ECONOMIC GEOLOGY RESEARCH UNIT

University of the Witwatersrand
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SOME ASPECTS OF ORE RESERVE ESTIMATION

by

F. MENDELSOHN

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ABSTRACT

Global ore reserve estimates for tabular bodies must provide the best estimate of what is present, having regard for the amount of data available and the cost of the data. Grade is more sensitive, of more immediate importance, and more difficult to estimate than tonnage. Geological factors must be given prime importance, but beyond them or in the absence of knowledge about them, the mathematical relation between intersections is the basis for reserve estimation. As the density of intersections increases so the variance of estimates decreases and the results by various methods approach each other. The grade, thickness, and accumulation (thickness x grade) are the three basic variables of which only the thickness and one other can be independent variables. Correlation between grade and thickness affects the relation between these variables and has a significant effect on reserve estimation.

Small (trace) amounts of the mineral containing the element of interest will tend to be lognormally distributed (positive skew), medium amounts normal, and large amounts negatively skewed. The amount of probable range of content suggest the distribution and probable degree of difficulty likely to be experienced.

The four main groups of estimation methods are geometric, mathematical, statistical, and geostatistical.

Geometric methods are based on an assumption of areal influence of intersections or linear change of accumulation between intersections. Grade estimates are affected by correlation. The 'Isted' integration allows for the assumption of a linear change in grade. Geometric methods, with these provisions, can still be usefully applied to many ore reserve estimation problems, though they do not permit the calculation of confidence limits.

Statistical methods give an equal influence to each intersection, are quick and relatively simple to apply, unbiased, understandable, provide comparable confidence limits, and form a useful and important means of estimating global and other reserves. The normal distribution and, in particular, arithmetic means are applicable in the 'percent' range of metal/mineral content.

Geostatistical models take into account the spatial relationship of regionalized variables, and estimates are unbiased and provide confidence limits, but are applicable mainly to second stage estimation involving block evaluation.

Eight case studies of base mineral deposits, six from the Zambian Copperbelt and two from North America, provide data that generally support the above conclusions but provide the raw data that can be further tested and examined.

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SOME ASPECTS OF ORE RESERVE ESTIMATION

1. INTRODUCTION

Ore reserve estimates, i.e. quantitative assessments of the tonnage and grade (content) of ore, potential ore, or 'mineralized material' are basic pre-requisites for decision-making and planning during exploration, development, and mining. Each estimate represents a compromise between the need to reach a level of accuracy or reliability sufficient for the particular purpose so as to avoid possible disaster, and the need to keep costs down by using the smallest possible number of intersections. The best compromise is reached by the most efficient method of reserve estimation, particularly one that has good performance characteristics at low levels of information.

There have been many developments in ore reserve estimation in recent times, particularly in the use of statistical, geostatistical, and computerized methods, but it nevertheless seems worth while considering some of the basic elements, principles, and concepts involved, as well as comparing various methods or groups of methods of estimation. There are undoubtedly many mines, particularly smaller ones, that do not have the facilities or the trained personnel for the use of the sophisticated methods, or where their use is not warranted on technical or financial grounds. Also, the mere fact that a method is sophisticated does not automatically mean that it is better. It is also perhaps risking going against some modern trends to suggest that where a simple method will give an acceptable result, there is no need, indeed no point, in using a complicated one.

With most orebodies it is not possible to achieve a precise measure of the amount of ore that has been mined out, owing to geologic characteristics and to mining factors such as incomplete recovery and dilution by external waste or low-grade material not included in the reserve. Thus it is not often possible to obtain a direct assessment of efficiency of estimation, and indirect assessments have to be made.

Many of the improvements made in the past have been empirical, formulae or methods being devised that accounted for observed characteristics of particular ores, to be followed later by the rationale behind them, or were made on imperfectly understood grounds.

The discussions that follow mainly concern tabular bodies, those in which two dimensions are large compared with the third, a common situation in mineral deposits and one that is more readily handled and understood; it is believed that the principles will extend to deposits of other shapes.

II. SAMPLING, SAMPLE PREPARATION, ASSAYING

All ore reserve estimates are based on samples that are measured, collected, prepared, and assayed, and no ore reserve estimate, no matter by what method it is achieved, is any better than the quality of this work and its results. This whole field of endeavour is a most important one, not always given its proper emphasis, and one on which much has and could be written. The present exercise concerne the methodology of treatment of the results obtained from these procedures and for simplicity it is therefore assumed that all data or results utilized are free from error. In practice an important part of ore reserve estimation would be to test these procedures and to apply corrections if required.

There are sources of error, nearly all having a bias, in all these procedures and it is therefore important that they be understood, appreciated, and prevented or allowed for. Some of the factors that can cause problems and (generally) bias are :

In core drilling : loss of core, even on a minor scale, erosion of softer parts of the core during drilling, collection or non-collection of sludge, and for non-coring drilling (or the sludge from core drilling), the manner of collection of sludge, losses of sludge in drilling, etc.

In hand sampling : uneven sampling, such as more of soft than hard portions, or more of rich than poor (or vice versa), poor selection of sampling sites, accessibility problems, poor measurement, etc.

In sample preparation : losses due to poor equipment or the incorrect use of equipment, loss of fines in dust-collection apparatus, introduction of impurities from grinding or other equipment, poor procedures, etc.

In the analytical laboratory : the problems of precision, accuracy, and bias from various causes are well known and are guarded against as part of normal procedure in any well-run laboratory.

The deliberate introduction of material (salting) at any stage of the sampling procedure must of course always be guarded against.

III. REQUIREMENTS

The purpose of ore reserve estimates is to outline the amount, quality and distribution of material that it profitably minable under defined conditions, using the term minable in its widest sense. A mineralized body that is outlined and measured to within similar limits before the viability is established, usually in order that its viability can be judged, should be referred to as constituting a mineral (or coal, copper, gold, etc.) reserve, the term ore reserve being applied only when it is known or shown to be economic.

For such purposes, what is required is a global estimate of the whole body or bodies, or of portions thereof that show significant differences. When for detailed planning or mining purposes it becomes necessary to estimate the reserves within specific blocks, certain other considerations become involved. For example, minor irregularities may not be important for global estimates but could be significant in day-to-day mining and metallurgical operations, and greater detail is needed for block estimates.

The purpose of ore reserve estimation is to make the best estimate of what is actually present, having due regard for the amount of data available and the cost of obtaining data.

Once a certain tonnage has been established, enough to support operations for between 10 and 20 years in most cases, fluctuations in global tonnage within fairly wide limits have little significance compared with those in grade. For example, an increase of 10% in the amount of ore reserves will allow operations to continue an extra year or two at the end of the mine's life, but would have no effect on today's cash flow and little on the present value of the property. On the other hand, an increase of 10% in grade would, assuming metallurgical plant can cope, lead to a 10% increase in production, which would create an almost immediate increase of 10% in cash flow and, since the operating costs would be little effected, an even greater increase in profitability. The effect of a decrease in ore reserve grade would be equally drastic compared with a tonnage decrease, leading to a decrease in cash flow and a lower profit or even loss. Though less severe, the effect is similar to changes in price of the product, and the effects of price fluctuations are too evident to require detailing here. Within individual mining blocks, decreases in tonnages estimated could have a severe effect unless there is sufficient flexibility in operation, or reserve faces. Thus, although it is important to estimate both grade and tonnage as closely as possible to the true, the estimation of grades is of greater immediate import, and is also the more difficult problem.

It should be clearly understood and stated what is being measured, and its degree of reliability. The degree of reliability can be indicated by the traditional terms *potential*, *possible*, *probable*, and *proved* or *proven* provided that these terms are defined. To be preferred as being more specific and comparable is the concept used in statistics and geostatistics of the confidence limits and confidence level; on this basis the statement 1 000 tons ± 100 tons (confidence limits) at the 90% confidence level, indicates that there is a 90% chance of the tonnage lying between 900 and 1 100 tons.

It is important that there should not be bias (i.e. consistent distortion in one direction) in estimates, but the question of bias should be kept in proper perspective. If, for example, a grade is quoted as 2,5% ± 0,8% at the 95% confidence level, a bias of 0,1% is insignificant compared with the confidence limits and well within them. The bias should become significant only when it approaches the magnitude of the half confidence interval, but as opposed to this, for such an orebody a change of 0,25% in grade would be highly significant in economic terms, and could make the difference between viability and non-viability. It would seem, therefore, that it is desirable to avoid bias, but each case should be considered on its merits and if bias is small relative to the confidence limits and not high enough to be economically significant, it is not worth adopting a method of estimation purely to avoid this bias. Bias is, therefore, one of the factors to be considered in deciding on methods of estimation and precautions to be taken, but not the only or overriding factor.

IV. DEFINITIONS

The following definitions are proposed for some terms related to ore reserves or used in the text.

MINERAL DEPOSIT : A body of rock containing one or more minerals, elements, or mineral aggregates whose extraction is potentially economically feasible.

OREBODY : A body of rock (in its broadest sense) of sufficient mass and with a sufficient content of one of more minerals, elements, or mineral aggregates that their extraction is economically feasible under prevailing or predicted conditions.

ORE : Naturally occurring material from which one or more valuable minerals, elements, or mineral aggregates can be profitably extracted, generally synonymous with orebody, or the material when extracted.

ORE RESERVE : A body of rock whose quantity and quality (valuable substance content) has been estimated to a necessary and stated degree of accuracy and whose valuable substance (mineral, element, or mineral aggregate) can be profitably extracted.

MINERAL RESERVE : A body of rock whose quantity and quality (valuable substance content) has been estimated to a necessary and stated degree of accuracy but whose valuable constituent is not known to be profitably extractable; it could be expressed in terms of the valuable substance, such as a copper reserve, coal reserve, etc.

INTERSECTION : An intersection of an orebody is a complete sampling by any method of the whole thickness and is expressed as a thickness and grade. The thickness is often expressed as the true thickness (T.T.), perpendicular to the plane formed by the two major dimensions of a tabular orebody, but any thickness (e.g. horizontal) can be used. The grade is the average of the samples forming the intersection, each weighted by its length and if necessary the specific gravity.

SAMPLE : In the mining, geological, and/or ore reserve context, a sample is a portion of the deposit that is selected and taken so as to be as representative as possible of a particular portion of the deposit; in most cases an intersection includes a number of samples. This type of sample is differentiated from a statistical sample, which consists of a number of geological samples or intersections.

GRADE : Content of valuable mineral or element, expressed as a weight fraction.

CUT-OFF : The lowest grade material constituting all or part of an orebody that can be profitably treated.

V. GEOLOGICAL FACTORS

Where there are any geological factors that affect ore reserve estimation these must be taken into account and, in most cases, would become the governing factor(s) in any method of estimation, particularly as regards areas of influence and limits of areas or blocks, though the effect and the way in which they are catered for will vary for different methods. Some of these factors are :

Faults, which could cause duplication or loss; limit the body by throwing the opposite side out of reach; lead to a difference in the type or habit of mineralization if related to the mineralizing process.

Folds, which could cause duplication; change in attitude; change in thickness from one limb to another.

Change in type or habit of the mineralized body, such as change from massive to disseminated sulphide; zoning change in valuable components; change in host rock leading to change in habit; mineralogical changes or differences that require different metallurgical treatment.

Multiple bodies, where the intersections of different bodies should be kept separate and not combined as if part of the same body.

Layering, where layers within an orebody are differently mineralized, either by hypogene or supergene processes, giving different minerals, valuable components, or grades.

Difference in grade, where the possibility of selective mining or a suitable balance of mining might be necessary or worth considering.

Differences in specific gravity caused by the presence of particular components or their removal by leaching.

Scale, since some factors would be unimportant for a global assessment but would be important on the scale of blocks or stopes.

Ore shoots and trends, since these could require separate outlining and consideration, or could be important in assessing how a particular method of estimation copes with them or can be adjusted to cope with them.

Where no geological factors are known to interfere, the handling of changes between intersections is crucial to ore reserve estimation and is the *raison d'etre* of many methods. In a general way it can be said that geometric methods assume certain linear changes between adjacent intersections, statistical methods ignore such changes, and mathematical and geostatistical methods use other intersections in the area to influence and model the nature and rate of changes between intersections. It is the appreciation of what is involved in such changes, and their effects, that lies behind the assessment and use of methods of estimation.

VI. DENSITY OF DATA

The data used in ore reserve estimation are samples taken from exposed faces of a body or from cores or sludges obtained by drilling through the body. For each sample position the samples are combined to give a complete exposure or an intersection of the body. The thickness is a sum of the individual thicknesses and may be true (perpendicular to the plane of the two major dimensions), horizontal, or other convenient thickness; the average grade of the body over the full thickness is obtained by weighting each sample by its thickness. The resulting thickness and grade at each position or point is the intersection that becomes the data used in calculations, and the density of data refers to the number or spacing of these intersections.

Though each is costly, the 'point' intersection is a very small part of the total deposit, and even the sample, in the statistical sense, that is formed by all the intersections used is a very small part of the total orebody. Even 'bulk' samples used for special purposes are very small parts of the whole. Therefore, the smallest sample that will give an estimate to within a desired degree of confidence or accuracy is the important one, and much effort has been devoted to establishing this. Clearly, if sufficient samples are taken, a precise and accurate idea of the tonnage and grade will be obtained, the ultimate being the complete excavation and measuring of the body. However, as more and more samples are taken, a stage should be reached when all differences are cancelled out, all estimates and all methods give the same answer, and more samples will not change this answer; this should be the true estimate of reserves.

As the number of intersections increases, the distance between adjacent intersections decreases proportionately in such a way that for a doubling of intersections within an orebody the average distance between intersections is reduced by approximately one third. The variance of estimates will decrease, but not necessarily in the same ratio, since this will be influenced by local conditions. The difference between different methods of ore reserve estimates will also decrease: methods based on similar principles will approach each other more rapidly than those based on different principles, but as the density of data increases, a stage will be reached where all methods of estimation will give virtually identical results. This suggests the important concept that for sufficiently dense or closely-spaced intersections it is immaterial, from the point of view of the accuracy of results, what method is used but that, conversely, as the density of intersections decreases the choice of method becomes more important.

It is also useful in that estimates based on closely spaced intersections, particularly if several methods agree, can be an acceptably accurate estimate, and can also be used as an experimental standard against which to compare estimates of the same body with sparser and more widely distributed intersections (see Case Study D). If such estimates can be compared with reliable production data, then their absolute accuracy could also be tested.

VII. GRADE/ACCUMULATION

Two methods of expressing the value of ore are used: The one is the grade (or valuable component content) expressed as a weight fraction of the whole, such as percentage, ounces per ton, grams per ton, etc. Since the weight fraction is based on the rock volume and its specific gravity, it is an expression of the valuable component content per unit volume of the body. The other unit is the product of the grade and the thickness, commonly the true thickness (T.T.), expressed as foot-percent, inch-pennyweight, centimetre-gram, etc, referred to by the Matheron school of geostatistics as the accumulation, and is an expression of the valuable component content per unit area. The grade is an absolute expression of the quantity of valuable component in the rock that reaches the treatment plant or market, and would seem to be the more fundamental unit, but in terms of ore reserves it is meaningful only if it applies over a minimum thickness i.e. provides sufficient volume or tons to be worth working. The accumulation is useful in mining to derive such figures as the volume of rock that is mined by a specific face advance in a stope; it is used in ore reserves as the (or a) basic measure of value, though in this case a minimum grade limit must be set for it to be meaningful, and as an intermediate step in many methods of estimation.

Koch and Link (1971, p236) suggested that neither variable is inherently more fundamental and showed that for tabular orebodies thinner than the minimum possible stoping width of around 1 m, the accumulation (or mixed-unit) gives an unbiased estimate of the mined grade of ore, since it is not affected by the waste mined outside the ore as would be the grade. This perhaps explains the common use of this measure of value for thin bodies, such as the Witwatersrand, but does not apply to thicker bodies. Krige (1978), though he uses the accumulation throughout in his geostatistical approach, particularly for the Witwatersrand ores, showed that for the Prieska copper-zinc deposit the use of the grade instead of the accumulation is justified for practical reasons and because '...... in the valuation of a massive ore body such as at Prieska metal grades rather than accumulations are the main goal ......'. Since both grade and accumulation are used to measure the value of an orebody or of an intersection of an orebody, there seems to be no reason for ore reserve purposes in selecting either as the fundamental measure. It is however important to appreciate that in the relation between thickness, grade, and accumulation, if any two are fixed the third becomes automatically fixed, i.e. it is a dependent variable. The thickness is one independent variable and it should always be clearly understood whether the grade or accumulation is being used as the other independent variable and how the dependent variable is affected: for example, if the grade has or is assumed to have a particular rate of change from one intersection to another, the accumulation will have a fixed but different rate of change over the same interval. Some of this may sound obvious, but it has been found that, for example, the grade is stated or assumed to change linearly from one intersection to another, whereas the calculation is done in such a way that it is the accumulation that changes linearly, which would mean that the change in grade is actually non-linear. Thus, should it be established or assumed that the grade varies linearly, then grade should be the independent variable, and if the accumulation varies linearly, *it* should be the independent variable; their relation will be discussed in the next section. In most of the deposits considered in this paper there is a notable tendency for the statistical distribution of grades to be symmetrical and approaching normal, whereas that of the accumulation tends to be strongly positively skewed, approaching lognormal.

Should there be a relation between the mode of genesis and the present distribution of the valuable minerals it might be possible to decide which is the fundamental association. For example, if the genetic process was one where a fluid containing an element were to be evenly distributed beneath a particular formation and to migrate evenly and vertically through that formation, depositing its load over a greater or lesser thickness of rock, then there would be a fundamental relation of the element of the unit area. Some such origin has been suggested for tabular deposits such as White Pine (Cu), and seems likely for porphyry deposits. In the case of the Witwatersrand gold-uranium deposits it has been proposed that the deposition was of a placer type, with near-horizontal movement of fluids and an interplay of sedimentary factors.

Whatever the merits of particular genetic models, it is clearly not desirable to tie a method or principle of ore reserve estimation to any model, not least because seldom is there an approach towards agreement for a genetic model till the deposit concerned is old or worked out, and ideas change with remarkable, almost alarming regularity, in some cases in a cyclical manner. It is therefore essential that ore reserve estimates, which become important early in the life of a deposit, be based not on genetic ideas, but on measurable physical properties.

VIII. CORRELATION AND REGRESSION

There could be a correlation between various properties of an orebody, such as between different elements, or between the same element at different positions, but the correlation that is important in ore reserves is that between the metal or other valuable constituent content (grade) and the thickness of the body. This correlation could be positive, i.e. thickness and metal content increase and decrease together, or negative, with one decreasing while the other increases (Fig. 1). The degree of correlation, or the strength of the tendency to increase and decrease together or inversely could also be non-existent. The correlation coefficient r is expressed as a decimal ranging from +1 to -1: +1 would be a perfect positive correlation, -1 a perfect negative correlation, zero no correlation, and intermediate figures intermediate degrees of correlation. Thus $r = 0{,}1$ indicates a weak positive correlation, $r = 0{,}9$ a strong negative

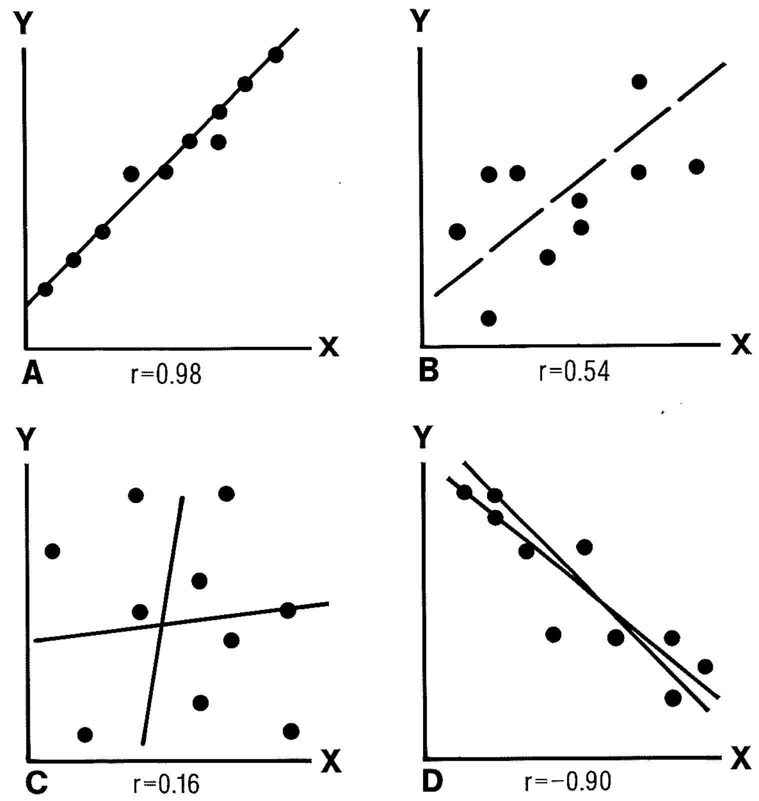


Figure 1 : *Examples of correlation between two variables. In A there is almost perfect positive correlation; B shows a good degree of positive correlation; C shows a poor positive correlation; D shows a high degree of negative correlation, where one variable decreases as the other increases. Lines of regression, best expressing the relationship between the two variables are shown for each (Hazen and Gladfelter, 1964).*

correlation, etc. The relation between two variables can be expressed by a line of regression expressing one (dependent) variable as a function of one or more (independent) variables, together with constant(s) and random fluctuations if necessary. In the present context, a line of regression represents the minimization by least squares of one variable about the other giving a straight line with the formula $y = a + bx$ (Fig. 1). There are two possible lines of regression, using each variable as a base; the two lines cross at the arithmetic means of the two variables, and the angle between the lines becomes smaller as the correlation coefficient increases (greater as it decreases). The correlation expresses the degree of dispersion about the line of regression, which can also be expressed in terms of variance. Regression analysis can be extended into two and three dimensions by means of planes and quadratic surfaces, some of which will be referred to later.

