\mathbf{C}	ontents	<pre>#define pob pop_back()</pre>		
1	Template 1 1.1 Default Code 1 1.2 .vimrc 1	#define F first #define S second		
2	Data Structure22.1 Binary Indexed Tree22.2 Disjoint Set Union-Find22.3 Segment Tree2	#define iceil(a, b) (((a) + (b) - 1) / (b)) #define tomax(a, b) ((a) = max((a), (b))) #define tomin(a, b) ((a) = min((a), (b)))		
3	Graph 3 3.1 Dijkstra 3 3.2 Floyd-Warshall 3 3.3 Kruskal 3 3.4 Tarjan SCC 3 3.5 SPFA 4	<pre>#define uni(a) a.resize(unique(iter(a)) - a. begin()) #define printv(a, b) {bool pvaspace=false; \ for(auto pva : a){ \ if(pvaspace) b << " "; pvaspace=true;\</pre>		
4	String 4 4.1 KMP 4 4.2 Z Value 4 4.3 Longest Palindromic Substring 5 4.4 Suffix Array 5	b << "\n";} //#define TEST		
5	Geometry 5 5.1 Vector Operations 5 5.2 Convex Hull 6	A		
	Number Theory 6 6.1 Prime Sieve 6 DP Trick 6 7.1 Dynamic Convex Hull 6	<pre>typedef long double ld; using pii = pair<int, int="">; using pll = pair<ll, ll="">;</ll,></int,></pre>		
8	Numbers and Math 7 8.1 Fibonacci 7 8.2 Catalan 7 8.3 Geometry 7 8.4 Prime Numbers 7 8.5 Number Theory 7	<pre>using tiii = tuple<int, int="" int,="">; const ll MOD = 1000000007; const ll MAX = 2147483647; template<typename a,="" b="" typename=""></typename></int,></pre>		
1	Template	return o << '(' << p.F << ',' << p.S << ') ';		
1.	1 Default Code #define NDEBUG	<pre>int main(){ StarBurstStream</pre>		
ļ	<pre>nclude <bits stdc++.h=""> nclude <bits extc++.h=""></bits></bits></pre>	<pre>return 0; }</pre>		
#d #d #d #d	<pre>efine StarBurstStream ios_base:: sync_with_stdio(false); cin.tie(0); cout. tie(0); efine iter(a) a.begin(), a.end() efine riter(a) a.rbegin(), a.rend() efine lsort(a) sort(iter(a)) efine gsort(a) sort(riter(a)) efine pb(a) push_back(a) efine eb(a) emplace_back(a) efine pf(a) push_front(a)</pre>	1.2 .vimrc :set nu :set ai :set cursorline :set tabstop=4 :set shiftwidth=4 :set mouse=a :set expandtab		
1	efine ef(a) emplace_front(a)	hi CursorLine cterm=none ctermbg=DarkMagenta		

2 Data Structure

2.1 Binary Indexed Tree

```
template < typename T>
struct BIT{
private:
  vector<T> bit;
  int lowbit(int x){
    return x & (-x);
public:
  explicit BIT(int sz){
    bit.resize(sz + 1);
  void modify(int x, T v){
    for(; x < bit.size(); x += lowbit(x))</pre>
   bit[x] += v;
  }
  T get(int x){
    T ans = T();
    for(; x; x -= lowbit(x)) ans += bit[x];
    return ans;
  }
};
```

2.2 Disjoint Set Union-Find

```
vector<int> dsu, rk;
void initDSU(int n){
  dsu.resize(n);
 rk.resize(n);
  for(int i = 0; i < n; i++) dsu[i] = i, rk[
   i] = 1;
}
int findDSU(int x){
  if(dsu[x] == x) return x;
  dsu[x] = findDSU(dsu[x]);
  return dsu[x];
void unionDSU(int a, int b){
  int pa = findDSU(a), pb = findDSU(b);
  if(rk[pa] > rk[pb]) swap(pa, pb);
  if(rk[pa] == rk[pb]) rk[pb]++;
  dsu[pa] = pb;
}
```

