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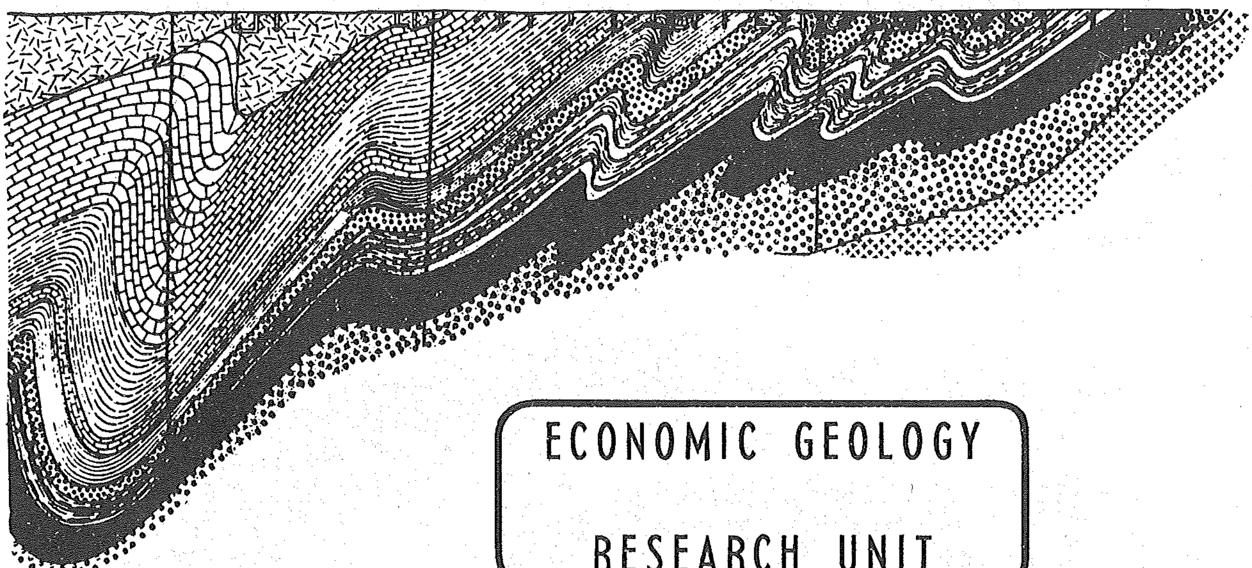

ECONOMIC GEOLOGY

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GEOMETRICAL CHARACTERISTICS OF SINUSOIDAL FOLDS

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by

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ABSTRACT

A method is outlined for determining the amount of compressive strain suffered by sinusoidal buckles and flattened sinusoidal buckles. The method is restricted to folds that have been formed as a result of a compressive stress acting in the plane of the bedding prior to buckling.

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GEOMETRICAL CHARACTERISTICS OF SINUSOIDAL FOLDS

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GEOMETRICAL CHARACTERISTICS OF SINUSOIDAL FOLDS

INTRODUCTION

Ramsay (1962) suggested that folds belong to two main types:

(a) flexure folds, in which the thickness of the bed remains constant when measured perpendicular to the bedding; this type of folding is probably brought about by a process of buckling;

(b) similar folds, in which the bed thickness, measured parallel to the shear (or flow) plane, remains constant; such folds can be generated by differential shear on parallel or sub-parallel planes, or by pure shear, or by a combination of the two.

Ramsay (1962) also described another group of folds which show complex variations in bed thickness. These are modified flexure folds in which an original flexural buckle has been further deformed, by a process of pure shear. This important group of folds can be used for determining the amount of flattening (ideally, pure shear) that they have suffered. The advantage of this approach is that it is concerned with the effect of pure shear on two geometrical elements of the fold - the thickness of the bed, and the angle that the bed makes with the axial plane at a particular point. It is not dependent on the location of the point in relation to the fold nor on the initial shape of the fold prior to flattening. However, the procedure is limited to folds which have their fold axis parallel to the b-kinematic axis.

Mukhopadhyay (1965) has developed various graphs for estimating the amount of compression from bed thickness variations of flattened flexure folds. These are based on the assumption that flexure folds can be represented by cylindrical surfaces with circular cross-sections, the difference in radii of two circles being a measure of the thickness of a particular bed. These cylindrical surfaces are flattened by pure shear to produce profiles of elliptical shape, from which thickness variations are derived for a particular bed subjected to varying amounts of compression. This method, applicable to certain ideally cylindrical folds, is limited to beds which have obtained a locked circular shape. Any other fold shapes which have not arrived at the locked position before being subjected to pure shear cannot be considered. The reason for this is that the functions used to derive the necessary graphs are based on equations of circles and ellipses.

