

Contents

1 Basic	1	5.11 Minimum Steiner Tree . . .	14
1.1 .vimrc	1	5.12 Count Cycles	14
1.2 Fast IO	1	6 Math	15
1.3 Random	1	6.1 Extended Euclidean Algo-	
1.4 PBDS Tree	1	rithm	15
1.5 Pragma	1	6.2 Floor & Ceil	15
1.6 SVG Writer	1	6.3 Legendre	15
2 Data Structure	2	6.4 Simplex	15
2.1 Heavy-Light Decomposition	2	6.5 Simplex Construction . . .	15
2.2 Link Cut Tree	2	6.6 DiscreteLog	16
2.3 Treap	2	6.7 Miller Rabin & Pollard Rho .	16
2.4 KD Tree	3	6.8 XOR Basis	16
2.5 Leftist Tree	3	6.9 Linear Equation	16
2.6 Convex 1D/1D	3	6.10 Chinese Remainder Theorem	17
2.7 Dynamic Convex Hull	4	6.11 Sqrt Decomposition	17
3 Flow & Matching	4	6.12 Floor Sum	17
3.1 Dinic	4	7 Polynomial	17
3.2 Bounded Flow	4	7.1 FWHT	17
3.3 MCMF	4	7.2 FFT	17
3.4 Min Cost Circulation	5	7.3 NTT	17
3.5 Gomory Hu	5	7.4 Polynomial Operation . . .	18
3.6 Stoer Wagner Algorithm . . .	5	7.5 Generating Function	20
3.7 Bipartite Matching	5	Ordinary Generating Func-	
3.8 Kuhn Munkres Algorithm . . .	6	tion	20
3.9 Max Simple Graph Matching	6	Exponential Generating	
3.10 Flow Model	6	Function	20
4 Geometry	7	7.6 Bostan Mori	20
4.1 Geometry Template	7	8 String	20
4.2 Polar Angle Comparator . . .	7	8.1 KMP Algorithm	20
4.3 Minkowski Sum	7	8.2 Manacher Algorithm	21
4.4 Intersection of Circle and		8.3 Lyndon Factorization	21
Convex Polygon	8	8.4 Suffix Array	21
4.5 Intersection of Circles . . .	8	8.5 Suffix Automaton	21
4.6 Tangent Line of Circles . . .	8	8.6 Z-value Algorithm	21
4.7 Intersection of Line and		8.7 Main Lorentz	22
Convex Polygon	8	8.8 AC Automaton	22
4.8 Intersection of Line and		8.9 Palindrome Automaton	22
Circle	8	8.10 Palindrome Partition	22
4.9 Point in Circle	8	9 Misc	23
4.10 Point in Convex	8	9.1 Cyclic Ternary Search	23
4.11 Half Plane Intersection . . .	8	9.2 Matroid	23
4.12 Minimum Enclosing Circle . .	9	9.3 Simulate Annealing	23
4.13 3D Point	9	9.4 Binary Search On Fraction . .	23
4.14 ConvexHull3D	9	9.5 Min Plus Convolution	23
4.15 Delaunay Triangulation . . .	10	9.6 SMAWK	23
4.16 Voronoi Diagram	10	9.7 Golden Ratio Search	23
4.17 Polygon Union	10	10 Notes	23
4.18 Tangent Point to Convex Hull	10	10.1 Geometry	23
4.19 Heart	11	Rotation Matrix	23
4.20 Rotating Sweep Line	11	Triangles	24
4.21 Vector In Poly	11	Quadrilaterals	24
4.22 Convex Hull DP	11	Spherical coordinates	24
4.23 Calculate Points in Triangle	11	Green's Theorem	24
5 Graph	11	Point-Line Duality	24
5.1 BCC	11	10.2 Trigonometry	24
5.2 SCC	12	10.3 Calculus	24
5.3 2-SAT	12	10.4 Sum & Series	24
5.4 Dominator Tree	12	10.5 Misc	25
5.5 Virtual Tree	13	10.6 Number	25
5.6 Fast DMST	13		
5.7 Vizing	13		
5.8 Maximum Clique	13		
5.9 Number of Maximal Clique	14		
5.10 Minimum Mean Cycle	14		

1 Basic

Default code: Basic 9c8f02, Debug 28c438

1.1 .vimrc [5f03d4]

```
sy on
se nu rnu bs=2 sw=4 ts=4 hls ls=2 si acd bo=all mouse=a
map <F9> :w<bar>!g++ "% " -o %:r -std=c++17 -Wall -
Wextra -Wshadow -O2 -Dzisk -g -fsanitize=address,
undefined<CR>
map <F8> :!./%:r<CR>
inoremap {<CR> {<CR>}<ESC>ko
ca Hash w !cpp -dD -P -fpreprocessed \\\ tr -d '[:space
:]' \\\ md5sum \\\ cut -c-6
inoremap fj <ESC>
vnoremap fj <ESC>
" Hash: dbf3ad (manual)
" -D_GLIBCXX_ASSERTIONS, -D_GLIBCXX_DEBUG
```

1.2 Fast IO [4f6f0e]

```
char readchar(){
```

```
const int N = 1<<20;
static char buf[N];
static char *p = buf , *end = buf;
if(p == end) {
    if((end = buf + fread(buf , 1 , N , stdin)) == buf)
        return EOF;
    p = buf;
}
return *p++;
}

const int buf_size = 524288;
struct Writer {
    char buf[buf_size]; int size = 0, ret;
    void flush() { ret = write(1, buf, size); size = 0; }
    void _flush(int sz) { if (sz + size > buf_size) flush
        (); }
    void write_char(char c) { _flush(1); buf[size++] = c;
        }
    void write_int(int x) {
        const int len = 20;
        _flush(len); int ptr = 0;
        if (x < 0) buf[size++] = '-'; x = -x;
        if (x == 0) buf[size + (ptr++)] = '0';
        else for (; x; x /= 10) buf[size + (ptr++)] = '0' +
            x % 10;
        reverse(buf + size, buf + size + ptr);
        size += ptr;
    }
}; // remember to call flush
```

1.3 Random [4cf9ed]

```
mt19937 rng(chrono::system_clock::now().
    time_since_epoch().count());
```

1.4 PBDS Tree [9e57e3]

```
#include <bits/extc++.h>
using namespace __gnu_pbds;
using Tree = tree<int, null_type, less<>, rb_tree_tag,
    tree_order_statistics_node_update>;
// .find_by_order(x)
// .order_of_key(x)
```

1.5 Pragma [6006f6]

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target("popcnt,abm,mmx,avx,arch=skylake")
__builtin_ia32_ldmxcsr(__builtin_ia32_stmxcsr()|0x8040)
```

1.6 SVG Writer [7adcc8]

```
class SVG {
    void p(string_view s) { o << s; }
    void p(string_view s, auto v, auto... vs) {
        auto i = s.find('$');
        o << s.substr(0, i) << v, p(s.substr(i + 1), vs...)
            ;
    }
    ofstream o; string c = "red";
public: // SVG svg("test.svg", 0, 0, 100, 100)
    SVG(auto f, auto x1, auto y1, auto x2, auto y2) : o(f)
    {
        p("<svg xmlns='http://www.w3.org/2000/svg' "
            "viewBox='0 0 100 100'"
            "<style>{*stroke-width:0.5px;}</style>\n",
            x1, -y2, x2 - x1, y2 - y1); }
    ~SVG() { p("</svg>\n"); }
    void color(string nc) { c = nc; }
    void line(auto x1, auto y1, auto x2, auto y2) {
        p("<line x1='$' y1='$' x2='$' y2='$' stroke='$'>\n"
            ,
            x1, -y1, x2, -y2, c); }
    void circle(auto x, auto y, auto r) {
        p("<circle cx='$' cy='$' r='$' stroke='$' "
            "fill='none'/>\n", x, -y, r, c); }
    void text(auto x, auto y, string s, int w = 12) {
        p("<text x='$' y='$' font-size='$px'>$</text>\n",
            x, -y, w, s); }
};
```

2 Data Structure

2.1 Heavy-Light Decomposition [f2dbca]

```
struct HLD{ // 1-based
    int n, ts = 0; // ord is 1-based
    vector<vector<int>> g;
    vector<int> par, top, down, ord, dpt, sub;
    explicit HLD(int _n): n(_n), g(n + 1),
        par(n + 1), top(n + 1), down(n + 1),
        ord(n + 1), dpt(n + 1), sub(n + 1) {}
    void add_edge(int u, int v){ g[u].pb(v); g[v].pb(u); }
    void dfs(int now, int p){
        par[now] = p; sub[now] = 1;
        for(int i : g[now]){
            if(i == p) continue;
            dpt[i] = dpt[now] + 1;
            dfs(i, now);
            sub[now] += sub[i];
            if(sub[i] > sub[down[now]]) down[now] = i;
        }
    }
    void cut(int now, int t){
        top[now] = t; ord[now] = ++ts;
        if(!down[now]) return;
        cut(down[now], t);
        for(int i : g[now]){
            if(i != par[now] && i != down[now])
                cut(i, i);
        }
    }
    void build(){ dfs(1, 1), cut(1, 1); }
    int query(int a, int b){
        int ta = top[a], tb = top[b];
        while(ta != tb){
            if(dpt[ta] > dpt[tb]) swap(ta, tb), swap(a, b);
            // ord[ta], ord[b]
            tb = top[b = par[tb]];
        }
        if(ord[a] > ord[b]) swap(a, b);
        // ord[a], ord[b]
        return a; // lca
    }
};
```

2.2 Link Cut Tree [502ab1]

