Contents

1 Basic

1.1 Default Code

```
#include <bits/stdc++.h>
#include <bits/extc++.h>

#define StarBurstStream ios_base::
    sync_with_stdio(false); cin.tie(0);
    cout.tie(0);
#define iter(a) a.begin(), a.end()
#define riter(a) a.rbegin(), a.rend()
#define lsort(a) sort(iter(a))
#define gsort(a) sort(riter(a))
#define pb(a) push_back(a)
#define eb(a) emplace_back(a)
#define pf(a) push_front(a)
#define ef(a) emplace_front(a)
#define pob pop_back()
#define pof pop_front()
#define mp(a, b) make_pair(a, b)
#define F first
#define S second
#define mt make_tuple
#define gt(t, i) get<i>(t)
#define iceil(a, b) (((a) + (b) - 1) / (
    b))
#define tomax(a, b) ((a) = max((a), (b))
    )
#define tomin(a, b) ((a) = min((a), (b))
    )
#define topos(a) ((a) = (((a) % MOD +
    MOD) % MOD))
#define uni(a) a.resize(unique(iter(a))
    - a.begin())
#define printv(a, b) {bool pvaspace=
    false; \
for(auto pva : a){ \
  if(pvaspace) b << " "; pvaspace=true;\
  b << pva;\
}\
b << "\n";}

using namespace std;
using namespace __gnu_pbds;

typedef long long ll;
typedef unsigned long long ull;
typedef long double ld;

using pii = pair<int, int>;
using pll = pair<ll, ll>;
using pdd = pair<ld, ld>;
using tiii = tuple<int, int, int>;

const ll MOD = 1000000007;
const ll MAX = 2147483647;

template<typename A, typename B>
ostream& operator<<(ostream& o, pair<A,
    B> p){
  return o << '(' << p.F << ',' << p.S
    << ')';
}

int main(){
  StarBurstStream
```

```
  return 0;
}
```

1.2 .vimrc

```
:set nu
:set ai
:set cursorline
:set tabstop=4
:set shiftwidth=4
:set mouse=a
:set expandtab
hi CursorLine cterm=none ctermbg=
    DarkMagenta
```

1.3 PBDS

```
tree<int, int, less<>, rb_tree_tag,
    tree_order_statistics_node_update> tr
    ;
tr.order_of_key(123);
tr.find_by_order(123);
```

1.4 Random

```
mt19937 rnd(chrono::steady_clock::now().
    time_since_epoch().count());
uniform_int_distribution<int> dis(1,
    100);
cout << dis(rnd) << "\n";
```

1.5 Clock

```
int st = clock();

int ed = clock();
if(ed - st >= CLOCKS_PER_SEC * 1);
```

2 Data Structure

2.1 Binary Indexed Tree

```
template<typename T>
struct BIT{

private:
  vector<T> bit;
  int lowbit(int x){
    return x & (-x);
  }

public:
  explicit BIT(int sz){
    bit.resize(sz + 1);
  }

  void modify(int x, T v){
    for(; x < bit.size(); x += lowbit(x)
    ) bit[x] += v;
  }

  T get(int x){
    T ans = T();
    for(; x; x -= lowbit(x)) ans += bit[
    x];
    return ans;
  }
};
```

2.2 Disjoint Set Union-Find

```
vector<int> dsu, rk;

void initDSU(int n){
  dsu.resize(n);
  rk.resize(n);
  for(int i = 0; i < n; i++) dsu[i] = i,
    rk[i] = 1;
}

int findDSU(int x){
  if(dsu[x] == x) return x;
  dsu[x] = findDSU(dsu[x]);
  return dsu[x];
}

void unionDSU(int a, int b){
  int pa = findDSU(a), pb = findDSU(b);
  if(rk[pa] > rk[pb]) swap(pa, pb);
  if(rk[pa] == rk[pb]) rk[pb]++;
  dsu[pa] = pb;
}
```

