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Extreme event dynamics in the formation of galaxy-sized dark matter structures

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ABSTRACT

The search for turbulent-like patterns in nonlinear gravitational clustering has recently advanced due to *N*-body simulations based on the cold dark matter scenario. In this work we present a computational statistical analysis of the formation of galaxy halos by gravitational collapse in *N*-body simulation from the Virgo Consortium Data. We find that rescaled data points of gravitational energy for different redshifts collapse into similar patterns, well approximated by a Generalized Extreme Value (GEV) distribution. Once similar statistical behavior was found for chaotic advection, this result is discussed in the context of non-dissipative turbulent-like behavior. From our analysis the unstable gravity field itself behaves as a chaotic advecting flow where the particles (galaxies) can be interpreted as turbulent tracers.

1. Introduction

In recent years, computational simulation and high resolution astrophysical observations have shown that under the mutual attraction of gravity the initially cosmological smooth distribution of matter develops inhomogeneous large scale structures. Thus, from a dynamical point of view, in the early universe gravitational instabilities were large enough to produce galaxies and clusters of galaxies observed today.

Although the theoretical understanding of the nonlinear gravitational clustering has greatly advanced in the last decades, in particular by the outstanding improvement on numerical *N*-body simulations, the physics behind this process is not fully discerned. Recently, a detailed percolation analysis of the Virgo data for different redshifts has shown that the gravitational clustering of dark matter may admit a turbulent-like representation [1]. From this approach, local dynamical scaling processes involving unstable gravitational galaxy interactions can act to form the significantly nonhomogeneous structures observed in low redshifts (see Fig. 1). In this approach, gravitational energy spectra for cluster-sized halos and galaxy-sized halos, for redshifts from 10 to 0, have been interpreted as a phenomenological signature of a possible turbulent-like mechanism characterized by combinations of small-scale eddies and larger flow-like structures due to the nonlinear gravitational galaxy–galaxy local interactions. However, we must note that since the cosmological simulated system is non-dissipative, a conservative turbulent-like mechanism must be addressed [2,3].

Recently, the non-dissipative concentration fluctuation in Lagrangian turbulence (or *chaotic advection* in the phase space [4]) has been studied using techniques of extreme value theory to predict extreme concentrations by modeling the upper tail of the probability density function of the fluid inhomogeneties [5]. Data from large eddy dynamics obtained from both the simulations and experiments have been analysed and validate by calculating the maximum concentration normalized by the local mean concentration (or by the local r.m.s. of concentration fluctuation).

In probability theory and computational statistics, non-Gaussian fluctuations usually are characterized taking into account generalized probability distributions [6]. The generalized extreme value distribution (GEV) is a family of continuous probability distributions developed within extreme value theory to combine the Gumbel, Fréchet and Weibull families also known as type I, II and III

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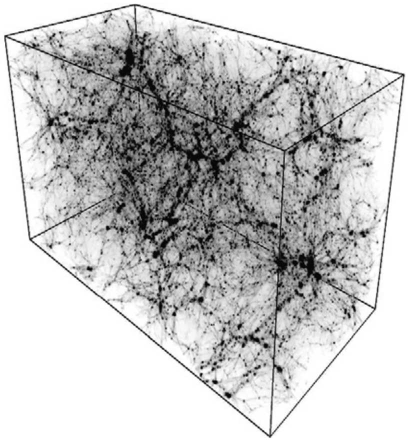


Fig. 1. A snapshot of structural cold dark matter pattern obtained from the Virgo consortium data, for $z = 0.1$ (box size scale of 141,3 Mpc/h with 256^3 dark matter particles).

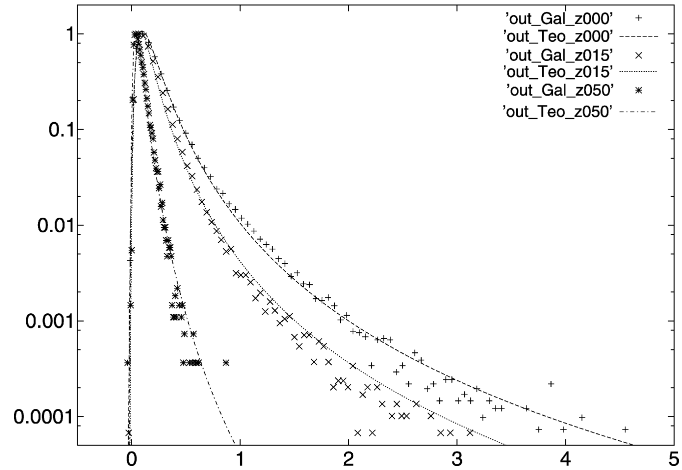


Fig. 2. Empirical and theoretical (GEV model) rescaled energy histograms, for redshifts $z = 0$, 1.5 and 5.0.

