

Concordia University
Department of Computer Science and Software
Engineering
SOEN 331 - S
Formal Methods for Software Engineering

Assignment 1: Fundamentals

Witnick-Hans Joseph

ID: 29348743

Alexandra Zana

ID: 40131077

Alexandre Eid

ID: 40155833

October 14, 2021.

Contents

1	General information	3
2	Introduction	3
3	Ground rules	3
4	Problems	4
4.1	Propositional logic (10 pts)	4
4.1.1	Propositional logic answer	4
4.2	Predicate logic 1 (10 pts)	6
4.2.1	Problem 2 answer	6
4.3	Predicate logic 2 (10 pts)	7
4.4	Ordered structures (10 pts)	9
4.5	Unordered structures and type declarations (10 pts)	10
4.6	Relational calculus 1 (25 pts)	12
4.7	Relational calculus 2 (25 pts)	14
5	What to submit	19

1 General information

Date posted: Thursday 30 September, 2021.

Date due: Thursday, 14 October, 2021, by 23:59.

Weight: 15% of the overall grade.

2 Introduction

You should form a team of **three** members. Each team should designate a leader who will submit the assignment electronically. In case you cannot find a team, please contact me and I will assign you to one. There are **7** problems in this assignment, with a total weight of **100** points. You must prepare all your solutions in L^AT_EX and produce a single pdf file. Name the file after the Concordia id of the person who will submit, e.g. 123456.pdf.

3 Ground rules

This is an assessment exercise. You may not seek any assistance while expecting to receive credit. **You must work strictly within your team and seek no assistance for this assignment ((e.g. from the teaching assistants, fellow classmates and other teams or external help)).** Please note that you should **not** discuss the assignment during tutorials. I am available to discuss clarifications in case you need any.

All team members are expected to work relatively equally on each problem. The team leader has the responsibility to ensure that the team does not violate this rule. **In your submission, you must include only the names of those team members who contributed to the assignment.** Accommodating someone who did not contribute will result in a penalty.

If there is any problem in the team (such as lack of contribution, etc.), the team leader must contact the instructor as soon as the problem appears.

4 Problems

4.1 Propositional logic (10 pts)

You are shown a set of four cards placed on a table, each of which has a **letter** on one side and a **symbol** on the other side. The visible faces of the cards show the letters **L** and **A**, and the symbols \square , and \diamond .

Which card(s) must you turn over in order to test the truth of the proposition that “*If a card has a consonant on one side, then it has the symbol \diamond on the other side*”? Explain your reasoning in detail by deciding for each card whether it should be turned over and why. In your answers, apply any and all appropriate validating or non-validating patterns where applicable.

4.1.1 Propositional logic answer

We have two statements:

p: The card has a consonant

q: The card has symbol \diamond

$$p \rightarrow q, \neg q \rightarrow \neg p$$

- Card \diamond : Implication does not necessarily entail causation. So even if the card is a \diamond the other side value could be L or A. It is not the card to turn.
- Card \square : Since this card is a vowel, we can check the validity of the contrapositive $\neg q \rightarrow \neg p$. This should be a card to turn. This card does not satisfy the second statement that the card is a \diamond . So we cannot evaluate the second statement. It is not the card to turn.
- Card L: Since this card is a consonant, we can check the validity of $p \rightarrow q$. We can check the back to validate q. This should be a card to turn.

- Card A: This card is like $\neg p$, "A" is not a consonent, and therefore does not help in showing $p \rightarrow q$ or $\neg q \rightarrow \neg p$

4.2 Predicate logic 1 (10 pts)

In the domain of all people, consider the predicate $disclosed(a, b)$ that is interpreted as “ a has disclosed a secret to b .”

1. How are the following two expressions translated into plain English? Are the two expressions logically equivalent $\forall a \exists b asks(a, b)$, and $\exists b \forall a asks(a, b)$.
 - $\forall a \exists b disclosed(a, b)$.
 - $\exists b \forall a disclosed(a, b)$.
2. Can we claim that $\forall a \exists b disclosed(a, b) \rightarrow \exists b \forall a disclosed(a, b)$? Discuss in detail.
3. Can we claim that $\exists b \forall a disclosed(a, b) \rightarrow \forall a \exists b disclosed(a, b)$? Discuss in detail.