2.3 Segment Tree

```
template < typename T >
struct Node{
  T v = 0, tag = 0;
  int sz = 1, l = -1, r = -1;
  T rv(){
```

```
return v + tag * sz;
  void addTag(T t){
    tag += t;
};
template < typename T >
T pullValue(T b, T c){
  return b + c;
template < typename T>
void pull(Node<T> &a, Node<T> &l, Node<T> &r
  a.v = pullValue(1.rv(), r.rv());
  a.sz = 1.sz + r.sz;
template < typename T>
void push(Node<T> &a, Node<T> &1, Node<T> &r
   ) {
  1.addTag(a.tag);
  r.addTag(a.tag);
  a.v = a.rv();
  a.tag = 0;
template < typename T>
struct SegmentTree{
  vector < Node < T >> st;
  int cnt = 0;
  explicit SegmentTree(int sz){
    st.resize(4 * sz);
  int build(int 1, int r, vector<T>& o){
    int id = cnt++;
    if(1 == r){
      st[id].v = o[1];
      return id;
    }
    int m = (1 + r) / 2;
    st[id].1 = build(1, m, o);
    st[id].r = build(m + 1, r, o);
    pull(st[id], st[st[id].1], st[st[id].r])
    return id;
  void modify(int 1, int r, int v, int L,
   int R, int id){
    if(1 == L \&\& r == R){
      st[id].addTag(v);
      return;
    }
    int M = (L + R) / 2;
    if(r \le M) modify(l, r, v, L, M, st[id].
```

```
else if (1 > M) modify (1, r, v, M + 1, R,
    st[id].r);
    else{
      modify(1, M, v, L, M, st[id].1);
      modify(M + 1, r, v, M + 1, R, st[id].r
   );
    }
    pull(st[id], st[st[id].1], st[st[id].r])
  }
  T query(int 1, int r, int L, int R, int id
    if(l == L && r == R) return st[id].rv();
    push(st[id], st[st[id].1], st[st[id].r])
    int M = (L + R) / 2;
    if(r <= M) return query(l, r, L, M, st[</pre>
   id].1);
    else if(1 > M) return query(1, r, M + 1,
    R, st[id].r);
    else{
      return pullValue(query(1, M, L, M, st[
   id].1), query(M + 1, r, M + 1, R, st[id].
   r));
    }
  }
};
```

3 Graph

3.1 Dijkstra

```
//The first element in pair should be edge
   weight, and the second should be vertex
vector<vector<pii>> g;
int n;
int dijkstra(int start, int end){
  priority_queue<pii, vector<pii>, greater<</pre>
   pii>> q;
  for(pii p : g[start]){
    q.push(p);
  q.push(mp(0, start));
  vector<int> dis(n, -1);
  dis[start] = 0;
  vector<int> visit(n);
  while(q.size()){
    int v = q.top().S;
    int d = q.top().F;
    if(v == end) break;
    q.pop();
    if(visit[v]) continue;
    visit[v] = true;
    for(pii p : g[v]){
      if(visit[p.S]) continue;
      if(dis[p.S] == -1 \mid \mid d + p.F < dis[p.S]
   ]){
        dis[p.S] = d + p.F;
```

```
q.push(mp(dis[p.S], p.S));
     }
}
return dis[end];
}
```

3.2 Floyd-Warshall

```
vector<vector<int>> g;
int n;

void floydwarshall(){
  for(int k = 0; k < n; k++)
    for(int i = 0; i < n; i++)
      for(int j = 0; j < n; j++)
        if(g[i][k] != -MAX && g[k][j] != -
      MAX && (g[i][j] == -MAX || g[i][k] + g[k]
      ][j] < g[i][j]))
        g[i][j] = g[i][k] + g[k][j];
}</pre>
```

3.3 Kruskal

```
int kruskal(){
   int ans = 0;
   lsort(e);
   initDSU();
   for(auto& i : e){
      int a = i.S.F, b = i.S.S;
      if(findDSU(a) == findDSU(b)) continue;
      ans += i.F;
      unionDSU(a, b);
   }
   return ans;
}
```

3.4 Tarjan SCC

```
vector<vector<int>> g;
vector<int> st;
vector<bool> inst;
vector<int> scc;
vector < int > ts, low;
int tmp = 0;
int sccid = 0;
void initSCC(int n){
  tmp = 0;
  sccid = 0;
  st.clear();
  g.clear();
  g.resize(2 * n + 1);
  inst.clear();
  inst.resize(2 * n + 1);
  scc.clear();
  scc.resize(2 * n + 1);
  ts.clear();
  ts.resize(2 * n + 1, -1);
  low.clear();
  low.resize(2 * n + 1);
}
```