Some other function is required to describe, in more general terms, the shapes of folds. Instead of using cylindrical profiles, sine curves are considered, as these have certain geometrical advantages, and because various workers have employed sine wave forms for the theoretical explanation of buckling (Biot, 1961; Biot, Odé, and Roever, 1961; Currie, Patnode, and Trump, 1962; Ramberg, 1964).

SINUSOIDAL FOLDS WITH PARALLEL FABRIC AND KINEMATIC AXES

It will be assumed that the particular layer to be buckled lies in the X-Z-plane of an orthogonal coordinate system comprising X-, Y-, and Z-axes. A compressive stress acts along the X-axis to produce buckles which may be described by the simple function $Y = R \sin X$, the fold axis of the buckle being parallel to the Z-axis. The geometry of the folded surface can now be studied in the X-Y-plane (Figure 1). To simplify further calculations, it is assumed that the surface $Y = R \sin X$ occurs in the middle of the buckled layer which has a thickness 2d.

It is desired to find the coordinates of a point (Xa, Ya) occurring at an orthogonal distance d from a point (X, R Sin X) on the median curve. The point (Xa, Ya) occurs above the median curve (Figure 2).

From Figure 2, the following relationships can be deduced:

$$\operatorname{Tan} \theta = R \operatorname{Cos} X$$

$$\operatorname{Sin} \theta = \frac{R \operatorname{Cos} X}{\sqrt{1 + R^2 \operatorname{Cos}^2 X}}$$

but

$$\operatorname{Sin} \theta = \frac{\Delta X}{d}$$

therefore

$$\frac{\Delta X}{d} = \frac{R \operatorname{Cos} X}{\sqrt{1 + R^2 \operatorname{Cos}^2 X}}$$

and

$$\Delta X = \frac{d\ R \operatorname{Cos} X}{\sqrt{1 + R^2 \operatorname{Cos}^2 X}}$$

$$Xa = X - \Delta X = X - \frac{d\ R}{\sqrt{1 + R^2 \operatorname{Cos}^2 X}}$$

similarly

$$\operatorname{Cos} \theta = \frac{1}{\sqrt{1 + R^2 \operatorname{Cos}^2 X}}$$

$$\operatorname{Cos} \theta = \frac{\Delta Y}{d}$$

$$\Delta Y = \frac{d}{\sqrt{1 + R^2 \operatorname{Cos}^2 X}}$$

$$Ya = R \sin X + \Delta Y$$

$$= R \sin X + \frac{d}{\sqrt{1 + R^2 \cos^2 X}}$$

For points lying below the median curve at an orthogonal distance d, the following relationships can similarly be obtained:

$$Xa' = X + \frac{d\,R}{\sqrt{1 + R^2 \cos^2 X}}$$

$$Ya' = R \sin X - \frac{d}{\sqrt{1 + R^2 \cos^2 X}}$$

Since it is possible to apply symmetry operations to one-fourth of a wavelength to produce a single wavelength, only a quarter of the fold will be considered from this stage onwards, i.e. only as far as $X = \pi/2$.

Having obtained equations for coordinates of the upper and lower surfaces of the buckled layer, a computer program was designed for ascertaining the geometrical characteristics of the fold. It was found that the most diagnostic feature of the fold shape was the difference in Y-value between the upper and lower surfaces of the fold for a particular value of X, i.e. the bed thickness measured parallel to the axial plane, or the T-value of Ramsay (1962). This value, T, was established by obtaining coordinates of points lying above and below the median fold surface ($Y = R \sin X$) at 50 intervals of X for the range $X = 0$ to $X = \pi/2$. With coordinates taken at such close intervals, it was possible to make a straight-line extrapolation between two adjacent points on the upper or lower surface, which was almost parallel to the tangent of the curve between the two points.

The computer program was designed to give different values of T at 50 intervals of X from 0 to $\pi/2$, for differing values of R (fold amplitude) and differing values of d (half bed thickness measured perpendicular to the plane of the bedding). Instead of using T (the axial planar bed thickness), the ratio T/T' was employed, where T' is the particular value of the bed thickness in the hinge of the fold, i.e. 2d at $X = \pi/2$.