In the Witwatersrand gold-uranium deposits there are, within the broad sedimentary depositional system, some processes that would lead to positive correlation (e.g. concentration of a rich deposit in a channel), some that would lead to negative correlation (e.g. erosion by winnowing of a deposit on a ridge with residual concentration), so that while either could occur locally, in general the relation between grade and thickness would be random (D.A. Pretorius, pers. comm.). In the Bushveld Igneous Complex, if it is assumed that the concentration of the platinum group metals and sulphides in the Merensky Reef are the product of settling from a single layer of minerals or elements, than, depending on whether the process went to completion or was arrested by freezing, a negative correlation would be expected, and there is evidence of such a relation (M.J. Viljoen, pers. comm.). Also, if (as seems likely) the original material was evenly mineralized, the accumulation for the whole layer should be approximately constant. For stratiform deposits of the Central African Copperbelt, with the complex interplay of mineral zoning, orebody thickness, conditions of sedimentation and diagenesis, quite apart from subsequent structural effects, it might be expected that there would be some tendency towards correlation, but that this would vary for different bodies and might vary even within some of the more extensive bodies. Also any broadly lenslike bodies where the tendency is to a thinning and decrease in grade towards the extremities would have a positive correlation, and many deposits could have this general characteristic.

Even this brief review indicates that there could be a wide range in the type and degree of correlation between ore deposits and within ore deposits, both locally and regionally. It seems likely that many ore deposits would show some degree of correlation, but it should not be accepted that seemingly general conclusions based on deposits with one degree of correlation would necessarily apply to those with a different degree. The degree of correlation that is significant in terms of ore reserve estimation will depend on a number of factors, but it seems that a fairly low degree can be significant, as shown in the case studies.

In addition, artificial restraints, such as minimum stoping width (thickness) or minable grade, that restrict the limits of orebodies within mineral deposits could influence these relations. It is in the main orebodies or potential orebodies with which ore reserve estimates are concerned.

Hazen and Gladfelter (1964) showed the effect of correlation in an exercise using artificial data for a phosphate bed with a group of intersections ranging from 15,0 to 23,9% P_2O_5 and from 2,7 to 8,1 feet in thickness; a summary of the results is recorded below. The arithmetic mean of phosphate content and thickness remain the same, but the weighted mean phosphate assays and the accumulation means vary somewhat; at zero correlation the weighted and arithmetic means are the same :

	Correlation -0,062			Correlation -0,987			Correlation +0,988		
	Assays P_2O_5 %	Thickness Feet	Accumulation Feet %	Assays P_2O_5 %	Thickness Feet	Accumulation Feet %	Assays P_2O_5 %	Thickness Feet	Accumulation Feet %
Mean	19,68	5,52	108,29	19,68	5,44	103,17	19,68	5,44	110,90
Weighted Mean	19,63			18,96			20,39		
Variance	6,61	2,44	1 057,68	6,61	2,40	313,39	6,61	2,40	1 948,70
Standard Deviation	2,57	1,56	32,52	2,57	1,55	17,70	2,57	1,55	44,14
C.I.	±0,66	±0,40	±8,40	±0,66	±0,40	±4,58	±0,66	±0,40	±11,40

Krige (1978, p.48) described how, in a section of the Prieska copper-zinc massive sulphide body, the correlation between copper grades and thicknesses is r = 0,2 whereas the copper accumulations (m%) are highly correlated with thicknesses. At Chambishi (Case Study E) where there is a negative correlation of -0,1881 between grade and thickness, there is a strong positive correlation of +0,7550 between accumulation (m%) and thickness. In these and other cases, by using the thickness in both expressions, the strong positive correlation introduced swamps any negative correlation between grade and thickness, particularly where the thickness is numerically greater than the grade. Therefore, it must be concluded that the use of accumulations will introduce a positive correlation, except that a strong negative correlation might be merely reduced, or will exaggerate an existing positive correlation.

The correlation and regression of normally distributed variables is described and illustrated by Krige (1978, pp. 14-21). He showed that where correlation between widely-spaced and later, closely-spaced, samples is not perfect, for positive correlation there is over-evaluation of high-grade blocks and under-evaluation of low-grade blocks in the first estimate made on the widespread samples. He concluded that this regression effect is present in all imperfect ore reserve valuations for all minerals but that it could be insignificant in some cases. It seems that this effect will be greatest on global estimates where a substantial proportion of the mineralized body is close to the pay limit and blocks could be included in, or excluded from, ore reserves according to their assessment. Blocks of a mean value are not affected by this regression effect. The regression effect would also be marginal where, as is commonly the case in base metal deposits, a change in pay limit affects the thickness of ore in a block by including or excluding marginal ore on the top or bottom of the orebody and does not cause the block as a whole to be included in or excluded from reserves. This correlation/regression effect is distinct from, and would be applied in addition to, that described above for correlation between thickness and grade.

IX. TWO INTERSECTIONS

The effect of correlation, the relationship between grade and accumulation, and the use of these different relations to obtain the average grade is shown in the following example, considering two intersections (Figure 2A), AB: 6 m at 3% metal, and CD: 10 m at 5% at a distance of x m, a case of positive correlation. For simplicity it is dealt with in two dimensions so that it concerns areas rather than volumes or tons, and it is conventionally assumed that the influence of each intersection extends half way. Though the arithmetic is not difficult, all steps are shown for clarity.

(a) *To obtain the area and the average (true) thickness (T):* The area, as conventionally calculated, consists of the sum of the two areas CD x $x/2$ + AB x $x/2$ = $3x + 5x = 8x$ m^2.

From simple geometry the areas of the two triangles a and b are the same, each being equal to (CD - AB)/2 x $\frac{1}{2}x/2$ = (10 - 6)/2 x $x/4$ = $\frac{1}{2}x$ m^2.
so that the area added to the CD area of influence is the same as that subtracted from the AB area. Therefore, the area of the two rectangles AB $x/2$ + CD $x/2$ is the same as that of the figure ABDC (10 + 6) $x/2$ = $8x$ m2, and the average thickness = $8x/x$ = 8 m, which is the same as the arithmetic mean thickness (CD + AB)/2, = (10 + 6)/2 = 8 m.

(b) *To obtain the average grade:*

(i) The usual method of obtaining the average grade by weighting the grade of each intersection by the thickness would give 10 x 5/2 + 6 x 3/2 = 68/2 = 34 m%, from which the grade = ΣT%/ΣT = mean T%/mean T = 34/8 = 4,25%.

(ii) To calculate the grade from areas in a similar manner to that used for the thickness, the average grade would be the sum of the area % divided by the total area = (5 x 10 x $x/2$ + 3 x 6 x $x/2$)/8 = (50 + 18)/2 ÷ 8 = 34 x%/$8x$ = 4,25%. It will be noted that the area b is at 5% and the area a is at 3%.

The average accumulation and the accumulation at the mid-point between intersections are both 34 m% (Figure 2B).

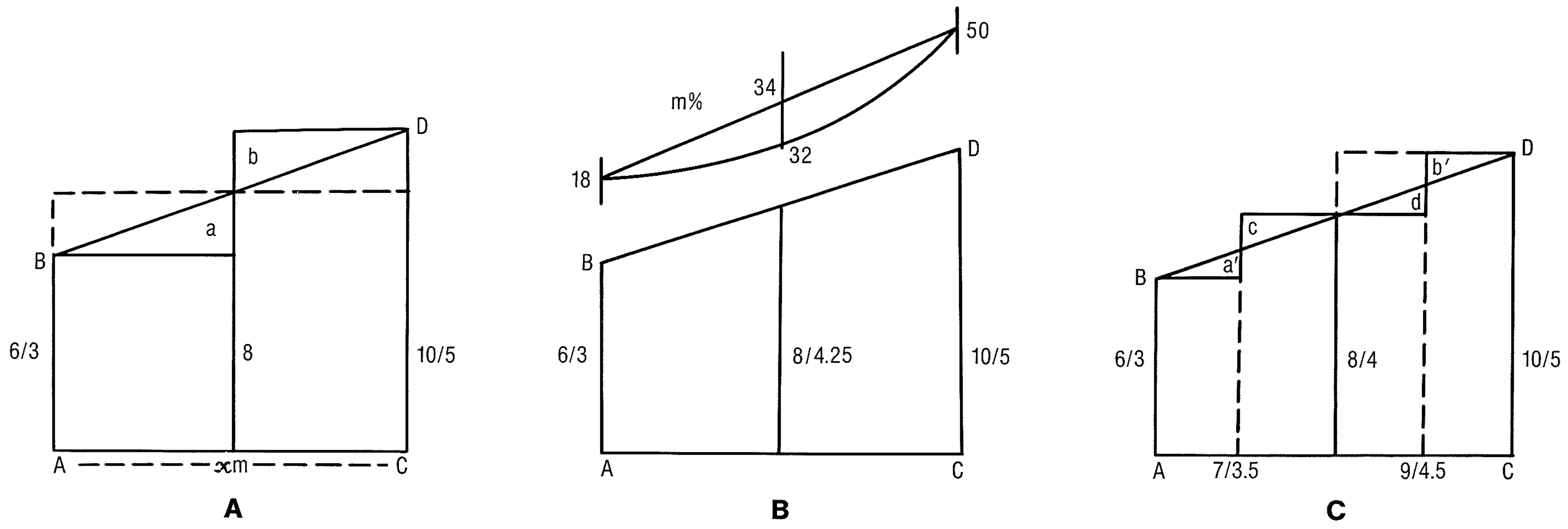


Figure 2 : *Two intersections, AB: 6 m at 3% and CD: 10 m at 5% (see text). 2A shows the relations and effect of estimating the total area and average thickness between intersections. 2B illustrates the grade estimation by the normal weighting method; the upper part shows the equivalent linear change of accumulation or m% between intersections (straight line), and the way in which the accumulation would vary for a linear change of grade between intersections (curve). 2C shows the effect of introducing a hypothetical intersection mid-way between AB and CD based on a linear change of thickness and of grade between these intersections.*

Since this is the same as the accumulation obtained by the weighting method (i) it is evident that weighting by thickness is what might be called an 'accumulation' method that is based on a linear change in the accumulation between intersections.

(iii) If it is now assumed that there is instead a linear change in the grade as well as thickness between the intersections, then at the mid-point there would be a hypothetical intersection of 8 m at 4%. This would give an accumulation of 32 m%, compared with 34 m% in (ii) above (see Figure 2B). If we now calculate a weighted mean grade using this hypothetical intersection, the influence of the mid-intersection is doubled (Figure 2C), and the grade becomes ((10 x 5) + (8 x 4) + (8 x 4) + (6 x 3))/32 = 132/32 = 4,125%.

Because triangles c and d are at the same grade and balance each other, the different-grade triangles a and b of Figure 2A have now become the triangles a' and b' in Figure 2C, which are one quarter the size, and this accounts for the lower grade now obtained. By adding more and more intermediate intersections using the linear change in Thickness and Grade, the triangles a' and b' become smaller and smaller, and the grade changes as shown in the following tabulation:

Intersections	Areas	Grade %
2	1	4,25
3	2	4,12
5	4	4,0939
17	16	4,0840
257	256	4,08333588
∞	∞	4,08333.....

This is an integration, 4,08333..... being the limiting value when the areas at different grades become become vanishingly small. Isted (Isted and Mendelsohn, 1967) carried out the integration and showed that the same results could be obtained by the formula: Weighted mean grade =

$$1/3\ (\Sigma T\%/\Sigma T + \Sigma\%) = 1/3\ (68/16 + 8) = 4{,}08333...\%.$$

(iv) The arithmetic mean grade is a straight arithmetic mean of the two percentages (5 +3)/2 = 4,0%.

(v) For the same figures arranged to present the case of negative correlation, the two intersections would be 6 m at 5% and 10 m at 3%, the results, similarly obtained, are listed below together with those already derived:

	+ve Correlation	-ve Correlation
Average thickness	8 m	8 m
Weighted average grade (m%)	4,25%	3,75%
'Isted' average grade (%)	4,0833.....	3,9166.....
Arithmetic mean grade	4,00	4,00

To summarize, the relationships that emerge are:

1. The acceptance of an areal influence for intersections is in fact the acceptance of linear change of certain properties between intersections.

2. All methods that involve weighting of intersection grades by their respective thicknesses, volumes, or tonnages is an acceptance and use of a linear change of thickness and accumulation between intersections.

3. The 'Isted' integration method is based on a linear change of thickness and grade between intersections.

4. For the case of no correlation there would be no difference between the three methods of estimating average grade.

5. For the case of positive correlation, the accumulation method would give the highest result, and the arithmetic mean grade the lowest. For the case of negative correlation, these relations would be reversed.

6. If it is established or assumed that there is a linear change of accumulation between intersections, then the accumulation or thickness-weighted method will give the best grade estimate; however, should there be a linear change of grade between intersections, then the grade or Isted method would give the best grade estimate. This should not be confused with the situation where the volume (tonnage) and grade for particular blocks have been established. In this case the average grade of two or more blocks would be correctly estimated by weighting the grade of each block by the volume or tonnage of the block.

7. If there is a higher degree of correlation between grade and volume of influence (Area of influence x thickness) than there is between grade and thickness, then any method based on this relation would give results that are relatively higher (+ ve) or lower (- ve).

8. In a general way, as the correlation co-efficient increases the differences between estimates made by these methods increases, though they are not directly proportional and the exact relation has not been established. Mendelsohn and Wiik (1970) suggested that co-variance would be a more effective measure of the relationship between the variables than the correlation co-efficient, and this is a field into which further research would be of interest.

9. In practice it would be expected that changes between intersections, for grade more than for thickness, would in general be irregular rather than linear, but this will be related to the nature of particular orebodies. For example, in Roan Basin West (Case Study B) a linear change of both grade and thickness would be a close approximation of the conditions indicated.

X. MULTIPLE INTERSECTIONS

For three intersections forming a triangle, similar relations pertain. This is because a plane surface can be generated between the three points for any given characteristic. Given a well-proportioned triangle, for a linear change in accumulation the thickness-weighted method would be appropriate. The Isted formula becomes $1/4\ (\Sigma T\%/\Sigma T + \Sigma\%)$ for a linear change in grade (Isted and Mendelsohn, 1967). For positive correlation this would give a lower average grade than the accumulation method, with the arithmetic mean still lower.

In the case of a rectangle with intersections at the corners, the relations are still similar, but there is a complication in that it is no longer possible to generate plane surfaces for any properties between the four corners. An example of such a situation is shown in Figure 3, with examples of contouring of thickness shown in 3A, 3B, and 3C. In the absence of geological trends, the col (Figure 3C) is the best overall mathematical solution, being intermediate between the extremes shown in Figure 3A and 3B. It is analogous to the linear change for two and three intersections.

In the example shown, there is a strong positive correlation, and the results are listed below. The previous exercises suggest that a general Isted formula would be $(1/(N + 1))\ (\Sigma T\%/\Sigma T + \Sigma\%)$ which here becomes $1/5\ \Sigma T\%/\Sigma T + \Sigma\%)$.

Method	Average grade %
Accumulation	3,79
Isted	3,15(714)
Arithmetic mean	3,01

If the area is divided into four triangles with an assumed intersection at the centre based on the col interpretation, the Isted formula applied to each triangle gives an average for the whole rectangle of 3,15475%. The small difference is caused by the fact that a plane surface cannot be generated between the intersections, but the general Isted formula gives a close approximation, and it is worth considering its

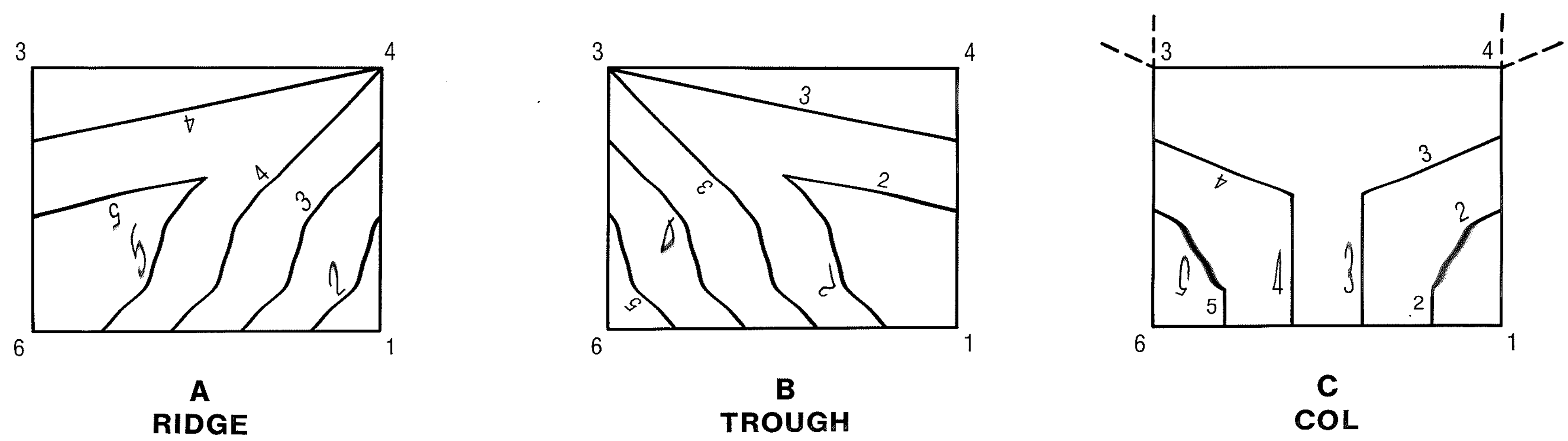


Figure 3 : *Four intersections at the corners of a rectangle, showing thickness. 3A shows a ridge interpretation of the data, giving a maximum volume; in 3B the trough interpretation gives the minimum volume; in C the col interpretation gives the middle volume, which is the best interpretation, i.e. on average the least in error, where no trend is known or indicated.*

completely general application. As the number of intersections N increases, the first term inside the bracket ($\Sigma T\%/\Sigma T$) becomes small compared with the second term ($\Sigma\%$) and the average grade approaches the arithmetic mean grade. With N at about 100 or more, the Isted and arithmetic mean are, for practical purposes, identical. Thus, for small numbers of intersections the general Isted forumla can provide an approximation for a linear change in grade, but for larger numbers it approaches the arithmetic mean grade; is this an appropriate and fundamentally correct relation?

XI. RANGE

The valuable element content in mineral deposits has a very wide range, from 1:20 000 000 or even 1:50 000 000 in the case of diamonds to nearly 100% in the case of iron or coal (Figure 4). If the valuable element is present in a mineral, it is the content of the mineral with which we are concerned. For example, copper constitutes only 1/3 of chalcopyrite, but it is the amount of chalcopyrite that governs the amount of copper and which is therefore statistically important; similarly the major iron minerals contain about 70% iron, so that even 100% of the mineral will give only about 70% Fe.

With this wide range in valuable mineral content it would be surprising if there were not differences in behaviour and in effects on ore reserve estimation. For example, in a gold ore, a rich sample could easily be 1 000 or 10 000 times the normal grade (of say 5-20 gpt) whereas a low sample could only possibly go a little below normal, say one quarter or one tenth; the usual cut-off grade would in practice place a lower limit on the possible range. In a copper deposit containing 1-5% metal, rich samples or patches could be no more than perhaps 10, or an absolute limit of 50 times the normal, and again a lower limit of perhaps one tenth would be the lowest detectable by routine methods of analysis, and again in ore a cut-off grade would still further restrict the range below normal to perhaps one quarter of the normal grade. A manganese or lead/zinc deposit in the 20% class, is likely to range from about one fifth to four to five times this figure, i.e. not too different above and below. In the case of an iron deposit containing about 60% iron, the possible range above normal would be very small, and the potential range below the normal would be restricted to perhaps one tenth, though mining limits would again probably restrict this to less.

Thus there is also something of an 'index of difficulty' in grade estimation, decreasing from trace to major elements. For example gold deposits are more likely than base metals to suffer from over-evaluation, quite apart from geological factors, and it is this problem that led to the early emphasis on gold deposits. On the other hand it would be difficult to over-value iron deposits substantially and not too great an under-evaluation would be expected. As against this, however, a small change in the iron content of ore mined or delivered could be rather significant.

XII. ELEMENT/MINERAL DISTRIBUTION

Some fifteen to twenty years ago there was extensive interest in, and discussion of, the statistical distribution of elements in igneous rocks. At that time Rogers and Adams (1963) produced a rather elegant demonstration of the distribution that might be expected of random elements in a homogeneous rock. They showed that for a broad middle range of composition (about 20-80%) an element will tend to be normally distributed. Above this, as the content approaches 100%, the distribution will be negatively skewed, and as the content approaches zero (i.e. trace elements) the distribution will be lognormal. Similarly, the binomal distribution

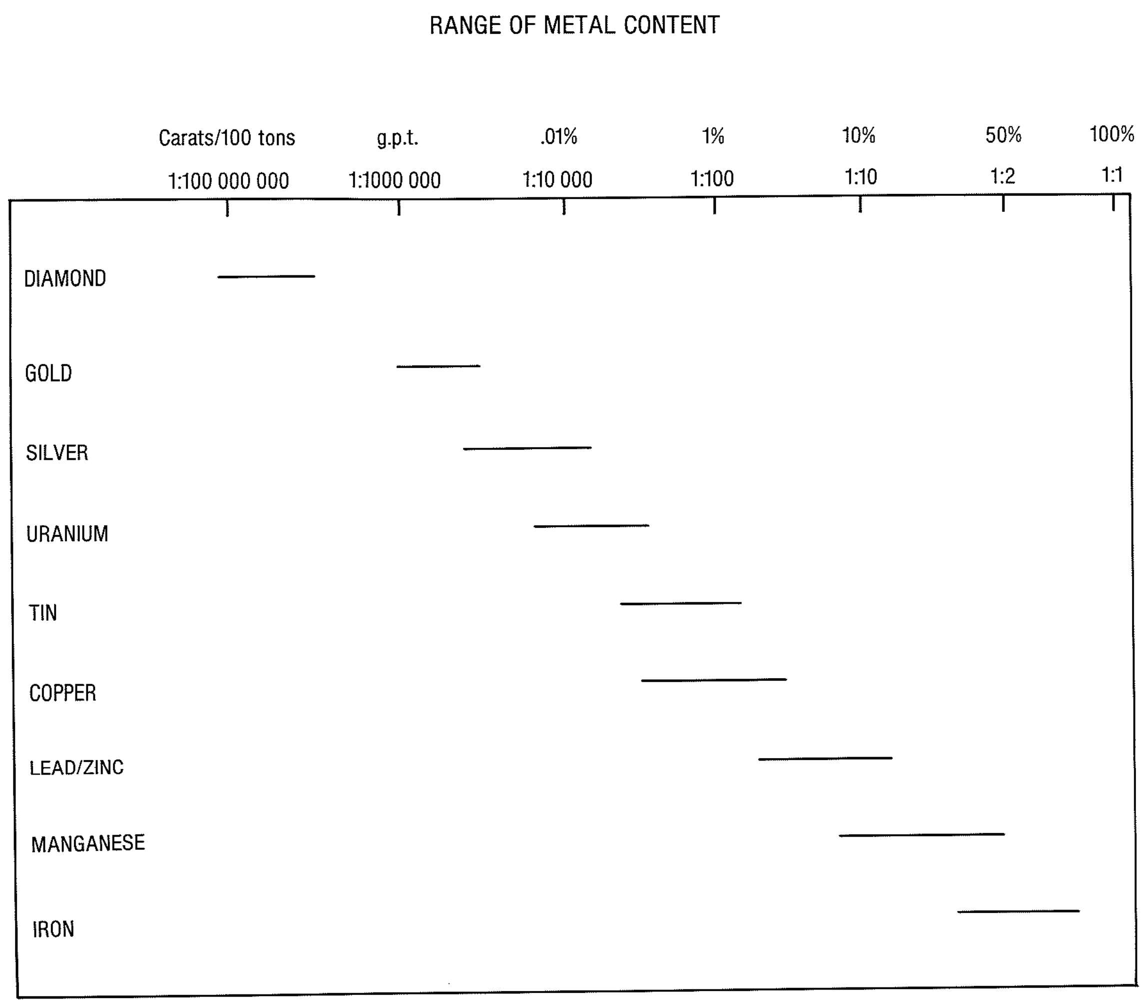


Figure 4 : *The range of valuable component contents characteristic of the ores of a number of different metals or minerals, also illustrating the range of contents met in estimating ore reserves. With such a wide range, differences in their characteristics, behaviour, and treatment in ore reserve estimation should not be wholly unexpected.*

is symmetrical and equivalent to normal for a mineral content of 50%. Above 50% the distribution becomes negatively skewed, and below 50% positively skewed. For this distribution, however, as N increases skewness decreases and the distribution approaches normal.