```
// 1-based
// == 43515a ==
template <typename Val, typename SVal> struct LCT {
    struct node {
        int pa, ch[2]; bool rev; int size;
        Val v, sum, rsum; SVal sv, sub, vir;
        node() : pa{0}, ch{0, 0}, rev{false}, size{1}, v{},
            sum{}, rsum{}, sv{}, sub{}, vir{} {}
    };
    #define cur o[u]
    #define lc cur.ch[0]
    #define rc cur.ch[1]
    vector<node> o;
    bool is_root(int u) const {
        return o[cur.pa].ch[0] != u && o[cur.pa].ch[1] != u; }
    bool is_rch(int u) const {
        return o[cur.pa].ch[1] == u && !is_root(u); }
    void down(int u) {
        for (int c : {lc, rc}) if (c) {
            if (cur.rev) set_rev(c);
        }
        cur.rev = false;
    }
    void up(int u) {
        cur.sum = o[lc].sum + cur.v + o[rc].sum;
        cur.rsum = o[rc].rsum + cur.v + o[lc].rsum;
        cur.sub = cur.vir + o[lc].sub + o[rc].sub + cur.sv;
        cur.size = o[lc].size + o[rc].size + 1;
    }
    void set_rev(int u) {
        swap(lc, rc), swap(cur.sum, cur.rsum);
        cur.rev ^= 1;
    }
    // == 3a186b ==
    void rotate(int u) {
```

```
int f = cur.pa, g = o[f].pa, l = is_rch(u);
if (cur.ch[l ^ 1]) o[cur.ch[l ^ 1]].pa = f;
if (not is_root(f)) o[g].ch[is_rch(f)] = u;
o[f].ch[l] = cur.ch[l ^ 1], cur.ch[l ^ 1] = f;
cur.pa = g, o[f].pa = u; up(f);
}
vector<int> stk;
void splay(int u) {
    stk.clear(); stk.pb(u);
    while (not is_root(stk.back()))
        stk.push_back(o[stk.back()].pa);
    while (not stk.empty())
        down(stk.back()), stk.pop_back();
    for (int f = cur.pa; not is_root(u); f = cur.pa) {
        if (!is_root(f))
            rotate(is_rch(u) == is_rch(f) ? f : u);
        rotate(u);
    }
    up(u);
}
void access(int x) {
    for (int u = x, last = 0; u; u = cur.pa) {
        splay(u);
        cur.vir = cur.vir + o[rc].sub - o[last].sub;
        rc = last; up(last = u);
    }
    splay(x);
}
int find_root(int u) {
    int la = 0;
    for (access(u); u; u = lc) down(la = u);
    return la;
}
void split(int x, int y) { chroot(x); access(y); }
void chroot(int u) { access(u); set_rev(u); }
// == a238c2 ==
LCT(int n = 0) : o(n + 1) { o[0].size = 0; }
void set_val(int u, const Val &v) {
    splay(u); cur.v = v; up(u); }
void set_sval(int u, const SVal &v) {
    access(u); cur.sv = v; up(u); }
Val query(int x, int y) {
    split(x, y); return o[y].sum; }
SVal subtree(int p, int u) {
    chroot(p); access(u); return cur.vir + cur.sv; }
bool connected(int u, int v) {
    return find_root(u) == find_root(v); }
void link(int x, int y) {
    chroot(x); access(y);
    o[y].vir = o[y].vir + o[x].sub;
    up(o[x].pa = y);
}
void cut(int x, int y) {
    split(x, y); o[y].ch[0] = o[x].pa = 0; up(y); }
#undef cur
#undef lc
#undef rc
};
```

2.3 Treap [2ac37e]

```
mt19937 rng(880301);
// == fb4359 ==
struct node {
    ll data; int sz;
    node *l, *r;
    node(ll k = 0) : data(k), sz(1), l(0), r(0) {}
    void up() {
        sz = 1;
        if (l) sz += l->sz;
        if (r) sz += r->sz;
    }
    void down() {}
};
node pool[1000010]; int pool_cnt = 0;
node *newnode(ll k){ return &(pool[pool_cnt++]) = node(k); }
int sz(node *a) { return a ? a->sz : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (int(rng() % (sz(a) + sz(b))) < sz(a))
        return a->down(), a->r = merge(a->r, b), a->up(),
            a;
```

```

    return b->down(), b->l = merge(a, b->l), b->up(), b;
}
// a: key <= k, b: key > k
void split(node *o, node *&a, node *&b, ll k) {
    if (!o) return a = b = 0, void();
    o->down();
    if (o->data <= k)
        a = o, split(o->r, a->r, b, k), a->up();
    else b = o, split(o->l, a, b->l, k), b->up();
}
// a: size k, b: size n - k
void split2(node *o, node *&a, node *&b, int k) {
    if (sz(o) <= k) return a = o, b = 0, void();
    o->down();
    if (sz(o->l) + 1 <= k)
        a = o, split2(o->r, a->r, b, k - sz(o->l) - 1);
    else b = o, split2(o->l, a, b->l, k);
    o->up();
}
// == e9f4d8 ==
node *kth(node *o, ll k) { // 1-based
    if (k <= sz(o->l)) return kth(o->l, k);
    if (k == sz(o->l) + 1) return o;
    return kth(o->r, k - sz(o->l) - 1);
}
int Rank(node *o, ll key) { // num of key < key
    if (!o) return 0;
    if (o->data < key)
        return sz(o->l) + 1 + Rank(o->r, key);
    else return Rank(o->l, key);
}
bool erase(node *o, ll k) {
    if (!o) return 0;
    if (o->data == k) {
        node *t = o;
        o->down(), o = merge(o->l, o->r);
        return 1;
    }
    node *&t = k < o->data ? o->l : o->r;
    return erase(t, k) ? o->up(), 1 : 0;
}
void insert(node *o, ll k) {
    node *a, *b;
    split(o, a, b, k),
    o = merge(a, merge(new node(k), b));
}
tuple<node*, node*, node*> interval(node *o, int l,
    int r) { // 1-based
    node *a, *b, *c; // b: [l, r]
    split2(o, a, b, l - 1), split2(b, b, c, r - l + 1);
    return make_tuple(a, b, c);
}

```

2.4 KD Tree [375ca2]

```

namespace kdt {
    int root, lc[maxn], rc[maxn], xl[maxn], xr[maxn],
    yl[maxn], yr[maxn];
    point p[maxn];
    int build(int l, int r, int dep = 0) {
        if (l == r) return -1;
        function<bool(const point &, const point &)> f =
            [dep](const point &a, const point &b) {
                if (dep & 1) return a.x < b.x;
                else return a.y < b.y;
            };
        int m = (l + r) >> 1;
        nth_element(p + l, p + m, p + r, f);
        xl[m] = xr[m] = p[m].x;
        yl[m] = yr[m] = p[m].y;
        lc[m] = build(l, m, dep + 1);
        if (~lc[m]) {
            xl[m] = min(xl[m], xl[lc[m]]);
            xr[m] = max(xr[m], xr[lc[m]]);
            yl[m] = min(yl[m], yl[lc[m]]);
            yr[m] = max(yr[m], yr[lc[m]]);
        }
        rc[m] = build(m + 1, r, dep + 1);
        if (~rc[m]) {
            xl[m] = min(xl[m], xl[rc[m]]);
            xr[m] = max(xr[m], xr[rc[m]]);
            yl[m] = min(yl[m], yl[rc[m]]);
            yr[m] = max(yr[m], yr[rc[m]]);
        }
    }
}

```

```

    }
    return m;
}
bool bound(const point &q, int o, long long d) {
    double ds = sqrt(d + 1.0);
    if (q.x < xl[o] - ds || q.x > xr[o] + ds ||
        q.y < yl[o] - ds || q.y > yr[o] + ds)
        return false;
    return true;
}
long long dist(const point &a, const point &b) {
    return (a.x - b.x) * 111 * (a.x - b.x) +
        (a.y - b.y) * 111 * (a.y - b.y);
}
void dfs(
    const point &q, long long &d, int o, int dep = 0)
{
    if (!bound(q, o, d)) return;
    long long cd = dist(p[o], q);
    if (cd != 0) d = min(d, cd);
    if ((dep & 1) && q.x < p[o].x ||
        !(dep & 1) && q.y < p[o].y) {
        if (~lc[o]) dfs(q, d, lc[o], dep + 1);
        if (~rc[o]) dfs(q, d, rc[o], dep + 1);
    } else {
        if (~rc[o]) dfs(q, d, rc[o], dep + 1);
        if (~lc[o]) dfs(q, d, lc[o], dep + 1);
    }
}
void init(const vector<point> &v) {
    for (int i = 0; i < v.size(); ++i) p[i] = v[i];
    root = build(0, v.size());
}
long long nearest(const point &q) {
    long long res = 1e18;
    dfs(q, res, root);
    return res;
}
} // namespace kdt

```

2.5 Leftist Tree [e91538]