Figure 3 is a plot of the variation in the bed thickness ratio for 50 intervals from the hinge to a position half-way down one of the limbs of the fold, i.e. 50 positions at equal intervals over

$\frac{1}{4}$ of the wavelength of the buckle. The variation of bed thickness ratio is shown for different values of the amplitude of the fold, R. The effect of the fourth variable, d, is not shown in Figure 3, but in Figure 4. Here, the departure of the $^{T}/T'$ ratio from a "standard value" is plotted for different bed thickness/wavelength ratios, and for different values of R. The "standard value" of $^{T}/T'$ is the value at which R and d have no influence on the ratio. The departure of the $^{T}/T'$ ratio from the "standard value" is also shown for different positions in the $\frac{1}{4}$-wavelength segment of the fold investigated. These points are situated at X = 0, 0.3141, 0.6283, and 1.0990 radians. Figure 4 shows that, for large values of R and d (or large bed thickness/ wavelength ratios), there is a significant departure from the "standard value". The curves are therefore inaccurate. A horizontal line has been drawn on each diagram equivalent to a value of Y = 0.010 on Figure 4. This is an empirical cut-off value where the data of Figure 3 are reliable. Figure 3 can thus be used only when the fold characteristics fall in the region below the cut-off value of Figure 4. The value of 0.010 on Figure 4 was choosen as it probably represents the accuracy at which it is possible to plot data on Figure 3. It is equivalent to a single division of the Y-axis in Figure 3.

As a generalization, therefore, folds in which R is greater than 0.6 cannot be studied by this method, if the bed thickness/wavelength ratio is $\frac{1}{4}$. Similarly, values of R greater than 0.9 cannot be used when the bed thickness/wavelength ratio is 1/6.7. When the bed thickness/wavelength ratio is 1/10, the curves of Figure 3 are reliable as far as R = 1.3. For smaller bed thickness/wavelength ratios, the curves of Figure 3 are reliable over the whole range of R values, for all practical purposes.

THE AMPLITUDE OF SINUSOIDAL BUCKLES

If a competent layer in a rock sequence has been deformed into a sinusoidal wave with the characteristics described above, then it is possible to determine the amplitude of the fold by measuring bed thicknesses parallel to the axial plane. The procedure to be adopted is as follows:

1. Obtain a profile of the fold which is orthogonal to the fold axis.

2. Define the mid-position of the one limb of the fold. This can be done by drawing a line joining the anticlinal crests and another line joining the synclinal troughs. The upper surface of the layer would be used for the anticlines, and the lower surface for the synclines. A line parallel to these two lines, occurring mid-way

between them, would intersect the fold limbs at the desirable $\frac{1}{4}$-wavelength position, i.e. where X = 0 on a sine curve.

3. Having located this position, divide the X-axis into an aliquot of 50.

4. At each value of X, measure the axial planar bed thickness, T.

5. Calculate the ratio $^{T}/T'$, where T' is the bed thickness in the hinge of the fold.

6. Plot the ratio $^{T}/T'$ for the particular position of X on the scale of 50 divisions of Figure 3. The resulting graph will give the value of R.

If the estimate of the bed thickness/wavelength ratios falls beyond the safe areas of Figure 4, the curves of Figure 3 cannot be used, and the method is not applicable.

Data on natural fold characteristics are few. The work of Currie, Patnode, and Trump (1962), which was based on field examples, showed a remarkable straight-line correlation between bed thickness of the competent member and the wavelength of the fold. The ratio was established as 1/27. Within this range, the graphs are applicable for all values of R up to 1.5.

Little is known about the limits to which a fold will maintain sinusoidal characteristics. Two schools of thought exist. One maintains that concentric folds will attain a circular profile when the layer is shortened by approximately 36 per cent. The second believes that, even when the amplitude is increased with time, the fold characteristics are sinusoidal.

THE AMOUNT OF COMPRESSION IN SINUSOIDAL FLEXURE FOLDS

Assuming that the buckles have not been subjected to pure shear, the value of R in a particular fold is related to the amount of compression, or shortening. For sinusoidal folds of the type described above, the relationship Y = R Sin X is established, and R is known. Now, if S is equal to the arc length of the fold across the wavelength, then the percentage shortening will be given by:

$$\%\ \text{shortening} = \frac{S - 2\pi}{S} \times 100$$

where S = arc length.

The conventional formula for arc length is:

$$S = \int (1 + (\frac{dY}{dX})^2)^{\frac{1}{2}} dX$$

which, in the present example, becomes

$$S = \int_0^{2\pi} (1 + R^2 \operatorname{Cos}^2 X)^{\frac{1}{2}} dX$$

Using these relationships, R can be related to the percentage compression, as shown in Figure 5.

THE AMOUNT OF PURE SHEAR IN FLATTENED SINUSOIDAL FOLDS

Let the amount of compression produced by pure shear be represented by K. In Figure 6, the change in coordinate values of points due to a pure shear deformation would be:

$$X' = {}^{X}/K$$

$$Y' = KY$$

This applies to the case where the compressive axis is the X-axis, and is at right angles to the axial plane of the fold.