2.3 Segment Tree

```
template<typename T>
struct Node{
  T v = 0, tag = 0;
  int sz = 1, l = -1, r = -1;
  T rv(){
    return v + tag * sz;
  }
  void addTag(T t){
    tag += t;
  }
};

template<typename T>
T pullValue(T b, T c){
  return b + c;
}

template<typename T>
void pull(Node<T> &a, Node<T> &l, Node<T
    > &r){
```

```
  a.v = pullValue(l.rv(), r.rv());
  a.sz = l.sz + r.sz;
}

template<typename T>
void push(Node<T> &a, Node<T> &l, Node<T
    > &r){
  l.addTag(a.tag);
  r.addTag(a.tag);
  a.v = a.rv();
  a.tag = 0;
}

template<typename T>
struct SegmentTree{
  vector<Node<T>> st;
  int cnt = 0;

  explicit SegmentTree(int sz){
    st.resize(4 * sz);
  }

  int build(int l, int r, vector<T>& o){
    int id = cnt++;
    if(l == r){
      st[id].v = o[l];
      return id;
    }
    int m = (l + r) / 2;
    st[id].l = build(l, m, o);
    st[id].r = build(m + 1, r, o);
    pull(st[id], st[st[id].l], st[st[id
   ].r]);
    return id;
  }

  void modify(int l, int r, int v, int L
   , int R, int id){
    if(l == L && r == R){
      st[id].addTag(v);
      return;
    }
    int M = (L + R) / 2;
    if(r <= M) modify(l, r, v, L, M, st[
   id].l);
    else if(l > M) modify(l, r, v, M +
   1, R, st[id].r);
    else{
      modify(l, M, v, L, M, st[id].l);
      modify(M + 1, r, v, M + 1, R, st[
   id].r);
    }
    pull(st[id], st[st[id].l], st[st[id
   ].r]);
  }

  T query(int l, int r, int L, int R,
    int id){
    if(l == L && r == R) return st[id].
   rv();
    push(st[id], st[st[id].l], st[st[id
   ].r]);
    int M = (L + R) / 2;
    if(r <= M) return query(l, r, L, M,
   st[id].l);
    else if(l > M) return query(l, r, M
   + 1, R, st[id].r);
    else{
      return pullValue(query(l, M, L, M,
    st[id].l), query(M + 1, r, M + 1, R,
    st[id].r));
    }
  }

};
```