Many ore deposits are broadly homogeneous, and data on the statistical distribution of the valuable element, mineral, or compound are presented in Table 1. The approximate distributions listed are in most cases those given by the respective authors. The distribution can be summarized as follows (examples are shown in Figure 5):

Trace to 8%	-	Lognormal
0,5% to 55%	-	Normal
50% to 70%	-	Negative skew.

That the distribution is independent of the metal or element concerned is shown by the manganese examples quoted, where at lower than about 8% of the mineral the distribution is lognormal, at 16% it is normal, and at 50% it is negatively skewed. The gold deposits are all in the lognormal group, and bearing in mind the remarks under RANGE, it is evident why most of the earlier work in this field was on gold deposits and why the main emphasis has been on the lognormal distribution. Another point of interest is that the 'percent' range, between about 0,5% and 10%, which includes most of the world's base metal deposits, lies in the overlap region between lognormal and normal distribution.

There are complications in this picture: for example it is well known that the gold distribution in the Witwatersrand deposits is not truly lognormal, but can be transformed to lognormal by adding suitable constants (Krige, 1962). Also Sichel (1973) has developed a new family of Poisson distributions for the very long-tailed distribution of discrete particles, such as diamonds, which represent a special problem. Similar problems could arise, for example, with sea-floor nodules. It has also been shown that some distributions are not normal nor lognormal, but the question of appropriate transformations to normalize distributions could perhaps be of assistance in some of these, and thus increase the scope of statistical treatment.

However, the main point that emerges, and one that is now widely accepted, is that in ore deposits there is positive skew or lognormality in the distribution of small amounts, normal distribution in the moderate to middle ranges of mineral content, and negative skew in the high ranges. These relations suggest that the statistical distribution of the mineral content of an ore deposit can give valuable information as to how the data could best be handled.

TABLE 1

ELEMENT/MINERAL DISTRIBUTION

Deposit	Element or Compound	Range	N	Approx. Average or Mean Content: Metal	Approx. Average or Mean Content: Mineral	Approximate Statistical Distribution	Reference
Getchell, Nevada	Au	0-1,3 oz/ton	956			Positive Skew	Link *et al.* (1971)
Homestake, S. Dakota	Au	0-2,60 ppm	900			Lognormal	Link *et al.* (1971)
City Deep, South Africa	Au		1536	4 dwt/ton (1.92ID)		Lognormal	Link *et al.* (1971)
Blyvooruitzicht, South Africa	Au	0-35000	28334	700 inch/dwt		Lognormal	Krige (1962)
Hollinger	Au	0-7000		4-6 dwt/ton		Lognormal	Jones (1943)
Shamva, Zimbabwe	Au	0-257	442	5 dwt/ton		Lognormal	Truscott (1937)
Kolar, Mysore (India)	Au	1-3500	10517	500 inch/dwt		Lognormal	Sarma (1969)
Comet, Arizona	Ag	0-81	127	5 oz/ton		Lognormal	Becker and Hazen (1961)
Coyote Creek	Sb	0-0,85	62	0,1%		Lognormal	Becker and Hazen (1961)
Zonia, Arizona	Cu	0-1,8	309	0,2%		Lognormal	Becker and Hazen (1961)
Porphyry Deposits	Mo	0,1-1,0	47	0,3%		Lognormal	de Geoffroy and Wignall (1973)
Climax, Colorado	MoS_2	0-1,2	224		0,45%	Normal	Becker and Hazen (1961)
Porphyry Deposits	Cu	0,1-1,8	58	0,6%		Lognormal	de Geoffroy and Wignall (1973)
Scully Mine, Labrador	Mn			0,5/0,7%	1%	Lognormal	O'Leary (1979)
South Crofty	Sn	0,4-10		3%	3,5%	Lognormal	Kuskevic *et al.* (1972)
Cartersville, Georgia	Mn	0-8	117	2,1%	3%	Lognormal	Becker and Hazen (1961)
Copperbelt, Zambia	Cu	0,3-20		2,3-5,5%	7-9%	Normal	Harju (1966)
Maggie Canyon, Arizona	Mn	0-32	406	5,4%	8%	Lognormal	Hazen (1958)
Cebolla Creek, Colorado	TiO_2	3-17	176	8,4%	16%	Normal ?	Berkenkotter and Hazen (1963)
Cebolla Creek, Colorado	Fe	8-19	176	12%	17%	Normal ?	Berkenkotter and Hazen (1963)
Aroostook County, Maine	Mn	4-15	46	10%	16%	Normal ?	Becker and Hazen (1961)
Phosphoria, Idaho	P_2O_5	6-30	199		18%	Normal	Hazen (1964)
Phosphoria, Idaho	P_2O_5	14-34	224		25%	Normal	Hazen (1964)
Phosphoria, Idaho	P_2O_5	26-36	42		32%	Normal	Hazen (1964)
Central Florida	Phosphate	20-48	239		35%	Normal	Berkenkotter (1964)
Black Rock, etc., Nevada	Mn	1-46	102	32%	50%	Negative Skew	Becker and Hazen (1961)
Scully Mine, Labrador	Fe			36/41%	55%	Normal	O'Leary (1979)
Arkansas Bauxite	Al_2O_3	40-58	123		50%	Normal	Becker and Hazen (1961)
Iron County, Utah	Fe	25-64		50%	70%	Negative Skew	Becker and Hazen (1961)
Iron Mountain	Fe	40-87	277	50%	70%	Negative Skew	Becker and Hazen (1961)

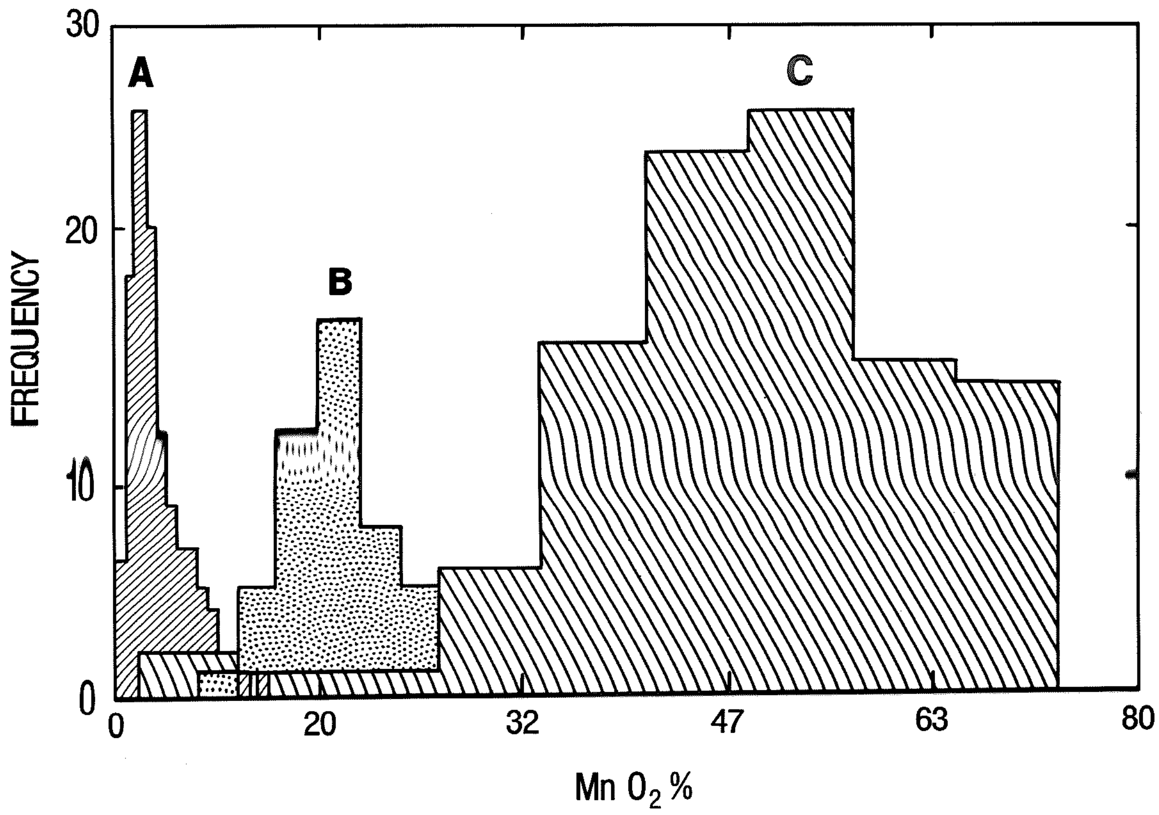


Figure 5 : *The range of manganese content, expressed in MnO_2, in different manganese deposits, showing, at different ranges: A - positive skew distribution, B - symmetrical distribution, and C - negative skew distribution (after Hazen, 1967).*

XIII. METHODS

There are four main groups of methods of ore reserve estimation, which are here labelled geometric, statistical, mathematical, and geostatistical, based on the underlying concepts.

The geometric methods include the various methods that have been used for many years, such as triangle, polygon, block (rectangle), and perhaps also contour methods. They are based on the solid geometry of the body, and assume a sphere or area of influence for each intersection, usually to half-way towards the adjoining intersections. This involves a linear change of two variables (thickness and either accumulation or grade) between intersections.

Statistical methods ignore areal distribution, giving equal weight or influence to each intersection in an attempt to obtain overall averages.

Mathematical methods attempt to express surfaces representing variables by mathematical expressions or processes. The rate of exchange of these variables is not linear but is based on the relations expressed mathematically, which give influence on this rate to other intersections and can separate regional from local trends.

Geostatistical methods are those that involve the application of statistical/mathematical methods or models that take into account the spatial relationships of 'regionalized' variables and quantify the confidence level of estimates.

Although there are substantial areas of overlap between the various approaches it is convenient to consider them separately.

A. Geometric Methods

Geometric methods are based on the solid geometry of the orebody, allocating to each intersection an area of influence, i.e. assuming or accepting linear changes between adjacent intersections. Considering two intersections in isolation, the assumption that will be least in error overall is that there are linear changes in parameters between them. This does not necessarily mean that it is the most accurate in a particular case, but merely that it is the best compromise. This, furthermore, is another way of saying that the influence of each intersection extends half-way to the next intersection, which is the way it is usually phrased. In addition, this approach is objective, reproducible, and mathematically readily handled and visualized. The weakness of these methods is that we do not normally deal with two intersections in isolation, and it is the influence of other intersections that has led to many of the recent developments in ore reserve estimation. However, geometric methods still have many good points and useful applications.

The calculation is done in two main steps:-

1. The average metal content (grade) and true thickness, and total volume for each of a series of blocks through the orebody are calculated. From the volume the tonnage is readily calculated by a factor based on the specific gravity of the rock.
2. The tonnages in each block at their respective grades are combined to provide an overall total tonnage and average grade.

The second stage is straightforward and independent of the first, so that it is the first stage, the manner of estimating the tonnage and grade of the individual blocks, that is of primary interest in considering methods of estimation of ore reserves.

In all the following descriptions it is assumed that for each interesection the average grade, if applicable the average specific gravity, and the thickness has been obtained. It is common to take all areal measurements in the plane of the orebody, i.e. the plane of the two major dimensions, and the true thickness measured perpendicular to this plane, but it is equally correct to use, for example, the horizontal thickness and areal measurements made in a vertical plane and plotted on a vertical longitudinal projection parallel to the strike of the orebody, or other appropriate combinations.

These methods have the disadvantage that widely-spaced and therefore less well-known interesections have agreater weight of influence in the final total than closely spaced intersections. Should such interspections be significantly different from the average, there is distortion of the result.

Geometric methods give acceptable and unbiased estimates for tonnage, and, if there is no correlation between grade and thickness or volume, also for grades. However, as has been shown under CORRELATION, if there is correlation, even if only low degree, the results become biased and unacceptable, the direction of bias being related to the nature of the correlation and the nature of the change shown or assumed between intersections. For the usual application of these methods, a linear change in accumulation is applied. Appropriate methods can be applied to reduce or remove the effect of correlation.

The greatest problem with geometric methods is that they are point estimates that do not provide any confidence limits to allow an impartial assessment of their quality. Although it is often stated that the estimates are correct to within 10% or 20%, these are subjective and arbitrary statements of opinion, not mathematical relationships. It seems that it should be possible for a statistician to devise a method of making assessments of confidence limits for such methods as the triangle and polygon, with appropriate allowances being made for factors such as weighting and the use of intersections in several triangles.

1. Triangle (Figure 6A)

In the standard triangle method, the average thickness is one-third the sum of thickness at the three apical intersections, and the average grade is the sum of the three accumulations divided by the sum of the three thicknesses.

The area of the triangle in the plane of measurement is calculated by means of a simple geometric formula, such as half-base x height, or from the polar co-ordinates of the corners for calculation by computer or sophisticated calculator. The volume is the area multiplied by the average thickness and the tonnage is the volume multiplied by the density factor. This method assumes a linear variation of accumulation and thickness, and an equal area of influence within the triangle for each intersection. It is simple to operate and programme for computer application. As triangles depart from equilateral so the influence of interesections varies from the ideal of half the distance to adjacent intersections, but unless the departure is substantial, the effect is not significant. If the triangles are of poor shape, this can be compensated by weighting according to the included angles, though this is seldom warranted (see Case Study D). The ideal number of triangles meeting at any intersection is six, and if there are more or less, the influence of the triangle is correspondingly increased or decreased. This too can be compensated by angular weighting. The manner in which triangles are drawn is in many cases an interpretation of the geology and their orientation should conform to the known geology. Individual triangles could extend from one geologic environment to another.

The Isted triangle formula for grade estimation, based on a linear change in grade, is

$$1/4\ (\Sigma\text{Thickness x Grade}/\Sigma\text{Thickness} - \Sigma\text{Grade}).$$

2. Polygon (Figure 6B)

The area of influence of each polygon is defined by lines perpendicular to, and half-way between, adjacent intersections, thus giving each intersection its true areal influence. Each polygon is based on a single intersection, and the thickness and grade of this intersection are applied to the whole polygon, so that no calculation of grades is necessary. The area is the sum of areas of the triangles forming the polygon, is planimetered, or it is programmed for computer calculation, an exercise that can be laborious, but has been done. Volume is the area times thickness, and tonnage = volume x factor. A linear change of accumulation and thickness is assumed and applied. Polygons are somewhat less likely to extend across geological boundaries than triangles. It can be shown that the polygon calculation and that for perfect triangles or those with angular weighting is identical, and for practical purposes the standard triangle and polygon methods give the same results if due allowance is made for the handling of outer intersections and border delineation.

3. Rectangle (or Block) (Figure 6C)

This is a special case and application of the polygon. The thickness and grade of the intersection again applies over the whole block. The area of influence extends the usual half-way to adjacent interesections, but only in two mutually perpendicular directions. Blocks can be erected about drillholes that are isolated or so distributed that triangles or polygons cannot be drawn, and the size of the block can be modified or defined as required. Junctions of blocks at corners can provide a problem, but it is relatively easy to match blocks with geologic boundaries. The case of rectangles with intersections at the corners, discussed under correlation, is not often used in ore reserve estimation.

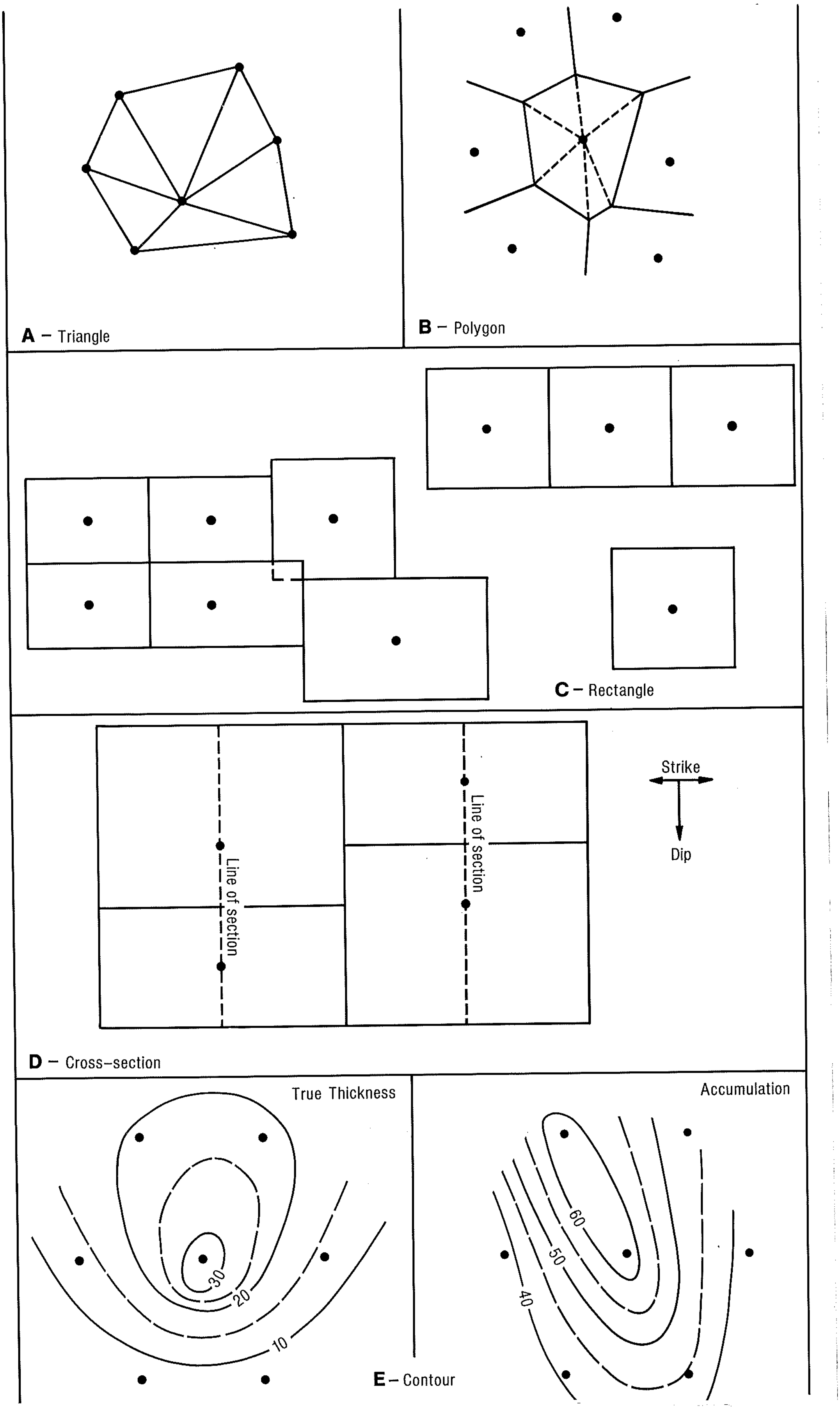


Figure 6 : Geometric methods of ore reserves determination. See text for description. A - Triangle. B - Polygon. C - Rectangle, showing regular and irregular continuous blocks, and isolated block. D - Cross-section. E - Contour for thickness and accumulation.

4. *Cross-Section* (Figure 6D)

The cross-section method is a particular application of the rectangle method. The influence of each intersection extends half-way to adjacent intersections along the section (normally down-dip) and the influence of the section extends half-way to adjacent sections. It is used for the geologically convenient case where data is obtained along a series of cross-sections. On each cross-section, the influence of each intersection is expressed as a cross-sectional area i.e. thickness x distance. Horizontal thickness and vertical distance can be used instead of thickness and incline distance if desired. On each section, the average grade is the area%/area, and the average thickness is total area/total dip-distance. The volume is the cross-sectional area x strike distance, the strike distance being the sum of half the distance to both adjacent sections (or one section at the extremes). The practice of using successive pairs of adjacent sections is mathematically identical.

5. *Contour* (Figure 6E)

Contour methods are really somewhere between the geometric and mathematical methods, since they can and do involve elements of both, depending in part on the approach used. For the direct hand-drawn method, as suggested by Gilmour (1964; Case Study A), contours for true thickness and accumulation are drawn. The areas between successive pairs of contours, and above the highest contour, are measured with a planimeter. The area between two successive contours is taken at the mid-value (thickness or accumulation); the area above the highest contour is taken at that value, or at the value of the intersection(s) within it. The volume between each pair of contours is calculated and the total volume found by summation. In the same way the total volume-percent is obtained. The average grade is the volume-percent/volume.

The drawing of contours is subjective and is an interpretation of trends, and previous remarks on this subject will apply (Figure 3). There is a human tendency to overemphasize highs, resulting in overestimates. There is an over-evaluation in taking the area and volume between two contours at the mid value, since less than half the area and volume is at the higher value, but this is likely to be small except under special circumstances. Positive correlation will give a high grade, but since contours will usually be intermediate between intersections (and the contour spacing can be reduced as desired) the effect is less than with other geometric methods.