The critical value in defining the fold shape is the ${}^{T}/T'$ ratio. In Figure 7, referring to the fold which has not suffered pure shear, this would be:

$$\frac{Y_1 - Y_2}{Y_3 - Y_4} = {}^{T}/T' \quad \text{at the point Xa.}$$

If this fold is subjected to a compression which is represented by K, the deformed ${}^{T}/T'$ ratio would be:

$$\frac{Y_1' - Y_2'}{Y_3' - Y_4'}$$

but $\qquad Y_1' - Y_2' = K (Y_1 - Y_2)$

and $\qquad Y_3' - Y_4' = K (Y_3 - Y_4)$

thus $$\frac{Y_1' - Y_2'}{Y_3' - Y'_4} = \frac{Y_1 - Y_2}{Y_3 - Y_4}$$

The original $^T/T'$ ratio is thus quite independent of the amount of pure shear, and therefore Figure 3 can be used for both unflattened and flattened sinusoidal buckles. They can both be treated in the same way, and will give the value of R. The flattened sinusoidal buckle will give its original value of R, prior to flattening, and thus the original amount of shortening prior to flattening (Figure 5). From Figure 6 the percentage shortening caused by pure shear can be obtained. By ascertaining the R value of the flattened flexure fold, the original fold geometry is fixed prior to flattening, i.e.

$$R = {}^Y/X$$

Now, in the flattened fold

$$X' = {}^X/K$$

$$Y' = YK$$

thus $${}^{Y'}/X' = K^2\ {}^Y/X$$

$$= K^2\ R$$

$$K = \sqrt{{}^{Y'}/RX'}$$

R is known and Y' and X' can be measured on the fold. By obtaining the midway position on the limb of the fold, the $\frac{1}{4}$-wavelength can be examined. By drawing a line through the midway position perpendicular to the axial plane, X' can be measured. Y' can also be measured along the axial plane from where the midway horizontal line intersects the axial plane. K can thus be determined.

In order to obtain the amount of shortening caused by pure shear, the following relationship can be used:

$$\%\ \text{shortening (pure shear)} = \frac{K - 1}{K} \times 100$$

Using this value in conjunction with the shortening caused by buckling (Figure 5), the total amount of compression can be obtained.

SINUSOIDAL FOLDS WITH NON-PARALLEL FABRIC AND KINEMATIC AXES

A second condition to be considered is that where the original plate is parallel to the orthogonal X-axis of the coordinate system, but makes an acute angle φ with the Z-axis. If the buckle still deforms according to the $Y = R \sin X$ relationship in the X-Y-plane, then the relationship between measurements made in the X-Y-plane, parallel to the Y-axis, and those made orthogonally to the axis of the fold which makes an angle φ with the Z-axis of the coordinate system, but parallel to the axial plane of the fold, is simply a $\cos \varphi$ relationship. In other words, if a distance, a, is measured in the X-Y-plane parallel to the Y-axis, and the corresponding distance when measured parallel to the axial plane in the orthogonal section at right-angles to the fold axis is b, then a is related to b by:

$$b = a \cos \varphi$$

Since $\cos \varphi$ is a constant:

$$b = a \text{ (constant)}$$

If the distance, a, corresponds to R in the X-Y-plane where the buckle has the form $Y = R \sin X$, any corresponding value of Y in the orthogonal section would be:

$$\begin{aligned} Y' &= (R \sin X)(\cos \varphi) \\ &= R \sin X \cdot \text{constant} \\ &= R' \sin X. \end{aligned}$$

Thus the sine rule would hold for these oblique sections. This method thus has an added advantage over those based on folds having circular cross-sections, where oblique sections of the type considered here have elliptical shapes. In the presently described analytical procedure, the sinusoidal and $^{T}/T'$ characteristics are maintained, and Figure 3 can be employed. To obtain the correct amount of compressive strain, all studies would have to be made in the a-c-kinematic plane of the fold. The only assumption is that the fold shape is sinusoidal when observed in a section orthogonal to the fold axis.

CONCLUSIONS

It is suggested that, if folds deform according to a sinusoidal rule, then it is possible to calculate the amount of compression that the buckle has experienced by using the procedure outlined. The restriction placed on the application of this method is that the compressive stress axis must occur in the plane of the layer being buckled. If this restriction is not met, then the procedure cannot be applied. It cannot, therefore, be used with asymmetrical folds. Another restriction is due to the fact that, during buckling, there occur both layer shortening and buckling. The value obtained from the presently described technique does not take layer shortening into account, and is thus a minimum value.