2.4 Treap

```
mt19937 rnd(chrono::steady_clock::now().
   time_since_epoch().count());

template<typename T>
struct Node{
  int l = -1, r = -1, pri = rnd(), sz =
    1;
  T v, sum, tag;
  T rsum(){
    return sum + tag * sz;
  }
};

template<typename T>
struct Treap{
  vector<Node<T>> tr;
  int ts = 0;
  explicit Treap(int sz){
    tr.resize(sz);
  }

  int node(int v){
    int r = ts++;
    tr[r].v = v;
    tr[r].sum = 0;
    tr[r].tag = 0;
    return r;
  }

  void pull(int r){
    if(r != -1){
      tr[r].sz = 1;
      tr[r].sum = tr[r].v;
      if(tr[r].l != -1){
        tr[r].sum += tr[tr[r].l].rsum();
```

```
      tr[r].sz += tr[tr[r].l].sz;
    }
    if(tr[r].r != -1){
      tr[r].sum += tr[tr[r].r].rsum();
      tr[r].sz += tr[tr[r].r].sz;
    }
  }
}

void push(int r){
  if(r == -1) return;
  if(tr[r].l != -1){
    tr[tr[r].l].tag += tr[r].tag;
  }
  if(tr[r].r != -1){
    tr[tr[r].r].tag += tr[r].tag;
  }
  tr[r].sum = tr[r].rsum();
  tr[r].v += tr[r].tag;
  tr[r].tag = 0;
}

void merge(int a, int b, int& r){
  push(a);
  push(b);
  if(a == -1 && b == -1) r = -1;
  else if(a == -1) r = b;
  else if(b == -1) r = a;
  else{
    if(tr[a].pri > tr[b].pri){
      r = a;
      merge(tr[a].r, b, tr[a].r);
    }
    else{
      r = b;
      merge(a, tr[b].l, tr[b].l);
    }
  }
  pull(r);
}

void split1(int a, int k, int& r1, int
 & r2){
  if(a == -1){
    r1 = r2 = -1;
    return;
  }
  push(a);
  if(tr[a].v < k){
    r1 = a;
    split1(tr[a].r, k, tr[a].r, r2);
  }
  else{
    r2 = a;
    split1(tr[a].l, k, r1, tr[a].l);
  }
  pull(a);
}

void split2(int a, int k, int& r1, int
 & r2){
  if(a == -1){
    r1 = r2 = -1;
    return;
  }
  push(a);
  if(k == 0){
    r1 = -1;
    r2 = a;
    return;
  }
  if(tr[a].l == -1 || tr[tr[a].l].sz <
  k){
    r1 = a;
    if(tr[a].l != -1) split2(tr[a].r,
 k - tr[tr[a].l].sz - 1, tr[a].r, r2);
    else split2(tr[a].r, k - 1, tr[a].
 r, r2);
  }
  else{
    r2 = a;
    split2(tr[a].l, k, r1, tr[a].l);
  }
  pull(a);
}

void printtr(int now){
  if(now == -1) return;
  printtr(tr[now].l);
  cerr << now << "," << tr[now].v + tr
 [now].tag << "," << tr[now].rsum() <<
  "," << tr[now].tag << " ";
  printtr(tr[now].r);
}

void print(int r){
  printtr(r);
  cerr << "\n";
}

};
```

3 Graph

3.1 Dijkstra

```
//The first element in pair should be
   edge weight, and the second should be
    vertex
vector<vector<pii>> g;
int n;

int dijkstra(int start, int end){
```

```
  priority_queue<pii, vector<pii>,
   greater<pii>> q;
  for(pii p : g[start]){
    q.push(p);
  }
  q.push(mp(0, start));
  vector<int> dis(n, -1);
  dis[start] = 0;
  vector<int> visit(n);
  while(q.size()){
    int v = q.top().S;
    int d = q.top().F;
    if(v == end) break;
    q.pop();
    if(visit[v]) continue;
    visit[v] = true;
    for(pii p : g[v]){
      if(visit[p.S]) continue;
      if(dis[p.S] == -1 || d + p.F < dis
   [p.S]){
        dis[p.S] = d + p.F;
        q.push(mp(dis[p.S], p.S));
      }
    }
  }
  return dis[end];
}
```

3.2 Floyd-Warshall

```
vector<vector<int>> g;
int n;

void floydwarshall(){
  for(int k = 0; k < n; k++)
    for(int i = 0; i < n; i++)
      for(int j = 0; j < n; j++)
        if(g[i][k] != -MAX && g[k][j] !=
    -MAX && (g[i][j] == -MAX || g[i][k]
   + g[k][j] < g[i][j]))
          g[i][j] = g[i][k] + g[k][j];
}
```

3.3 Kruskal

```
int kruskal(){
  int ans = 0;
  lsort(e);
  initDSU();
  for(auto& i : e){
    int a = i.S.F, b = i.S.S;
    if(findDSU(a) == findDSU(b))
   continue;
    ans += i.F;
    unionDSU(a, b);
  }
  return ans;
}
```