Standard computer contouring packages are available, and one such was used in the Muliashi (Case Study D) exercise. A computer contour map is prepared from the observed thickness data, and the volume (and tonnage) estimates are obtained by numerical integration of the thickness surface. For the accumulation a similar estimate is prepared and the average grade is obtained from the two as for other methods. In the case of Muliashi, for the grade version, grade and thickness data of intersections were processed by the programme to produce interpolated values for each on a regular (100 feet) grid over the orebody area. The product of these then provided a set of accumulation grid values different to the direct contouring of original intersection accumulations as described. Numerical integration of the grid thickness and indirectly obtained accumulation values then gives the orebody tonnage and grade estimates. Computer contouring is objective, so that overemphasis on highs would not occur, but known trends should be allowed for and the basis of the contouring should be evaluated. It seems likely that some sort of geostatistically-based areal interpolation procedure developed with regard to particular mineral fields or types of deposit would provide better results.

Effective interpolating between the known data points can also be achieved by weighted moving average techniques, which are also not subject to the symmetry-restrictions of mathematical functional trend surfaces. However, a weighted moving average surface is not a true surface but is made up of discrete points.

B. Statistics

Statistics has long been applied to the handling of numerical data, and for many years has been used for various aspects of mining operations, including ore reserve estimation, though in the latter field the recent upsurge in popularity of geostatistics has overshadowed it. Excellent descriptions of statistical methods, approaches, and calculations with respect to ore reserves have been given by Sichel (1952, 1973), Krige (1962, 1978), Hazen (1967), and Koch and Link (1970, 1971), and the reader is referred to these and to some specific and general statistical applications in geology listed in the Bibliography. The complete study of these publications and those in the purer statistical field would be a full-time occupation and tends to scare off many potential users. However, sufficient of the basic principles and applications can be understood and successfully utilized by most geologists.

It has been shown under the headings RANGE and ELEMENT/MINERAL DISTRIBUTION that the proportion of the valuable element or mineral in ore deposits is related to their statistical distribution. The gold deposits that first led to the investigation of statistical approaches are in the lognormal range, and most emphasis has been on the lognormal distribution. Substantial and major constituents tend to have a normal distribution, and the 'percent' range (about 0,5 to 10%) lies in the fringe area between normal and lognormal distributions. It is this range that is the main concern here.

Simply stated, the statistical approach is that no weighting should be applied to intersections, i.e. that each intersection is as important as every other intersection. This would immediately eliminate all effects of correlation, and it was, in fact, the investigation of whether the arithmetic mean was not for this reason a better estimate of grade that led to the Isted approach. Statistical manipulation requires a random distribution of intersections, but the majority of exploration programmes achieve what is in practice a close approximation to random spatial distribution. With physical limitations as to drill sites, sheer inability to aim drillholes to intersect at desired points, financial limitations, etc, it is in most cases difficult to avoid a somewhat random spatial distribution, and grid intersections also give a random or close approximation to random distribution. The first and obvious danger is that concentration of drilling in one area, for example a high-grade ore shoot, at the expense of lower grade surrounding areas leads to distortion. Should there be such intentional concentration of drilling in an area, then that area should be treated separately, as a different population or stratum. As long as there is a random distribution of sampling points, the arithmetic mean is an unbiased estimate of grade or other characteristic.

1. *Concepts and Definitions*

Only some of the significant and relevant statistical concepts and definitions are discussed briefly:

Sample. A group of geological samples forms a statistical sample that is used to give an approximation of the population (in this case an orebody)from which it is drawn. The number of individual items forming the statistical sample is usually referred to as N.

Mean ($\bar{x}$). The arithmetic mean or average of all items in the sample or population.

Trend. A change in average value, sharp or gradual, from one place to another and persistent over some distance or area.

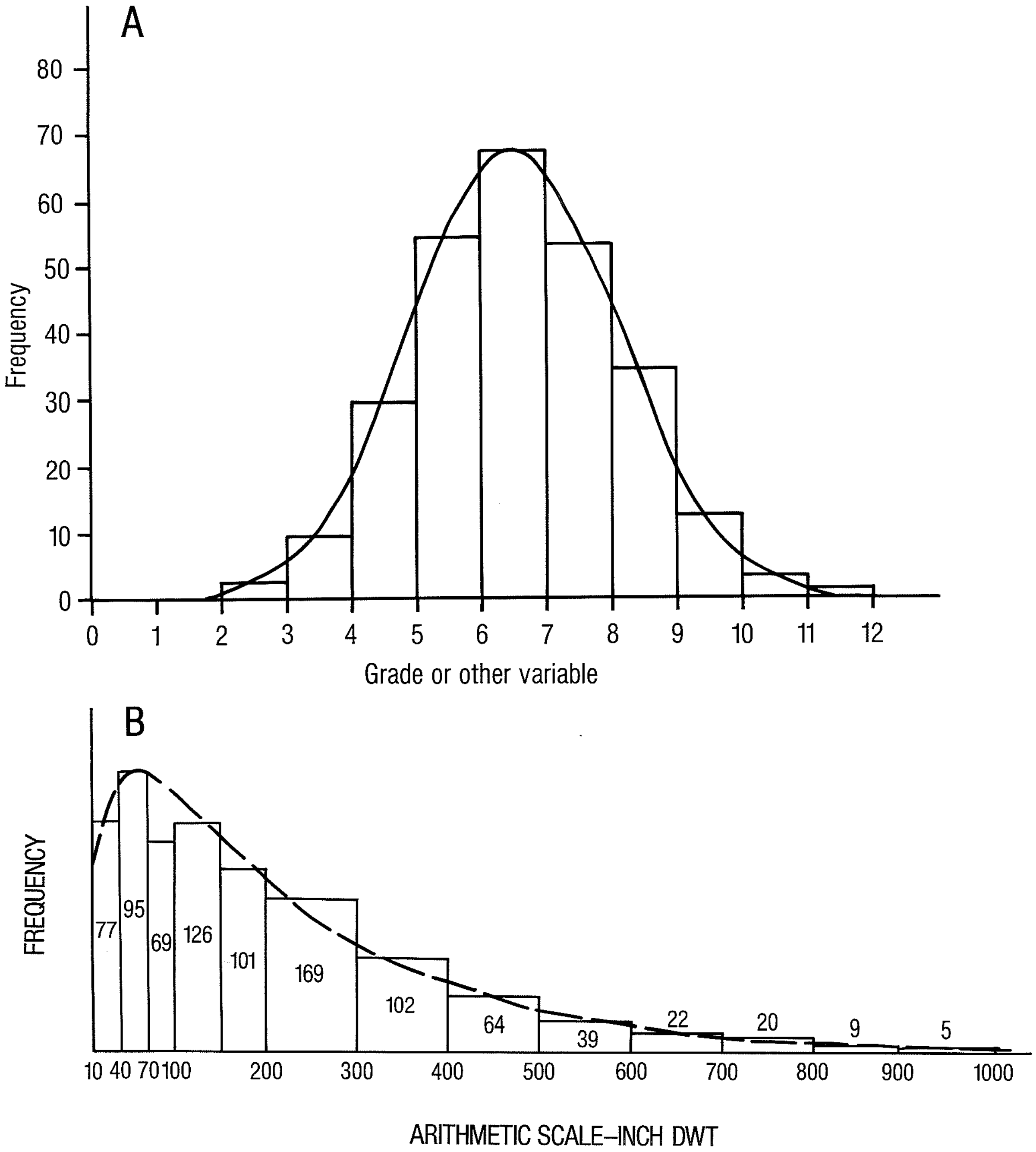


Figure 7. : *Statistical frequency distributions: A – Normal curve and histogram. B – Lognormal curve and histogram (Krige, 1962).*

Normal Distribution. A distribution whose frequency is expressed by the normal curve, the limit of the binomial distribution as N approaches infinity, and described by a specific equation. The curve is bell-shaped and symmetrical and is the distribution for a set of means based on data, as shown by the central limit theorem. A frequency distribution, graphically illustrated by a histogram and curve (Figure 7), can be tested for goodness of fit to a normal distribution, and the closest normal curve can be fitted to the distribution. The normal distribution has been extensively studied and there are many standard relations, approaches, and programmes that make its application relatively simple.

Lognormal Distribution. A variable is lognormally distributed if the logarithms of the variable are normally distributed. The geometric mean is the same as the (antilog of) the mean of the logarithms. See Figure 7.

Variance. The variance is the mean of the squares of the differences between each value and the mean of the values.

Standard Deviation (s). The square root of the variance (root mean square or quadratic mean). For a normal distribution, approximately 68% of values lie within one standard deviation of their mean, 95% within two standard deviations, and 99.7% within three standard deviations.

Standard Error of the Mean. The standard deviation divided by the square root of the number of values.

The Co-efficient of Variation is the standard deviation expressed as a percentage of the mean.

Level of Confidence. A 95% level of confidence means that in similar judgements of the same kind for 100 deposits, 95 would be within the limits set, and five would be outside these limits, in other words the statement would be correct for 95 of the 100 deposits. This is often loosely interpreted as meaning that there are 95 chances in one hundred (or 19 in 20) that the true average lies within the limits quoted.

Confidence (Fiducial) Interval: Confidence Limits. The confidence interval is the interval that defines the confidence limits within which the true average is contained at the specified confidence level. The half confidence interval is obtained by multiplying the standard error of the mean by the Student t factor. Sichel (1952, 1966) has shown that if the data distribution is skewed, the Student t factor is not applicable. For positively skewed (lognormal) data, particularly for small samples (N< about 20) the arithmetic mean can be distorted by few high values. Sichel's t estimator is a more efficient estimator of the mean, and with the aid of tables, confidence limits can be determined.

Student t Factor. This factor is based on "Fishers" Student t distribution obtained from tables, and is dependent on the confidence level selected and on the number of values (N). The factor is highly variable for small samples (N = 1 to 20) but above N = 100 it becomes a constant, depending only on the confidence level.

Transformation of Data. Skewed data can be transformed to a normal distribution or at least to a symmetrical or unskewed distribution that can be more readily handled. In addition to the logarithmic transformation already described, Koch and Link (1970) suggested the possibilities of normal-score transformation, and Mancey and Howarth (1980) commended the use of the Box-Cox powers transform for the efficient 'de-skewing' of data, even large data sets.

Precision. Since the confidence interval and limits are related to sample size and standard deviation, it is possible to calculate the approximate number of additional intersections needed to achieve particular confidence limits at a specified confidence level, assuming that the standard deviation would remain constant (Hazen, 1967). The formula is:

$$N = (s/CI/2 \times tCL)^2$$

Where N = Number of intersections

Where s = Standard deviation

CI/2 = Half the Confidence Interval

tCL = t factor at the desired confidence level

David (1977) pointed out that for this to be valid the intersections have to be independent of each other; even if they are not, this would give a useful approximation.

2. *Application*

Statistics has been applied to ore reserve and allied problems for many years, with highly significant contributions from Sichel (1947, 1966) and Krige (1962) on the lognormal distribution, based on the Witwatersrand gold-uranium deposits.

One of the first studies of the application of statistics on the Central African Copperbelt was that of Harju (1946 - Case Studies B and C). In Case Study B he showed that the distribution of individual sample grades, both unweighted and weighted by length, for each of four stratified zones and for the whole body is close to normal. The almost-pure zones (bornite and chalcopyrite) give Pearson Type IV curves for the weighted samples, more leptokurtic or peaked than normal, and the frequency distribution for the set of all samples is best represented by a Pearson Type I curve (see Figure 9). Harju (1966) concluded that a normal curve approximation for the populations of these zones is acceptable for all practical purposes. The distribution of complete intersection grades, both weighted by length and unweighted, show (graphically) approximations to normal though they are not as close as for the individual samples, and also eliminate the trends found within most intersections. Since the mean grades for both are rather close, he concludes that complete intersections are best used for estimation of grades. This means that statistical exercises can be carried out with the data normally calculated for each intersection of the orebody by drillhole or other means, and avoids elaborate exercises with large numbers of data that are individually meaningless. At Maggie Canyon manganese deposit (Hazen, 1958; Case Study H), the individual samples were used in all the statistical estimates, but one limited exercise (Table 15) also suggests that complete intersections provide suitable estimation data.

The distribution of grades in other Copperbelt deposits is, in most cases, symmetrical, and in several of these probably close to normal. A further interesting aspect is that thicknesses tend to be even or to have a slight positive skew, whereas accumulations generally have a strong positive skew, approaching lognormal (by inspection) in several. Should this relation be more general, it could account for the widespread acceptance of the lognormal association and the use of the accumulation in ore reserve estimation.

Harju (1966 - Case Studies B and C) showed that the arithmetic mean grade is as acceptable as the average grade obtained by geometric methods, and the geometric methods here give good estimates because they are closely spaced from detailed sampling in underground workings. Even where there is substantial stratification (Case Study B) the mean is acceptable for each stratum and for the whole deposit. In Case Study C, another portion of the Roan deposit, Harju (1966) showed that the arithmetic mean grade is similarly acceptable as a grade estimator. Similarly, the thickness mean, as represented by the tonnage is, in most cases studied by Harju, acceptable, though in one instance the difference between statistical and geometric tonnages is as high as 12%. In the more extended study of Table 4, it becomes apparent that for thickness, means should be sought only in areas limited to particular populations, a feature that is apparent on geologic or other grounds. The symmetrical distribution of grades and positive skew in thickness seem to be in accord with these results. Other Copperbelt results are in general agreement with those mentioned above.

Even at Maggie Canyon manganese deposit (Hazen, 1958), where the distribution of grades is closer to lognormal than normal, the arithmetic mean is an acceptable estimator of grades and also of thickness. It is interesting, though possibly co-incidental, that after subjective re-assessment and recalculation of all intersections with fairly substantial differences resulting, the arithmetic mean grade alone remains unchanged.

Where there is positive correlation, the arithmetic mean grade is lower than that obtained by geometric methods. The use of the mean of accumulation and thickness has been tested in most case studies, and it is usually rather close to the arithmetic mean grade but with a similar relationship to that between arithmetic mean grades and geometric averages. These differences increase as the degree of correlation increases and as the data density decreases.

The use of statistical methods, and in particular the normal distribution and the arithmetic mean, is an effective way of estimating ore reserve grades in the 'percent' range, as suggested by theoretical considerations and as demonstrated by the cases studied. Where there is significant positive correlation, perhaps as low as 0,1 and certainly around 0,2, it will give a lower and what is considered to be an improved estimate. The arithmetic mean of the accumulation divided by the arithmetic mean thickness is an acceptable alternative where it is considered that this better expresses the nature of the variation between intersections.

In addition, the estimator is unbiased, objective, and reproducible, and its efficiency and confidence limits and confidence level can be measured and compared with others. The ability to assess the approximate number of additional intersections to achieve a particular level of confidence and specified confidence limits is also most useful in assessing exploration programmes.

The similar use of statistics for the estimation of average thicknesses is somewhat less reliable but nevertheless is likely to be within acceptable limits. The area over which ore reserves are estimated is defined by the geologist on the basis selected by him, so that the tonnage is a simple calculation of the volume (thickness x area) and a density factor.

The definition of the limits of the area is subjective and could be governed by considerations such as the geologic habit, persistence and trend of ore shoots, limitation to the area outlined by the outer intersections, extension for a stated amount beyond these intersections, etc. With geometric methods, the limits are normally inherent in the method selected, though the method can be suitably (but with some difficulty) modified to allow for particular preference, whereas with statistics the area can be simply defined and the appropriate intersections used for thickness estimates.

C. Geostatistics

Geostatistics in ore reserve estimation developed from statistics through the efforts of de Wijs (1951; 1953), Krige (1962, 1966, 1976, 1978), Matheron (1963), and more recently David (1977), Royle (1977, 1980), and others in an endeavour to measure and utilize the spatial relations of intersections so as to obtain the most meaningful ore reserve estimates. Unfortunately, "....there is still an enormous gap between the practising mine engineer or geologist faced with the day-to-day problems of ore reserve estimation and grade control and the complex ideas (of geostatistics) ...." (J. O'Leary, in a review of Rendu, 1978, in Mining Magazine, April 1979). Because of this somewhat esoteric mathematical basis, there are the twin possible dangers of 'believers' applying the methods without appreciating the principles, and of a paucity of critical appraisal of the principles or practice. The principles and practice of geostatistics are fully described by the various authors referred to above (e.g. David, 1977; Krige, 1978; Rendu, 1978; Royle, 1977) and no analysis or re-description is necessary, though a simplified explanation would be a useful addition to the literature for the uninitiated. All that will be attempted is to briefly consider certain aspects of geostatistics and its application to global ore reserve estimation.

A substantial part of geostatistics is devoted to the estimation of blocks within an ore deposit with the least bias and lowest estimation variance, particularly with kriging methods for weighting intersections within the block. Though the block size can be varied, and a combination of blocks within the orebody will provide a global estimate, the complications of this indirect method as well as knowing and using the various procedures might not be warranted unless there is some distinct advantage in the global estimate itself. Royle (1977) had also pointed out that the estimation of small blocks on sparsely distributed data is meaningless.

Royle (1977) proposed the use of a random stratified grid for global reserve estimates as an improvement on geometric methods. A pattern of equal, square, suitably sized panels is located so that as far as possible one intersection lies within each panel and those panels containing 'ore' intersections constitute the orebody. In his example, of 104 intersections, 82 are within the 79 panels that form the RSG and outline the ore. Sixty five panels have one intersection, eight have two each, and six have none. The arithmetic mean thickness and accumulation for the 79 panels are calculated and the mean metal content is obtained from their dividend. Variograms were determined from underground exploration data because the surface drillhole intersections were too far apart to be used for this purpose, and were used to obtain the estimation variances. Kriging of the panels led to a decreased estimate for the tonnage of 5,2% and metal content of 0,9%.

Comment: Although no data are given, it seems unlikely that there would be any significant change in the results if a direct arithmetic mean were taken for the 82 intersections of the thickness and metal content (percent). The RSG method is an accumulation method with what that involves in possible bias (see 'TWO INTERSECTIONS'). If variograms could not be determined from 104 (or 82) intersections, a substantial number for an exploration programme, it indicates possibly severe limitations to the use of geostatistics in exploration. The calculation of an estimation variance on the area seems of doubtful validity, since the area is defined by selection of the grid, and also includes and excludes panels whose choice might be questioned on mining grounds. No advantage is seen in the kriging exercise, since the difference in grade is so small and the tonnage difference is not economically significant, and estimates for both are well within confidence limits. Thus it would seem, though in the absence of the data cannot be proved, that the following exercise would give equally acceptable results, and certainly in a fraction of the time: 1. define boundaries of the area on geological/mining grounds, 2. find the straight arithmetic mean of thickness and grades of included intersections to apply over the defined area, or use the mean of accumulations for the grade, and 3. calculate confidence limits.

At Roan Basin West semivariograms (Case Study C, Figure 11) reflect and give expression to the geologic conditions, but do not seem to show anything that cannot be demonstrated visually and with less effort by a direct examination and hand-contouring of the data already available on the vertical longitudinal projection. Owing to the special relation of grade and thickness in this part of the orebody, the semivariograms would not apply to other parts of the deposit, such as that in 6 Limb, discussed in Case Study B, but would have to be recalculated for each and would presumably be quite different.

The random distribution, small range, and symmetrical distribution of grades suggest that arithmetic mean grades would be reliable indicators of average grade. Similarly, the areal distribution of the thickness and the statistical and geostatistical relations show that the thickness and accumulation data should be treated with some care, and preferably restricted to relatively small and selected portions of the area. No further investigation was undertaken of geostatistical evaluation of these deposits.

D. Mathematical Methods

It is not proposed to attempt to discuss these methods in detail, not least because of their scope and need for mathematical expertise. Their basis is that all the intersections exert an influence, in general with near intersections exerting a greater effect, and the mathematical surfaces providing an approximation of the surface defined by the variables under consideration.

For the polynomial trend surface, multiple regression is used to set up a linear mathematical relationship between the dependent geologic variable, e.g. thickness, grade, or accumulation, and a polynomial expansion of the intersection co-ordinates as independent variables. Ore reserve estimates can be obtained by fitting polynomial trend surfaces to the true thickness and accumulation values respectively. Numerical integration of these two mathematical surfaces provides estimates of orebody volume and metal content of the orebody, and from these, orebody tonnage and average grade are found as usual. Trend surfaces of a high (>4th) order with a large number of terms may be required for a complex surface, and these can be difficult to handle.

The 'Fourier' trend surface technique, fitting a double Fourier series, is broadly similar but, because harmonic functions replace the polynomial terms, are often better suited for portraying the variability of natural phenomena than low order polynomial surfaces.

These mathematical functions are fitted to the data points by a least squares regression technique as a means of interpolating between the points. If the residuals, the difference between the actual and computed values, are independent and normally distributed, statistical tests can be made for reliability. Regional trends can be extracted to leave only the local variations.

At Muliashi, Case Study D, it was found that the Fourier trend surface was rather accurate at high density but at low data density it was unreliable because unreasonable highs and lows were generated, particularly where the data was sparsest. The polynomial trend surface higher than 4th order could not be calculated owing to computer limitations and it was not further tested. It is also difficult to take geologic factors into account with these methods.

Rolling mean techniques for defining trend surfaces have been proposed by Krige (1966) and Williamson and Thomas (1972), and they seem to have achieved some success but limited acceptance and no proper consideration has been given to them nor do I have the expertise to comment further. Intuitively it would be expected that there should be some place for this technique.

E. Discussion

It has been shown at Muliashi (Case Study D) that the triangle and polygon methods give closely comparable grade results for all sequences and at all levels of data density. For triangles, the weighting of corners to correct for variations from the equilateral shape is not warranted in this case, and would not normally be worth using unless there are a number of extremely poorly shaped triangles, when its use in these triangles might be justified. With equilateral triangles, there would be six triangles meeting at, and dependent on, each point, except at the edge of the orebody. If the number of triangles meeting at an intersection is less or more than six, so the influence of that intersection is less or more than the desirable norm. This too can be adjusted by the use of corner weighting, but since it is random, the effect is not likely to be sufficient to warrant corner weighting. In the Muliashi study the angular weighted triangle method did not give results that were detectably better than other geometric methods. One feature that can cause problems in geometric methods is drillhole spacing: one intersection in a large 'open' area has a disproportionate weight in terms of tonnage and grade; if such an intersection is thicker than average and also has an unusually high or low grade, the effect on the overall grade can be significant.