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Acknowledgements

The author is indebted to Mr. M. Gould of the Department of Mining, of the University of the Witwatersrand, who wrote and ran the computer programs.

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Key to Figures

Figure 1 : Characteristics of the median surface of a sinusoidally buckled layer. The amplitude of the fold is equal to R.

Figure 2 : Details of the geometry of a point (Xa, Ya) situated at a normal distance, d, from a point (X, R Sin X) on the curve Y = R Sin X.

Figure 3 : The relationship of axial planar bed thickness to the position on the fold and to amplitude of the fold. The curves 0.1 to 2.0 represent the amplitude of the folds with values of R from 0.1 to 2.0. The X-axis has been divided into 50 divisions, and represents the right-angular distance from the hinge of a fold, from the axial plane to a point midway along the limb of the fold. There are 50 divisions of X, from X = 0 to X = $\pi/2$, on a fold, the median surface of which is Y = R Sin X. The Y-axis is a plot of the bed thickness, measured parallel to the axial plane divided by the bed thickness in the hinge of the fold.

Figure 4 : The departure of the axial planar bed thickness ratio from a constant value, with variation in R (the X-axis) and bed thickness (the various curves). Bed thickness is expressed in terms of bed thickness/wavelength ratios, viz. 1/4, 1/5, 1/6.7, 1/10, and 1/20. Four separate diagrams are presented to show the variations at four particular positions of the fold, viz. X = 0, 0.3141, 0.6283, and 1.0990 radians, on the X-axis of Figure 1.

Figure 5 : Graph showing the relationship of percentage shortening to R, the amplitude of a sinusoidal buckle.

Figure 6 : Sinusoidal flexure fold (thin line) which has been subjected to pure shear (heavy line). It is assumed that the axial plane remains stationary.

Figure 7 : Characteristics of a sinusoidal buckle that has been shortened. This diagramatically portrays the change in dimension of the axial planar bed thickness from an original position (heavy fold) at Xa to a deformed position, after the fold has been flattened by pure shear (dotted fold) at Xb.

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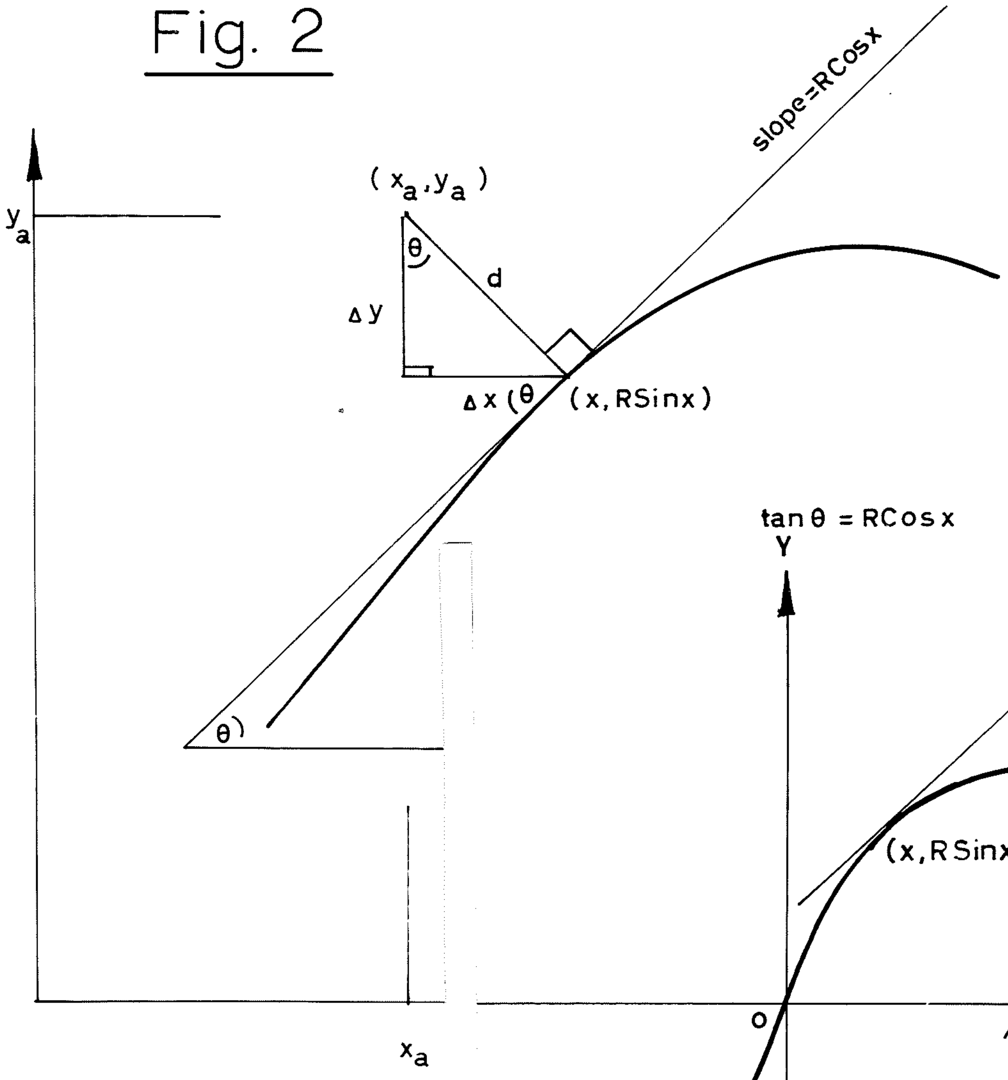
Fig. 2