4.2.1 Problem 2 answer

1. translation

$\forall a \exists b disclosed(a, b)$: For all a , there exist a b such that a has asked b out on a date.

$\exists b \forall a disclosed(a, b)$: There exists a b such that all a asked b out on a date.

2. Can we claim that $\forall a \exists b disclosed(a, b) \rightarrow \exists b \forall a disclosed(a, b)$? Discuss in detail.
3. Can we claim that $\exists b \forall a disclosed(a, b) \rightarrow \forall a \exists b disclosed(a, b)$? Discuss in detail.

4.3 Predicate logic 2 (10 pts)

Consider the subject “x is a person” and the predicate “x is a mortal”, together with the following list of categorical propositions:

- “No person is immortal.”
- “All people is immortal.”
- “Some people are mortal.”
- “Some people are not mortal.”

1. “Identify each categorical statement with its name (i.e. letter description)”.

answer:

”No person is immortal.”: A Form

”All people are immortal.”: E Form

”Some people are mortal.”: I Form

”Some people are not mortal.”: O Form

2. “Identify universal statements.”

answer:

”No person is immortal.” and ”All people are immortal.”

3. “Identify particular statements.”

answer:

”Some people are mortal.” and ”Some people are not mortal.”

4. “Identify affirmative statements”

answer:

”No person is immortal.” and ”Some people are mortal.”

5. "Identify negative statements."

answer:

"All people are immortal." and "Some people are not mortal."

6. "Identify statements with opposite truth values"

answer:

7. "Identify statements that cannot both be true, but could both be false."

answer:

8. "Identify statements that cannot both be false but could both be true."

answer:

9. "Identify pairs of super-subaltern statements."

answer:

4.4 Ordered structures (10 pts)

Consider a list $\Lambda = \langle w, x, y, z \rangle$, deployed to implement a stack Abstract Data Type.

1. Let the head of Λ correspond to the topmost position of the Stack. Implement the body of operations `push(el, Λ)` and `pop(Λ)` (let return element be held in variable `topmost`) using list construction operations. In both cases a) we assume that appropriate preconditions exist, and b) we can refer to Λ' as the state of the list upon successful termination of one of its operations.

answer:

`push(el, Λ)` can be written as $\Lambda' = cons(el, \Lambda) = \langle el, w, x, y, z \rangle$.

`pop(Λ)` can be written as $head(\Lambda) = element = w$ and $\Lambda' = tail(\Lambda) = \langle x, y, z \rangle$,

2. Let the last element of Λ correspond to the topmost position of the Stack. Implement the body of both operations as above. When applicable, use control flow statements in your answer.

4.5 Unordered structures and type declarations (10 pts)

Consider the sets

- Laptop = {Apple, IBM, Sony, HP, Acer, Dell, LG}, and
- Favorite = {Apple, Sony, Dell}.

Answer the following questions:

1. How do we interpret the expression Favorite : $\mathbb{P}\text{Laptop}$?

answer:

The variable Favorite can assume any value supported by $\mathbb{P}\text{Laptop}$ (ie: the var Favorite can be any value of the PowerSet of Laptop)

2. Is $\mathbb{P}\text{Laptop}$ a legitimate type?

answer:

Yes; Favorite is of type $\mathbb{P}\text{Laptop}$. The powerset Laptop is a legitimate type.

3. What is the nature of the variable in Favorite : $\mathbb{P}\text{Laptop}$? (i.e. Atomic or composite? If composite, what type?)

answer:

It is a composition. the variable Favorite: $\mathbb{P}\text{Laptop}$ is a set of elements, so it is a composition.

4. Is $\text{Apple} \in \mathbb{P}\text{Laptop}$? Explain why or why no

answer:

No; the $\mathbb{P}\text{Laptop}$ is a list of all possible subsets we can produce from the set Laptop, and the atomic element Apple is not an element of $\mathbb{P}\text{Laptop}$.

5. Is $\{\text{Apple}\} \in \mathbb{P}\text{Laptop}$? Explain why or why not.

answer:

Yes; since $\{\text{Apple}\}$ is a valid element of the $\mathbb{P}\text{Laptop}$ set. The list element of any atomic element is within the PowerSet of its type.

6. Is $\{\{\}\} \in \mathbb{P}\text{Laptop}$?

answer:

No, because list Laptop itself does not contain the empty element $\{\}$, it's power set can't contain the element $\{\{\}\}$.