At Muliashi the relationship between the grade and accumulation contour methods is similar to that between the Isted and standard triangle methods: the grade method gives a lower result with positive correlation and a higher result with negative correlation, though the differences are small. In fact, there are bigger differences between the grade contour and triangle methods, and the grade contour is closer to the standard triangle than to the Isted triangle method, to which, because they are both dependent on a regular change in grade rather than accumulation, it might be expected to be a closer equivalent. In sequence 4 (Table 8) with negative correlation the Isted triangle and computer contour grade methods are similar and are close to the 'true', whereas all other geometric methods are low, and this is particularly noticeable in stage 1. In sequence 5, also with negative correlation, the relationship is similar, but here the other methods are close to the 'true' and these two are high. In general it can be said that with a low correlation, all geometric methods are rather close, and that differences become apparent with significant correlation, as is predicted by the considerations under the heading CORRELATION. At Mufilira (Case Study F) the highest known correlation of +0,5680 occurs in all or a major part of the C orebody as known in 1961. Here, the original triangle estimate gave an acceptable volume, but the grade was 5,319% CU as against a subsequent blocked out estimate, after sub-development and detailed sampling and assessment, of 4,501%. This is an over-evaluation of about 18%, a highly significant amount, and is related to the high degree of positive correlation between thickness and grade in addition to the Krige regression effect.

Royle (1980) compared the estimation of global reserves and made a number of useful observations and comparisons. He concluded that all methods other than kriging are empirical and that they are liable to, and must continually be sounded for, inherent bias though previous remarks (see REQUIREMENTS) indicate that only significant bias is important. Classical statistics can be correct for spatially sparse data. Geo-statistical kriging methods call for a lot more work and mathematics than the empirical methods but give unbiased estimates, enable variances to be calculated, and allow the derivation of unbiased grade-tonnage curves. The prime requirement is that variograms of the variables of interest can be obtained to represent their spatial variation, which is, seemingly, not often possible for global estimates. The presence of drift as shown for instance at Roan Basin West (Case Study C, Figure 11) introduces complications that can be allowed for by more sophisticated techniques. Where they are applicable, geostatistical methods would seem to have their greatest benefit in stages subsequent to global estimates, where evaluation of blocks is beneficial or required. At low data density present methods can introduce bias and other problems (Watson, 1977).

It has been shown in the various case studies that statistical methods give results (particularly for grade estimates) that are as good as or better than the geometric methods, and where data are few or sparse they are virtually mandatory. Confidence limits and variance can be calculated for statistical methods and results are unbiased except under special conditions for which simple precautions can be taken. In addition they are simple to apply or to modify with additional data, readily comparable, and normal distribution statistics are probably applicable to most base metal deposits and form a suitable first approximation. The suitability of normal, lognormal, or transformed distribution can be fairly readily established.

The traditional geometric ore reserve methods still have an application, particularly if, as has been suggested, confidence limits could be calculated, and are in many cases convenient to apply and are 'visual' methods that can be readily appreciated, even by non-technical persons. They must be applied with caution where there is correlation between grade and thickness, particularly where this is a positive correlation. For positive correlation, the safest (and more conservative) assumption is that there is a linear change of grade (*not* accumulation) between intersections, which in the case of sections or triangles suggests the use of the Isted integration approach. Since they are simple to calculate and apply, arithmetic mean grades and thicknesses (hence volumes and tonnages) could and should be used as a convenient check where statistics is not the primary method of estimation.

Geostatistics has important applications, particularly to second stage estimates where evaluation of blocks becomes necessary, but does not seem to be most suitable for global ore reserve estimates.

XIV. CASE STUDIES

The development of theoretical concepts is an essential part of the study of ore reserve estimation, but it is the extent to which they can be successfully applied in practice, and also to a significant degree the extent to which they are based on practice that is important. It is unfortunate that, probably for reasons of space or security, the raw data is not given for nearly all case studies in the literature, which means that checks or alternative methods cannot be applied for comparison. For this reason a number of case studies are summarized below. The data available vary considerably. Where appropriate the data are listed, but for reasons of space some are omitted where they are available elsewhere. Copies or the source could be provided on request. The level of data available varies considerably, but all cases have some lesson(s) or useful information. The approach here is to list the data and the results, with descriptive comments where necessary. Further discussion and questions of interpretation can be found in discussions of the various methods in the body of the report.

A. Gilmour Copper-Zinc Deposit

This tabular deposit, presumably of volcanic affiliation, was discussed by Gilmour (1964) in advocating a contour method of estimated reserves. Contours of thickness are used to obtain the volume (and tonnage), which gives an average thickness when divided by the area. Accumulation contours are similarly used to obtain an average accumulation; the average accumulation (Ft%) divided by the average thickness (Ft) gives the average grade. For Table 2 the thickness and accumulation for each intersection was estimated from the contour plans, and the grade calculated from these data. The resulting data are considered to be sufficiently accurate for this exercise. Gilmour (1964) stated that the polygon method gives grades that are too high where there is positive correlation between grade and thickness, and that the contour method minimizes this by inserting steps (as many as may be desired) between intersections, but unfortunately does

TABLE 2

GILMOUR DATA

Drillhole No.	T.T. ft	Cu %	Ft%	Drillhole No.	T.T. ft	Cu %	Ft%
1	2,0	8,5	17,0	20	18,6	11,0	205,0
2	7,9	11,4	90,0	21	4,0	4,6	17,5
3	5,5	3,7	20,0	22	2,0	8,5	17,0
4	7,0	5,0	35,0	23	7,5	8,0	60,0
5	6,0	7,5	45,0	24	10,0	4,8	48,0
6	8,0	6,3	50,0	25	9,5	6,9	68,0
7	20,2	3,0	60,0	26	14,0	5,3	75,0
8	18,0	4,1	74,0	27	12,0	4,6	55,0
9	26,2	7,3	190,0	28	11,5	4,8	55,0
10	3,5	5,0	17,5	29	12,2	2,3	28,0
11	2,0	8,7	17,5	30	10,0	6,0	60,0
12	2,0	8,8	17,5	31	12,0	5,8	70,0
13	35,2	9,1	320,0	32	6,0	2,9	17,5
14	17,5	11,2	195,0	33	10,0	4,8	48,0
15	22,0	10,4	230,0	34	16,0	6,5	105,0
16	16,0	3,4	55,0	35	14,3	4,2	60,0
17	12,0	10,0	120,0	36	6,2	3,6	22,0
18	8,0	6,0	48,0	37	10,0	5,3	53,0
19	20,5	10,7	220,0				

not quote the grade obtained by the polygonal method. As the contour interval is made smaller the results by the contour method would approach that by the mean accumulation method. This method gives a grade that is significantly lower than Gilmour's contour method.

A summary of the results is :

Method	Av. T.T. (ft)	Av. Cu%
1. Gilmour Contour	10,15	6,79
2. Arithmetic mean	11,49	6,46
3. Isted general formula	-	6,47
4. Arithmetic mean accumulation method $\frac{\Sigma Ft\%}{\Sigma TT}$ or $\frac{\bar{x}\ Ft\%}{\bar{x}\ TT}$	-	6,53

Correlation co-efficient r = +0,1825

Least squares regression line of grade on thickness y = 5,71 + 0,0655x

i.e. intercept = 5,74 and slope = +0,0655x

Since the area is defined by a fault and by a 17,5 ft% cut-off, the tonnage is proportional to the thickness and is not specifically calculated. An interesting feature is that although the statistical distribution of grades is rather flat with a barely detectable positive skew, and the thickness has a distinct but slight positive skew, the accumulation (Ft%) has a strong positive skew. If the additional eight low-grade intersections outside the limits of the defined ore are added, the distribution becomes somewhat modified: by definition, the eight intersections outside the limits of the ore are below 17,5 ft%, and if it is assumed that these are distributed evenly between 0 and 4% and 0 and 9 feet, then the grade becomes roughly symmetrically distributed, the thickness strongly positively skewed, and the accumulation has a very positive skew, approximately (by inspection) lognormal.

B. Roan Extension - 6 Limb

The first study carried out by Harju (1966) was on an area of the Roan Antelope mine containing a complex mineral zoning and copper grade distribution but a simple structure. In addition to the broad zoning shown in Figure 8 there is a stratigraphic zoning, but this is of minor significance and has been ignored, since the only effect is to cause minor distortions in some distributions, and there is no practical difference. Harju (1966) prepared histograms and calculated frequency distributions for the various blocks and for each of the four major zones, bornite, bornite-chalcopyrite, chalcopyrite, and chalcopyrite-pyrite, and for the whole area, using: 1. individual sample grades each weighted by its length; 2. unweighted individual sample grades; 3. complete intersections, each weighted by its true thickness, and 4. unweighted complete intersections. The average grade for each intersection was obtained in the usual way as a weighted average of the constituent samples.

Stratification was carried out by linking into one zone all blocks of uniform grade, and these were found to correlate closely with mineralogical zones. Since the total sulphide content is constant at about 9%, in itself a feature of some geological interest, and the sulphides are evenly disseminated, no specific gravity differences affect the picture.

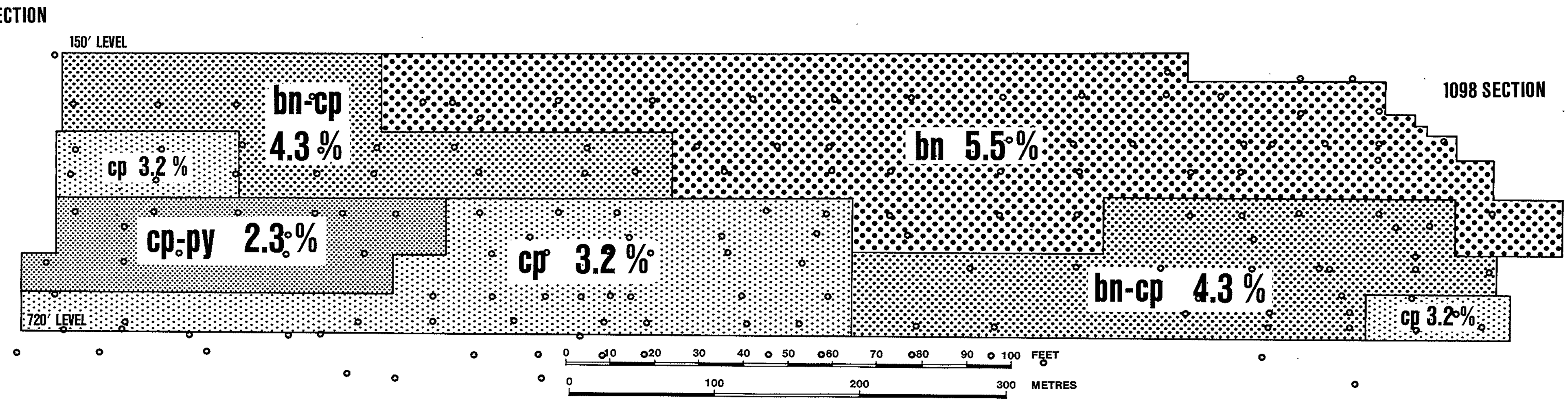


Figure 8 : *Roan Antelope 6 limb : Vertical longitudinal projection showing major mineral zones with their copper contents and positions of intersections. Though there is some mixing and overlap, the minerals in the different blocks are the dominant ones throughout the orebody in these blocks. On the upper right corner, adjacent to the bornite zone, is a barren stromatolitic biohermal body (Harju, 1966).*

The monomineralic zones gave unimodal frequency distributions for individual samples and the bimineralic zones bimodal distributions, as might be expected. The peaks in all cases reflected the mineral(s) present. The two unimodal distributions are somewhat leptokurtic and symmetrical, the slight distortions being caused by minor amounts of the other mineral, and Pearson type IV curves are the closest fit. For the frequency distribution of all samples, Pearson Type I curve is a close fit (see Figure 9). All distributions are symmetrical and close to normal, and a normal curve approximation for their populations is acceptable for all practical purposes. Complete exposures were also found graphically to have a close-to-normal distribution. Fiducial intervals were found to be uniformally low at around ± 0,2%. True thickness histograms were found to be symmetrical, particularly for the total intersections, and were used without further tests.

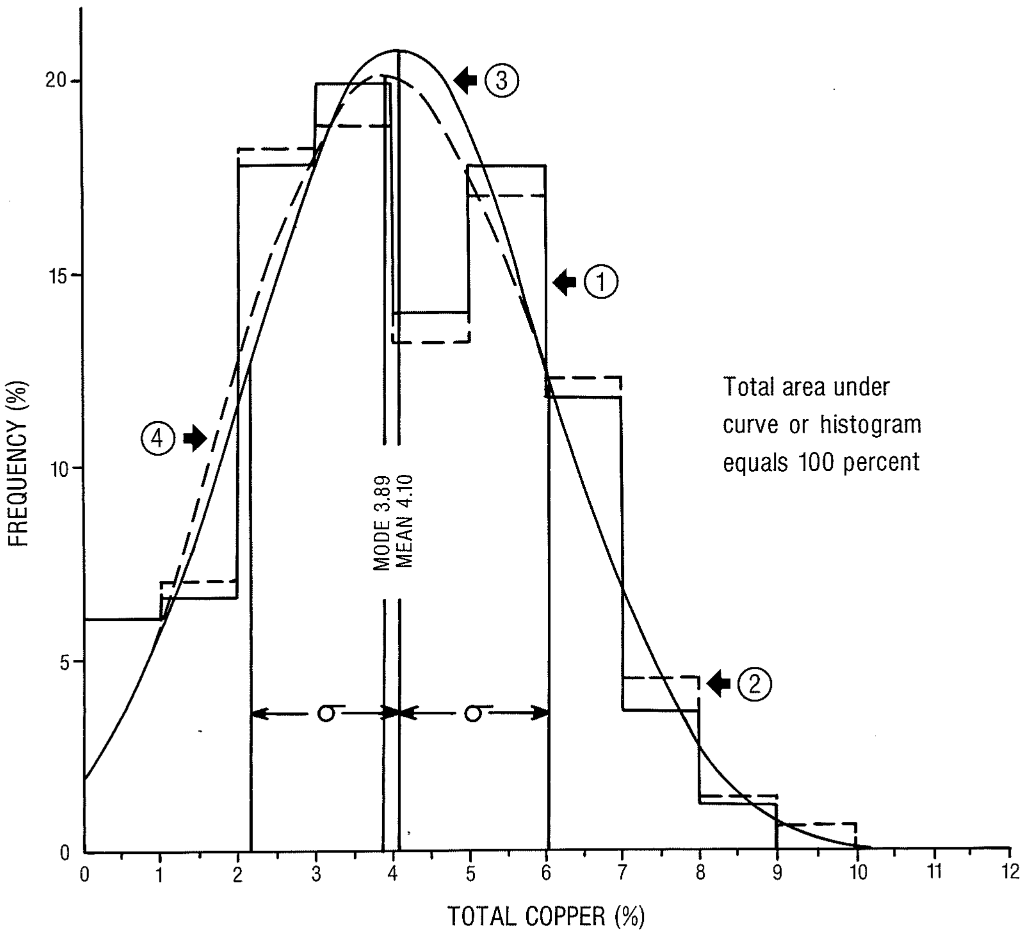


Figure 9 : *Roan Antelope 6 Limb. Frequency distribution for 2 177 samples in 163 intersections showing histograms, the derived Pearson Type I curve and its close similarity to the fitted normal curve (Harju, 1966).*

1. *Histogram of individual samples weighted by their length.*
2. *Histogram of unweighted individual samples.*
3. *Fitted normal curve.*
4. *Fitted Pearson type I curve.*

TABLE 3

ROAN BASIN - 6 LIMB RESULTS

ORE ZONE	STATISTICAL						MOD. RECTANGLE		TRIANGLE	
	N		TT	TONS	GRADE	FIDUCIAL INTERVAL ±	TONS	GRADE	TONS	GRADE
bn	596 596 49 49	1a 1b 2a 2b	39,1	2 993 590	5,54 5,54 5,50 5,56	0,12 0,13 0,17 0,18	2 992 920	5,46	3 026 540	5,55
bn-cp	634 634 46 46	1a 1b 2a 2b	47,8	3 153 150	4,47 4,48 4,30 4,38	0,13 0,14 0,26 0,24	3 281 430	4,29	3 253 490	4,37
cp	609 609 50 50	1a 1b 2a 2b	43,9	2 242 020	3,26 3,33 3,28 3,25	0,12 0,13 0,23 0,23	1 996 900	3,30	1 752 230	3,13
cp-py	337 337 18 18	1a 1b 2a 2b	51,9	1 042 680	2,38 2,56 2,32 2,39	0,18 0,18 0,23 0,34	1 049 640	2,33	916 740	2,31
TOTAL UNSTRATIFIED	2177 2177 163 163	1a 1b 2a 2b	44,1	9 439 910	4,10 4,15 4,05 4,17	0,08 0,08 0,20 0,20				
TOTAL EACH ZONE WEIGHED BY TONNAGE	2177 163	1a 1b 2a 2b		9 431 450	4,26 4,33 4,22 4,27		9 320 980	4,23	8 949 000	4,32

1a Individual samples weighted by length.
1b Individual samples unweighted.

2a Complete exposures weighted by thickness.
2b Complete exposures unweighted

A summary of the results obtained for each zone, for the total where each zone is weighted by its tonnage, and of the total unstratified sample are listed in Table 3, and are compared with an estimate by the triangular method, and with the mine estimate, made by a modified rectangle system.

The points that emerge from this study are :

1. The use of complete intersections is as effective as the use of all samples and is far more practical and simpler in use.
2. The weighted (accumulation) and unweighted average grades are closely comparable and there is no advantage in weighting.
3. The arithmetic mean grade and thickness (tonnage) are as close to the mine figures as are the 'triangle' figures, and are therefore acceptable alternatives.
4. Correlation was found between successive samples in an intersection.

C. Roan Basin West

In a similar study of another part of the same deposit that is fairly uniform mineralogically, and could be treated as a single population, Harju (1966) found that the arithmetic mean unweighted grade and true thickness are virtually identical with those obtained by the modified rectangle (mine) and triangle methods, as shown in the tabulation below.

TABLE 4

ROAN BASIN WEST - HARJU RESULTS

Statistical				Modified Rectangle				Triangle			
TT ft	Tons	Grade %	N	TT ft	Tons	Grade %	N	TT ft	Tons	Grade %	N
15,8	3 116 750	3,70	187								
16,5	3 227 070	3,66	183	16,1	3 156 590	3,66	183	16,4	3 204 803	3,66	183

Reconstruction of the data of Harju's exercise, which differs in some degree from the original because of uncertainty of the exact limits of the area and of the specific intersections used, shows the following :

N	=	173	
Thickness $\bar{x}$	=	16,1 ft;	s = 6,2 ft
Grade $\bar{x}$	=	3,70%	s = 0,37%
Accumulation $\bar{x}$	=	61,94 ft%	
Grade from accumulation	=	ΣFt%/ΣFt	= 3,84%
Correlation : r	=	-0.1341	
Regression line : $y = a + bx$	=	3,8338	$-0,0081x$

Recently, by courtesy of Roan Consolidated Mines Limited, it has been possible to obtain the sample data over a more extensive area containing around 15 million tons of ore and 391 intersections, and this has been further studied. The data are listed in Table 5 and the positions of intersections are shown in Figure 10.

The grades (Cu%) have a symmetrical distribution, and thicknesses and accumulations have moderate positive skew. The thickness decreases generally towards the east, with a series of fairly distinct highs and lows trending slightly below horizontal, as illustrated by the contours in Figure 10. The grades show a somewhat similar directional trend and there is a decrease towards both east and west but the differences are small and less marked owing to the small range. The results of the relevant calculations are listed below in Table 6.

Semivariograms were calculated along the 1385, 1440, 1650, 1740, 1930, and 2220 levels and the 606 and 646 sections, in each case using between 16 and 24 intersections, and are shown in Figure 11. The grade semivariograms show a random distribution throughout, reflecting the lack of both correlation between intersections and of areal influence of intersections. The semivariograms of both thickness and accumulation along strike are of the linear type, except for the 1930 level, which is random, and the 2220 level, which is of the transitive type. The reasons are not difficult to find: the 1930 level happens to be at a position where there is no trend at all in thickness, and the 2220 level at one where there is a distinct trend in the thickness. The average semivariograms are also approximately linear. Two sections, 606 and 646, both showed distinct transitive semivariograms, and again this can be seen to be because they cross the regional trends and have their own distinct trend. This exercise was not pursued any farther.

