

**ECONOMIC GEOLOGY
RESEARCH UNIT**

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Johannesburg

THE USE OF FACTOR ANALYSIS IN THE
PROCESSING OF GEOCHEMICAL DATA FROM
THE BASAL REEF, ORANGE FREE STATE
GOLDFIELD, SOUTH AFRICA

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ABSTRACT

Simplification of a complex system of geochemical variables - obtained from an auriferous ore-horizon of the Witwatersrand System - is attempted by means of factor analysis. Nine variables (Co, Ni, Cu, Zn, Pb, Ag, Au, U₃O₈, and S) are expressed in terms of three theoretical variables or factors. The factors indicate that the gold and the uraninite in the ore-body were deposited as placer minerals, whereas the pyrite was formed by precipitation and sulfidization in situ as well as being deposited by sedimentary processes. The chalcopyrite was apparently emplaced exclusively by precipitation. Comparison of these results with the well-established modified placer theory of the genesis of the Witwatersrand deposit demonstrates the usefulness of the method in the elucidation of metallogenetic problems.

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INTRODUCTION

Geochemical investigation of the Witwatersrand banket is still in its infancy. Two recent papers have, however, been published, one by Hiemstra (1967) who studied the contents of certain trace elements in the Dominion Reef, and one by Saager (1969a) who investigated the silver/gold content of the Basal Reef.

The geochemical data (Cu, Pb, Zn, Co, Ni, Ag, Au, U₃O₈, and S values) used in this paper were obtained from stope samples collected at the Free State Geduld Mine during a comprehensive geochemical and mineralogical investigation of the Basal Reef by one of the authors (R.S.).

The present authors were interested in the linear association between the measured variables, with a view to throwing further light on the origin of the ore-body and elaborating the use of geochemical data for generally defining pay areas in the Witwatersrand conglomerates. Since simple and multiple linear regression analyses yield complicated results, it was hoped that factor analysis would make the system more comprehensible. Furthermore, the technique of factor analysis has been applied to the present suite of variables because the mode of formation of the Witwatersrand deposit is well-established and enables reliable comparisons to be made between the results of this study and results obtained from sedimentary, structural, and mineralogical investigations.

DEVELOPMENT AND COMPOSITION OF THE BASAL REEF

A. MACROSCOPIC DESCRIPTION

The Basal Reef is, economically, the most important ore-horizon in the Orange Free State Goldfield. It lies in the upper half of the Main-Bird Series within the lower portion of the Upper Witwatersrand System. Macroscopically the Basal Reef is a greyish to dark-greenish metamorphosed conglomerate composed of waterworn, rounded to elliptical, sometimes bladed pebbles which are usually firmly cemented by a fine-grained matrix of quartz and micaceous minerals. The thickness of the Basal Reef varies between 3 inches and 10 feet, with an average of 2 feet. In the area investigated, the Basal Reef is represented by a basal conglomerate, sometimes forming a single line of scattered pebbles. The pebbles consist mainly of white vein quartz, smoky quartz, black chert and, to a lesser extent, dark-grey orthoquartzites in a fine-grained matrix of quartz, sericite and chlorite. The size of the pebbles varies between 1 and 30 mm.

B. MICROSCOPIC DESCRIPTION

The majority of the ore-minerals occur in the matrix. Microscopic investigations showed that pebbles of quartz and chert are generally free from enclosed minerals except for dust- and stringer-like inclusions of secondary pyrite.

By far the most abundant ore-mineral is pyrite which generally constitutes over 90 per cent of the ore-minerals. It occurs predominantly as rounded, waterworn, detrital grains which are accompanied by secondary pyrite pseudomorphs, concretions, and subhedral crystals. In many cases secondary pyrite forms fracture fillings or overgrowths on older detrital constituents. A detailed description of the different forms of pyrite encountered in the Basal Reef has been given by Saager (1969b) and will, therefore, not be given further attention here.