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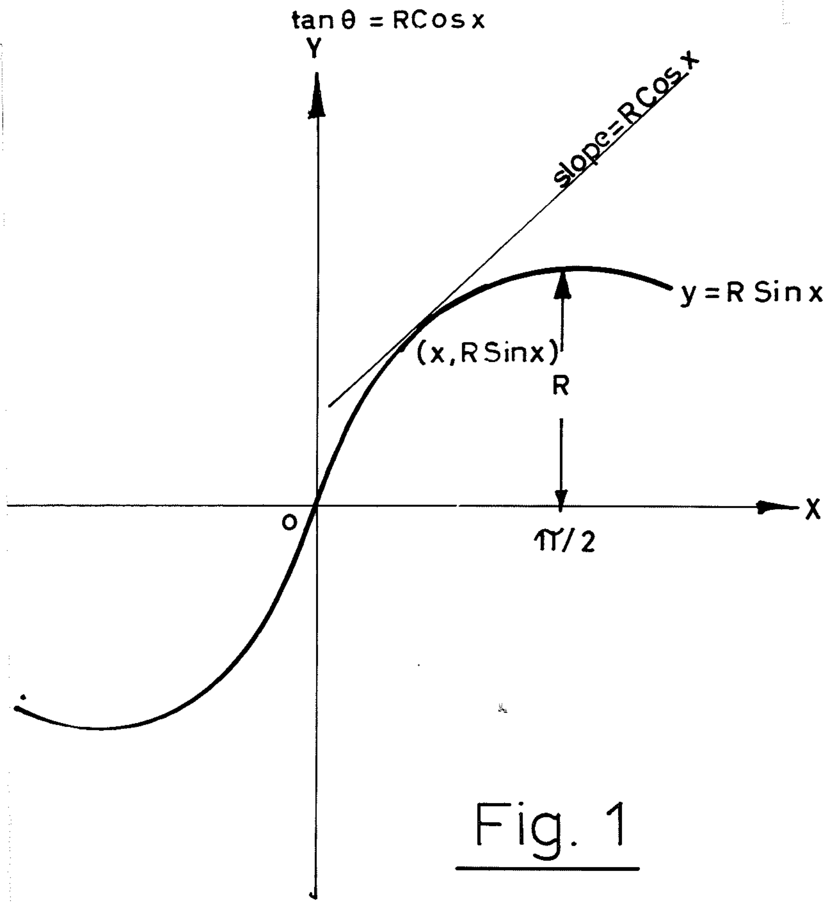
(xa , ya)
ya
θ
d
Δy
Δx (θ
(x, RSinx)
Y = RSinx
θ
xa