3.4 Tarjan SCC

```
vector<vector<int>> g;
vector<int> st;
vector<bool> inst;
vector<int> scc;
vector<int> ts, low;
int tmp = 0;
int sccid = 0;

void initSCC(int n){
  tmp = 0;
  sccid = 0;
  st.clear();
  g.clear();
  g.resize(2 * n + 1);
  inst.clear();
  inst.resize(2 * n + 1);
  scc.clear();
  scc.resize(2 * n + 1);
  ts.clear();
  ts.resize(2 * n + 1, -1);
  low.clear();
  low.resize(2 * n + 1);
}

void dfs(int now){

  st.eb(now);
  inst[now] = true;
  ts[now] = ++tmp;
  low[now] = ts[now];

  for(int i : g[now]){
    if(ts[i] == -1){
      dfs(i);
      low[now] = min(low[now], low[i]);
    }
    else if(inst[i]) low[now] = min(low[
   now], ts[i]);
  }

  if(low[now] == ts[now]){
    sccid++;
    int t;
    do{
      t = st.back();
      st.pob;
      inst[t] = false;
      scc[t] = sccid;
    }
    while(t != now);
  }

}
```

3.5 SPFA

```
const ll INFINITE = 2147483647;

int n;
vector<vector<pii>> g;

int spfa(int start, int end){

  vector<int> dis(n, INFINITE);
  int start;
  cin >> start;
  dis[start] = 0;

  queue<int> q;
  q.push(start);
  vector<bool> inq(n);
  inq[start] = true;
  vector<int> cnt(n);

  while(!q.empty()){
    int v = q.front();
    q.pop();
    inq[v] = false;
    for(pii p : g[v]){
      if(!(dis[p.F] == INFINITE || dis[v
   ] + p.S < dis[p.F])) continue;
      cnt[p.F]++;
      if(cnt[p.F] >= n) return -INFINITE
   ; //negetive cycle
      dis[p.F] = dis[v] + p.S;
      if(!inq[p.F]){
        inq[p.F] = true;
        q.push(p.F);
      }
    }

  }
  return dis[end];
}
```

3.6 Block-cut Tree

```
#include <bits/stdc++.h>

#define eb(a) emplace_back(a)

using namespace std;

// tg is the origin graph, g is the
   result
vector<vector<int>> tg, g;
int bcc; // = n+1, initially
vector<int> low, in;
int tts = 1;
stack<int> st;
vector<vector<int>> c;
vector<bool> iscut;

void dfsbcc(int now, int p){ //
   calculate low
  low[now] = in[now] = tts++;
  for(int i : tg[now]){
    if(i == p) continue;
    if(in[i]) low[now] = min(low[now],
   in[i]);
    else{
      dfsbcc(i, now);
      low[now] = min(low[now], low[i]);
      c[now].eb(i);
    }
    if(low[i] >= in[now] && now != 1)
   iscut[now] = true;
  }
  if(now == 1 && c[now].size() > 1)
    iscut[now] = true;
}

void dfsbcc2(int now, int p){ // build
   block-cut tree
  st.push(now);
  for(int i : c[now]){
    dfsbcc2(i, now);
  }
  if(now == 1){
    if(st.size() > 1){
      while(!st.empty()){
        g[st.top()].eb(bcc);
        g[bcc].eb(st.top());
        st.pop();
      }
      bcc++;
    }
  }
  else if((p != 1 && low[now] >= in[p])
   || (p == 1 && c[p].size() > 1)){
    while(!st.empty()){
      int t = st.top();
      g[st.top()].eb(bcc);
      g[bcc].eb(st.top());
      st.pop();
      if(t == now) break;
    }
    g[bcc].eb(p);
    g[p].eb(bcc);
    bcc++;
  }
}
```

4 String

4.1 KMP

```
vector<int> f;
void build(string& t){
  f.clear();
```

```
  f.resize(t.size());
  int p = -1;
  f[0] = -1;
  for(int i = 1; i < t.size(); i++){
    while(p != -1 && t[p + 1] != t[i]) p
    = f[p];
    if(t[p + 1] == t[i]) f[i] = p + 1;
    else f[i] = -1;
    p = f[i];
  }
}

int kmp(string& s, string& t){
  int ans = 0;
  int p = -1;
  for(int i = 0; i < s.size(); i++){
    while(p != -1 && t[p + 1] != s[i]) p
    = f[p];
    if(t[p + 1] == s[i]) p++;
    if(p + 1 == t.size()){
      ans++;
      p = f[p];
    }
  }
  return ans;
}
```