```

struct node {
    ll v, data, sz, sum;
    node *l, *r;
    node(ll k)
        : v(0), data(k), sz(1), l(0), r(0), sum(k) {}
};
ll sz(node *p) { return p ? p->sz : 0; }
ll V(node *p) { return p ? p->v : -1; }
ll sum(node *p) { return p ? p->sum : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (a->data < b->data) swap(a, b);
    a->r = merge(a->r, b);
    if (V(a->r) > V(a->l)) swap(a->r, a->l);
    a->v = V(a->r) + 1, a->sz = sz(a->l) + sz(a->r) + 1;
    a->sum = sum(a->l) + sum(a->r) + a->data;
    return a;
}
void pop(node *o) {
    node *tmp = o;
    o = merge(o->l, o->r);
    delete tmp;
}

```

2.6 Convex 1D/1D [a449dd]

```

template<class T>
struct DynamicHull {
    struct seg { int x, l, r; };
    T f; int C; deque<seg> dq; // range: 1~C
    explicit DynamicHull(T _f, int _C): f(_f), C(_C) {}
    // max t s.t. f(x, t) >= f(y, t), x < y, maintain max
    int intersect(int x, int y) {
        int l = 0, r = C + 1;
        while (l + 1 < r) {
            int mid = (l + r) / 2;
            if (f(x, mid) >= f(y, mid)) l = mid;
            else r = mid;
        }
        return l;
    }
};

```

```

void push_back(int x) {
    for (int i; !dq.empty() &&
        (i = dq.back().l, f(dq.back().x, i) < f(x, i));
        )
        dq.pop_back();
    if (dq.empty()) return dq.pb(seg({x, 1, C})), void
    ();
    dq.back().r = intersect(dq.back().x, x);
    if (dq.back().r + 1 <= C) dq.pb(seg({x, dq.back().r
        + 1, C}));
}
int query(int x) {
    while (dq.front().r < x) dq.pop_front();
    return dq.front().x;
}
};

```

2.7 Dynamic Convex Hull [b45ebc]

```

// only works for integer coordinates!! maintain max
struct Line {
    mutable ll a, b, p;
    bool operator<(const Line &rhs) const { return a <
        rhs.a; }
    bool operator<(ll x) const { return p < x; }
};
struct DynamicHull : multiset<Line, less<>> {
    static const ll kInf = 1e18;
    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = kInf; return 0; }
        if (x->a == y->a) x->p = x->b > y->b ? kInf : -kInf
            ;
        else x->p = iceil(y->b - x->b, x->a - y->a);
        return x->p >= y->p;
    }
    void addline(ll a, ll b) {
        auto z = insert({a, b, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y =
            erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    ll query(ll x) {
        auto l = *lower_bound(x);
        return l.a * x + l.b;
    }
};

```

3 Flow & Matching

3.1 Dinic [801a71]

```

struct Dinic { // 0-based,  $O(V^2E)$ , unit flow:  $O(\min(V
    ^{2/3}E, E^{3/2}))$ , bipartite matching:  $O(\sqrt{V}E)$ 
    struct edge {
        ll to, cap, flow, rev;
    };
    int n, s, t;
    vector<vector<edge>> g;
    vector<int> dis, ind;

    void init(int _n) {
        n = _n;
        g.assign(n, vector<edge>());
    }
    void reset() {
        for (int i = 0; i < n; ++i)
            for (auto &j : g[i]) j.flow = 0;
    }
    void add_edge(int u, int v, ll cap) {
        g[u].pb(edge{v, cap, 0, SZ(g[v]))};
        g[v].pb(edge{u, 0, 0, SZ(g[u]) - 1});
        //change g[v] to cap for undirected graphs
    }
    bool bfs() {
        dis.assign(n, -1);
        queue<int> q;
        q.push(s), dis[s] = 0;
        while (!q.empty()) {
            int cur = q.front(); q.pop();
            for (auto &e : g[cur]) {

```

```

                if (dis[e.to] == -1 && e.flow != e.cap) {
                    q.push(e.to);
                    dis[e.to] = dis[cur] + 1;
                }
            }
        }
        return dis[t] != -1;
    }
    ll dfs(int u, ll cap) {
        if (u == t || !cap) return cap;
        for (int &i = ind[u]; i < SZ(g[u]); ++i) {
            edge &e = g[u][i];
            if (dis[e.to] == dis[u] + 1 && e.flow != e.cap) {
                ll df = dfs(e.to, min(e.cap - e.flow, cap));
                if (df) {
                    e.flow += df;
                    g[e.to][e.rev].flow -= df;
                    return df;
                }
            }
        }
        dis[u] = -1;
        return 0;
    }
    ll maxflow(int _s, int _t) {
        s = _s; t = _t;
        ll flow = 0, df;
        while (bfs()) {
            ind.assign(n, 0);
            while ((df = dfs(s, INF))) flow += df;
        }
        return flow;
    }
};

```

3.2 Bounded Flow [758826]

```

struct BoundedFlow : Dinic {
    vector<ll> tot;
    void init(int _n) {
        Dinic::init(_n + 2);
        tot.assign(n, 0);
    }
    void add_edge(int u, int v, ll lcap, ll rcap) {
        tot[u] -= lcap, tot[v] += lcap;
        g[u].pb(edge{v, rcap, lcap, SZ(g[v]))};
        g[v].pb(edge{u, 0, 0, SZ(g[u]) - 1});
    }
    bool feasible() {
        ll sum = 0;
        int vs = n - 2, vt = n - 1;
        for (int i = 0; i < n - 2; ++i)
            if (tot[i] > 0)
                add_edge(vs, i, 0, tot[i]), sum += tot[i];
            else if (tot[i] < 0) add_edge(i, vt, 0, -tot[i]);
        if (sum != maxflow(vs, vt)) sum = -1;
        for (int i = 0; i < n - 2; ++i)
            if (tot[i] > 0)
                g[vs].pop_back(), g[i].pop_back();
            else if (tot[i] < 0)
                g[i].pop_back(), g[vt].pop_back();
        return sum != -1;
    }
    ll boundedflow(int _s, int _t) {
        add_edge(_t, _s, 0, INF);
        if (!feasible()) return -1;
        ll x = g[_t].back().flow;
        g[_t].pop_back(), g[_s].pop_back();
        return x - maxflow(_t, _s); // min
        //return x + maxflow(_s, _t); // max
    }
};

```

3.3 MCMF [671e14]

```

struct MCMF { // 0-base
    struct Edge {
        ll from, to, cap, flow, cost, rev;
    };
    int n, s, t;
    vector<vector<Edge>> g;
    vector<Edge*> past;
    vector<ll> dis, up, pot;

```

```

explicit MCMF(int _n): n(_n), g(n), past(n), dis(n),
    up(n), pot(n) {}
void add_edge(ll a, ll b, ll cap, ll cost) {
    g[a].pb(Edge{a, b, cap, 0, cost, SZ(g[b]))};
    g[b].pb(Edge{b, a, 0, 0, -cost, SZ(g[a]) - 1});
}
bool BellmanFord() {
    vector<bool> inq(n);
    fill(iter(dis), INF);
    queue<int> q;
    auto relax = [&](int u, ll d, ll cap, Edge *e) {
        if (cap > 0 && dis[u] > d) {
            dis[u] = d, up[u] = cap, past[u] = e;
            if (!inq[u]) inq[u] = 1, q.push(u);
        }
    };
    relax(s, 0, INF, 0);
    while (!q.empty()) {
        int u = q.front();
        q.pop(), inq[u] = 0;
        for (auto &e : g[u]) {
            ll d2 = dis[u] + e.cost + pot[u] - pot[e.to];
            relax(e.to, d2, min(up[u], e.cap - e.flow), &e);
        }
    }
    return dis[t] != INF;
}
pair<ll, ll> solve(int _s, int _t, bool neg = true) {
    s = _s, t = _t; ll flow = 0, cost = 0;
    if (neg) BellmanFord(), pot = dis;
    for (; BellmanFord(); pot = dis) {
        for (int i = 0; i < n; ++i)
            if (dis[i] != INF) dis[i] += pot[i] - pot[s];
        flow += up[t], cost += up[t] * dis[t];
        for (int i = t; past[i]; i = past[i]->from) {
            auto &e = *past[i];
            e.flow += up[t], g[e.to][e.rev].flow -= up[t];
        }
    }
    return {flow, cost};
}
};

```

3.4 Min Cost Circulation [47cf18]