TABLE 5

ROAN BASIN WEST - DATA

Level	Sect.	No.	T.T. ft	Cu %	Ft%	Level	Sect.	No.	T.T. ft	Cu %	Ft%
840	708	1	16,7	3,05	50,9	1360	658	72	19,9	2,53	50,3
870	694	2	14,3	3,75	53,6		610	73	10,9	4,14	45,1
	658	3	13,6	3,34	45,4		602	74	14,1	4,15	58,5
	646	4	15,3	3,73	57,1	1385	722	75	13,9	4,55	63,3
900	708	5	14,0	3,89	54,4		714	76	23,0	3,25	74,8
	704	6	16,6	3,22	53,5		708	77	25,1	3,61	90,6
	678	7	15,2	3,29	50,0		702	78	25,5	3,68	93,8
	668	8	14,7	3,45	50,7		692	79	20,7	3,75	77,6
	656	9	15,4	3,11	47,9		656	80	19,1	2,67	51,0
	646	10	18,9	3,60	68,0		646	81	28,5	3,79	108,0
	634	11	15,4	2,34	36,0		634	82	18,9	3,67	69,4
903	646	12	15,4	3,90	60,1		632	83	16,1	4,58	73,7
960	714	13	14,7	3,60	52,9		624	84	13,1	3,97	52,0
	658	14	16,3	2,81	45,8		610	85	12,5	4,70	58,8
990	708	15	16,9	4,04	68,3		602	86	13,6	3,70	50,3
	702	16	17,0	3,25	55,3		596	87	13,6	4,20	57,1
	692	17	17,3	3,36	58,1		592	88	12,0	3,80	45,6
	678	18	14,7	3,39	49,8		584	89	10,0	3,80	38,0
	668	19	15,4	3,35	51,6		582	90	10,8	3,60	38,9
	656	20	15,4	2,91	44,8		574	91	9,4	3,90	36,7
	634	21	15,4	3,64	56,1	1385	558	92	5,6	3,70	20,7
1020	648	22	15,4	3,20	49,3		542	93	10,4	3,70	38,5
	646	23	16,5	3,00	49,5		528	94	7,7	3,60	27,7
	624	24	16,1	3,29	53,0		514	95	8,8	3,00	26,4
	610	25	17,2	3,67	63,1		504	96	8,0	3,60	28,8
	602	26	16,0	3,46	55,4		492	97	7,6	3,40	25,8
1080	722	27	12,4	3,94	48,9	1440	722	98	18,8	3,40	63,9
	714	28	15,4	3,27	50,4		710	99	25,6	3,72	95,2
	708	29	16,8	3,55	59,6		700	100	29,3	3,74	109,6
1080	704	30	15,0	3,47	52,1		694	101	29,9	3,46	103,5
	694	31	15,0	3,61	54,2		684	102	27,3	2,83	77,3
	678	32	15,0	3,23	48,5		674	103	28,0	3,07	86,0
	688	33	15,4	3,18	49,0		664	104	22,1	2,77	61,2
1110	712	34	15,6	4,14	64,6		652	105	32,0	3,55	113,6
	660	35	15,3	3,18	48,7		644	106	26,1	3,69	96,3
1120	634	36	15,4	2,55	39,3		634	107	16,1	4,58	73,7
	624	37	19,3	2,65	51,5		624	108	11,6	4,59	53,2
	610	38	14,3	3,81	54,5		610	109	10,6	3,40	36,0
	602	39	15,7	3,58	56,2		598	110	15,2	3,70	56,2
1140	712	40	9,0	4,76	42,8		590	111	11,6	4,00	46,4
	648	41	15,3	3,37	51,6		588	112	11,9	3,80	45,2
	646	42	18,3	2,95	54,0		587	113	10,9	3,80	41,4
1170	702	43	18,2	3,64	66,2		586	114	11,7	3,70	43,3
	692	44	13,9	3,74	52,0		585	115	12,2	3,80	46,4
	678	45	15,6	3,62	56,5		582	116	13,4	3,70	49,6
	668	46	15,1	3,03	45,8		581	117	13,2	3,80	50,2
	658	47	15,1	3,47	52,4		580	118	11,7	3,60	42,1
1200	722	48	19,4	3,30	64,0		578	119	10,9	3,60	39,2
	714	49	13,0	3,95	51,4		575	120	11,0	3,50	38,5
	708	50	18,5	3,27	60,5		572	121	9,5	3,80	36,1
	658	51	17,5	3,21	56,2		570	122	8,4	3,90	32,8
	646	52	19,0	3,54	67,3		566	123	10,1	3,50	35,4
	624	53	19,0	3,84	73,0	1440	560	124	10,2	3,70	37,7
	610	54	13,6	3,66	49,8		556	125	11,4	3,40	38,8
	602	55	17,3	4,17	72,1		554	126	10,7	3,30	35,3
1260	704	56	18,5	3,41	63,1		552	127	10,9	3,60	39,2
1290	722	57	18,6	3,13	58,2		550	128	10,9	3,60	39,2
	714	58	22,0	3,45	75,9		546	129	11,0	3,50	38,5
	708	59	18,9	2,62	49,5		544	130	11,1	3,60	40,0
	704	60	19,3	2,36	45,5		540	131	12,0	3,30	39,6
	692	61	21,0	3,04	63,8		538	132	10,7	3,40	36,4
1290	678	62	20,3	3,37	68,4		534	133	10,7	3,60	38,5
	668	63	19,3	3,93	75,8		532	134	10,8	3,30	35,6
	658	64	19,3	3,30	63,7		528	135	9,0	3,20	28,8
	646	65	18,0	1,82	32,8		526	136	8,0	3,20	25,6
	632	66	18,4	3,25	59,8		524	137	7,1	3,60	25,6
	624	67	14,4	3,75	54,0		522	138	8,1	3,20	25,9
	610	68	12,1	4,43	53,6		520	139	8,0	3,20	25,6
	602	69	15,4	4,51	69,5		516	140	6,9	3,20	22,1
1360	680	70	19,3	3,31	63,9		514	141	8,2	3,40	27,9
	668	71	19,5	3,92	76,4		512	142	6,2	3,10	19,2

TABLE 5 (Continued)

Level	Sect.	No.	T.T. ft	Cu %	Ft%	Level	Sect.	No.	T.T. ft	Cu %	Ft%
	508	143	7,7	3,00	21,1		636	214	19,4	3,98	77,2
	504	144	7,0	3,20	22,4		628	215	15,9	4,33	68,8
	502	145	9,0	2,80	25,2		620	216	13,4	3,31	44,4
	500	146	8,5	2,90	24,7		610	217	16,4	4,09	67,1
	494	147	9,3	3,20	29,8		604	218	16,3	3,76	61,3
	490	148	7,4	3,80	28,1		592	219	13,9	3,43	47,7
1470	722	149	19,2	3,43	65,9		586	220	13,4	4,00	53,6
	714	150	24,7	3,44	85,0		580	221	12,3	3,88	47,7
	708	151	25,8	3,29	84,9		570	222	13,0	3,62	47,1
	693	152	30,3	3,14	95,1		558	223	13,7	3,70	50,7
	688	153	30,0	3,33	99,9		542	224	11,5	3,32	38,2
	631	154	18,6	4,01	74,6		534	225	12,7	3,81	48,4
	627	155	16,0	3,75	60,0	1760	638	226	19,1	3,62	69,1
	624	156	15,2	3,93	59,7		610	227	16,4	4,09	67,1
	678	157	28,8	3,38	97,3		603	228	16,3	3,76	61,3
	670	158	17,8	3,16	56,2		592	229	13,9	3,43	47,7
	660	159	31,4	3,41	107,1		584	230	13,4	4,00	53,6
	654	160	30,6	3,38	103,4		578	231	12,3	3,88	47,7
	646	161	21,5	3,78	81,3	1790	706	232	23,3	3,41	79,5
	638	162	18,3	3,00	54,9		696	233	21,8	3,75	81,8
	630	163	24,1	3,36	81,0		688	234	22,0	3,55	78,1
	628	164	15,6	4,08	63,6		680	235	23,8	3,84	91,4
	624	165	15,6	3,78	59,0		666	236	23,6	3,74	88,3
	616	166	15,6	4,44	69,3		661	237	25,1	3,93	98,6
	610	167	8,4	3,17	26,6		654	238	24,2	3,83	92,7
	604	168	14,6	4,07	59,4		646	239	19,9	3,73	74,2
	596	169	11,2	4,24	47,5	1800	630	240	21,0	4,05	85,1
	594	170	10,4	4,53	47,1		624	241	18,1	3,73	67,5
	593	171	11,2	4,13	46,3		596	242	16,5	3,17	52,3
1570	625	172	15,6	3,31	51,6	1840	644	243	20,7	3,32	68,7
	624	173	12,2	3,45	42,1	1850	590	244	15,6	4,02	62,7
	616	174	14,7	3,97	58,4		586	245	12,9	3,96	51,1
	603	175	11,4	3,58	40,8		580	246	15,9	3,82	60,7
	597	176	11,5	3,58	41,2		570	247	12,6	3,97	50,0
	593	177	12,8	4,11	52,6		554	248	16,9	3,85	65,1
	584	178	12,1	4,48	54,2		548	249	14,6	3,70	54,2
1580	578	179	11,9	3,83	45,6		532	250	12,9	3,93	50,7
	570	180	10,9	3,89	42,4		520	251	13,4	3,32	44,5
	556	181	12,1	3,65	44,2	1870	722	252	25,0	3,49	87,3
	544	182	9,3	3,91	36,4		714	253	19,2	3,06	58,8
	533	183	10,4	3,84	39,9		708	254	17,3	2,92	50,5
	522	184	9,5	3,26	31,0		702	255	21,7	3,03	65,8
	512	185	8,4	3,45	29,0		688	256	14,5	4,36	63,2
1650	722	186	28,1	3,07	86,3		678	257	15,0	3,41	51,2
	714	187	25,1	3,22	80,8		670	258	10,2	3,84	39,2
	708	188	28,1	3,76	105,7		662	259	19,4	3,35	65,0
	702	189	29,0	3,21	93,1		654	260	21,2	3,27	69,3
	688	190	34,3	3,78	129,7		634	261	18,8	3,61	67,9
	682	191	33,8	3,83	129,5		630	262	19,2	3,83	73,5
	670	192	20,0	4,24	84,8		614	263	16,3	4,47	72,9
	662	193	27,1	3,37	91,3		604	264	15,1	4,38	66,1
	653	194	24,8	3,52	87,3		598	265	15,6	3,81	59,4
	646	195	26,5	3,59	95,1	1900	542	266	14,3	3,71	53,1
	638	196	14,8	4,33	64,1	1930	702	267	16,4	3,43	56,3
1650	631	197	18,8	3,53	66,4		692	268	13,4	3,28	44,0
	627	198	18,1	3,36	60,8		685	269	13,7	3,09	42,3
	624	199	16,3	3,70	60,3		672	270	13,0	3,35	43,6
	616	200	16,9	4,10	69,3		644	271	10,5	3,71	39,0
	606	201	13,8	4,42	61,0		634	272	15,0	3,83	57,5
	603	202	12,1	4,08	49,4		624	273	12,0	3,56	42,7
	596	203	12,9	4,28	55,2		608	274	15,0	3,67	55,1
	594	204	12,0	3,88	46,6		602	275	15,3	3,94	60,3
1740	722	205	22,3	3,46	77,2		596	276	14,0	3,69	51,7
	718	206	23,4	3,60	84,2		588	277	12,0	3,82	45,8
	702	207	28,8	3,30	95,0		582	278	13,4	3,95	52,9
	688	208	28,5	3,44	98,0		566	279	11,6	3,51	40,7
	678	209	30,0	3,55	106,5	1930	554	280	10,5	4,01	42,1
	668	210	28,6	3,74	107,0		540	281	11,5	3,54	40,7
	662	211	21,6	3,70	79,9		530	282	13,6	3,88	52,8
	654	212	22,3	3,71	82,7		522	283	8,7	4,30	37,4
	646	213	20,1	3,69	74,2		520	284	15,0	3,79	56,9

TABLE 5 (Continued)

Level	Sect.	No.	T.T. ft	Cu %	Ft%	Level	Sect.	No.	T.T. ft	Cu %	Ft%
	514	285	14,2	3,15	44,7		708	339	36,5	3,56	129,9
	510	286	15,4	3,71	57,1		704	340	22,8	2,98	67,9
1970	718	287	32,5	3,73	121,2		688	341	23,6	3,34	78,8
	708	288	25,3	4,08	103,2		678	342	17,5	3,22	56,4
	698	289	20,4	3,56	72,6		666	343	12,1	3,58	43,3
	698	290	20,3	3,77	76,5		662	344	10,4	3,59	37,3
	688	291	17,9	3,56	63,7		652	345	10,6	3,11	33,0
	678	292	13,1	4,15	54,4		644	346	8,0	3,48	27,8
	662	293	13,5	3,06	41,3		636	347	11,4	3,30	37,6
	654	294	17,3	3,41	59,0	2220	706	348	29,4	3,24	95,3
2000	718	295	32,1	3,46	111,1		698	349	29,4	2,91	85,6
	708	296	36,5	3,38	123,4		688	350	29,7	3,02	89,7
	704	297	24,4	3,43	83,7		678	351	22,5	3,89	87,5
	698	298	31,2	3,62	112,9		662	352	10,9	3,62	39,5
	688	299	2,11	3,78	79,8		660	353	9,3	2,75	25,6
	678	300	14,9	3,73	55,6		646	354	10,3	3,15	32,4
	674	301	12,9	3,40	43,9		636	355	9,25	3,40	31,5
	662	302	12,5	3,22	40,3		626	356	10,5	3,14	33,0
	642	303	12,4	3,64	45,1		602	357	13,0	3,45	44,9
	636	304	14,3	3,49	49,9		594	358	13,2	4,03	53,2
	624	305	14,2	3,69	52,3		588	359	14,5	3,24	47,0
	588	306	14,5	3,85	55,8		582	360	13,9	3,55	49,3
	582	307	16,4	3,28	53,8		572	361	13,2	3,60	47,5
	560	308	14,8	3,66	54,2		560	362	14,5	3,48	50,5
	548	309	16,0	3,98	63,7		548	363	13,6	3,34	45,4
	536	310	19,3	4,41	85,1		542	364	13,7	3,38	46,3
	530	311	15,8	3,49	55,1		526	365	14,6	3,70	54,0
	518	312	14,5	3,07	44,5		514	366	13,1	3,63	47,6
2030	702	313	32,6	4,05	132,0	2245	584	367	11,9	3,39	40,3
	656	314	13,1	3,09	40,5		574	368	13,0	3,60	46,8
	644	315	14,1	2,96	41,7		568	369	12,1	3,27	39,8
	628	316	14,1	3,89	54,8		560	370	12,7	3,67	46,6
	618	317	12,8	3,93	50,3		552	371	9,1	3,45	31,4
	722	318	24,6	3,30	81,2		544	372	10,8	3,71	40,1
2090	718	319	22,2	3,42	75,9		536	373	11,7	3,78	44,2
	708	320	24,5	3,18	77,9		522	374	9,4	4,04	38,0
	702	321	28,8	3,54	102,0		514	375	10,2	3,73	38,3
	692	322	24,5	3,68	90,2		508	376	11,1	3,56	39,5
	688	323	21,8	3,57	77,8		496	377	10,0	3,65	36,5
	674	324	15,6	3,57	55,7		490	378	17,6	3,41	60,0
	662	325	11,3	3,36	38,0	2280	674	379	28,2	3,80	107,2
	652	326	12,0	2,75	33,0		664	380	16,1	3,75	60,4
	644	327	12,4	3,63	45,0		628	381	9,0	3,65	32,9
	636	328	11,5	3,62	41,6		626	382	37,5	3,61	135,4
	632	329	9,2	3,13	28,8		594	383	7,7	4,34	33,4
2100	594	330	17,9	3,43	61,4		554	384	11,0	3,64	40,0
	588	331	13,2	3,29	43,4		522	385	12,3	3,96	48,7
	582	332	13,4	3,09	41,4		514	386	12,4	3,73	46,3
	548	333	13,7	3,73	51,1		500	387	13,9	3,98	55,3
	538	334	12,4	3,63	45,0		494	388	13,9	3,54	49,2
	530	335	11,6	4,25	49,3		494	389	12,4	3,75	46,5
	518	336	15,4	3,82	58,8		490	390	27,7	3,17	87,8
2140	636	337	13,1	4,00	52,4	2340	674	391	26,9	2,72	73,2
2190	720	338	33,5	3,3!	110,9						

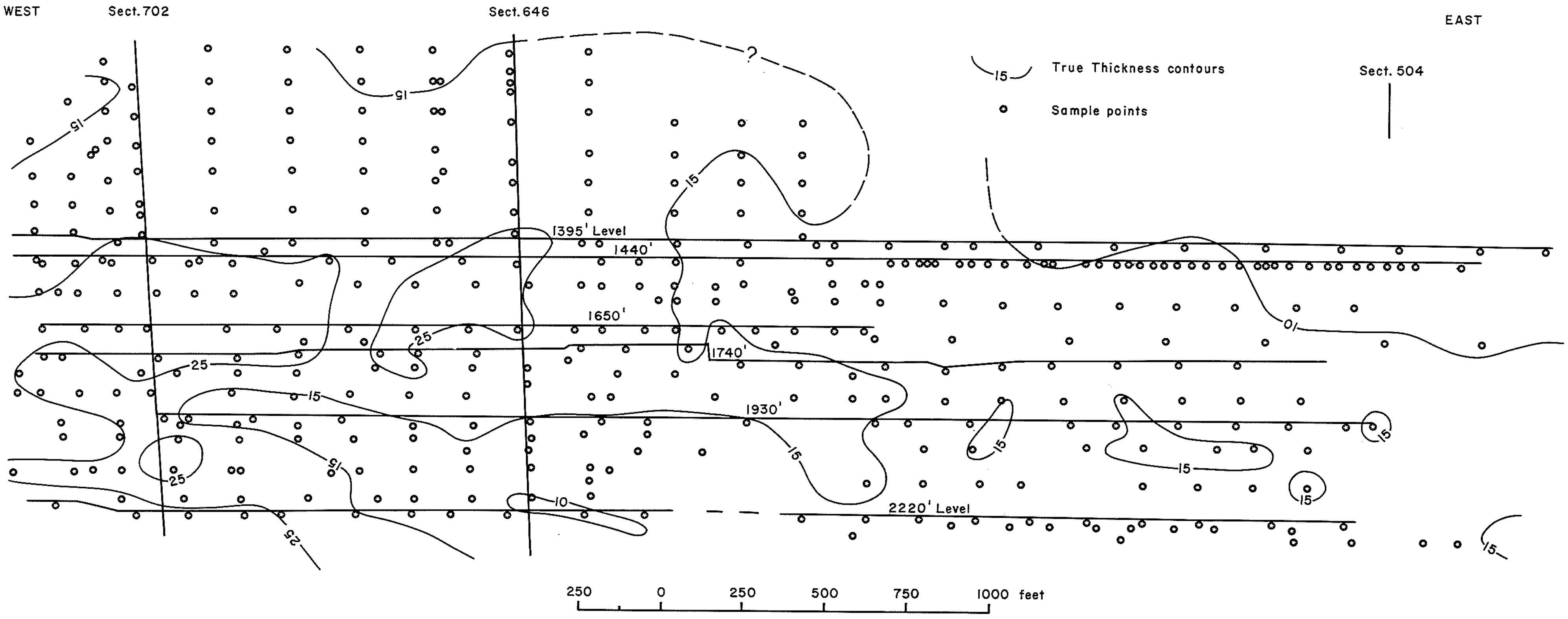


Figure 10 : *Roan Basin West. Vertical longitudinal projection showing levels, sections, the positions of intersections listed in Table 5, and true thickness contours in feet; grade shows little variation or trend. (data courtesy R.C.M. Limited).*

TABLE 6

ROAN BASIN WEST - RESULTS

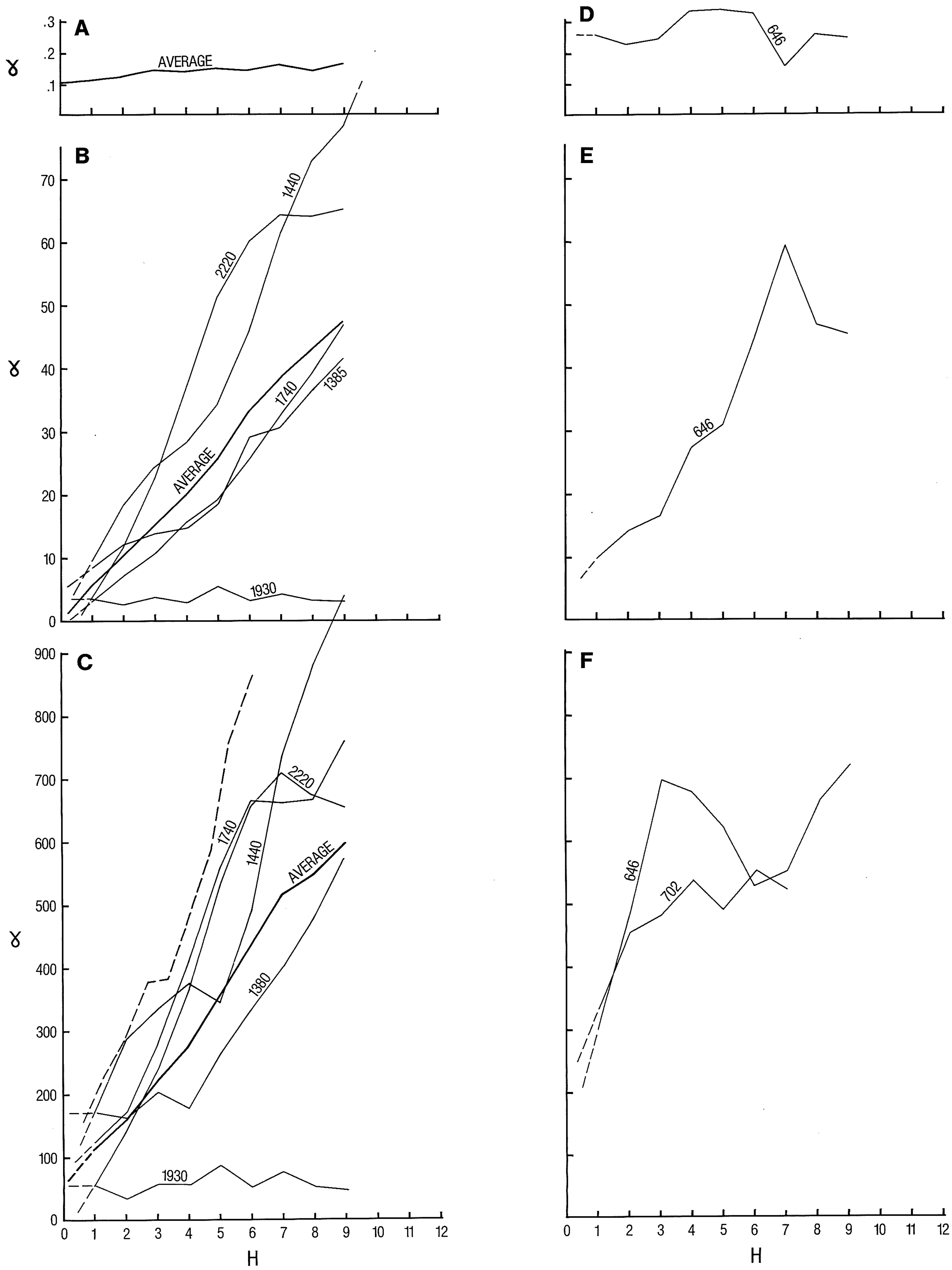
Area	N	Grade				Thickness			Corrln	Regression y = a + bx	
		Cu%	s	95%CL	Acc'n	Ft	s	CL		a	b
East 1385-2280 L	132	3,64	0,60	±0,21	3,65	11,9	3,8	±1,3	+0,14	3,47	0,01
West 1385-2280 L	188	3,60	0,40	±0,12		19,7	7,0	±2,0	-0,17	3,79	-0,01
West 820-2290 L (Includes somewhat sparser data above 1385)	259	3,55	0,56	±0,14	3,53	18,7	6,2	±1,5	-0,14	3,73	-0,01
Whole area	391	3,58	0,40		3,55	16,5	6,3		-0,13	3,71	-0,01

D. Muliashi

An extensive exercise was carried out over a portion of the Muliashi orebody, in the western part of the Roan Antelope synclinorium. This body is structurally fairly simple, and more important it is fairly shallow and had been drilled at close intervals for mine planning purposes, but the orebody is a different one to the other Roan bodies studied, and has been affected by some supergene activity (Mendelsohn and Wiik, 1970).