4.2 Z Value

```
vector<int> z;

void build(string s, int n){
  z.clear();
  z.resize(n);
  int l = 0;
  for(int i = 1; i < n; i++){
    if(l + z[l] >= i) z[i] = min(z[l] +
   l - i, z[i - l]);
    while(i + z[i] < n && s[z[i]] == s[i
    + z[i]]) z[i]++;
    if(i + z[i] > l + z[l]) l = i;
  }
}
```

4.3 Longest Palindromic Substring

```
#define T(x) ((x) % 2 ? s[(x) / 2] : '.'
   )

string s;
int L;

int ex(int l, int r){
  int i = 0;
  while(l - i >= 0 && r + i < L && T(l -
    i) == T(r + i)) i++;
  return i;
}

int lps(string ss){
  s = ss;
  L = 2 * s.size() + 1;

  int mx = 0;
  int center = 0;
  vector<int> r(L);
  int ans = 1;
  r[0] = 1;
  for(int i = 1; i < L; i++){
    int ii = center - (i - center);
    int len = mx - i + 1;
    if(i > mx){
      r[i] = ex(i, i);
      center = i;
      mx = i + r[i] - 1;
    }
    else if(r[ii] == len){
      r[i] = len + ex(i - len, i + len);
      center = i;
      mx = i + r[i] - 1;
    }
    else r[i] = min(r[ii], len);
    ans = max(ans, r[i]);
  }

  return ans - 1;
}
```

4.4 Suffix Array

```
#include <bits/stdc++.h>

#define eb(a) emplace_back(a)

using namespace std;

vector<int> sa(string s){
  s += '$';
  int n = s.size();
  int t = __lg(n) + 1;

  vector<vector<int>> rk(t + 1, vector<
   int>(n)), b;

  vector<vector<int>> c1(27);
  for(int i = 0; i < n; i++) c1[s[i] ==
   '$' ? 0 : s[i] - 'a' + 1].eb(i);;
  for(int i = 0; i < 27; i++){
    if(!c1[i].empty()) b.eb(c1[i]);
  }
  b.resize(n);
  for(int i = 0; i < n; i++){
    for(int k : b[i]) rk[0][k] = i;
  }
```

```
  for(int i = 1; i <= t; i++){
    vector<vector<int>> tb(n);
    for(int j = 0; j < n; j++){
      for(int k : b[j]){
        int tmp = ((k - (1 << (i - 1)))
   % n + n) % n;
        int now = rk[i - 1][tmp];
        tb[now].eb(tmp);
      }
    }
    b = tb;
    int cnt = -1;
    for(int j = 0; j < n; j++){
      int lst = -1;
      for(int k : b[j]){
        int now = rk[i - 1][(k + (1 << (
   i - 1))) % n];
        if(now != lst) cnt++;
        rk[i][k] = cnt;
        lst = now;
      }
    }
  }

  return rk[t];
}
```

5 Math and Geometry

5.1 Vector Operations

```
template<typename T>
pair<T, T> operator+(pair<T, T> a, pair<
    T, T> b){
  return mp(a.F + b.F, a.S + b.S);
}

template<typename T>
pair<T, T> operator-(pair<T, T> a, pair<
    T, T> b){
  return mp(a.F - b.F, a.S - b.S);
}

template<typename T>
pair<T, T> operator*(pair<T, T> a, T b){
  return mp(a.F * b, a.S * b);
}

template<typename T>
pair<T, T> operator/(pair<T, T> a, T b){
  return mp(a.F / b, a.S / b);
}

template<typename T>
T dot(pair<T, T> a, pair<T, T> b){
  return a.F * b.F + a.S * b.S;
}

template<typename T>
T cross(pair<T, T> a, pair<T, T> b){
  return a.F * b.S - a.S * b.F;
}

template<typename T>
T abs2(pair<T, T> a){
  return a.F * a.F + a.S * a.S;
}
```