extreme value distributions. Its importance arises from the fact that it is the limit distribution of the maxima of a sequence of independent and identically distributed random variables. Due to this property, the GEV is used as an approximation to model the maxima of long (finite) sequences of random variables [7]. Interestingly, in recent approaches the GEV has been the most refined statistical model able to explain the chaotic advection of passive tracer in realistic time-dependent turbulent stratospheric flow [8]. Hence, the aim of this paper is to investigate GEV conditions which allow statistical model to discuss the hypothesis of turbulent-like behavior in the large scale structure formation using N-body simulations.

2. Data and methodology

Four versions of the cold dark matter model are available from the intermediate scale simulation of the Virgo Consortium project,[1] reported in [9] from which we chose the ΛCDM one, with cosmological parameters $(\Omega_M, \Omega_\Lambda, h, \sigma_8) = (0.3, 0.7, 0.7, 0.9)$.[2] The ΛCDM is the simplest cosmological model that is in general agreement with observed data.

Among the additional (numerical) parameters that characterize the simulation are a simulation box of side $239.5h^{-1}$ Mpc and the number of particles, 256^3, with individual masses of $6.86 \times 10^{10} h^{-1} M_\odot$. Comoving spatial coordinates and periodic boundary conditions were also used [1,9,10]. A detailed description about the basic procedures of the simulation, our data mining process, and the methodology for the identification and characterization of dark matter structures (galaxy and cluster halos) is given in [1,11].

The gravitational energy from our data, in each z-snapshot, is calculated as a function of the halo distance so that the cumulative values are used for computing probability distribution of mean values for different redshifts. Ten snapshots are available, at redshifts: 10.0, 5.0, 3.0, 2.0, 1.5, 1.0, 0.5, 0.3, 0.1 and 0.0.

Recently, it was found that the energy spectra for Virgo simulated galaxy-sized scale follows closely the turbulent $k^{-5/3}$ scaling in the wave number range $0.02 \leqslant k \leqslant 0.07$ (from 15 to $50h^{-1}$ Mpc) [1]. Therefore, here we have analysed the rescaled energy probability distributions exclusively for the galaxy-sized halos scales.

A probability distribution algorithm [12] calculates probability distribution by Fourier transform inversion of the characteristic function, and a probability distribution output device outputs the calculated probability distribution. Here, the GEV is used as an approximation to model the maxima of long (finite) sequences of mean energies. The GEV distributions typically arise in the analysis of extreme events. Extreme events are those phenomena described by the tails of the probability distribution [7,13,14]. One of the main analytical results of Extreme Value theory states that the statistics of exceedances over a sufficiently high threshold of an independent, identically distributed random variable converge to a GEV distribution [7]. The GEV distribution has a cumulative distribution function given by $F(x; \mu, \xi, \sigma) = \exp\{-[1 - (1 + \xi(x - \mu)/\sigma)]^{-1/\xi}\}$, for $\xi \neq 0$. For $\xi = 0$, a Gumbel (light-tailed) distribution is recovered with $F(x; \mu, \xi, \sigma) = \exp\{-\exp[-(x - \mu)/\sigma]\}$.

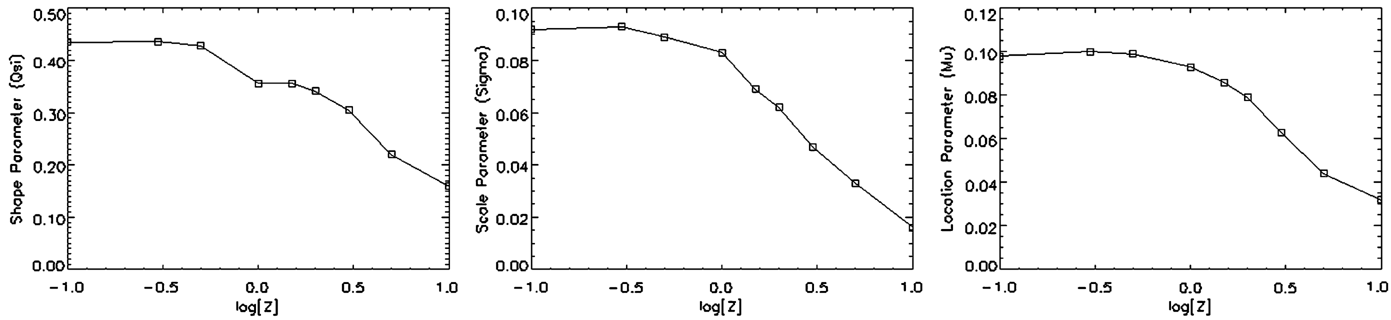
3. Results and interpretation

Our results indicate that rescaled energy data points for different redshifts collapse into similar patterns, well approximated by a Generalized Extreme Value (GEV) distribution, as shown in Fig. 2 where the statistical fitting is shown for three representative redshifts $z = 0$, 1.5 and 5.0. The variation with z of the corresponding shape (ξ), scale (σ) and location (μ) parameters are given in Figs. 3a, 3b and 3c, respectively.