Of a total of 176 drillhole intersections, 140 are within the limits of the orebody selected for mining (Figure 12). All drillholes are listed in Table 7, with an indication of which are used in the N = 14 set. Eight methods of estimating reserves were tested on a series of different drillhole sequences at various drillhole densities. There were eight sequences, namely the actual chronological sequence, four random sequences and three exploration models generated by geologists. Each sequence was studied and reserves measured after 10, 20, 40, 60, 80, 100, 120, and 140 drillholes, except that the geologists' models ceased after the third (40 drillhole) stage, when the body was essentially outlined. The figures calculated with all 140 intersections by several methods were accepted as the norm, or 'correct' figures, namely 11,5 M tons at 2,53% Cu. Table 8 shows three of the sequences, numbers 2, 3, and 5. Table 9 shows the range and middle value of the 10-drillhole results for all sequences.

The tonnage increased rapidly to around 11 M tons after the third (40 hole) stage, and is fairly consistent, except that those calculated by statistical (TT) methods are lower than the others till the last stages. The study was not entirely conclusive because there is little difference between the grades throughout and differences between sequences are greater than those between different methods in the same sequence. All geometric methods, including computer contour, gave closely similar results and are virtually identical at a high data density. Statistical methods, both percentage and foot-percent, are lower for positive correlation and higher for negative correlation, but the differences are not proportional to the degree of



<u>*Figure 11*</u> : *Roan Basin West. Semivariograms along levels and sections shown in Figure 10.*

A. Levels - Grade, %. Lag 1 = H1 = 60 ft.

B. Levels - Thickness, ft. Lag 1 = H1 = 60 ft.

C. Levels - Accumulation, ft%. Lag 1 = H1 = 60 ft.

D. Sections - Grade, %. Lag 1 = H1 = 30 ft.

E. Sections - Thickness, ft. Lag 1 = H1 = 30 ft.

F. Sections - Accumulation, ft%. Lag 1 = H1 = 30 ft.

The type of semivariogram is dependent on the position and orientation of the line of intersections relative to the thickness distribution and trends.

TABLE 7

MULIASHI DATA

N = 176 DH No.	T.T. ft	Cu %	Ft%	N = 140 DH No.	N = 176 DH No.	T.T. ft	Cu %	Ft%	N = 140 DH No.
1	29,70	2,00	59,40	1	71	38,10	2,35	89,83	66
2	22,87	2,25	51,45	2	72	25,10	2,14	53,71	67
3	8,70	3,31	21,79	-	73	38,40	3,02	115,96	68
4	3,90	4,72	18,40	-	74	41,20	2,27	93,52	69
5	3,90	1,82	7,09	-	75	21,40	2,52	53,92	70
6	14,30	3,11	44,47	-	76	26,00	2,55	66,30	71
7	19,10	1,56	29,79	3	77	25,60	2,54	65,02	72
8	26,80	1,94	51,99	4	78	19,90	2,93	58,30	72
9	18,40	2,64	48,57	5	79	46,70	1,68	78,45	74
10	18,60	2,24	41,66	6	80	24,60	2,79	68,37	75
11	19,20	2,19	42,02	7	81	51,20	2,74	140,28	76
12	3,98	1,50	5,97	-	82	26,70	2,91	77,69	77
13	40,50	2,42	98,01	8	83	18,90	2,34	44,22	78
14	29,70	1,59	47,22	9	84	27,00	1,67	45,09	79
15	39,20	2,64	103,48	10	85	34,70	2,41	83,62	80
16	36,00	2,43	87,48	11	86	35,00	2,34	81,90	81
17	21,90	2,42	52,99	12	87	26,40	4,34	114,51	82
18	23,40	2,30	53,82	13	88	37,90	4,10	155,39	83
19	50,50	2,96	149,48	14	89	28,80	3,11	89,56	84
20	4,20	1,70	7,14	15	90	19,70	2,27	44,71	85
21	11,90	2,52	29,98	16	91	20,80	2,90	60,32	86
22	17,70	2,02	35,75	17	92	33,70	1,78	59,98	87
23	35,80	2,56	91,64	18	93	28,90	2,42	69,93	88
24	32,80	3,80	124,64	19	94	31,40	2,00	62,80	89
25	72,50	3,27	237,07	20	95	45,10	2,18	98,31	90
26	31,00	2,70	83,70	21	96	13,20	2,59	34,18	91
27	28,00	2,21	61,88	22	97	40,90	2,34	95,70	92
28	34,10	1,74	59,33	23	98	29,50	2,79	82,30	93
29	26,20	2,37	62,09	24	99	16,50	5,09	83,98	94
30	48,40	2,32	112,28	25	100	9,20	2,60	23,92	95
31	32,50	3,01	97,82	26	101	27,40	1,93	52,88	96
32	12,20	4,26	51,97	27	102	29,50	2,09	61,65	97
33	5,10	1,28	6,52	28	103	24,20	1,74	42,10	98
34	28,20	2,63	74,16	29	104	30,10	1,99	59,89	99
35	15,80	2,07	32,70	30	105	6,90	1,35	9,31	100
36	27,90	3,04	84,81	31	106	10,40	0,90	9,36	101
37	47,10	2,33	109,74	32	107	37,10	2,02	74,94	102
38	36,90	2,80	103,32	33	108	15,00	2,60	39,00	103
39	30,00	2,40	72,00	34	109	23,60	1,77	41,77	104
40	30,90	2,35	72,61	35	110	22,30	1,10	24,53	105
41	23,60	3,93	92,74	26	111	30,00	2,67	80,10	106
42	30,30	2,45	74,23	37	112	30,20	2,59	78,21	107
43	31,50	2,29	72,13	38	113	22,80	2,28	51,98	108
44	5,00	1,88	9,40	39	114	31,00	3,19	98,89	109
45	6,60	2,10	13,87	40	115	27,10	2,34	63,41	110
46	23,90	1,37	32,74	41	116	48,30	2,29	110,60	111
47	17,20	3,12	53,66	42	117	24,80	2,05	50,84	112
48	41,90	2,66	111,45	43	118	9,60	3,52	33,79	113
49	38,60	3,13	120,81	44	119	14,60	4,38	63,94	114
50	31,80	3,06	97,30	45	120	25,60	2,64	67,58	115
51	33,70	3,23	108,85	46	121	37,10	3,22	119,46	-
52	33,30	3,11	103,56	47	122	11,20	2,89	32,36	-
53	45,40	2,23	101,24	48	123	13,00	1,67	21,71	-
54	38,60	2,90	111,94	49	124	10,70	1,57	16,79	116
55	32,50	2,59	84,17	50	125	21,75	1,63	35,45	117
56	27,20	1,63	44,33	51	126	22,50	2,06	46,35	118
57	23,50	2,20	51,70	52	127	17,10	1,57	26,84	119
58	55,18	2,22	122,32	53	128	21,30	1,64	34,93	120
59	15,60	1,99	31,04	54	129	31,60	3,40	107,44	121
60	21,50	2,23	47,94	55	130	48,70	2,75	133,92	122
61	20,00	3,13	62,60	56	131	51,40	2,47	126,95	123
62	38,89	2,24	87,11	57	132	27,40	1,49	40,82	124
63	94,30	2,12	199,91	58	133	20,00	2,16	43,20	125
64	70,90	2,40	170,16	59	134	14,00	3,18	44,52	126
65	38,60	2,75	106,15	60	135	22,60	2,40	54,24	127
66	17,50	2,55	44,62	61	136	21,80	2,26	49,26	128
67	37,50	3,28	123,00	62	137	18,80	2,45	46,06	129
68	33,20	3,30	109,56	63	138	18,40	2,37	43,60	-
69	32,80	3,26	106,92	64	139	12,60	1,34	16,88	-
70	30,60	2,60	79,56	65	140	14,70	3,39	49,83	-

TABLE 7 (Continued)

N = 176 DH No.	T.T. ft	Cu %	Ft%	N = 140 DH No.	N = 176 DH No.	T.T. ft	Cu %	Ft%	N = 140 DH No.
141	14,40	1,71	24,62	130	159	28,10	1,84	51,70	-
142	22,40	2,29	51,29	131	160	14,90	3,30	49,17	-
143	12,10	2,41	29,16	132	161	23,40	2,11	49,37	-
144	20,70	1,88	38,91	133	162	11,50	1,93	22,08	-
145	13,20	2,56	33,79	134	163	28,70	2,75	78,90	-
146	14,80	2,40	35,52	135	164	24,00	3,40	81,60	-
147	13,50	2,71	36,58	136	165	18,40	2,00	36,80	-
148	10,70	2,88	30,81	137	166	14,50	2,00	29,00	-
149	12,00	2,36	28,32	-	167	14,10	1,29	18,18	-
150	12,20	2,34	28,54	139	168	23,20	2,57	59,62	-
151	16,50	1,24	20,46	139	169	12,80	2,91	37,24	-
152	8,40	3,02	25,36	140	170	15,00	2,37	35,55	-
153	18,70	1,99	37,28	-	171	16,80	2,02	33,93	-
154	23,60	2,55	60,18	-	172	14,90	1,99	29,65	-
155	21,30	1,43	30,45	-	173	5,40	2,00	10,80	-
156	13,70	2,19	30,00	-	174	17,10	1,70	29,07	-
157	15,10	2,18	32,91	-	175	31,00	2,35	72,85	-
158	13,60	2,45	33,32	-	176	14,00	1,86	34,93	-

TABLE 8

MULIASHI - RESULTS

Stage	N	Corrln	Stats %	Stats Ft%	TRIANGLE Isted	TRIANGLE Un-Wted.	TRIANGLE Cor. Wtng.	Polygon	Comp. %	Comp. Ft%
					SEQUENCE 2					
1	10	,6124	2,458	2,629	2,602	2,675	2,697	2,671	2,634	2,656
2	20	,3620	2,273	2,361	2,441	2,473	2,517	2,464	2,491	2,501
3	40	,2763	2,394	2,484	2,514	2,605	2,625	2,646	2,587	2,643
4	60	,2165	2,445	2,523	2,624	2,652	2,638	2,613	2,636	2,633
5	80	,1780	2,456	2,516	2,560	2,594	2,588	2,580	2,602	2,600
6	100	,0974	2,487	2,522	2,550	2,576	2,554	2,560	2,523	2,559
7	120	,0920	2,497	2,528	2,536	2,541	2,555	2,550	2,543	2,541
8	140	,1048	2,461	2,494	2,522	2,525	2,534	2,535	2,530	2,534
					SEQUENCE 3					
1	10	,0516	2,223	2,232	2,222	2,230	2,115	2,141	2,163	2,167
2	20	,0867	2,273	2,296	2,288	2,300	2,329	2,320	2,313	2,309
3	40	,1194	2,357	2,386	2,383	2,401	2,401	2,390	2,374	2,308
4	60	,0388	2,509	2,523	2,588	2,591	2,605	2,586	2,596	2,608
5	80	,0143	2,492	2,497	2,576	2,577	2,592	2,573	2,587	2,589
6	100	,0363	2,451	2,462	2,534	2,533	2,543	2,543	2,541	2,546
7	120	,0643	2,477	2,498	2,559	2,559	2,550	2,551	2,548	2,558
8	140	,1048	2,461	2,494	2,522	2,525	2,534	2,535	2,530	2,534
					SEQUENCE 5					
1	10	- ,3804	2,741	2,463	2,676	2,471	2,496	2,543	2,775	2,541
2	20	- ,2770	2,641	2,508	2,667	2,511	2,468	2,476	2,703	2,499
3	40	- ,0641	2,630	2,607	2,681	2,611	2,604	2,585	2,629	2,568
4	60	- ,0641	2,635	2,613	2,677	2,611	2,587	2,593	2,638	2,588
5	80	,0441	2,558	2,574	2,600	2,593	2,583	2,596	2,601	2,596
6	100	,0566	2,507	2,527	2,551	2,556	2,565	2,566	2,561	2,565
7	120	,0832	2,483	2,511	2,520	2,529	2,529	2,547	2,551	2,557
8	140	,1048	2,461	2,494	2,522	2,525	2,534	2,535	2,530	2,534

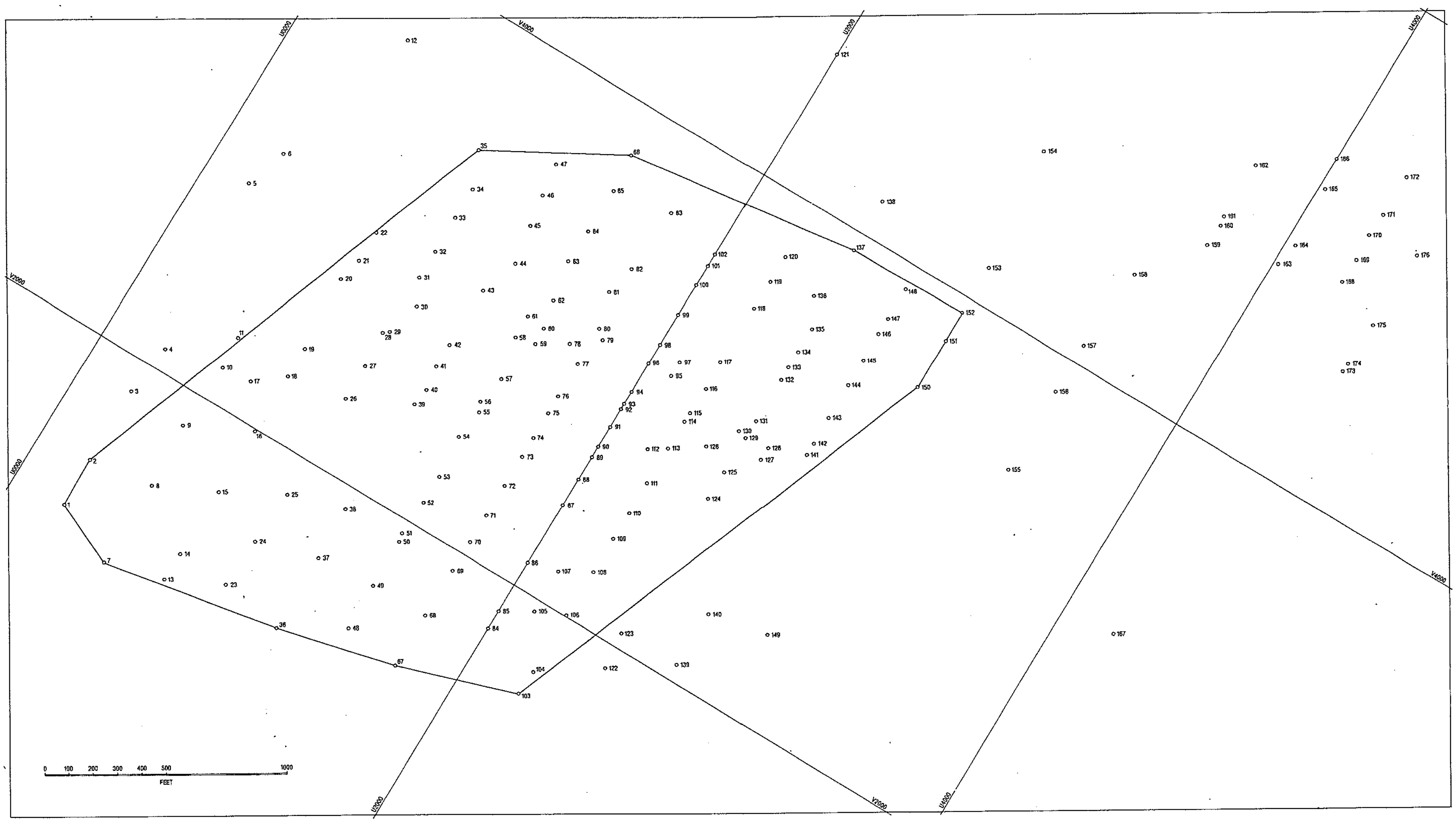


Figure 12 : *Muliashi. Plan showing positions and numbers of intersections used, and selected limits of orebody (data courtesy R.C.M. Limited).*

correlation. Even for very low to no correlation they are generally lower and remain so to the end stage, where they are about 0,07% Cu lower than the rest. One interesting but seemingly inexplicable feature is that nearly all sequences reach a maximum at 60 intersections, then decrease again at higher drilling densities. The Polynomial trend surface could not be adequately tested for technical reasons. Fourier trend surface produced accurate models of thickness and accumulation variability over the orebody when fitted to the 140 boreholes when the harmonic surfaces and numeric integration of these surfaces gave tonnage and grade estimates that were in good agreement with those from conventional methods. However, at lower levels of data density results were unreliable because unrealistic extremes were developed in places.

TABLE 9

MULIASHI - STAGE I

Sequence	Stage 1 - 10 DD holes		
	Grade		
	Range%	Mid Pt%	Tons (M) Range
1. Actual	2,48 - 2,64	2,56	3,4 - 3,7
2. Random	2,46 - 2,70	2,58	3,8 - 4,6
3. Random	2,11 - 2,23	2,19	5,2 - 6,0
4. Random	2,18 - 2,60	2,39	4,7 - 5,0
5. Random	2,47 - 2,78	2,63	5,1 - 6,7
6. Geol.	2,34 - 2,56	2,45	5,5 - 6,2
7. Geol.	2,45 - 2,60	2,53	5,4 - 5,8
8. Geol.	2,42 - 2,71	2,67	5,5 - 6,5
9. Geol.	2,44 - 2,51	2,48	4,9 - 5,4

E. Chambishi

Although the full details and results of the investigation carried out by Koch (1975) are not quoted, this example is of interest because the data (Table 10) and their spatial distribution are known, and results of average grade (copper content) estimates by several methods are given (Table 11). There are 33 intersections within the reserve block, and six more in the general area. The intersections, results and reserves estimated within the mining area above this block are not given (Figure 13).

TABLE 10

CHAMBISHI - DATA

Drillhole No.	T.T. m	Cu %	m%	Drillhole No.	T.T. m	Cu %	m%
1	13,62	2,12	28,87	20	8,56	3,43	29,36
2	19,81	1,13	23,39	21	15,06	3,63	54,67
3	8,20	1,30	10,66	22	11,64	3,74	43,53
4	3,05	2,93	8,94	23	9,72	5,64	54,82
5	18,23	2,20	40,11	24	8,14	2,02	16,44
6	8,00	2,83	21,84	25	7,18	2,84	20,39
7	8,05	2,95	23,75	26	7,01	3,94	27,62
8	3,38	2,28	7,71	27	8,47	2,28	19,31
9	7,60	2,08	15,81	28	8,81	2,96	26,08
10	4,33	2,22	9,61	29	7,96	2,43	19,34
11	6,55	2,85	18,67	30	2,50	4,49	11,23
12	10,49	2,43	25,49	31	7,50	3,76	28,20
13	18,01	2,73	49,17	32	7,80	3,09	24,10
14	13,87	2,04	28,29	33	8,44	3,09	26,08
15	6,90	3,27	22,56	34	1,64	2,93	4,81
16	10,94	2,64	28,88	35	4,57	3,88	17,73
17	14,45	2,99	43,21	36	3,60	1,41	5,08
18	8,75	2,20	19,25	37	2,10	4,05	8,51
19	11,89	3,90	46,37	38	3,78	2,06	7,79

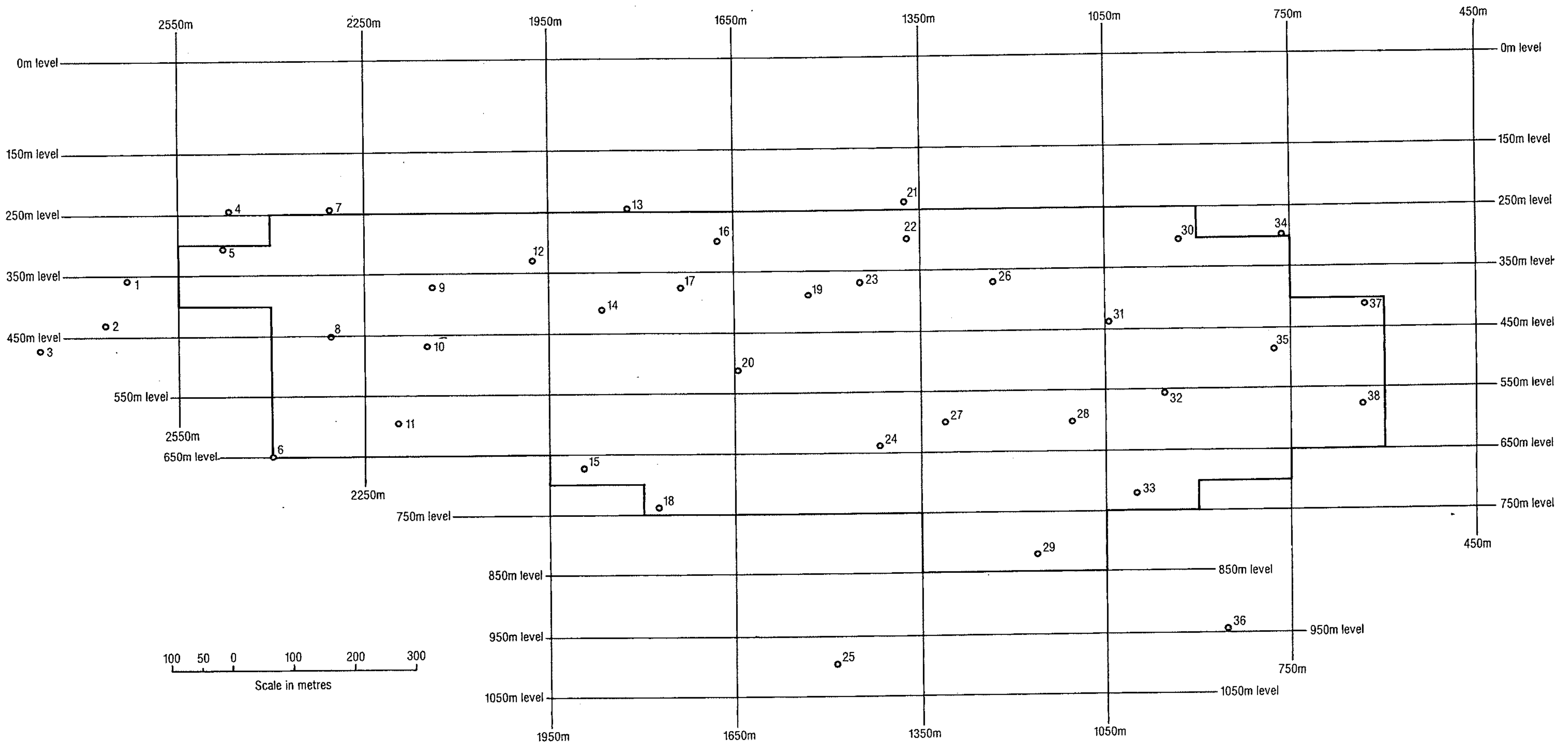


Figure 13 : *Chambishi. Vertical longitudinal projection showing positions and numbers of intersections used and outline of ore reserve (data courtesy R.C.M. Limited).*

TABLE 11

CHAMBISHI - RESULTS

Source	No.	Method	Cu %	N	T.T.	Tons
Link	1	Geologist's estimate	2,94	33?	-	-
Link	2	G P C P	3,03	33?	-	-
Link	3	Quadratic Regression	3,00	33?	-	-
Mine	4	Table in Link's Figure 3	2,86		-	26 000 000
FM	5 (a)	Statistical Cu%	2,86	38	8,7	
	(b)	Statistical Accumulation	2,75	38		
FM	6 (a)	Statistical Cu%	3,03	32	8,6	
	(b)	Statistical Accumulation	2,95	32		

Notes on methods :

1. No indication of method used.

2. General Purpose Contouring Package - Computer.

4. Series of blocks each given a value based on the included and surrounding intersections by an undescribed regression method.