```

struct MinCostCirculation { // 0-based, O(VE * ELogC)
    struct edge {
        ll from, to, cap, fcap, flow, cost, rev;
    };
    int n;
    vector<edge*> past;
    vector<vector<edge*>> g;
    vector<ll> dis;
    void BellmanFord(int s) {
        vector<int> inq(n);
        dis.assign(n, INF);
        queue<int> q;
        auto relax = [&](int u, ll d, edge *e) {
            if (dis[u] > d) {
                dis[u] = d, past[u] = e;
                if (!inq[u]) inq[u] = 1, q.push(u);
            }
        };
        relax(s, 0, 0);
        while (!q.empty()) {
            int u = q.front();
            q.pop(), inq[u] = 0;
            for (auto &e : g[u])
                if (e.cap > e.flow)
                    relax(e.to, dis[u] + e.cost, &e);
        }
    }
    void try_edge(edge &cur) {
        if (cur.cap > cur.flow) return ++cur.cap, void();
        BellmanFord(cur.to);
        if (dis[cur.from] + cur.cost < 0) {
            ++cur.flow, --g[cur.to][cur.rev].flow;
            for (int i = cur.from; past[i]; i = past[i]->from) {
                auto &e = *past[i];
                ++e.flow, --g[e.to][e.rev].flow;
            }
        }
    }
};

```

```

    }
    ++cur.cap;
}
void solve(int mxlg) { // mxlg >= Log(max cap)
    for (int b = mxlg; b >= 0; --b) {
        for (int i = 0; i < n; ++i)
            for (auto &e : g[i])
                e.cap *= 2, e.flow *= 2;
        for (int i = 0; i < n; ++i)
            for (auto &e : g[i])
                if (e.fcap >> b & 1)
                    try_edge(e);
    }
}
void init(int _n) {
    n = _n;
    past.assign(n, nullptr);
    g.assign(n, vector<edge*>());
}
void add_edge(ll a, ll b, ll cap, ll cost) {
    g[a].pb(edge{a, b, 0, cap, 0, cost, SZ(g[b]) + (a
        == b)});
    g[b].pb(edge{b, a, 0, 0, 0, -cost, SZ(g[a]) - 1});
}
};

```

3.5 Gomory Hu [82d968]

```

void GomoryHu(Dinic &flow) { // 0-based
    int n = flow.n;
    vector<int> par(n);
    for (int i = 1; i < n; ++i) {
        flow.reset();
        add_edge(i, par[i], flow.maxflow(i, par[i]));
        for (int j = i + 1; j < n; ++j)
            if (par[j] == par[i] && ~flow.dis[j])
                par[j] = i;
    }
}

```

3.6 Stoer Wagner Algorithm [a9917b]

```

struct StoerWagner { // 0-based, O(V^3)
    int n;
    vector<int> vis, del;
    vector<ll> wei;
    vector<vector<ll>> edge;
    void init(int _n) {
        n = _n;
        del.assign(n, 0);
        edge.assign(n, vector<ll>(n));
    }
    void add_edge(int u, int v, ll w) {
        edge[u][v] += w, edge[v][u] += w;
    }
    void search(int &s, int &t) {
        vis.assign(n, 0); wei.assign(n, 0);
        s = t = -1;
        while (1) {
            ll mx = -1, cur = 0;
            for (int i = 0; i < n; ++i)
                if (!del[i] && !vis[i] && mx < wei[i])
                    cur = i, mx = wei[i];
            if (mx == -1) break;
            vis[cur] = 1, s = t, t = cur;
            for (int i = 0; i < n; ++i)
                if (!vis[i] && !del[i]) wei[i] += edge[cur][i];
        }
    }
    ll solve() {
        ll ret = INF;
        for (int i = 0, x=0, y=0; i < n-1; ++i) {
            search(x, y), ret = min(ret, wei[y]), del[y] = 1;
            for (int j = 0; j < n; ++j)
                edge[x][j] = (edge[j][x] += edge[y][j]);
        }
        return ret;
    }
};

```

3.7 Bipartite Matching [5bb9be]

// $O(E \sqrt{V})$, $O(E \log V)$ for random sparse graphs


```

struct BipartiteMatching { // 0-based
    int nl, nr;
    vector<int> mx, my, dis, cur;
    vector<vector<int>> g;
    bool dfs(int u) {
        for (int &i = cur[u]; i < SZ(g[u]); ++i) {
            int e = g[u][i];
            if (!~my[e] || (dis[my[e]] == dis[u] + 1 && dfs(
                my[e])))
                return mx[my[e] = u] = e, 1;
        }
        dis[u] = -1;
        return 0;
    }
    bool bfs() {
        int ret = 0;
        queue<int> q;
        dis.assign(nl, -1);
        for (int i = 0; i < nl; ++i)
            if (!~mx[i]) q.push(i), dis[i] = 0;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (int e : g[u])
                if (!~my[e]) ret = 1;
                else if (!~dis[my[e]]) {
                    q.push(my[e]);
                    dis[my[e]] = dis[u] + 1;
                }
        }
        return ret;
    }
    int matching() {
        int ret = 0;
        mx.assign(nl, -1); my.assign(nr, -1);
        while (bfs()) {
            cur.assign(nl, 0);
            for (int i = 0; i < nl; ++i)
                if (!~mx[i] && dfs(i)) ++ret;
        }
        return ret;
    }
    void add_edge(int s, int t) { g[s].pb(t); }
    void init(int _nl, int _nr) {
        nl = _nl, nr = _nr;
        g.assign(nl, vector<int>());
    }
};

```

3.8 Kuhn Munkres Algorithm [683e0a]

```

struct KM { // 0-based, maximum matching,  $O(V^3)$ 
    int n, ql, qr;
    vector<vector<int>> w;
    vector<int> hl, hr, slk;
    vector<int> fl, fr, pre, qu, vl, vr;
    void init(int _n) {
        n = _n;
        // -INF for perfect matching
        w.assign(n, vector<int>(n, 0));
        pre.assign(n, 0);
        qu.assign(n, 0);
    }
    void add_edge(int a, int b, int wei) {
        w[a][b] = wei;
    }
    bool check(int x) {
        if (vl[x] = 1, ~fl[x])
            return (vr[qu[qr++] = fl[x]] = 1);
        while (~x) swap(x, fr[fl[x] = pre[x]]);
        return 0;
    }
    void bfs(int s) {
        slk.assign(n, INF); vl.assign(n, 0); vr.assign(n, 0);
        ql = qr = 0, qu[qr++] = s, vr[s] = 1;
        for (int d; ;) {
            while (ql < qr)
                for (int x = 0, y = qu[ql++]; x < n; ++x)
                    if (!vl[x] && slk[x] >= (d = hl[x] + hr[y] -
                        w[x][y])) {
                        if (pre[x] = y, d) slk[x] = d;
                        else if (!check(x)) return;
                    }
        }
    }
};

```

```

    }
    d = INF;
    for (int x = 0; x < n; ++x)
        if (!vl[x] && d > slk[x]) d = slk[x];
    for (int x = 0; x < n; ++x) {
        if (vl[x]) hl[x] += d;
        else slk[x] -= d;
        if (vr[x]) hr[x] -= d;
    }
    for (int x = 0; x < n; ++x)
        if (!vl[x] && !slk[x] && !check(x)) return;
    }
};
int solve() {
    fl.assign(n, -1); fr.assign(n, -1); hl.assign(n, 0);
    ; hr.assign(n, 0);
    for (int i = 0; i < n; ++i)
        hl[i] = *max_element(iter(w[i]));
    for (int i = 0; i < n; ++i) bfs(i);
    int res = 0;
    for (int i = 0; i < n; ++i) res += w[i][fl[i]];
    return res;
}
};

```

3.9 Max Simple Graph Matching [907d7c]