Positive shape parameters, like the ones estimated for our data ($0.15 < \xi < 0.45$), correspond to a Fréchet (heavy-tailed) distribution, indicating the presence of rare, intense events, which in the present context corresponds to the presence of Galaxy-sized halos or even larges lumps of cold dark matter. Light-tailed distributions ($\xi = 0$) decrease too steeply to model properly the events in the low-probability extremes of the empirical histograms. In practice, this result is particularly relevant because natural systems which exhibit extreme event dynamics are not well described by central values and typical statistical fluctuations [15]. In this class of systems, the largest values dominate their long term trends as much as the largest terms dominate a sum of random variables with a power-law probability distribution function (provided its tails decay slow enough). In order to test the utility of GEV statistical modeling, in the cosmological context, its shape, scale and location parameters variation were plotted as a function of the redshift (0.1, 0.3, 0.5, 1.0, 1.5, 2.0, 3.0, 5.0, 10.0) having error bars less than 0.02. Thus, the curves shown in Fig. 3 constitute a set of statistical quantities for measuring chaotic advection structural properties that can be validate from future high resolution observational data.

[1] Available from the web-page http://www.mpa-garching.mpg.de/Virgo/data_download.html.

[2] $\Omega_M = \bar{\rho}/\rho_c = 8\pi G\bar{\rho}/(3H^2)$; $\Omega_\Lambda = \Lambda c^2/(3H^2)$; $h = H_0/100$ km s^{-1} Mpc^{-1}; and σ_8 is the r.m.s. mass fluctuation within a top-hat radius of $8h^{-1}$ Mpc.



(a) (b) (c)

Fig. 3. GEV parameters as a function of the redshift (Log[z]): (a) Shape [ξ], (b) Scale [σ] and (c) Location [μ].

4. Concluding remarks

Our analysis covers additional aspects of the turbulent-like hypothesis for cosmological structure formation, particularly covering the case of advection by random unidirectional shear, and action of small-scale velocity fields on large-scale local fluctuations in the range of 15–50h^{-1} Mpc (the galaxy-sized scale range).

While the existence of such structures is clear from visual observations, their mathematical description is far more difficult. Several measurements have been proposed and used to describe coherent structures in Lagrangian turbulence. Usually, there is no robust characterization. For example, coherent structures are sometimes interpreted as regions of high vorticity or low shearing dynamics. In the gravitational frame it is interesting to note equivalent ambiguous phenomena from the simulations of multi-galaxies collision (see for example [16]).

In the present case, the PDF tails we found imply that the inhomogeneous large fluctuations are important because these rare events (formation of dense galaxy-sized halos and multi-galaxy clumps) have much higher probabilities than random events governed by Gaussian tails.

From the statistical GEV analysis we can gain some insight as to how the concentration fluctuations depend on gravitational instability in different scales. In our interpretation the gravity field itself behaves as an advecting flow where the galaxies are tracers describing an extreme event dynamics usually found in Lagrangian turbulence.

The interpretation reported here may be extended in various ways, some of which are relevant to cosmological considerations. For example, the presence of typical turbulent asymmetries can be studied by using the gradient pattern analysis methodology [17–19]. In a recent work, using the same data analysed here, such analysis has showed a particular turbulent-like pattern in redshift $z = 4$ [20]. From a theoretical point of view, it would be interesting to formulate a cosmological model having a complementary Navier–Stokes approach in the classical analytical framework composed by both Einstein and Friedman equations. In a such investigation appropriate spatio-temporal chaotic equations [21,22] and multifractal modeling [23,24] can also be addressed constructing special equivalent parametrization for the nonlinear gravitational field.

In summary, we have obtained complementary results that reveal turbulent-like structures that would normally remain hidden in instantaneous density plots simulated from the ΛCDM model. In addition to the characterization we have shown in this paper, further investigation can apply the GEV statistics to distinguish different versions of the cold dark matter model including possible more realistic ΛCDM extensions (for example, to allow quintessence rather than a cosmological constant).

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