5.2 Convex Hull

```
template<typename T>
pair<T, T> operator-(pair<T, T> a, pair<
    T, T> b){
  return mp(a.F - b.F, a.S - b.S);
}

template<typename T>
T cross(pair<T, T> a, pair<T, T> b){
  return a.F * b.S - a.S * b.F;
}

template<typename T>
vector<pair<T, T>> getConvexHull(vector<
    pair<T, T>>& pnts){

  int n = pnts.size();
  lsort(pnts);

  vector<pair<T, T>> hull;
  hull.reserve(n);

  for(int i = 0; i < 2; i++){
    int t = hull.size();
    for(pair<T, T> pnt : pnts){
      while(hull.size() - t >= 2 &&
    cross(hull.back() - hull[hull.size()
    - 2], pnt - hull[hull.size() - 2]) <=
     0){
        hull.pop_back();
      }
      hull.pb(pnt);
    }
    hull.pop_back();
    reverse(iter(pnts));
  }

  return hull;
}
```

5.3 Prime Sieve

```
vector<int> prime;
vector<int> p;
```

```
void sieve(int n){
  prime.resize(n + 1, 1);
  for(int i = 2; i <= n; i++){
    if(prime[i] == 1){
      p.push_back(i);
      prime[i] = i;
    }
    for(int j : p){
      if((ll)i * j > n || j > prime[i])
   break;
      prime[i * j] = j;
    }
  }
}
```

5.4 XOR Basis

```
const int mxdigit = 50;

vector<ll> b(mxdigit + 1);

void add(ll t){
  for(int i = mxdigit; i >= 0; i--){
    if(!(1LL << i & t)) continue;
    if(b[i] != 0){
      t ^= b[i];
      continue;
    }
    for(int j = 0; j < i; j++){
      if(1LL << j & t) t ^= b[j];
    }
    for(int j = i + 1; j <= mxdigit; j
   ++){
      if(1LL << i & b[j]) b[j] ^= t;
    }
    b[i] = t;
    break;
  }
}
```

6 DP Trick

6.1 Dynamic Convex Hull

```
const ll INF = 1LL << 60;

template<typename T>
struct Line{
  mutable T a, b, r = 0;

  Line(T a, T b) : a(a), b(b){}

  bool operator<(Line<T> l)const{
    return a < l.a;
  }

  bool operator<(T v)const{
    return r < v;
  }
};

template<typename T>
T divfloor(T a, T b){
  return a / b - ((a ^ b) < 0 && a % b);
}

template<typename T>
struct DynamicHull{
  multiset<Line<T>, less<>> s;

  int size(){
    return s.size();
  }

  bool intersect(typename set<Line<T>>::
   iterator a, typename set<Line<T>>::
   iterator &b){
    if(b == s.end()){
      a->r = INF;
      return false;
    }
    if(a->a == b->a){
      if(a->b > b->b) a->r = INF;
      else a->r = -INF;
    }
    else{
      a->r = divfloor(b->b - a->b, a->a
   - b->a);
    }
    return a->r >= b->r;
  }

  void insert(T a, T b){
    Line<T> l(a, b);
    auto it = s.insert(l), after = next(
   it), before = it;
    while(intersect(it, after)) after =
   s.erase(after);
    if(before != s.begin() && intersect
   (--before, it)){
      it = s.erase(it);
      intersect(before, it);
    }
    while((it = before) != s.begin() &&
   (--before)->r >= it->r) intersect(
   before, it = s.erase(it));
  }

  T query(T v){
    Line<T> l = *s.lower_bound(v);
    return l.a * v + l.b;
  }
};
```