Irregular masses of pyrrhotite, marcasite, chalcopyrite, galena, sphalerite, arsenopyrite, pentlandite, linnaeite, cobaltite, skutterudite, and safflorite occur in much smaller amounts interstitially in the matrix and replace the non-opaque minerals to a varying degree. Very often these minor sulfides are intergrown with secondary pyrite or have entered fractured pebbles without major replacement. Except for cobaltite, linnaeite, and arsenopyrite, none of the minor sulfides show any detrital morphology; they were clearly formed in situ during the various periods of

metamorphism to which the Witwatersrand sediments were subjected.

The gold in the Basal Reef occurs in a manner similar to that of other ore-horizons in the Witwatersrand Goldfield. Macroscopically visible gold is only very rarely encountered. Gold forms irregular, jagged, particles among the matrix constituents of the conglomerates and is intergrown with reconstituted sulfides or fills fractures and cavities in pebbles. Detailed descriptions of the development of the gold fraction in the Basal Reef and other reefs of the Orange Free State Goldfield have been given by Schidlowski (1968) and Saager (1969a). A large percentage of the gold occurs at the bottom contact of the Basal Reef, which clearly suggests that the distribution of the metal must be governed by sedimentary processes. It further shows, that the secondary reconstitution which took place during periods of metamorphism never seriously affected the primary gold distribution pattern of the placer.

Uraninite generally occurs in the form of rounded grains, concentrated, like gold, in the lower portion of the ore-horizon. It appears that uraninite has exerted a precipitating effect on migrating hydrocarbons, resulting in a close relationship between uraninite and thucholite (Liebenberg, 1955). Usually the uraninite is dusted by radiogenic galena, which also occasionally forms aureoles around it or fills fractures in cataclastic uraninite.

Subrounded grains of chromite, and less abundant rounded-to-prismatic grains of zircon, are relatively common in the matrix of the conglomerate. Rutile has also been observed as rounded allogenic components, but is encountered more often as irregular aggregates formed in situ or as fine needles in the interstitial micaceous minerals and quartz.

SAMPLING AND TREATMENT OF DATA

109 ore-samples were collected from the Basal Reef in currently accessible stopes in the Free State Geduld Mine, near the town of Welkom in the Orange Free State, the exact localities being accurately recorded. The silver and gold contents of the ore-samples were determined by the conventional fire assay method to an accuracy of 0.05 dwt. per ton. Cobalt, nickel, copper, zinc, and lead were determined by the atomic absorption method to an accuracy of 10 ppm. The sulfur values were obtained by wet chemical analysis to an accuracy of 0.01 per cent. U_3O_8 values were determined by measuring the total radioactivity, the accuracy being 0.01 lb. per ton. It is realized that the value of the latter can be affected by the presence of other radioactive minerals such as monazite. However, a microscopic investigation of several hundred polished- and thin-sections revealed that the amounts of radioactive minerals other than uraninite are extremely small and thus affect the values obtained for the U_3O_8 only insignificantly. All analyses were carried out by the Anglo American Research Laboratory in Johannesburg.

For the processing of the analytical values, various statistical techniques, including trend surface analysis, factor analysis, and graphic methods, have been used. The programs employed were written in FORTRAN for the IBM 360 model 50 computer at the University of the Witwatersrand and the IBM 360 model 40 computer of Anglo American Corporation, Johannesburg.

OUTLINE OF FACTOR ANALYSIS

A. HISTORICAL BACKGROUND

Factor analysis is a branch of multivariate statistics but has been used and developed largely by psychologists. The principal contributors to the theory include Pearson (1901), Spearman (1904), Thurstone (1931), and Holzinger (1944), while a comprehensive modern account can be found in Harman (1960). In recent years geologists have begun to use factor analysis as an aid in the interpretation of complicated problems.

Imbrie and Purdy (1962), and Krumbein and Imbrie (1963) have applied the technique to sedimentary problems while Imbrie and Van Andel (1964) have analysed heavy mineral data. Modifications to suit the needs of geologists have been attempted in some of these papers, and a few computer programs have been published, notably by Imbrie (1963), and Manson and Imbrie (1964).