tan θ = RCos x
Y
slope=RCos x
y = R Sin x
(x, R Sinx)
R
o
X
π/2

Fig. 1

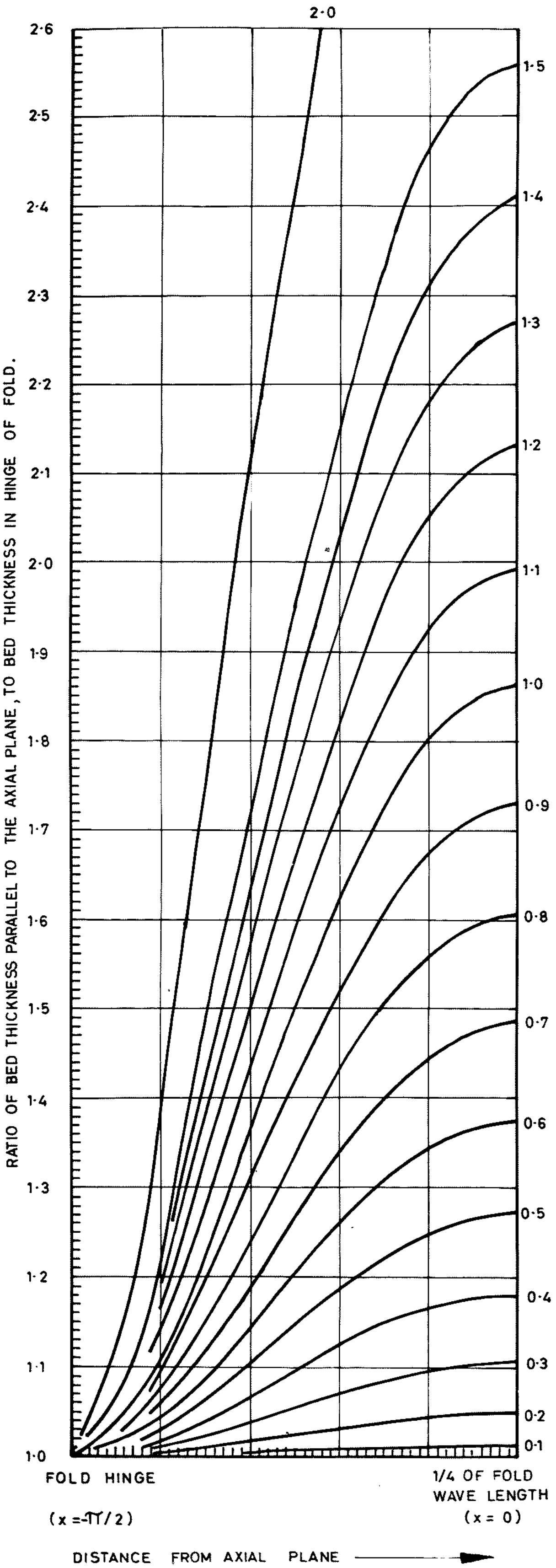


Fig. 3

Fig. 6

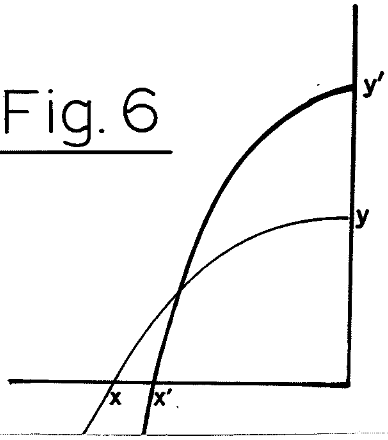


Fig. 7

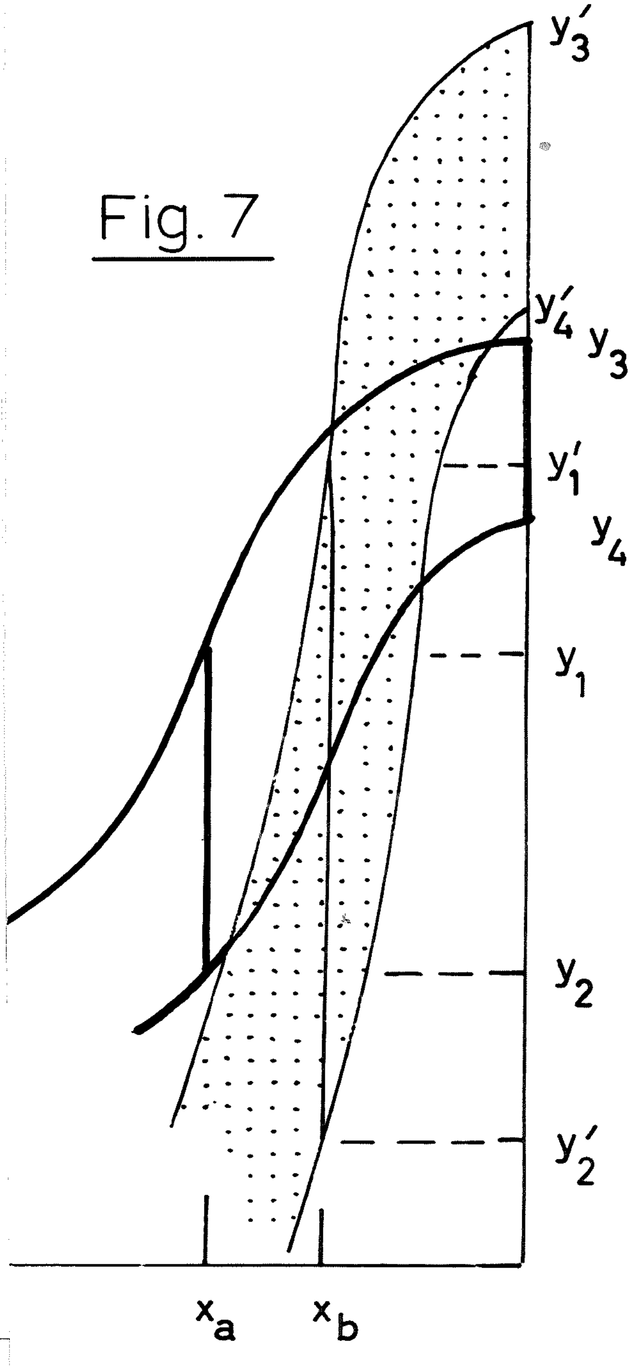