7 Numbers and Math Formulae

7.1 Fibonacci

$$f(n) = f(n-1) + f(n-2)$$

$$\begin{bmatrix} f(n) \\ f(n-1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

1	1	1	2	3
5	5	8	13	21
9	34	55	89	144
13	233	377	610	987
17	1597	2584	4181	6765
21	10946	17711	28657	46368
25	75025	121393	196418	317811
29	514229	832040	1346269	2178309
33	3524578	5702887	9227465	14930352

$f(45) \approx 10^9$
$f(88) \approx 10^{18}$

7.2 Catalan

$$C_0 = 1, C_n = \sum_{i=0}^{n-1} C_i C_{n-1-i}$$

$$C_n = C_n^{2n} - C_{n-1}^{2n}$$

0	1	1	2	5
4	14	42	132	429
8	1430	4862	16796	58786
12	208012	742900	2674440	9694845

7.3 Geometry

- Heron's formula:
  The area of a triangle whose lengths of sides is a,b,c and $s = (a+b+c)/2$ is $\sqrt{s(s-a)(s-b)(s-c)}$.
- Vector cross product:
  $v_1 \times v_2 = |v_1||v_2| \sin\theta = (x_1 \times y_2) - (x_2 \times y_1)$.
- Vector dot product:
  $v_1 \cdot v_2 = |v_1||v_2| \cos\theta = (x_1 \times y_1) + (x_2 \times y_2)$.

7.4 Prime Numbers

First 50 prime numbers:

1	2	3	5	7	11
6	13	17	19	23	29
11	31	37	41	43	47
16	53	59	61	67	71
21	73	79	83	89	97
26	101	103	107	109	113
31	127	131	137	139	149
36	151	157	163	167	173
41	179	181	191	193	197
46	199	211	223	227	229

Very large prime numbers:
1000001333 1000500889 2500001909
2000000659 900004151 850001359

7.5 Number Theory

- Inversion:
  $aa^{-1} \equiv 1 \pmod m$. a^{-1} exists iff $\gcd(a,m) = 1$.
- Linear inversion:
  $a^{-1} \equiv (m - \lfloor \frac{m}{a} \rfloor) \times (m \bmod a)^{-1} \pmod m$
- Fermat's little theorem:
  $a^p \equiv a \pmod p$ if p is prime.
- Euler function:
  $\phi(n) = n \prod_{p|n} \frac{p-1}{p}$
- Euler theorem:
  $a^{\phi(n)} \equiv 1 \pmod n$ if $\gcd(a,n) = 1$.
- Extended Euclidean algorithm:
  $ax + by = \gcd(a,b) = \gcd(b, a \bmod b) = \gcd(b, a - \lfloor \frac{a}{b} \rfloor b) = bx_1 + (a - \lfloor \frac{a}{b} \rfloor b)y_1 = ay_1 + b(x_1 - \lfloor \frac{a}{b} \rfloor y_1)$
- Divisor function:
  $\sigma_x(n) = \sum_{d|n} d^x$. $n = \prod_{i=1}^r p_i^{a_i}$.
  $\sigma_x(n) = \prod_{i=1}^r \frac{p_i^{(a_i+1)x}-1}{p_i^x-1}$ if $x \neq 0$. $\sigma_0(n) = \prod_{i=1}^r (a_i+1)$.
- Chinese remainder theorem:
  $x \equiv a_i \pmod{m_i}$.
  $M = \prod m_i$. $M_i = M/m_i$. $t_i = M_i^{-1}$.
  $x = kM + \sum a_i t_i M_i$, $k \in \mathbb{Z}$.

7.6 Combinatorics

- $P_k^n = \frac{n!}{(n-k)!}$
- $C_k^n = \frac{n!}{(n-k)!k!}$
- $H_k^n = C_k^{n+k-1} = \frac{(n+k-1)!}{k!(n-1)!}$