5. (a) Straight arithmetic mean of grades and true thicknesses within blocks. Correlation coefficient r = -0,1881. Regression line (grade on thickness) intercept = 3,1921, slope = -0,0382. The correlation coefficient between accumulation and thickness is +0,7550, the regression line intercept is 4,489 and the slope +2,2599.

   (b) Accumulation method - grade from arithmetic mean of m%'s divided by the mean of the thicknesses within blocks.

6. (a) Straight arithmetic mean of grades and true thicknesses of all intersections. Correlation coefficient r = -0,1198. Regression line (grade on thickness) intercept 3,2367, slope = -0,0242.

   (b) Accumulation method - grade from arithmetic means of accumulation (m%) divided by the mean thickness of all intersections.

   For 5 and 6, two intersections that are very close have been averaged and taken as one (No. 7 in Table 10) but this does not have an appreciable effect.

All results are within rather close limits, and it is not certain which is the most acceptable. No. 5 (Table 11) includes several intersections of lower grade and/or thickness beyond the limits of acceptable ore and the results would be expected to be somewhat low.

The distribution of grades is symmetrical and approximately normal, as demonstrated graphically. That of the thickness is roughly symmetrical, and of the accumulations even, but with a slight tail on the high side.

F. Mufulira

The data forming Table 12 is extracted from a figure in Powell (1962), reproduced as Figure 14, and refers to all or a major part of the 'C' orebody as known at that time.

Results:

Arithmetic mean thickness = 30,97 feet

Arithmetic mean grade = 3,05%

Mean grade by accumulation method = 3,36%

'C' Orebody quoted in 1965 Annual Report:

Tonnage = 104 M tons: Average Grade = 3,2% Cu.

Correlation co-efficient = +0,5680

Least squares regression line of thickness on grade :

intercept = 12,6336, slope + 6,0058

Least squares regression line of grade on thickness:

intercept = 1,3894, slope = + 0,537

Should there be perfect correlation between grade and thickness (r = 1), the regression lines would merge to form only one line that would be exactly half-way between them. The equation for this line in this case would be, for thickness on grade: intercept = 12,3139, slope = + 6,6199

Powell (1962) recorded the following results of ore reserve estimates from one particular block:

Original triangle estimate: 34,9 m feet^3 (about 3 M tons) at 5,319%

Subsequent blocked out estimate: 35,6 M feet^3 at 4,501%

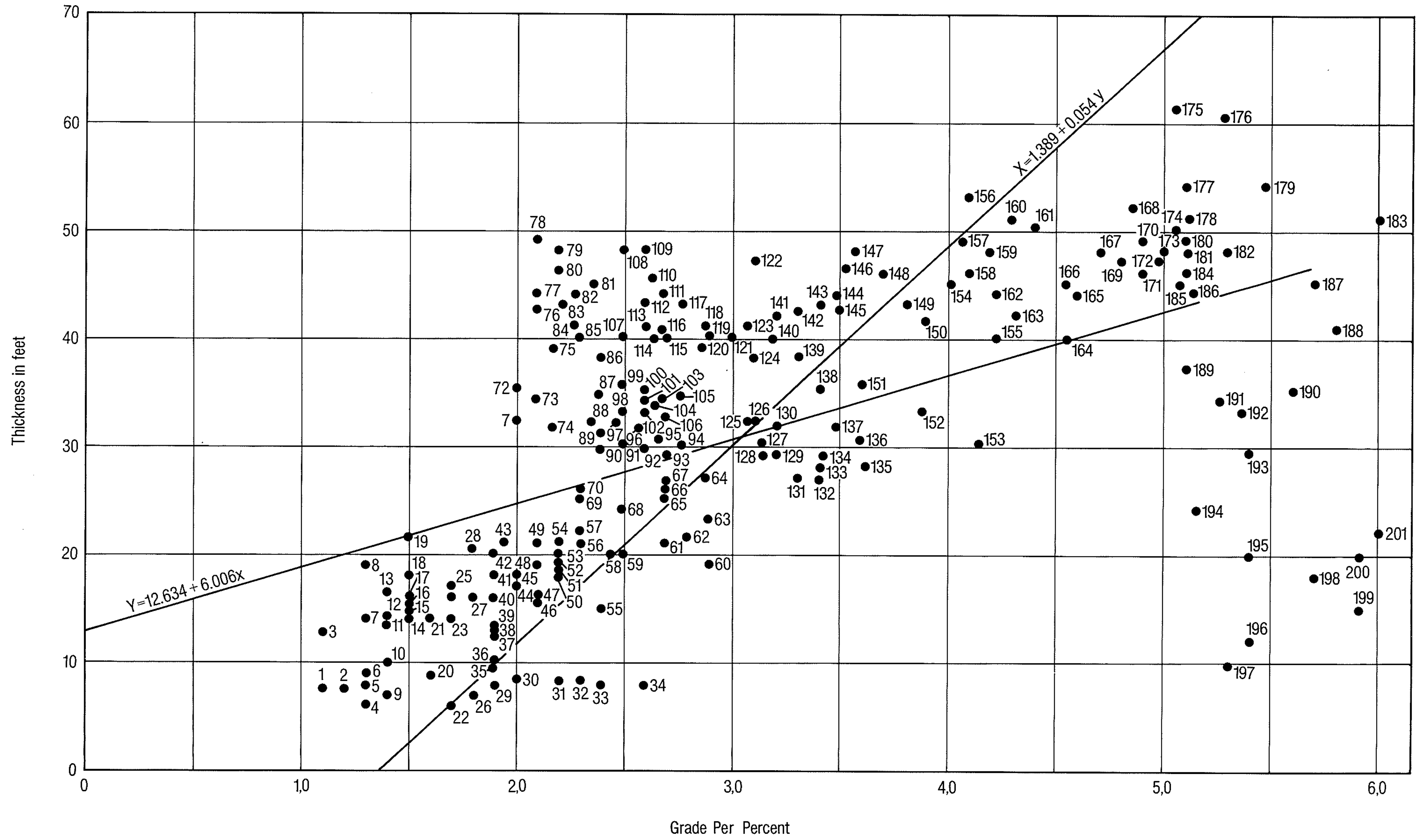


Figure 14. : *Mufilira C Orebody: Scatter diagram of thickness and copper content of 201 intersections. The linear regression lines are based on all intersections; there is a possibility that there are three separate and overlapping populations (after Powell, 1962, data courtesy R.C.M. Ltd.).*

TABLE 12

MUFULIRA - DATA

Drillhole No.	T.T.	Cu %	Ft%	Drillhole No.	T.T.	Cu %	Ft%
1	7,5	1,1	8,3	21	14,0	1,6	22,4
2	7,5	1,2	9,0	22	6,0	1,7	10,2
3	13,0	1,1	14,3	23	14,0	1,7	23,8
4	6,0	1,3	7,8	24	16,0	1,7	27,2
5	8,0	1,3	10,4	25	17,0	1,7	28,9
6	9,0	1,3	11,7	26	7,0	1,8	12,6
7	14,0	1,3	18,2	27	16,0	1,8	28,3
8	19,0	1,3	24,7	28	21,0	1,8	37,8
9	7,0	1,4	9,8	29	8,0	1,9	15,2
10	10,0	1,4	14,0	30	9,0	2,0	18,0
11	14,0	1,4	19,6	31	14,0	2,2	17,1
12	14,0	1,4	19,6	32	14,0	2,3	32,2
13	17,0	1,4	23,3	33	13,0	2,4	31,2
14	14,0	1,5	21,0	34	13,0	2,6	33,8
15	15,0	1,5	22,5	35	10,0	1,9	19,0
16	15,0	1,5	22,5	36	10,0	1,9	19,0
17	16,0	1,5	24,0	37	13,0	1,9	24,7
18	1,0	1,5	27,0	38	13,0	1,9	24,7
19	22,0	1,5	32,0	39	13,0	1,9	24,7
20	9,0	1,6	14,4	40	16,0	1,9	30,4

TABLE 12 (Continued)

Drillhole No.	T.T.	Cu %	Ft%	Drillhole No.	T.T.	Cu %	Ft%
41	18,0	1,9	34,2	115	40,0	2,7	108,0
42	20,0	1,9	38,0	116	41,0	2,7	110,7
43	21,0	1,95	41,0	117	43,0	2,75	118,3
44	17,0	2,0	34,0	118	41,0	2,85	116,9
45	18,0	2,0	36,0	119	40,0	2,9	116,0
46	16,0	2,1	33,6	120	39,0	2,85	111,1
47	16,0	2,1	33,6	121	40,0	3,0	120,0
48	19,0	2,1	39,9	122	47,5	3,1	147,2
49	21,0	2,1	44,1	123	41,0	3,05	125,1
50	18,0	2,2	39,6	124	38,0	3,1	117,8
51	18,0	2,2	39,6	125	32,0	3,05	97,6
52	19,0	2,2	41,8	126	32,0	3,1	99,2
53	20,0	2,2	44,0	127	30,0	3,15	94,5
54	21,0	2,2	46,2	128	29,0	3,15	91,4
55	15,0	2,4	36,0	129	29,0	3,2	92,8
56	21,0	2,3	48,3	130	32,0	3,2	102,4
57	22,0	2,3	50,6	131	27,0	3,3	89,1
58	20,0	2,45	49,0	132	27,0	3,4	91,8
59	20,0	2,5	50,0	133	28,0	3,4	95,2
60	19,0	2,9	43,5	134	29,0	3,4	98,6
61	21,0	2,7	56,7	135	28,0	3,6	104,4
62	22,0	2,8	61,6	136	30,5	3,6	109,8
63	23,0	2,9	66,7	137	32,0	3,5	112,0
64	27,0	2,9	78,3	138	35,0	3,4	119,0
65	25,0	2,7	67,5	139	38,0	3,3	125,4
66	26,0	2,7	70,2	140	40,0	3,2	128,0
67	26,0	2,7	70,2	141	42,0	3,2	134,4
68	14,0	2,5	60,0	142	42,5	3,3	140,3
69	25,0	2,3	57,5	143	43,0	3,4	146,2
70	26,0	2,3	59,8	144	44,0	3,5	154,0
71	32,0	2,0	64,0	145	43,0	3,5	150,5
72	35,0	2,0	70,0	146	46,5	3,55	165,1
73	34,0	2,1	71,4	147	48,0	3,6	172,8
74	32,0	2,2	70,4	148	46,0	3,7	170,2
75	39,0	2,2	85,8	149	43,0	3,8	163,4
76	42,0	2,1	88,2	150	41,5	3,9	161,9
77	44,0	2,1	92,4	151	36,0	3,6	129,6
78	49,0	2,1	102,9	152	33,0	3,9	128,7
79	48,0	2,2	195,6	153	30,0	4,15	124,5
80	47,0	2,2	103,4	154	45,0	4,0	80,0
81	45,0	2,4	108,0	155	40,0	4,2	168,0
82	44,0	2,3	101,2	156	53,0	4,1	217,3
83	43,0	2,2	94,6	157	49,0	4,05	198,5
84	41,0	2,3	94,3	158	46,0	4,1	188,6
85	40,0	2,3	92,0	159	48,0	4,2	201,6
86	38,0	2,4	91,2	160	51,0	4,3	219,3
87	35,0	2,4	84,0	161	50,0	4,4	220,0
88	32,0	2,35	75,2	162	44,0	4,2	184,8
89	31,0	2,4	74,4	163	42,0	4,3	180,6
90	30,0	2,4	72,0	164	40,0	4,55	182,0
91	30,0	2,5	75,0	165	44,0	4,6	202,4
92	29,5	2,6	76,7	166	45,0	4,55	204,8
93	29,0	2,7	78,3	167	48,0	4,7	225,6
94	30,0	2,75	82,5	168	52,0	4,85	252,2
95	30,5	2,65	80,8	169	47,0	4,8	225,6
96	32,0	2,55	81,6	170	49,0	4,9	240,1
97	32,0	2,45	78,4	171	49,0	4,9	225,4
98	33,0	2,5	82,5	172	47,0	5,0	235,0
99	36,0	2,5	90,0	173	48,0	5,0	240,0
100	35,0	2,6	91,0	174	50,0	5,05	252,5
101	34,0	2,6	88,4	175	61,0	5,05	308,1
102	33,0	2,6	85,8	176	60,0	5,3	318,0
103	34,0	2,7	91,8	177	54,0	5,1	275,4
104	34,0	2,65	90,1	178	51,0	5,1	260,1
105	34,5	2,75	94,9	179	54,0	5,5	297,0
106	33,0	2,7	89,1	180	49,0	5,1	249,9
107	40,0	2,5	100,0	181	48,0	5,1	244,8
108	48,0	2,5	120,0	182	48,0	5,3	254,4
109	48,0	2,6	124,8	183	51,0	6,0	306,0
110	45,5	2,65	120,6	184	46,0	5,1	234,6
111	44,0	2,7	118,8	185	45,0	5,1	229,5
112	43,0	2,6	111,8	186	44,0	5,1	224,4
113	41,0	2,6	106,6	187	45,0	5,7	256,5
114	40,0	2,65	106,0	188	41,0	5,8	237,8

TABLE 12 (Continued)

Drillhole No.	T.T.	Cu %	Ft%	Drillhole No.	T.T.	Cu %	Ft%
189	37,0	5,1	188,7	196	12,0	5,4	64,8
190	35,0	5,6	196,0	197	10,0	5,3	53,0
191	34,5	5,25	181,1	198	18,0	5,7	102,6
192	33,0	5,5	181,5	199	15,0	5,9	88,5
193	28,5	5,4	159,3	200	20,0	5,9	118,0
194	24,0	5,15	123,6	201	22,0	6,0	132,0
195	20,0	5,4	108,0				

G. Unidentified COPPERBELT Deposit

This deposit was used as an example for the statistical evaluation of ore reserves by Wiik (1971), and no information other than that listed in Table 13 is available.

Results:

Arithmetic mean grade = 2,52%

Arithmetic mean thickness = 24,56 ft

Arithmetic mean accumulation = 52,57 ft%

Mean grade, accumulation method = 52,57/24,56 = 2,55%

Correlation co-efficient = +0,04

Least square regression line
(grade on thickness) :

intercept = 2,4719, slope + 0,0021

The statistical distributions are:

Grade: Symmetrical, approximately normal

Thickness: Even

Accumulation: Positive skew

TABLE 13

UNIDENTIFIED COPPERBELT DEPOSIT - DATA

Drillhole No.	T.T. ft	Cu %	Ft%	Drillhole No.	T.T. ft	Cu %	Ft%
1	17,80	1,49	26,52	28	15,30	2,33	48,76
2	12,60	1,96	24,69	29	32,20	3,01	96,92
3	17,90	2,23	39.91	30	19,70	3,47	68,35
4	31,00	3,17	98,27	31	11,20	2,76	30,91
5	25,80	2,19	56,50	32	6,00	2,86	17,16
6	5,80	2,06	11,94	33	3,00	5,91	17,73
7	25,60	2,85	72,96	34	40,30	2,58	103,97
8	5,10	1,03	5,25	35	64,50	2,40	154,80
9	39,80	2,59	103,08	36	25,80	2,80	72,24
10	24,50	2,30	56,35	37	42,90	2,92	125,26
11	5,00	2,31	11,55	38	43,40	2,59	112,40
12	19,00	2,31	43,89	39	22,90	1,99	45,57
13	12,10	2,75	33,27	40	34,00	2,54	86,36
14	38,60	2,35	90,71	41	8,70	3,03	26,36
15	30,10	4,26	128,22	42	27,10	2,87	77,77
16	41,10	2,34	96,17	43	3,40	1,65	5,61
17	11,20	3,98	44,57	44	37,20	2,71	100,81
18	17,30	1,93	33,38	45	27,90	2,67	74,49
19	3,50	1,56	5,46	46	45,90	2,37	108,78
20	26,80	1,67	44,75	47	35,00	2,87	100,45
21	20,10	2,14	43,01	48	41,70	2,12	88,40
22	16,30	2,04	33,25	49	4,10	5,32	21,81
23	21,00	0,96	20,16	50	23,90	2,41	57,59
24	21,90	1,65	36,13	51	40,20	2,96	118,99
25	11,80	2,57	30,32	52	10,10	3,83	38,68
26	16,00	2,88	46,08	53	34,90	2,60	90,74
27	5,30	1,73	9,16	54	35,00	2,10	73,50

TABLE 13 (Continued)

Drillhole No.	T.T. ft	Cu %	Ft%	Drillhole No.	T.T. ft	Cu %	Ft%
55	12,60	4,19	52,79	68	18,40	2,65	48,76
56	46,40	2,44	113,21	69	3,50	1,07	3,74
57	4,50	2,24	10,08	70	20,30	1,57	31,87
58	35,70	1,87	66,75	71	3,40	2,44	8,29
59	31,60	1,70	53,72	72	1,60	2,40	3,84
60	11,00	3,20	35,20	73	20,40	1,71	34,88
61	21,50	2,45	52,67	74	44,30	2,28	101,00
62	35,10	1,84	64,58	75	41,50	3,07	127,40
63	32,70	2,38	77,82	76	61,10	3,53	215,68
64	21,40	2,84	60,77	77	61,80	2,68	165,62
65	11,50	2,15	24,72	78	55,90	3,02	168,81
66	26,30	2,35	61,80	79	61,50	3,00	184,50
67	17,30	1,35	23,35				

H. Maggie Canyon

In 1958 Scott W. Hazen Jnr. of the United States Bureau of Mines undertook a detailed comparison of statistical and several geometric methods of ore reserve estimation on the Maggie Canyon manganese deposit in Arizona. Briefly, his conclusions are that grades computed by triangle, polygon, and statistical methods agree closely, whereas those from the cross-sectional method are rather different and likely to be less consistent. It can also be seen from the results that the tonnages vary by around 5% and the cross-sectional tonnage is also somewhat different from the other geometric methods. Polygon and triangle methods agree closely in both respects.

On the statistical side Hazen (1958) used individual samples (not complete intersections) throughout and concluded that the grades are lognormally distributed. The arithmetic mean is used as the best estimate of grade and thickness. Table 15 shows a comparison between the polygon method, Hazen's (1958) statistical method using the arithmetic mean of individual samples, and the arithmetic mean of complete intersections (not used by Hazen).

Another exercise had been carried out with independently re-assessed and recalculated complete intersections (Table 14) and this re-computed data is used to show another comparison between the triangle grade and the arithmetic mean grade calculated directly and via the accumulation (see Table 15). The results suggest that subjective personal differences or the meeting of somewhat different requirements could have a greater effect than different methods of estimation, except that the arithmetic mean grade remains consistent, a feature of possible significance if not coincidental in this case.

TABLE 14

MAGGIE CANYON - DATA

Drillhole No.	T.T. ft	Mn %	Ft%	Drillhole No.	T.T. ft	Mn %	Ft%
U-108	33,4	7,76	259,2	B-14	10,0	3,47	34,7
U-204	36,6	10,43	381,7	117	30,0	4,42	132,6
U-209	24,2	5,95	143,99	B-15	76,0	4,78	363,3
A-2	54,4	5,67	308,4	8	20,0	4,19	83,8
A-9	29,4	4,03	118,5	12	32,0	4,07	130,2
B3	10,0	3,63	36,3	B-13	85,5	4,13	353,1
A1	14,8	4,01	59,3	B-12	30,0	4,05	121,5
A4	39,6	7,06	279,6	103	10,0	3,46	34,6
A4	32,4	8,92	289,0	B5	49,0	5,92	290,1
A5	54,8	8,51	466,3	107	45,0	4,06	182,7
B2	80,0	5,51	412,0	11	40,0	4,80	192,0
13	51,0	5,30	270,3	B6	34,0	2,77	94,2
124	30,0	4,17	125,1	3	37,0	8,04	297,5
A-10	36,0	8,43	303,5	B8	15,0	4,29	64,4
A-11	49,6	6,17	306,0	113	10,0	3,01	30,1
B1	80,7	4,84	390,6	B7	45,0	5,66	254,7
132	64,0	4,13	264,3	114	15,0	4,29	64,4
B4	20,0	4,63	92,6	9	10,0	4,25	42,5
A7	7,1	3,17	22,5	27	45,0	11,98	539,1
A8	29,1	4,47	130,1	B17	60,0	5,95	357,0
118	37,0	4,94	182,8	22	56,0	7,35	411,6
7	36,0	13,50	486,0	B18	15,0	5,66	84,9
115	70,0	1,93	135,1	21	54,0	8,75	472,5

TABLE 15

MAGGIE CANYON - RESULTS

Polygon				Individual Samples		Complete Intersections	
				Hazen Statistics		Separate Areas and Weighted mean	Total Set
Area	N	Tons	Grade	Tons	Grade	$\bar{x}$ Grade	$\bar{x}$ Grade
		000		000			
S	26	7 184	5,54	6 877	5,89	5,85	
SE	14	10 591	4,24	11 232	4,14	4,18	
N	5	9 613	6,72	7 987	6,54	6,88	
TOTAL	45	27 388	5,45%	26 374	5,42%	5,45%	5,57%

Recalculated Data

All intersections recalculated.

Area	Triangle Tons 000	Grade	Statistical $\bar{x}$ Grade	$\bar{x}$ Ft% Grade
S	7 290	5,19		
SE	9 145	4,78		
N	9 763	8,01		
TOTAL	26 198	6,10%	5,57%	5,79%

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