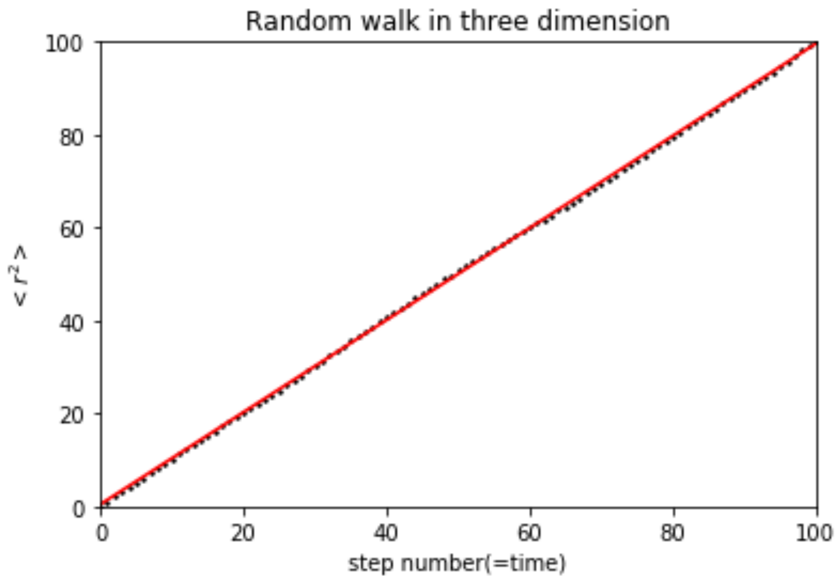
The figure shows that just like the situation of one dimension, its track is unpredicted.

## 2.3 Random walks in 3D



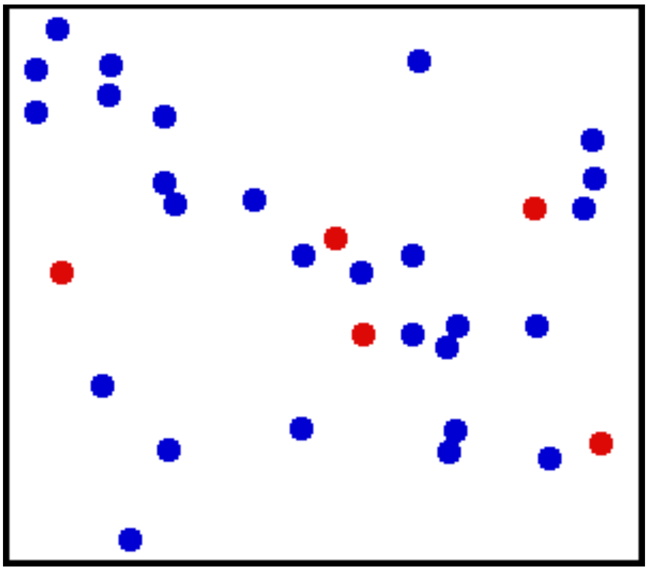

As we all know, we live in a 3D world,so this situation is the most universal situation in our daily life. codes




From the figures above, we can get the same conclusion as the random walks in 1D and 3D.

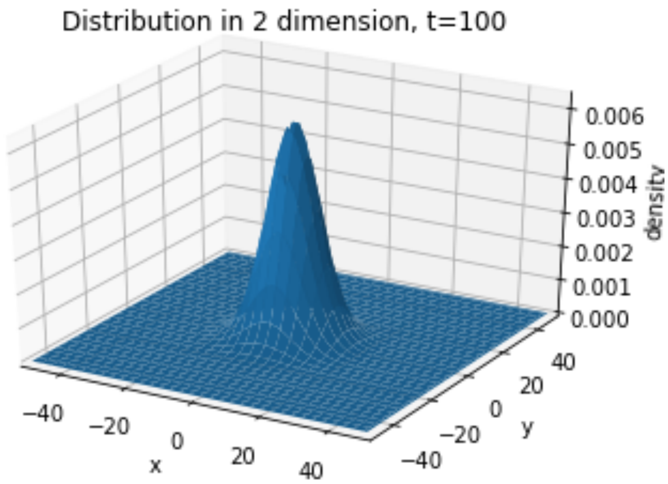
# 3 Diffusion



Diffusion is the net movement of molecules or atoms from a region of high concentration with high chemical potential to a region of low concentration with low chemical potential.

### Diffsion in 2D

In order to get closer to our lives, I will just show diffusions in 2D. codes




# 4 Conclusion

As the analysis and figures above, we can get the following coclusions:
1. As for the random walk, the mean displacement of the walker will fluctuate near 0, which is bacause the possibility of the walker to go left or right is same according to the theory.

2. Given the chance that we make infinite walks every for every step,the figure is just the straight line.
3. More generally, the average value of x increases linearly with the number of steps, and the average of squares has a second order relationship with the number of steps.

## 5 Reference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