4

```
dis[p.F] = dis[v] + p.S;
void dfs(int now){
                                                        if(!inq[p.F]){
                                                          inq[p.F] = true;
  st.eb(now);
                                                          q.push(p.F);
                                                       }
  inst[now] = true;
  ts[now] = ++tmp;
                                                     }
  low[now] = ts[now];
                                                   }
  for(int i : g[now]){
                                                   return dis[end];
    if(ts[i] == -1){
      dfs(i);
                                                     String
      low[now] = min(low[now], low[i]);
                                                 4.1 KMP
    else if(inst[i]) low[now] = min(low[now
   ], ts[i]);
                                                 vector<int> f;
                                                 void build(string& t){
  if(low[now] == ts[now]){
                                                   f.clear();
    sccid++;
                                                   f.resize(t.size());
    int t;
                                                   int p = -1;
    do{
                                                   f[0] = -1;
                                                   for(int i = 1; i < t.size(); i++){</pre>
      t = st.back();
      st.pob;
                                                     while (p != -1 \&\& t[p + 1] != t[i]) p = f
      inst[t] = false;
                                                     [p];
      scc[t] = sccid;
                                                     if(t[p + 1] == t[i]) f[i] = p + 1;
    }
                                                     else f[i] = -1;
    while(t != now);
                                                     p = f[i];
                                                   }
                                                 }
}
                                                 int kmp(string& s, string& t){
3.5
     SPFA
                                                   int ans = 0;
                                                   int p = -1;
const 11 INFINITE = 2147483647;
                                                   for(int i = 0; i < s.size(); i++){</pre>
                                                     while (p != -1 \&\& t[p + 1] != s[i]) p = f
int n;
vector<vector<pii>> g;
                                                     if(t[p + 1] == s[i]) p++;
                                                     if(p + 1 == t.size()){
int spfa(int start, int end){
                                                       ans++;
                                                       p = f[p];
  vector<int> dis(n, INFINITE);
  int start;
                                                   }
  cin >> start;
                                                   return ans;
  dis[start] = 0;
                                                 }
                                                 4.2 Z Value
  queue < int > q;
  q.push(start);
  vector < bool > inq(n);
                                                 vector<int> z;
  inq[start] = true;
  vector<int> cnt(n);
                                                 void build(string s, int n){
                                                   z.clear();
                                                   z.resize(n);
  while(!q.empty()){
    int v = q.front();
                                                   int 1 = 0;
                                                   for(int i = 1; i < n; i++){
    q.pop();
    inq[v] = false;
                                                     if(1 + z[1] >= i) z[i] = min(z[1] + 1 -
    for(pii p : g[v]){
                                                     i, z[i - 1]);
      if(!(dis[p.F] == INFINITE || dis[v] +
                                                     while(i + z[i] < n && s[z[i]] == s[i + z
   p.S < dis[p.F])) continue;</pre>
                                                     [i]]) z[i]++;
```

}

cnt[p.F]++;

negetive cycle

if(cnt[p.F] >= n) return -INFINITE; //

if(i + z[i] > 1 + z[1]) 1 = i;

4.3 Longest Palindromic Substring

```
#define T(x) ((x) % 2 ? s[(x) / 2] : '.')
string s;
int L;
int ex(int 1, int r){
 int i = 0;
 while(1 - i >= 0 && r + i < L && T(1 - i)
   == T(r + i)) i++;
 return i;
}
int lps(string ss){
 s = ss;
 L = 2 * s.size() + 1;
 int mx = 0;
 int center = 0;
 vector<int> r(L);
 int ans = 1;
 r[0] = 1;
 for(int i = 1; i < L; i++){
    int ii = center - (i - center);
    int len = mx - i + 1;
    if(i > mx){
     r[i] = ex(i, i);
     center = i;
     mx = i + r[i] - 1;
    else if(r[ii] == len){
     r[i] = len + ex(i - len, i + len);
     center = i;
     mx = i + r[i] - 1;
   else r[i] = min(r[ii], len);
   ans = max(ans, r[i]);
 return ans - 1;
```