B. BRIEF STATISTICAL DESCRIPTION

The prime objective of factor analysis is to simplify a complex set of variables by expressing the total variance of the set in as few "factors" or theoretical variables as possible. The analysis may proceed by examining the relationships between variables (R-mode of analysis) or between samples (Q-mode of analysis). In this paper, only the relationships between variables are examined.

Assume a p-variate system x_1, x_2, \dots, x_p each observed on n individuals. If x_{ij} is the jth observation on the ith individual the variables can be arranged in a matrix of dimension $n \times p$ and can easily be manipulated using the rules of matrix algebra. If it is possible to define the system with less than p variables it means that some of the variables must be linear combinations of some of the others. However, in general, it is not possible to effect a reduction of this sort, partly because of the error factor involved.

Factor analysis assumes that each variable x_j can be expressed in the form

$$x_j = a_{j1}f_1 + a_{j2}f_2 + \dots + a_{jm}f_m + a_{ju}u_j$$

where the f_j are common factors and the a_{j1} indicates the relative importance of the common factor f_1 . A common factor is a factor which is present in each variable. u_j is a factor which is unique to the particular variable in question.

Most problems are only concerned with the common factors. Since the readings are arranged so that the variance of each variable is unity, the sum of squares of the coefficients of the common factors indicate to what extent the common factors account for the variance of that variable and is known as the communality.

The dimensional reduction is carried out in an approximate way by choosing the first factor so that it accounts for as much of the variance as possible and then choosing each succeeding factor so that it also accounts for as much of the variance as possible and is uncorrelated with all preceding factors. If the first few factors account for a large proportion of the variation it may be possible to ignore the rest.

To take a hypothetical example, it might be found that the gold contents of eight reef horizons depend on two common factors such as (i) source material and, (ii) distance from the source, while each horizon was affected by a unique factor such as footwall configuration or reworking. All the factors might account for one hundred per cent of the variation in the data, but the common factors might explain only ninety per cent.

Various techniques exist which relate each variable to the statistically independent hypothetical factors. The method of principal components described in Harman (1960) is the one in general use and is probably the best. It results in what is known as an initial factor matrix which may be visualized geometrically as providing an orthogonal co-ordinate scheme in which the hypothetical factors are the reference axes and the variables are vectors in the space of dimension determined by the number of axes (Figure 1).

Unfortunately these reference axes are not related to the variables in such a way as to be readily interpreted geologically. They are rotated so that they correspond as closely as possible to any definite grouping of the observed variables, resulting in the rotated factor matrix wherein it is usually possible to identify each of the hypothetical factors with one or more of the observed variables (Figure 2).

Returning to the previous example, this means that it may be possible to identify the gold content of the first reef horizon with the source material while the gold content of the third may correspond to the distance from the source. Alternatively, both factors may only be explicable as a combination of two or more variables.

The R-mode of analysis proceeds from an analysis of the correlation matrix where the product-moment correlation coefficient between x_j and x_k is given by

$$r_{jk} = \frac{\sum (x_{ji} - \bar{x}_j)(x_{ki} - \bar{x}_k)}{\sqrt{\sum (x_{ji} - \bar{x}_j)^2 \sum (x_{ki} - \bar{x}_k)^2}}$$

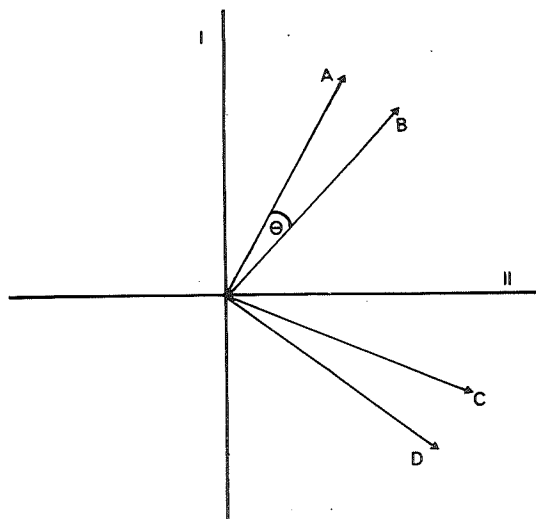


Figure 1 : Two-dimensional plot of four variables with two principal factor axes (I and II). The variables are represented as vectors of unit length with the cosine of the angle between any two vectors (θ) being proportional to the value of the correlation coefficient between them. After Rummell (1966).