7. Is $\{\} \in \mathbb{P}\text{Laptop}$? Explain why or why not.

answer:

Yes; since the null/empty element is an element of any power set.

8. If we define variable Favorite : $\mathbb{P}\text{Laptop}$, is $\{\}$ a legitimate value for variable Favorite? Explain why or why not.

answer:

Yes, because the $\mathbb{P}\text{Laptop}$ contains the empty list element. Ie, if we remove all the elements contained in Favorite, we will have $\{\}$ set.

9. Is Favorite $\in \mathbb{P}\text{Laptop}$? Explain.

answer:

Yes; the elements of Favorite are elements in $\mathbb{P}\text{Laptop}$, but the variable Favorite itself is not.

10. Is Favorite $\subset \mathbb{P}\text{Laptop}$? Explain.

answer:

No

4.6 Relational calculus 1 (25 pts)

Consider a system that assigns id's to locations. An id may represent a vehicle in a parking lot, a train in a station, a process in an operating system etc. A location may represent a parking spot, a train station platform, a core in a hardware system etc. The requirements of the system are as follows:

1. Id's are unique.
2. An id is assigned to a single location (and maybe re-assigned subsequently).
3. No multiple id's may be assigned to the same location.

The model of the system is captured by a relation assignment, which is represented as a set as shown below:

$$\begin{aligned} \text{assignment} = \\ \{ \\ \quad 001 \mapsto A, \\ \quad 002 \mapsto B, \\ \quad 003 \mapsto C \\ \} \end{aligned}$$

1. Define the precondition to operation add, that assigns a new id to a location.
2. Provide two alternative definitions for the main functionality of the operation.
3. What would be the result of calling the operation with $\text{id?} = 001$, and $\text{location?} = D$?
4. Let us get rid of the precondition. What would be the result of calling add with $\text{id?} = 004$, and $\text{location?} = C$? Would you accept or reject the call? If accepted then the pair should be added to the relation. If rejected, it should not be added to the relation.
5. Assume that in the absence of a precondition, we attempted to call operation add with some input parameters id? and location? . Under what conditions, if any, would this be acceptable?

6. Consider operation `reassign` that modifies the location of an existing id. Define the precondition to operation `reassign`.
7. Define the body of operation `reassign`.

4.7 Relational calculus 2 (25 pts)

Consider the following relation:

$$cellphones : Model \leftrightarrow Brand$$

where

$$cellphones = \{ \begin{array}{l} galaxyA21s \mapsto samsung, \\ mate10 \mapsto huawei, \\ mate10pro \mapsto huawei, \\ galaxyA01 \mapsto samsung, \\ iPhone12ProMax \mapsto apple, \\ iPhoneSE \mapsto apple, \\ galaxyJ2Core \mapsto samsung, \\ redmi6A \mapsto xiaomi, \\ redmi8 \mapsto xiaomi \end{array} \}$$

1. Given the above relation,

(a) Define the domain of cellphones.

answer:

$$\text{domain } cellphones = Model = \{galaxyA21s, mate10, mate10pro, galaxyA01, iPhone12ProMax, iPhoneSE, galaxyJ2Core, redmi6A, redmi8\}$$

(b) Define the range of cellphones.

answer:

$$\text{range } cellphones = Brand = \{samsung, huawei, apple, xiaomi\}$$

2. Given the following the expression:

$$\{galaxyA01, galaxyJ2Core, redmi8\} \triangleleft cellphones$$

(a) What is the meaning of operator \triangleleft ?

answer: \triangleleft is a domain restriction, it selects pairs according to the first element.

(b) What is the value of the expression?

answer:

$\{\text{galaxyA01}, \text{galaxyJ2Core}, \text{redmi8}\} \triangleleft \text{cellphones} = \{$
galaxyA01 \mapsto samsung,
galaxyJ2Core \mapsto samsung,
redmi8 \mapsto xiaomi
 $\}$

- (c) Where would you deploy the operator \triangleleft in the context of a database management system?

answer: This operator is used to model database queries.

3. Given the following the expression:

$\text{cellphones} \triangleright \{\text{samsung}, \text{xiaomi}\}$

- (a) What is the meaning of operator \triangleright ?

answer: \triangleright is a range restriction operator; It assigns pairs based on the second element.