4.4 Suffix Array

```
#include <bits/stdc++.h>
#define eb(a) emplace_back(a)
using namespace std;

vector<int> sa(string s){
   s += '$';
   int n = s.size();
   int t = __lg(n) + 1;

   vector<vector<int>> rk(t + 1, vector<int>(
        n)), b;

   vector<vector<int>> c1(27);
```

```
for(int i = 0; i < n; i++) c1[s[i] == '$'
 ? 0 : s[i] - 'a' + 1].eb(i);;
for(int i = 0; i < 27; i++){
  if(!c1[i].empty()) b.eb(c1[i]);
b.resize(n);
for(int i = 0; i < n; i++){
  for(int k : b[i]) rk[0][k] = i;
for(int i = 1; i <= t; i++){
  vector<vector<int>> tb(n);
  for(int j = 0; j < n; j++){
    for(int k : b[j]){
      int tmp = ((k - (1 << (i - 1))) \% n
 + n) % n;
      int now = rk[i - 1][tmp];
      tb[now].eb(tmp);
    }
  }
 b = tb;
  int cnt = -1;
 for(int j = 0; j < n; j++){
    int lst = -1;
    for(int k : b[j]){
      int now = rk[i - 1][(k + (1 << (i -
 1))) % n];
      if(now != lst) cnt++;
      rk[i][k] = cnt;
      lst = now;
    }
}
return rk[t];
```

5 Geometry

5.1 Vector Operations

```
template < typename T>
T dot(pair < T, T > a, pair < T, T > b) {
  return a.F * b.F + a.S * b.S;
}

template < typename T >
T cross(pair < T, T > a, pair < T, T > b) {
  return a.F * b.S - a.S * b.F;
}

template < typename T >
T abs2(pair < T, T > a) {
  return a.F * a.F + a.S * a.S;
}
```

5.2 Convex Hull

```
template < typename T>
pair<T, T> operator-(pair<T, T> a, pair<T, T
   > b){
  return mp(a.F - b.F, a.S - b.S);
}
template < typename T>
T cross(pair<T, T> a, pair<T, T> b){
  return a.F * b.S - a.S * b.F;
}
template < typename T>
vector<pair<T, T>> getConvexHull(vector<pair</pre>
   <T, T>>& pnts){
  int n = pnts.size();
  lsort(pnts);
  vector<pair<T, T>> hull;
  hull.reserve(n);
  for(int i = 0; i < 2; i++){
    int t = hull.size();
    for(pair<T, T> pnt : pnts){
      while(hull.size() - t >= 2 \&\& cross(
   hull.back() - hull[hull.size() - 2], pnt
   - hull[hull.size() - 2]) <= 0){
        hull.pop_back();
      hull.pb(pnt);
    hull.pop_back();
    reverse(iter(pnts));
  return hull;
}
```

6 Number Theory

6.1 Prime Sieve

```
vector<int> prime;
vector<int> p;
void sieve(int n){
  prime.resize(n + 1, 1);
  for(int i = 2; i <= n; i++){
    if(prime[i] == 1){
      p.push_back(i);
      prime[i] = i;
    }
  for(int j : p){
      if((ll)i * j > n || j > prime[i])
    break;
      prime[i * j] = j;
    }
}
```

7 DP Trick

7.1 Dynamic Convex Hull

```
const ll INF = 1LL << 60;</pre>
template < typename T>
struct Line{
  mutable T a, b, r = 0;
  Line(T a, T b) : a(a), b(b){}
  bool operator < (Line < T > 1) const{
    return a < 1.a;
  }
  bool operator<(T v)const{</pre>
    return r < v;
  }
};
template < typename T>
T divfloor(T a, T b){
  return a / b - ((a ^ b) < 0 && a % b);
template < typename T>
struct DynamicHull{
  multiset<Line<T>, less<>> s;
  int size(){
    return s.size();
  bool intersect(typename set<Line<T>>::
   iterator a, typename set<Line<T>>::
   iterator &b){
    if(b == s.end()){}
      a \rightarrow r = INF;
      return false;
    if(a->a == b->a){}
      if(a->b > b->b) a->r = INF;
      else a \rightarrow r = -INF;
```

```
}
    else{
      a \rightarrow r = divfloor(b \rightarrow b - a \rightarrow b, a \rightarrow a - b)
    ->a);
    return a->r >= b->r;
  }
  void insert(T a, T b){
    Line T > l(a, b);
    auto it = s.insert(1), after = next(it),
    before = it;
    while(intersect(it, after)) after = s.
    erase(after);
    if(before != s.begin() && intersect(--
    before, it)){
       it = s.erase(it);
       intersect(before, it);
    while((it = before) != s.begin() && (--
   before)->r >= it->r) intersect(before, it
     = s.erase(it));
  }
  T query(T v){
    Line<T> 1 = *s.lower_bound(v);
    return l.a * v + l.b;
};
```