```

struct Matching { // 0-based,  $O(V^3)$ 
    queue<int> q; int n;
    vector<int> fa, s, vis, pre, match;
    vector<vector<int>> g;
    int Find(int u) {
        if (u == fa[u]) return u;
        return fa[u] = Find(fa[u]);
    }
    int LCA(int x, int y) {
        static int tk = 0; tk++; x = Find(x); y = Find(y);
        for (; swap(x, y)) if (x != n) {
            if (vis[x] == tk) return x;
            vis[x] = tk;
            x = Find(pre[match[x]]);
        }
    }
    void Blossom(int x, int y, int l) {
        for (; Find(x) != l; x = pre[y]) {
            pre[x] = y, y = match[x];
            if (s[y] == 1) q.push(y), s[y] = 0;
            for (int z : {x, y}) if (fa[z] == z) fa[z] = l;
        }
    }
    bool Bfs(int r) {
        iota(iter(fa), 0); fill(iter(s), -1);
        q = queue<int>(); q.push(r); s[r] = 0;
        for (; !q.empty(); q.pop()) {
            for (int x = q.front(); int u : g[x])
                if (s[u] == -1) {
                    if (pre[u] = x, s[u] = 1, match[u] == n) {
                        for (int a = u, b = x, last;
                            b != n; a = last, b = pre[a])
                            last = match[b], match[b] = a, match[a] =
                                b;
                        return true;
                    }
                    q.push(match[u]); s[match[u]] = 0;
                }
                else if (!s[u] && Find(u) != Find(x)) {
                    int l = LCA(u, x);
                    Blossom(x, u, l); Blossom(u, x, l);
                }
        }
        return false;
    }
    Matching(int _n) : n(_n), fa(n + 1), s(n + 1), vis(n
        + 1), pre(n + 1, n), match(n + 1, n), g(n) {}
    void add_edge(int u, int v) {
        g[u].pb(v), g[v].pb(u);
    }
    int solve() {
        int ans = 0;
        for (int x = 0; x < n; ++x)
            if (match[x] == n) ans += Bfs(x);
        return ans;
    }
    // match[x] == n means not matched
};

```

3.10 Flow Model

- Maximum/Minimum flow with lower bound / Circulation problem

- Construct super source S and sink T .
- For each edge (x, y, l, u) , connect $x \rightarrow y$ with capacity $u - l$.
- For each vertex v , denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
- If $in(v) > 0$, connect $S \rightarrow v$ with capacity $in(v)$, otherwise, connect $v \rightarrow T$ with capacity $-in(v)$.
 - To maximize, connect $t \rightarrow s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T . If $f \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
 - To minimize, let f be the maximum flow from S to T . Connect $t \rightarrow s$ with capacity ∞ and let the flow from S to T be f' . If $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, f' is the answer.
- The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
 - Redirect every edge: $y \rightarrow x$ if $(x, y) \in M$, $x \rightarrow y$ otherwise.
 - DFS from unmatched vertices in X .
 - $x \in X$ is chosen iff x is unvisited.
 - $y \in Y$ is chosen iff y is visited.
- Minimum cost cyclic flow
 - Construct super source S and sink T
 - For each edge (x, y, c) , connect $x \rightarrow y$ with $(cost, cap) = (c, 1)$ if $c > 0$, otherwise connect $y \rightarrow x$ with $(cost, cap) = (-c, 1)$
 - For each edge with $c < 0$, sum these cost as K , then increase $d(y)$ by 1, decrease $d(x)$ by 1
 - For each vertex v with $d(v) > 0$, connect $S \rightarrow v$ with $(cost, cap) = (0, d(v))$
 - For each vertex v with $d(v) < 0$, connect $v \rightarrow T$ with $(cost, cap) = (0, -d(v))$
 - Flow from S to T , the answer is the cost of the flow $C + K$
- Maximum density induced subgraph
 - Binary search on answer, suppose we're checking answer T
 - Construct a max flow model, let K be the sum of all weights
 - Connect source $s \rightarrow v$, $v \in G$ with capacity K
 - For each edge (u, v, w) in G , connect $u \rightarrow v$ and $v \rightarrow u$ with capacity w
 - For $v \in G$, connect it with sink $v \rightarrow t$ with capacity $K + 2T - (\sum_{e \in E(v)} w_e) / 2w(v)$
 - T is a valid answer if the maximum flow $f < K|V|$
- Minimum weight edge cover
 - Let $w'(u, v) = w(u, v) - \mu(u) - \mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v .
 - Find the minimum weight matching M with w' . The answer is $\sum \mu(v) + w'(M)$.
- Project selection problem
 - If $p_v > 0$, create edge (s, v) with capacity p_v ; otherwise, create edge (v, t) with capacity $-p_v$.
 - Create edge (u, v) with capacity w with w being the cost of choosing u without choosing v .
 - The mincut is equivalent to the maximum profit of a subset of projects.
- Dual of minimum cost maximum flow
 - Capacity c_{uv} , Flow f_{uv} , Cost w_{uv} , Required Flow difference for vertex b_u .
 - If all w_{uv} are integers, then optimal solution can happen when all p_u are integers.

$$\min \sum_{uv} w_{uv} f_{uv} \quad \min \sum_u b_u p_u + \sum_{uv} c_{uv} \max(0, p_v - p_u - w_{uv})$$

$$-f_{uv} \geq -c_{uv} \Leftrightarrow \sum_v f_{vu} - \sum_v f_{uv} = -b_u \quad p_u \geq 0$$

4 Geometry

4.1 Geometry Template [86f0f1]

```
using ld = ll;
using pdd = pair<ld, ld>;
#define X first
#define Y second
// ld eps = 1e-7;
```

```
pdd operator+(pdd a, pdd b)
{ return {a.X + b.X, a.Y + b.Y}; }
pdd operator-(pdd a, pdd b)
{ return {a.X - b.X, a.Y - b.Y}; }
pdd operator*(ld i, pdd v)
{ return {i * v.X, i * v.Y}; }
pdd operator*(pdd v, ld i)
{ return {i * v.X, i * v.Y}; }
```

```
pdd operator/(pdd v, ld i)
{ return {v.X / i, v.Y / i}; }
ld dot(pdd a, pdd b)
{ return a.X * b.X + a.Y * b.Y; }
ld cross(pdd a, pdd b)
{ return a.X * b.Y - a.Y * b.X; }
ld abs2(pdd v)
{ return v.X * v.X + v.Y * v.Y; }
ld abs(pdd v)
{ return sqrt(abs2(v)); }
int sgn(ld v)
{ return v > 0 ? 1 : (v < 0 ? -1 : 0); }
// int sgn(ld v){ return v > eps ? 1 : (v < -eps ? -1 : 0); }
int ori(pdd a, pdd b, pdd c)
{ return sgn(cross(b - a, c - a)); }
bool collinearity(pdd a, pdd b, pdd c)
{ return ori(a, b, c) == 0; }
bool btw(pdd p, pdd a, pdd b)
{ return collinearity(p, a, b) && sgn(dot(a - p, b - p)) <= 0; }

bool seg_intersect(pdd p1, pdd p2, pdd p3, pdd p4){
    if(btw(p1, p3, p4) || btw(p2, p3, p4) || btw(p3, p1, p2) || btw(p4, p1, p2))
        return true;
    return ori(p1, p2, p3) * ori(p1, p2, p4) < 0 && ori(p3, p4, p1) * ori(p3, p4, p2) < 0;
}
pdd intersect(pdd p1, pdd p2, pdd p3, pdd p4){
    ld a123 = cross(p2 - p1, p3 - p1);
    ld a124 = cross(p2 - p1, p4 - p1);
    return (p4 * a123 - p3 * a124) / (a123 - a124);
}
pdd perp(pdd p1)
{ return pdd(-p1.Y, p1.X); }
pdd projection(pdd p1, pdd p2, pdd p3)
{ return p1 + (p2 - p1) * dot(p3 - p1, p2 - p1) / abs2(p2 - p1); }
pdd reflection(pdd p1, pdd p2, pdd p3)
{ return p3 + perp(p2 - p1) * cross(p3 - p1, p2 - p1) / abs2(p2 - p1) * 2; }
pdd linearTransformation(pdd p0, pdd p1, pdd q0, pdd q1, pdd r) {
    pdd dp = p1 - p0, dq = q1 - q0, num(cross(dp, dq), dot(dp, dq));
    return q0 + pdd(cross(r - p0, num), dot(r - p0, num)) / abs2(dp);
} // from line p0--p1 to q0--q1, apply to r
```

4.2 Polar Angle Comparator [808e89]

```
// -1: a // b (if same), 0/1: a < b
int cmp(p11 a, p11 b, bool same = true){
    #define is_neg(k) (sgn(k.Y) < 0 || (sgn(k.Y) == 0 && sgn(k.X) < 0))
    int A = is_neg(a), B = is_neg(b);
    if(A != B)
        return A < B;
    if(sgn(cross(a, b)) == 0)
        return same ? abs2(a) < abs2(b) : -1;
    return sgn(cross(a, b)) > 0;
}
```

4.3 Minkowski Sum [b3028c]

```
void reorder_poly(vector<pdd>& pnts){
    int mn = 0;
    for(int i = 1; i < (int)pnts.size(); i++)
        if(pnts[i].Y < pnts[mn].Y || (pnts[i].Y == pnts[mn].Y && pnts[i].X < pnts[mn].X)) mn = i;
    rotate(pnts.begin(), pnts.begin() + mn, pnts.end());
}
vector<pdd> minkowski(vector<pdd> P, vector<pdd> Q){
    reorder_poly(P); reorder_poly(Q);
    int psz = P.size(), qsz = Q.size();
    P.pb(P[0]); P.pb(P[1]); Q.pb(Q[0]); Q.pb(Q[1]);
    vector<pdd> ans; int i = 0, j = 0;
    while(i < psz || j < qsz){
        ans.pb(P[i] + Q[j]);
        int t = sgn(cross(P[i + 1] - P[i], Q[j + 1] - Q[j]));
        if(t >= 0) i++; if(t <= 0) j++;
    }
}
```

```

    return ans;
}

```

4.4 Intersection of Circle and Convex Polygon [63653d]

```

double _area(pdd pa, pdd pb, double r){
    if(abs(pa)<abs(pb)) swap(pa, pb);
    if(abs(pb)<eps) return 0;
    double S, h, theta;
    double a=abs(pb),b=abs(pa),c=abs(pb-pa);
    double cosB = dot(pb,pb-pa) / a / c, B = acos(cosB);
    double cosC = dot(pa,pb) / a / b, C = acos(cosC);
    if(a > r){
        S = (C/2)*r*r;
        h = a*b*sin(C)/c;
        if (h < r && B < PI/2) S -= (acos(h/r)*r*r - h*sqrt(r*r-h*h));
    }
    else if(b > r){
        theta = PI - B - asin(sin(B)/r*a);
        S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
    }
    else S = .5*sin(C)*a*b;
    return S;
}

double areaPolyCircle(const vector<pdd> poly,const pdd
    &o,const double r){
    double S=0;
    for(int i=0;i<SZ(poly);++i)
        S+=_area(poly[i]-o,poly[(i+1)%SZ(poly)]-o,r)*ori(0,
            poly[i],poly[(i+1)%SZ(poly)]);
    return fabs(S);
}

```

4.5 Intersection of Circles [f7a2fe]