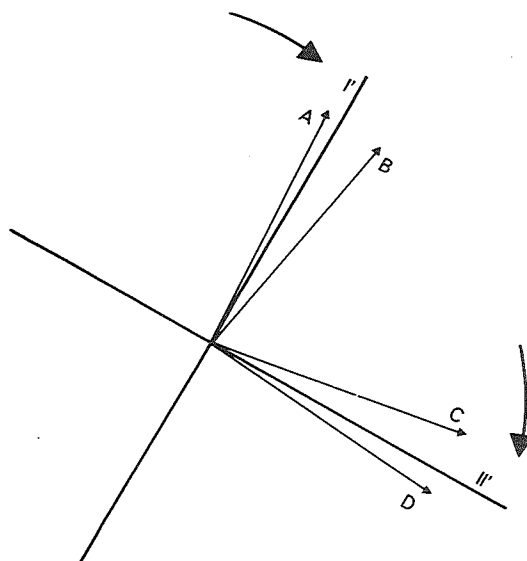


Figure 2 : A and B now correspond more closely to factor axis I' and C and D to factor axis II'. After Rummell (1966).

The correlation coefficient is a measure of the linear relationship between two variables and its value lies between +1 and -1, depending on whether the variables are related sympathetically or antipathetically. Various other coefficients of similarity have been used in a study of this sort; the product-moment correlation coefficient is used most often, however, and its statistical properties are known.

After the variables corresponding most closely to the factor axes have been determined they must be interpreted geologically. The most extreme variables are chosen because it is usually easier to identify them with some physical process. However, as the number of possible solutions of the factor analysis model is usually infinite, care must be taken to substantiate all conclusions arrived at solely as a result of a study of this sort.

RESULTS

The program used was an IBM package program, a listing of which appears in the system 360 scientific subroutine package (360 A-CM-03).

A. MEANS, STANDARD DEVIATION, AND COEFFICIENTS OF VARIATION

Means were calculated in the usual way and standard deviations were obtained using the formula

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{N - 1}}$$

The coefficient of variation is a measure of the relative dispersion of a variable and enables direct comparisons to be made between distributions having different units of measurement. It was computed using the formula

$$V = \frac{s}{\bar{x}}$$

The values of the mean, standard deviation, and coefficient of variation for the determined elements are given in Table 1.

TABLE 1

Means, Standard Deviations, and Coefficients of Variation of the Co,
Zn, Cu, Au, Ag, U₃O₈, S, Ni and Pb Values from the 109 Stope Samples.

	Co (ppm)	Zn (ppm)	Cu (ppm)	Ag (dwt/ton)	Ag (dwt/ton)	U ₃ O ₈ (lb/ton)	S (%)	Ni (ppm)	Pb (ppm)
\bar{x}	155.9	98.8	211.6	220.3	21.0	2.1	6.32	252.7	447.8
s	98.2	111.0	611.9	494.6	46.9	2.7	5.36	165.0	693.1
V	0.63	1.12	2.89	2.25	2.24	1.27	0.85	0.65	1.55

\bar{x} : mean; s : standard deviation; V : coefficient of variation

B. CORRELATION MATRIX

The correlation matrix is given in Table 2. The values shown are product-moment correlation coefficients (as previously defined).