8 Numbers and Math

8.1 Fibonacci

$$f(n) = f(n-1) + f(n-2)$$

$$\begin{bmatrix} f(n) \\ f(n-1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

1	1	1	2	3	5			
6	8	13	21	34	55			
11	89	144	233	377	610			
16	987	1597	2584	4181	6765			
21	10946	17711	28657	46368	75025			
26	121393	196418	317811	514229	832040			
31	1346269	2178309	3524578	5702887	9227465			
$f(45) \approx 10^9$								
$f(88) \approx 10^{18}$								

8.2 Catalan

$$C_0 = 1, C_n = \sum_{i=0}^{n-1} C_i C_{n-1-i}$$

$$C_n = C_n^{2n} - C_{n-1}^{2n}$$

1	1	2	5	14
42	132	429	1430	4862
16796	58786	208012	742900	2674440
9694845	35357670	129644790	477638700	1767263190
	16796	16796 58786	16796 58786 208012	16796 58786 208012 742900

8.3 Geometry

- Heron's formula: The area of a triangle whose lengths of sides is a,b,c and s = (a+b+c)/2 is $\sqrt{s(s-a)(s-b)(s-c)}$.
- Vector cross product: $v_1 \times v_2 = |v_1||v_2|\sin\theta = (x_1 \times y_2) - (x_2 \times y_1).$
- Vector dot product: $v_1 \cdot v_2 = |v_1||v_2|\cos\theta = (x_1 \times y_1) + (x_2 \times y_2).$

8.4 Prime Numbers

First 50 prime numbers:

```
29
                                            17
                                                  19
                                                         23
 1
      2
            3
                               11
                                      13
 11
      31
            37
                   41
                         43
                               47
                                      53
                                            59
                                                  61
                                                         67
                                                               71
 21
      73
            79
                               97
                                      101
                                                  107
                   83
                         89
                                            103
                                                         109
                                                               113
 31
      127
            131
                   137
                         139
                               149
                                      151
                                            157
                                                   163
                                                         167
                                                               173
 41
      179
            181
                   191
                         193
                               197
                                      199
                                            211
                                                  223
                                                         227
                                                               229
Very large prime numbers:
```

 $1000001333 \quad 1000500889 \quad 2000000659 \quad 900004151 \quad 850001359$

8.5 Number Theory

- Inversion: $aa^{-1} \equiv 1 \pmod{m}$. a^{-1} exists iff gcd(a, m) = 1.
- Linear inversion: $a^{-1} \equiv (m \lfloor \frac{m}{a} \rfloor) \times (m \mod a)^{-1} \pmod{m}$
- Fermat's little theorem: $a^p \equiv a \pmod{p}$ if p is prime.
- Euler function: $\phi(n) = n \prod_{p|n} \frac{p-1}{p}$
- Euler theorem: $a^{\phi(n)} \equiv 1 \pmod{n}$ if gcd(a, n) = 1.
- Extended Euclidean algorithm: $ax+by=\gcd(a,b)=\gcd(b,a\bmod b)=\gcd(b,a-\lfloor\frac{a}{b}\rfloor b)=bx_1+(a-\lfloor\frac{a}{b}\rfloor b)y_1=ay_1+b(x_1-\lfloor\frac{a}{b}\rfloor y_1)$
- Divisor function: $\sigma_x(n) = \sum_{d|n} d^x. \ n = \prod_{i=1}^r p_i^{a_i}.$ $\sigma_x(n) = \prod_{i=1}^r \frac{p_i^{(a_i+1)x} 1}{p_i^x 1} \text{ if } x \neq 0. \ \sigma_0(n) = \prod_{i=1}^r (a_i + 1).$
- Chinese remainder theorem: $x \equiv a_i \pmod{m_i}$. $M = \prod m_i$. $M_i = M/m_i$. $t_i = M_i^{-1}$. $x = kM + \sum a_i t_i M_i$, $k \in \mathbb{Z}$.