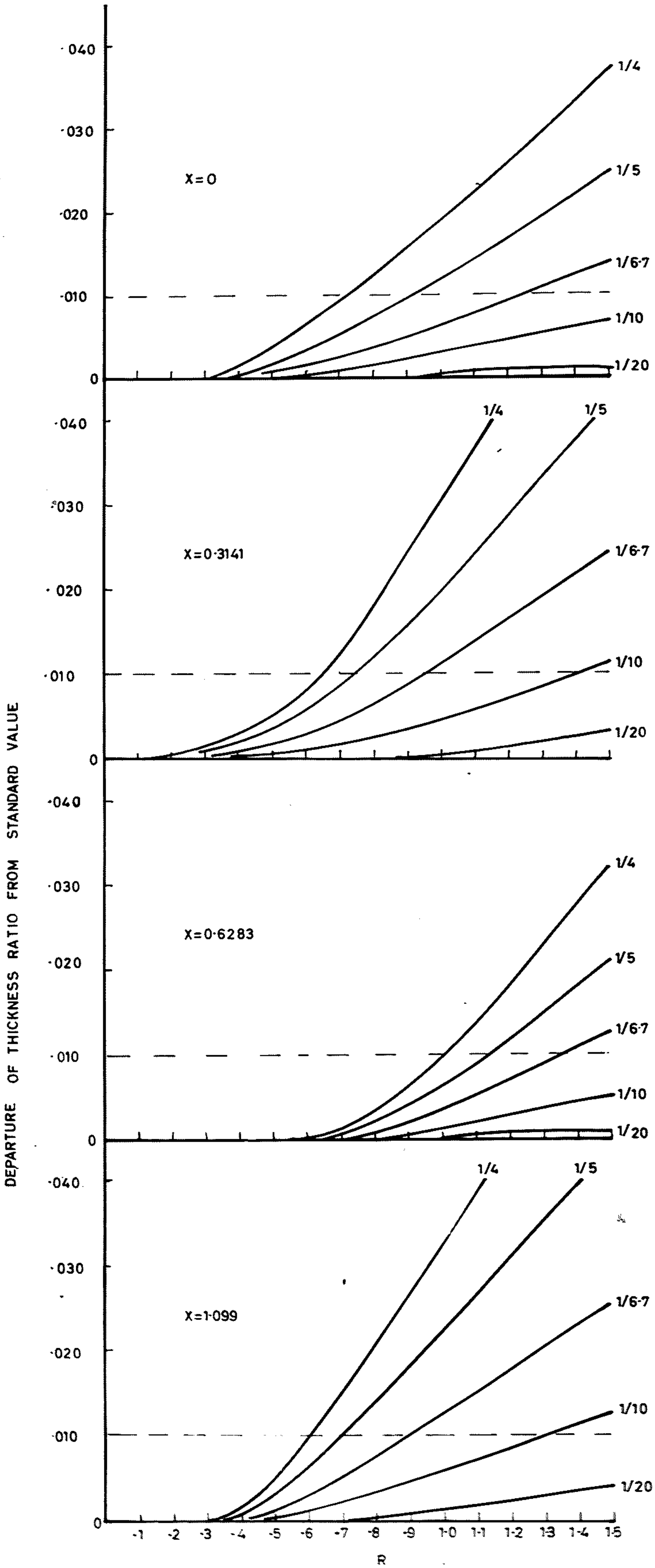


Fig. 4



DEPARTURE OF THICKNESS RATIO FROM STANDARD VALUE
X=0
X=0·3141
X=0·6283
X=1·099
·040
·030
·020
·010
0
1/4
1/5
1/6·7
1/10
1/20
·1
·2
·3
·4
·5
·6
·7
·8
·9
1·0
1·1
1·2
1·3
1·4
1·5
R

Fig. 5