```

bool CCinter(Cir &a, Cir &b, pdd &p1, pdd &p2) {
    pdd o1 = a.o, o2 = b.o;
    double r1 = a.R, r2 = b.R, d2 = abs2(o1 - o2), d =
        sqrt(d2);
    if(d < max(r1, r2) - min(r1, r2) || d > r1 + r2)
        return 0;
    pdd u = (o1 + o2) * 0.5 + (o1 - o2) * ((r2 * r2 - r1
        * r1) / (2 * d2));
    double A = sqrt((r1 + r2 + d) * (r1 - r2 + d) * (r1 +
        r2 - d) * (-r1 + r2 + d));
    pdd v = pdd(o1.Y - o2.Y, -o1.X + o2.X) * A / (2 * d2)
        ;
    p1 = u + v, p2 = u - v;
    return 1;
}

```

4.6 Tangent Line of Circles [c51d90]

```

vector<Line> CCTang(const Cir& c1, const Cir& c2,
    int sign1){
    vector<Line> ret;
    double d_sq = abs2(c1.o - c2.o);
    if(sgn(d_sq) == 0) return ret;
    double d = sqrt(d_sq);
    pdd v = (c2.o - c1.o) / d;
    double c = (c1.R - sign1 * c2.R) / d; // cos t
    if(c * c > 1) return ret;
    double h = sqrt(max(0.0, 1.0 - c * c)); // sin t
    for(int sign2 = 1; sign2 >= -1; sign2 -= 2){
        pdd n = pdd(v.X * c - sign2 * h * v.Y,
            v.Y * c + sign2 * h * v.X);
        pdd p1 = c1.o + n * c1.R;
        pdd p2 = c2.o + n * (c2.R * sign1);
        if(sgn(p1.X - p2.X) == 0 and
            sgn(p1.Y - p2.Y) == 0)
            p2 = p1 + perp(c2.o - c1.o);
        ret.pb(Line(p1, p2));
    }
    return ret;
}

```

4.7 Intersection of Line and Convex Polygon [157258]

```

int TangentDir(vector<p11> &C, p11 dir) {
    return cyc_tsearch(SZ(C), [&](int a, int b) {
        return cross(dir, C[a]) > cross(dir, C[b]);
    });
}

```

```

});
}

#define cml(i) sign(cross(C[i] - a, b - a))
pii lineHull(p11 a, p11 b, vector<p11> &C) {
    int A = TangentDir(C, a - b);
    int B = TangentDir(C, b - a);
    int n = SZ(C);
    if(cml(A) < 0 || cml(B) > 0)
        return pii(-1, -1); // no collision
    auto gao = [&](int l, int r) {
        for(int t = l; (l + 1) % n != r; ) {
            int m = ((l + r + (l < r ? 0 : n)) / 2) % n;
            (cml(m) == cml(t) ? l : r) = m;
        }
        return (l + !cml(r)) % n;
    };
    pii res = pii(gao(B, A), gao(A, B)); // (i, j)
    if(res.X == res.Y) // touching the corner i
        return pii(res.X, -1);
    if(!cml(res.X) && !cml(res.Y)) // along side i, i
        +1
        switch((res.X - res.Y + n + 1) % n) {
            case 0: return pii(res.X, res.X);
            case 2: return pii(res.Y, res.Y);
        }
    /* crossing sides (i, i+1) and (j, j+1)
    crossing corner i is treated as side (i, i+1)
    returned in the same order as the line hits the
    convex */
    return res;
} // convex cut: (r, l]

```

4.8 Intersection of Line and Circle [9183db]

```

vector<pdd> circleLineIntersection(pdd c, double r, pdd
    a, pdd b) {
    pdd p = a + (b - a) * dot(c - a, b - a) / abs2(b - a)
        ;
    double s = cross(b - a, c - a), h2 = r * r - s * s /
        abs2(b - a);
    if(sgn(h2) < 0) return {};
    if(sgn(h2) == 0) return {p};
    pdd h = (b - a) / abs(b - a) * sqrt(h2);
    return {p - h, p + h};
}

```

4.9 Point in Circle [ecf954]

```

// return q's relation with circumcircle of tri(p[0],p
    [1],p[2])
bool in_cc(const array<p11, 3> &p, p11 q) {
    __int128 det = 0;
    for(int i = 0; i < 3; ++i)
        det += __int128(abs2(p[i]) - abs2(q)) * cross(p[(i
            + 1) % 3] - q, p[(i + 2) % 3] - q);
    return det > 0; // in: >0, on: =0, out: <0
}

```

4.10 Point in Convex [82b81e]

```

bool PointInConvex(const vector<p11> &C, p11 p, bool
    strict = true) {
    int a = 1, b = SZ(C) - 1, r = !strict;
    if(SZ(C) == 0) return false;
    if(SZ(C) < 3) return r && btw(p, C[0], C.back());
    if(ori(C[0], C[a], C[b]) > 0) swap(a, b);
    if(ori(C[0], C[a], p) >= r || ori(C[0], C[b], p) <=
        -r)
        return false;
    while(abs(a - b) > 1) {
        int c = (a + b) / 2;
        (ori(C[0], C[c], p) > 0 ? b : a) = c;
    }
    return ori(C[a], C[b], p) < r;
}

```

4.11 Half Plane Intersection [dfb833]

```

p11 area_pair(Line a, Line b)
{ return p11(cross(a.Y - a.X, b.X - a.X), cross(a.Y - a
    .X, b.Y - a.X)); }
bool isin(Line l0, Line l1, Line l2) {
    // Check inter(l1, l2) strictly in l0
    auto [a02X, a02Y] = area_pair(l0, l2);
}

```



```

auto [a12X, a12Y] = area_pair(l1, l2);
if (a12X - a12Y < 0) a12X *= -1, a12Y *= -1;
return ((__int128) a02Y * a12X - ((__int128) a02X *
    a12Y > 0; // C^4
}
/* Having solution, check size > 2 */
/* --- Line.X --- Line.Y --- */
vector<Line> halfPlaneInter(vector<Line> arr) {
    sort(iter(arr), [&](Line a, Line b) -> int {
        if (cmp(a.Y - a.X, b.Y - b.X, 0) != -1)
            return cmp(a.Y - a.X, b.Y - b.X, 0);
        return ori(a.X, a.Y, b.Y) < 0;
    });
    deque<Line> dq(1, arr[0]);
    for (auto p : arr) {
        if (cmp(dq.back().Y - dq.back().X, p.Y - p.X, 0) ==
            -1)
            continue;
        while (SZ(dq) >= 2 && !isin(p, dq[SZ(dq) - 2], dq.
            back()))
            dq.pop_back();
        while (SZ(dq) >= 2 && !isin(p, dq[0], dq[1]))
            dq.pop_front();
        dq.pb(p);
    }
    while (SZ(dq) >= 3 && !isin(dq[0], dq[SZ(dq) - 2], dq
        .back()))
        dq.pop_back();
    while (SZ(dq) >= 3 && !isin(dq.back(), dq[0], dq[1]))
        dq.pop_front();
    return vector<Line>(iter(dq));
}

```

4.12 Minimum Enclosing Circle [5af6d5]

```

using ld = long double;
pair<pdd, ld> circumcenter(pdd a, pdd b, pdd c);
pair<pdd, ld> MinimumEnclosingCircle(vector<pdd> &pts){
    random_shuffle(iter(pts));
    pdd c = pts[0];
    ld r = 0;
    for(int i = 1; i < SZ(pts); i++){
        if(abs(pts[i] - c) <= r) continue;
        c = pts[i]; r = 0;
        for(int j = 0; j < i; j++){
            if(abs(pts[j] - c) <= r) continue;
            c = (pts[i] + pts[j]) / 2;
            r = abs(pts[i] - c);
            for(int k = 0; k < j; k++){
                if(abs(pts[k] - c) > r)
                    tie(c, r) = circumcenter(pts[i], pts[j], pts[
                        k]);
            }
        }
    }
    return {c, r};
}

```

4.13 3D Point [badbbd]

```

struct Point {
    double x, y, z;
    Point(double _x = 0, double _y = 0, double _z = 0): x
        (_x), y(_y), z(_z){}
    Point(pdd p) { x = p.X, y = p.Y, z = abs2(p); }
};
Point operator-(Point p1, Point p2)
{ return Point(p1.x - p2.x, p1.y - p2.y, p1.z - p2.z);
}
Point operator+(Point p1, Point p2)
{ return Point(p1.x + p2.x, p1.y + p2.y, p1.z + p2.z);
}
Point operator*(Point p1, double v)
{ return Point(p1.x * v, p1.y * v, p1.z * v); }
Point operator/(Point p1, double v)
{ return Point(p1.x / v, p1.y / v, p1.z / v); }
Point cross(Point p1, Point p2)
{ return Point(p1.y * p2.z - p1.z * p2.y, p1.z * p2.x -
    p1.x * p2.z, p1.x * p2.y - p1.y * p2.x); }
double dot(Point p1, Point p2)
{ return p1.x * p2.x + p1.y * p2.y + p1.z * p2.z; }
double abs(Point a)
{ return sqrt(dot(a, a)); }

```