- (b) What is the value of the expression? $\text{cellphones} \triangleright \{\text{samsung}, \text{xiaomi}\} =$
 $\{$

galaxyA21s \mapsto samsung,
galaxyA01 \mapsto samsung,
galaxyJ2Core \mapsto samsung,
redmi6A \mapsto xiaomi,
redmi8 \mapsto xiaomi

- (c) Where would you deploy such operator in the context of a database management system?

answer:

This operator is used to model database queries.

4. Given the following the expression:

$\{\text{mate10pro}, \text{iPhoneSE}, \text{galaxyJ2Core}\} \triangleleft \text{cellphones}$

- (a) What is the meaning of operator \triangleleft ?

answer:

It is a domain subtraction, it removes all specified element from the domain of the relation.

- (b) What is the value of the expression?

answer:

$\text{cellphones}' = \{\text{mate10pro}, \text{iphoneSE}, \text{galaxyJ2Core}\} \triangleleft \text{cellphones} = \{$
galaxyA21s \mapsto samsung,
mate10 \mapsto huawei,
galaxyA01 \mapsto samsung,
iphone12ProMax \mapsto apple,
redmi6A \mapsto xiaomi,
redmi8 \mapsto xiaomi
 $\}$

- (c) Where would you deploy such operator in the context of a database management system?

answer:

This operator would be deployed when we want to delete elements from the database.

5. Given the following the expression:

$\text{cellphones} \triangleright \{\text{apple}, \text{xiaomi}\}$

- (a) What is the meaning of operator \triangleright ?

answer:

\triangleright is a range subtraction operator, it removes all elements from the range of the relation.

- (b) What is the value of the expression?

answer:

$\text{cellphones}' = \text{cellphones} \triangleright \{\text{apple}, \text{xiaomi}\} = \{$
galaxyA21s \mapsto samsung,
mate10 \mapsto huawei,

$\text{mate10pro} \mapsto \text{huawei},$
 $\text{galaxyA01} \mapsto \text{samsung},$
 $\text{galaxyJ2Core} \mapsto \text{samsung},$
 $\}$

- (c) Where would you deploy such operator in the context of a database management system?

answer:

You would use \triangleright when wanting to modify the content of a database (deleting element, in the case of \triangleright).

6. Given the following the expression:

$\text{cellphones} \oplus \{\text{galaxyA51} \xrightarrow{7} \text{samsung}, \text{mate9} \xrightarrow{7} \text{huawei}\}$

- (a) What is the meaning of operator \oplus ?

answer:

\oplus is a relational override

- (b) What is the value of the expression?

answer:

$\text{cellphones} \oplus \{\text{galaxyA51} \mapsto \text{samsung}, \text{mate9} \mapsto \text{huawei}\} = \{$
 $\text{galaxyA21s} \mapsto \text{samsung},$
 $\text{mate10} \mapsto \text{huawei},$
 $\text{mate10pro} \mapsto \text{huawei},$
 $\text{galaxyA01} \mapsto \text{samsung},$
 $\text{iphone12ProMax} \mapsto \text{apple},$
 $\text{iphoneSE} \mapsto \text{apple},$
 $\text{galaxyJ2Core} \mapsto \text{samsung},$
 $\text{redmi6A} \mapsto \text{xiaomi},$
 $\text{redmi8} \mapsto \text{xiaomi},$
 $\text{galaxyA51} \mapsto \text{samsung},$
 $\text{mate9} \mapsto \text{huawei},$
 $\}$

- (c) Where would you deploy such operator in the context of a database management system?

answer:

This operator would be used in the event that we want to add or modify elements in the database.

- (d) Does the result of the expression have a permanent effect on the database (relation)? If not, describe in detail how would you ensure that such operation would have a permanent effect.

answer:

The expression doesn't have a permanent effect. If we want to have a permanent effect, we would need to have an assignment operation as such:

$\text{cellphones}' = \text{cellphones} \oplus \{\text{galaxyA51} \mapsto \text{samsung}, \text{mate9} \mapsto \text{huawei}\}$

We need to assign the value of the expression to cellphone to have permanent effect.

5 What to submit

Please submit your pdf file at the Electronic Assignment Submission portal

(<https://fis.encs.concordia.ca/eas>)

under **Theory Assignment 1**.