Ćwiczenia 09.05.2012

Całkowanie

$$\int_a^b f(x)\,dx$$

Wzór ~~prostokątów~~ trapezów

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$\langle x_0, x_1\rangle \langle x_1, x_2\rangle \dots \langle x_{n-1}, x_n\rangle$

$$\int_a^b f(x) = \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} f(x)\,dx \qquad h = \frac{b-a}{n}$$ szerokość każdego z podprzedziałów

$$\int_{x_{i-1}}^{x_i} f(x)\,dx \approx \int_{x_{i-1}}^{x_i} W(x)\,dx$$

$$\int_{x_{i-1}}^{x_i} W_1(x)\,dx = \frac{y_i + y_{i-1}}{2}\cdot h$$

$$\int_a^b f(x)\,dx = \sum_{i=1}^{n} \frac{y_i + y_{i-1}}{2}\cdot h = \frac{h}{2}\sum_{i=1}^{n} y_i + y_{i-1} = \frac{h}{2}(y_0 + y_1 + y_1 + y_2 + \dots + y_{n-1} +$$

$$+ y_{n-1} + y_{n-1} + y_n) = \frac{h}{2}\left(y_0 + y_n + 2\sum_{k=1}^{n-1} y_k\right)$$ – ogólny wzór trapezów

$$|E| \leq \frac{(b-a)^3}{12n^2}\cdot M \qquad M = \max_{x\in\langle a,b\rangle} |f''(x)|$$

$$\int_a^b f(x)\,dx \approx h\cdot\sum_{i=1}^{n} y_{(i-\frac{1}{2})}$$

$$|E| \leq \frac{(b-a)^3}{24n^2}\cdot M \qquad M = \max_{x\in\langle a,b\rangle} |f''(x)|$$ ogólny wzór prostokątów

$$\int_a^b f(x)\,dx \approx \frac{1}{3}h\sum_{i=1}^{n}\left(y_{2i-2} + 4y_{2i-1} + y_{2i}\right)$$

$$|E| \leq \frac{(b-a)^5}{180n^4}\cdot M \qquad M = \max_{x\in\langle a,b\rangle} |f^{(4)}(x)|$$ ogólny wzór Simpsona

ZAD.

Obl. $\int_0^2 \frac{1}{x^2+1}\,dx$ z dokładnością $E = 10^{-1}$

$$f'(x) = \frac{-2x}{(x^2+1)^2}$$

$$f''(x) \overset{Pd}{=} \dots = \frac{6x^2-2}{(x^2+1)^3} = g(x)$$

$$g'(x) = f^{(3)}(x) \overset{Pd}{=} \dots = \frac{-24x^3+24x}{(x^2+1)^4}$$

$$f^{(3)}(x) = 0 \iff -24x(x^2-1) = 0$$

$$x = -1 \lor x = 0 \lor x = 1$$

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