```

Point cross3(Point a, Point b, Point c)
{ return cross(b - a, c - a); }
double area(Point a, Point b, Point c)
{ return abs(cross3(a, b, c)); }
double volume(Point a, Point b, Point c, Point d)
{ return dot(cross3(a, b, c), d - a); }
//Azimuthal angle (Longitude) to x-axis in interval [-
    pi, pi]
double phi(Point p) { return atan2(p.y, p.x); }
//Zenith angle (Latitude) to the z-axis in interval [0,
    pi]
double theta(Point p) { return atan2(sqrt(p.x * p.x + p
    .y * p.y), p.z); }
Point masscenter(Point a, Point b, Point c, Point d)
{ return (a + b + c + d) / 4; }
pdd proj(Point a, Point b, Point c, Point u) {
    // proj. u to the plane of a, b, and c
    Point e1 = b - a;
    Point e2 = c - a;
    e1 = e1 / abs(e1);
    e2 = e2 - e1 * dot(e2, e1);
    e2 = e2 / abs(e2);
    Point p = u - a;
    return pdd(dot(p, e1), dot(p, e2));
}
Point rotate_around(Point p, double angle, Point axis)
{
    double s = sin(angle), c = cos(angle);
    Point u = axis / abs(axis);
    return u * dot(u, p) * (1 - c) + p * c + cross(u, p)
        * s;
}

```

4.14 ConvexHull3D [156311]

```

struct convex_hull_3D {
    struct Face {
        int a, b, c;
        Face(int ta, int tb, int tc): a(ta), b(tb), c(tc) {}
    }; // return the faces with pt indexes
    vector<Face> res;
    vector<Point> P;
    convex_hull_3D(const vector<Point> &_P): res(), P(_P) {
        // all points coplanar case will WA, O(n^2)
        int n = SZ(P);
        if (n <= 2) return; // be careful about edge case
        // ensure first 4 points are not coplanar
        swap(P[1], *find_if(iter(P), [&](auto p) { return sgn
            (abs2(P[0] - p)) != 0; }));
        swap(P[2], *find_if(iter(P), [&](auto p) { return sgn
            (abs2(cross3(p, P[0], P[1]))) != 0; }));
        swap(P[3], *find_if(iter(P), [&](auto p) { return sgn
            (volume(P[0], P[1], P[2], p)) != 0; }));
        vector<vector<int>> flag(n, vector<int>(n));
        res.emplace_back(0, 1, 2); res.emplace_back(2, 1, 0);
        for (int i = 3; i < n; ++i) {
            vector<Face> next;
            for (auto f : res) {
                int d = sgn(volume(P[f.a], P[f.b], P[f.c], P[i]))
                    ;
                if (d <= 0) next.pb(f);
                int ff = (d > 0) - (d < 0);
                flag[f.a][f.b] = flag[f.b][f.c] = flag[f.c][f.a]
                    = ff;
            }
            for (auto f : res) {
                auto F = [&](int x, int y) {
                    if (flag[x][y] > 0 && flag[y][x] <= 0)
                        next.emplace_back(x, y, i);
                };
                F(f.a, f.b); F(f.b, f.c); F(f.c, f.a);
            }
            res = next;
        }
    }
    bool same(Face s, Face t) {
        if (sgn(volume(P[s.a], P[s.b], P[s.c], P[t.a])) != 0)
            return 0;
        if (sgn(volume(P[s.a], P[s.b], P[s.c], P[t.b])) != 0)
            return 0;
        if (sgn(volume(P[s.a], P[s.b], P[s.c], P[t.c])) != 0)
            return 0;
        return 1;
    }
}

```

```

}
int polygon_face_num() {
    int ans = 0;
    for (int i = 0; i < SZ(res); ++i)
        ans += none_of(res.begin(), res.begin() + i, [&](
            Face g) { return same(res[i], g); });
    return ans;
}
double get_volume() {
    double ans = 0;
    for (auto f : res)
        ans += volume(Point(0, 0, 0), P[f.a], P[f.b], P[f.c
        ]);
    return fabs(ans / 6);
}
double get_dis(Point p, Face f) {
    Point p1 = P[f.a], p2 = P[f.b], p3 = P[f.c];
    double a = (p2.y - p1.y) * (p3.z - p1.z) - (p2.z - p1
        .z) * (p3.y - p1.y);
    double b = (p2.z - p1.z) * (p3.x - p1.x) - (p2.x - p1
        .x) * (p3.z - p1.z);
    double c = (p2.x - p1.x) * (p3.y - p1.y) - (p2.y - p1
        .y) * (p3.x - p1.x);
    double d = 0 - (a * p1.x + b * p1.y + c * p1.z);
    return fabs(a * p.x + b * p.y + c * p.z + d) / sqrt(a
        * a + b * b + c * c);
}
};
// n^2 delaunay: facets with negative z normal of
// convexhull of (x, y, x^2 + y^2), use a pseudo-point
// (0, 0, inf) to avoid degenerate case
4.15 Delaunay Triangulation [6a9916]
/* Delaunay Triangulation:
   Given a sets of points on 2D plane, find a
   triangulation such that no points will strictly
   inside circumcircle of any triangle. */
struct Edge {
    int id; // oidx[id]
    list<Edge>::iterator twin;
    Edge(int _id = 0):id(_id) {}
};
struct Delaunay { // 0-base
    int n;
    vector<int> oidx;
    vector<list<Edge>> head; // result udir. graph
    vector<p11> p;
    Delaunay(int _n, vector<p11> _p): n(_n), oidx(n),
        head(n), p(n) {
        iota(iter(oidx), 0);
        for (int i = 0; i < n; ++i) head[i].clear();
        sort(iter(oidx), [&](int a, int b)
            { return _p[a] < _p[b]; });
        for (int i = 0; i < n; ++i) p[i] = _p[oidx[i]];
        divide(0, n - 1);
    }
    void addEdge(int u, int v) {
        head[u].push_front(Edge(v));
        head[v].push_front(Edge(u));
        head[u].begin()->twin = head[v].begin();
        head[v].begin()->twin = head[u].begin();
    }
    void divide(int l, int r) {
        if (l == r) return;
        if (l + 1 == r) return addEdge(l, l + 1);
        int mid = (l + r) >> 1, nw[2] = {l, r};
        divide(l, mid), divide(mid + 1, r);
        auto gao = [&](int t) {
            p11 pt[2] = {p[nw[0]], p[nw[1]]};
            for (auto it : head[nw[t]]) {
                int v = ori(pt[1], pt[0], p[it.id]);
                if (v > 0 || (v == 0 && abs2(pt[t ^ 1] - p[it
                    .id]) < abs2(pt[1] - pt[0])))
                    return nw[t] = it.id, true;
            }
            return false;
        };
        while (gao(0) || gao(1));
        addEdge(nw[0], nw[1]); // add tangent
        while (true) {
            p11 pt[2] = {p[nw[0]], p[nw[1]]};
            int ch = -1, sd = 0;

```

```

            for (int t = 0; t < 2; ++t)
                for (auto it : head[nw[t]])
                    if (ori(pt[0], pt[1], p[it.id]) > 0 && (ch ==
                        -1 || in_cc({pt[0], pt[1], p[ch]}, p[it
                            .id])))
                        ch = it.id, sd = t;
            if (ch == -1) break; // upper common tangent
            for (auto it = head[nw[sd]].begin(); it != head[
                nw[sd]].end(); )
                if (seg_strict_intersect(pt[sd], p[it->id], pt[
                    sd ^ 1], p[ch]))
                    head[it->id].erase(it->twin), head[nw[sd]].
                        erase(it++);
            else ++it;
            nw[sd] = ch, addEdge(nw[0], nw[1]);
        }
    }
};

```

4.16 Voronoi Diagram [e4f408]

```

// all coord. is even, you may want to call
// halfPlaneInter after then
vector<vector<Line>> vec;
void build_voronoi_line(int n, vector<p11> &pts) {
    Delaunay tool(n, pts); // Delaunay
    vec.clear(), vec.resize(n);
    for (int i = 0; i < n; ++i)
        for (auto e : tool.head[i]) {
            int u = tool.oidx[i], v = tool.oidx[e.id];
            p11 m = (pts[v] + pts[u]) / 2LL, d = perp(pts[v]
                - pts[u]);
            vec[u].pb(Line(m, m + d));
        }
}

```

4.17 Polygon Union [9fbf66]