TABLE 2
Correlation Matrix

	Co	Zn	Cu	Au	Ag	U ₃ O ₈	S	Ni	Pb
Co	1.000	0.400	-0.027	0.067	0.080	0.341	0.870	0.865	0.318
Zn		1.000	0.111	0.071	0.065	0.052	0.370	0.446	0.255
Cu			1.000	0.087	0.096	0.039	-0.008	0.030	0.466
Au				1.000	0.994	0.605	0.021	0.063	0.471
Ag					1.000	0.631	0.031	0.064	0.490
U ₃ O ₈						1.000	0.178	0.276	0.696
S							1.000	0.856	0.205
Ni								1.000	0.241
Pb									1.000

C. ROTATED FACTOR MATRIX

The rotated factor matrix is shown in Table 3. Three factors explaining approximately 81 per cent of the total variation in the data have been extracted from the correlation matrix and the system has been rotated so that the three independent factor axes correspond with groupings present in the data. The grouped variables which correspond most closely to any particular factor axis may be obtained by scanning each column of the matrix for the highest values. It can be seen that factor 1 may be represented by cobalt, nickel, and sulfur; factor 2 by gold, silver, and U₃O₈ and factor 3 by copper.

TABLE 3
Rotated Factor Matrix (R-Mode)

Variable	Factor 1	Factor 2	Factor 3	Communality
Co	0.94	0.13	-0.02	0.91
Zn	0.55	0.00	0.31	0.40
Cu	-0.04	0.02	0.93	0.87
Au	-0.04	0.94	0.02	0.89
Ag	-0.03	0.95	0.03	0.91
U ₃ O ₈	0.23	0.81	0.09	0.72
S	0.93	0.03	-0.04	0.87
Ni	0.94	0.08	0.01	0.89
Pb	0.25	0.61	0.61	0.80

It is interesting to observe that lead also displays fairly high values for both factor 2 and factor 3, indicating a certain association. On a subsequent run, four factors were extracted from the correlation matrix, but it was felt that the additional factor did not aid the geologic interpretation of the results and is therefore not discussed here.

The column headed communality indicates what proportion of the variability of any particular variable has been explained by the three factors shown. It is obtained by squaring the values in each row (e.g. a communality of 0.91 for cobalt is obtained by adding $(0.94)^2 + (0.13)^2 + (0.02)^2$). As can be seen, the low communality of zinc suggests that this variable would probably be better contained in a space of higher dimensionality.

DISCUSSION

A. GENERAL

As can be seen from the means (Table 1), sulfur is the only element which has been determined in significant amounts in the present suite of stope samples (> 1 per cent). This is mainly due to the relatively high pyrite content of the Witwatersrand blanket; all other ore-minerals occur only in trace amounts, as pointed out previously.

The coefficient of variation (V), indicates clearly the wide dispersion of copper, silver, and gold and the much smaller dispersion of cobalt, nickel, and sulfur. This demonstrates that sample estimates of the population means of the latter three elements are likely to be better approximations than those of the former. U₃O₈, zinc, and lead take up a somewhat intermediate position (Table 1).

The most striking correlation obtained from the correlation matrix (Table 2) is that of Ag/Au which indicates almost perfect positive linearity. This feature can be explained by the fact that silver occurs almost exclusively in the native gold. Furthermore, the relationship is remarkable in view of the high coefficients of variation for both precious metals. The silver/gold relationship in the Basal Reef has been dealt with in great detail by Saager (1969a) in another publication.

Further high correlations have been obtained between sulfur and cobalt, sulfur and nickel, and nickel and cobalt (Table 2). These three elements also have the lowest coefficients of variation (Table 1). From microscopical and geochemical investigations it has been found that cobaltite and linnaeite are the only cobalt and nickel minerals occurring in any comparatively significant quantities (~ 0.05 per cent) in the investigated ore-horizon. Saager and Mihalik (1967) were able to show that some of the pyrite - which is by far the most common sulfide in the Basal Reef - carries relatively high trace element contents of cobalt and nickel. They reported amounts of 500 to 2300 ppm cobalt and 110 to 4300 ppm nickel. From these mineralogical facts the statistical behaviour of the three elements can be easily understood.

A correlation coefficient of +0.70 exists between U₃O₈ and lead (Table 2). This phenomenon can be explained by the fact that a large percentage of the galena - which is practically the only lead-bearing ore-mineral in the Basal Reef - is of radiogenic origin. Thus, in order to give the high correlation, it seems that the lead migrated only extremely short distances from the uraninite. Extremely short distances of transport during the metamorphic reconstitution of a portion of the ore-minerals in the Witwatersrand blanket have also been advocated by Liebenberg (1955) and Ramdohr (1958), who stated "all indications are that the pseudohydrothermal transport of material has been effective over only a very small distance and is to be measured in fractions of an inch rather than in hundreds of feet".

Since it is widely considered that in the Witwatersrand ore-deposit uraninite, as well as the gold-silver alloy are of placer origin, (Liebenberg, 1955; Ramdohr, 1958; Schidlowski, 1968), the relatively high correlation coefficients between U₃O₈ and gold (+0.60) and U₃O₈ and silver (+0.63), can be expected.

The three factors obtained in the analysis correspond most closely to the sets of variables indicated and may be explained by the following two hypotheses.

B. HYPOTHESIS ONE

It is suggested that the three factors represent chemical elements occurring in detrital minerals or in secondary sulfides formed in situ.

(a) Factor 1 (S, Ni, and Co) is regarded to be a combination of chemical and physical processes. From intensive microscopic investigation it is well-known that cobalt, nickel, and sulfur occur in

allogenic minerals which were introduced at the time of banket formation, as well as in authigenic minerals which were formed within the banket (Liebenberg, 1955; Ramdohr, 1958; Saager, 1968, and 1969b). The most important sulfur-bearing mineral is pyrite which forms generally about 90 per cent of the ore-minerals in the Basal Reef. Pyrite has been found in many portions of the reef to be predominantly of detrital origin, forming distinct waterworn compact grains (Saager, 1969b). Coetzee (1955) and von Rahden (1963) reported trace amounts of cobalt and nickel in detrital Witwatersrand pyrite. Allogenic cobalt and nickel-bearing minerals occurring in the Basal Reef are cobaltite, linnaeite, and cobalt-rich arsenopyrite. All these minerals, however, are found only in minute amounts.

The occurrence of unmobilized detrital pyrite, directly adjacent to mobilized pyrite formed in situ, has led Saager (1969b) to the assumption that the latter variety was formed from amorphous ferrous sulfide, precipitated shortly after the deposition of the conglomerates and mobilized and reconstituted during metamorphic periods. Saager (1969b) suggested the possibility that biogenic H_2S , present in pocket-like anaerobic portions of the sedimentation basin, acted as a precipitating agent. Not only might pyrite have been formed in such a way but also other metal sulfides, such as the authigenic constituents millerite, niccolite, skutterudite, bravoite, and linnaeite, found in small amounts in the ore. The presence of H_2S in the Witwatersrand basin is held to be responsible also for the formation of pyrite concretions and for the pyritization of iron-rich material within the banket (Liebenberg, 1955; Ramdohr, 1958). As mentioned earlier, Saager and Mihalik (1967), studying large concretionary pyrite grains from the Basal Reef, found that these consist of an isotropic and an anisotropic variety, the latter showing relatively high trace contents of nickel and cobalt. They suggested that such pyrite individuals were formed "from Fe-rich silica gels by the addition of H_2S ".

The extremely minute amounts of cobalt and nickel minerals amongst both authigenic and allogenic constituents allow the conclusion to be drawn that pyrite is possibly the largest contributor to the nickel and cobalt and also to the sulfur values obtained in the present investigation. The different genetic backgrounds of the authigenic and allogenic pyrite and of the cobalt- and nickel-bearing minerals in the Witwatersrand banket is the reason why factor 1 (Co, Ni, and S) is regarded as a combination of mainly sedimentary and precipitation/sulfidization processes.

(b) Factor 2 (Ag, Au, and U_3O_8) is easier to explain. It comprises elements which occur exclusively in heavy minerals, i.e. in the natural gold-silver alloy and in uraninite. The latter usually shows rounded, waterworn outlines, whereas the original detrital gold generally forms jagged, irregular grains which quite clearly were reconstituted during metamorphism of the sediments. Both minerals are usually concentrated along the footwall contact of the ore-horizon or occur less abundantly on false footwalls (Liebenberg, 1955; Ramdohr, 1958; Schidlowski, 1966). It is therefore logical to ascribe the distribution pattern of the gold-silver alloy and of uraninite to sedimentary processes. Factor 2 (Ag, Au, U_3O_8), therefore, can be regarded as representing detrital processes.

(c) Factor 3 (Cu) appears to indicate precipitation processes. The element Cu is found almost exclusively in the authigenic chalcopyrite which occurs as irregular, small, aggregates in the matrix of the conglomerate. Since there is no evidence for a detrital origin of this mineral, it is suggested that it resulted from precipitation by H_2S of copper in solution. The fact that small amounts of chalcopyrite occur as inclusions in detrital pyrite grains can only be of negligible significance. Factor 3 (Cu) thus indicates a pure precipitation process.

C. HYPOTHESIS TWO

Sedimentological investigations carried out by various authors (Brock and Pretorius, 1964; Winter, 1964) have shown that the source area of the Basal Reef lies to the southwest of the Orange Free State Goldfield. It is thought that the source area was drained and eroded by a major stream which entered the basin of deposition at a point approximately 15 miles to the south of the Free State Geduld Mine. Therefore, the second hypothesis, invoked to explain the three factors, is to ascribe them to source deposits of differing paragenetic composition.

(a) Factor 1 (S, Ni, and Co), for instance, would represent a mineral assemblage from a sulfide deposit. Such deposits composed predominantly of pyrite often carry small amounts of cobalt- and nickel-bearing minerals. Furthermore, it has been shown that the cobalt and nickel values in the present study are predominantly contained in the Witwatersrand pyrite. Investigations by Carstens (1941 and 1942), Hegemann (1943), and Mitchell (1968) have shown that pyrite from sulfide deposits commonly contains nickel and cobalt as trace elements in various amounts.

(b) Factor 2 (Ag, Au, and U₃O₈) would represent a source deposit containing gold-silver and uranium. Such a paragenesis, however, seems to be rather unlikely and is not known from classic paragenetic types. Recent investigations have revealed that the gold derives, most probably, from gold-quartz vein deposits (Ramdohr, 1958; Saager, 1968 and 1969b), whereas the relatively high thorium content of the Witwatersrand uraninite appears to indicate a pegmatitic origin for the latter mineral (Schidrowski, 1966). Gold-quartz veins are not known to carry uraninite nor are uraninite-bearing pegmatites known to carry gold in any significant amount. At best, it can be visualized that the three elements originate from geographically related ore-deposits.

(c) Factor 3 (Cu) could be explained by a source deposit containing predominantly copper-bearing minerals (e.g. a chalcopyrite-quartz vein deposit).

From the above discussion it becomes quite clear that the first hypothesis is simpler to invoke and that it also conforms to a large extent with the results of recent studies of the history of the Witwatersrand banket. The authors feel, therefore, that it should be used in preference to the second hypothesis. However, further plausible explanations as to the interpretation of the three factors are possible.

CONCLUSIONS

1. The interpretation of the results indicate that the mise en place of the gold and uranium content in the Witwatersrand conglomerates was largely controlled by sedimentary processes, whereas the emplacement of the pyrite and the cobalt- and nickel-bearing minerals was both chemical (precipitation, sulfidization) and sedimentary. The copper seems to be formed solely by precipitation from solution.

2. The present study reveals that a factor analysis on a few geochemical variables obtained from a relatively small number of samples, as well as from a restricted area within the ore-deposit, gives results which correspond closely with the generally accepted theory on the mode of formation of the Witwatersrand bankets.

3. It is considered that such an approach could be useful as a practical and relatively rapid method of formulating an initial concept of the mode of formation of similar ore-deposits. In view of the present results, it is planned to carry out a more detailed investigation utilizing the Q-mode method of analysis, which is generally held to be of greater value in geological problems.

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