```

ld rat(p11 a, p11 b) {
    return sgn(b.X) ? (ld)a.X / b.X : (ld)a.Y / b.Y;
} // all poly. should be ccw
ld polyUnion(vector<vector<p11>> &poly) {
    ld res = 0;
    for (auto &p : poly)
        for (int a = 0; a < SZ(p); ++a) {
            p11 A = p[a], B = p[(a + 1) % SZ(p)];
            vector<pair<ld, int>> segs = {{0, 0}, {1, 0}};
            for (auto &q : poly) {
                if (&p == &q) continue;
                for (int b = 0; b < SZ(q); ++b) {
                    p11 C = q[b], D = q[(b + 1) % SZ(q)];
                    int sc = ori(A, B, C), sd = ori(A, B, D);
                    if (sc != sd && min(sc, sd) < 0) {
                        ld sa = cross(D - C, A - C), sb = cross(D -
                            C, B - C);
                        segs.pb(sa / (sa - sb), sgn(sc - sd));
                    }
                    if (!sc && !sd && &q < &p && sgn(dot(B - A, D
                        - C)) > 0) {
                        segs.pb(rat(C - A, B - A), 1);
                        segs.pb(rat(D - A, B - A), -1);
                    }
                }
            }
            sort(iter(segs));
            for (auto &s : segs) s.X = clamp(s.X, 0.0, 1.0);
            ld sum = 0;
            int cnt = segs[0].second;
            for (int j = 1; j < SZ(segs); ++j) {
                if (!cnt) sum += segs[j].X - segs[j - 1].X;
                cnt += segs[j].Y;
            }
            res += cross(A, B) * sum;
        }
    return res / 2;
}

```

4.18 Tangent Point to Convex Hull [523bc1]

```

/* The point should be strictly out of hull
   return arbitrary point on the tangent line */
pii get_tangent(vector<p11> &C, p11 p) {
    auto gao = [&](int s) {

```

```

    return cyc_tsearch(SZ(C), [&](int x, int y)
    { return ori(p, C[x], C[y]) == s; });
};
return pii(gao(1), gao(-1));
} // return (a, b), ori(p, C[a], C[b]) >= 0

```

4.19 Heart [082d19]

```

pdd circenter(pdd p0, pdd p1, pdd p2) { // 156d1f
    p1 = p1 - p0, p2 = p2 - p0; // radius = abs(center)
    double x1 = p1.X, y1 = p1.Y, x2 = p2.X, y2 = p2.Y;
    double m = 2. * (x1 * y2 - y1 * x2);
    pdd center;
    center.X = (x1 * x1 * y2 - x2 * x2 * y1 + y1 * y2 * (
        y1 - y2)) / m;
    center.Y = (x1 * x2 * (x2 - x1) - y1 * y1 * x2 + x1 *
        y2 * y2) / m;
    return center + p0;
}
pdd incenter(pdd p1, pdd p2, pdd p3) { // radius = area
    / s * 2
    double a = abs(p2 - p3), b = abs(p1 - p3), c = abs(p1
        - p2);
    double s = a + b + c;
    return (a * p1 + b * p2 + c * p3) / s;
}
pdd masscenter(pdd p1, pdd p2, pdd p3)
{ return (p1 + p2 + p3) / 3; }
pdd orthcenter(pdd p1, pdd p2, pdd p3)
{ return masscenter(p1, p2, p3) * 3 - circenter(p1, p2,
    p3) * 2; }

```

4.20 Rotating Sweep Line [f5f689]

```

struct Event {
    pll d; int u, v;
    bool operator<(const Event &b) const {
        int ret = cmp(d, b.d, false);
        return ret == -1 ? false : ret; } // no tie-break
};
void rotatingSweepLine(const vector<pll> &p) {
    const int n = SZ(p);
    vector<Event> e; e.reserve(n * (n - 1));
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++) // pos[i] < pos[j] when
            // the event occurs
            if (i != j) e.pb(p[j] - p[i], i, j);
    sort(iter(e));
    vector<int> ord(n), pos(n);
    iota(iter(ord), 0);
    sort(iter(ord), [&](int i, int j) { // initial order
        return p[i].Y != p[j].Y ? p[i].Y < p[j].Y : p[i].
            X < p[j].X; });
    for (int i = 0; i < n; i++) pos[ord[i]] = i;
    // initialize
    for (int i = 0, j = 0; i < SZ(e); i = j) {
        // do something
        vector<pii> tmp;
        for (; j < SZ(e) && !(e[i] < e[j]); j++)
            tmp.pb(pii(e[j].u, e[j].v));
        sort(iter(tmp), [&](pii x, pii y){
            return pii(pos[x.ff], pos[x.ss]) < pii(pos[y.ff]
                , pos[y.ss]); });
        for (auto [x, y] : tmp) // pos[x] + 1 == pos[y]
            tie(ord[pos[x]], ord[pos[y]], pos[x], pos[y]) =
                make_tuple(ord[pos[y]], ord[pos[x]], pos[y],
                    pos[x]);
    }
}

```

4.21 Vector In Poly [c6d0fa]

```

// ori(a, b, c) >= 0, valid: "strict" angle from a-b to
// a-c
bool btwangle(pll a, pll b, pll c, pll p, int strict) {
    return ori(a, b, p) >= strict && ori(a, p, c) >=
        strict;
}
// whether vector{cur, p} in counter-clockwise order
// prv, cur, nxt
bool inside(pll prv, pll cur, pll nxt, pll p, int
    strict) {
    if (ori(cur, nxt, prv) >= 0)

```

```

    return btwangle(cur, nxt, prv, p, strict);
    return !btwangle(cur, prv, nxt, p, !strict);
}

```

4.22 Convex Hull DP [6dc001]

```

sort(iter(pts), [&](pll x, pll y) {
    return x.Y != y.Y ? x.Y < y.Y : x.X < y.X;
});
auto getvec = [&](pii x) { return pts[x.ss] - pts[x.ff]
    }; };
vector<pii> trans;
for (int j = 0; j < n; j++)
    for (int k = 0; k < n; k++)
        if (j != k) trans.pb(pii(j, k));
sort(iter(trans), [&](pii x, pii y) -> bool{
    int tmp = cmp(getvec(x), getvec(y), false);
    if (tmp != -1) return tmp;
    pll v = getvec(x);
    return dot(v, pts[x.ff]) > dot(v, pts[y.ff]);
});
// DP for convex hull vertices (no points on edges)
auto solve = [&](int bottom) { // O(n^3)
    // vector<ll> dp(n);
    for (int j = bottom + 1; j < n; j++) {
        // check whether bottom -> j is legal
        // init trans -> j
    }
    for (auto [i, j] : trans) {
        if (i <= bottom || j <= bottom ||
            ori(pts[bottom], pts[i], pts[j]) <= 0) continue
        ;
        // check whether i -> j is legal
        // normal trans i -> j
    }
    for (int j = bottom + 1; j < n; j++) {
        // check whether j -> bottom is legal
        // end trans j ->
    }
};
for (int i = 0; i < n; i++) solve(i);

```

4.23 Calculate Points in Triangle [bf746f]

```

// all points are distinct
// cnt[i][j] = # of point k s.t. strictly above ij, and
// i < k < j
// cnt2[i][j] = # of points k s.t. strictly in ij
// preprocess space: O(n^2), time: O(n^3), query time:
// O(1)
vector cnt(n, vector<int>(n)), cnt2(n, vector<int>(n));
for (int i = 0; i < n; i++)
    for (int j = 0; j < n; j++) {
        if (pts[i] >= pts[j]) continue;
        for (int k = 0; k < n; k++) {
            if (pts[i] < pts[k] && pts[k] < pts[j]) {
                int tmp = ori(pts[i], pts[j], pts[k]);
                if (tmp > 0) cnt[i][j]++; // only for i < j
                else if (tmp == 0) cnt2[i][j]++, cnt2[j][i]++;
            }
        }
    }
auto calc_tri = [&](array<int, 3> arr) { // strictly
    inside
    sort(iter(arr), [&](int x, int y) { return pts[x] <
        pts[y]; });
    auto [x, y, z] = arr;
    int tmp = ori(pts[x], pts[y], pts[z]);
    if (tmp == 0) return 0;
    else if (tmp < 0)
        return cnt[x][z] - cnt[x][y] - cnt[y][z] - cnt2[x][
            y] - cnt2[y][z] - 1;
    else return cnt[x][y] + cnt[y][z] - cnt[x][z] - cnt2[
        x][z];
};

```

5 Graph

5.1 BCC [d04ebe]

```

struct BCC { // 0-based, allow multi edges but not allow
    loops
    int n, m, cnt = 0;

```

```

// n:|V|, m:|E|, cnt:#bcc
// bcc i : vertices bcc_v[i] and edges bcc_e[i]
vector<vector<int>> bcc_v, bcc_e;
vector<vector<pii>> g; // original graph
vector<pii> edges; // 0-based
BCC(int _n, vector<pii> _edges):
n(_n), m(SZ(_edges)), g(_n), edges(_edges){
    for(int i = 0; i < m; i++){
        auto [u, v] = edges[i];
        g[u].pb(pii(v, i)); g[v].pb(pii(u, i));
    }
}
void make_bcc(){ bcc_v.pb(); bcc_e.pb(); cnt++; }
// modify these if you need more information
void add_v(int v){ bcc_v.back().pb(v); }
void add_e(int e){ bcc_e.back().pb(e); }
void build(){
    vector<int> in(n, -1), low(n, -1), stk;
    vector<vector<int>> up(n);
    int ts = 0;
    auto _dfs = [&](auto dfs, int now, int par, int pe)
        -> void{
        if(pe != -1) up[now].pb(pe);
        in[now] = low[now] = ts++;
        stk.pb(now);
        for(auto [v, e] : g[now]){
            if(e == pe) continue;
            if(in[v] != -1){
                if(in[v] < in[now]) up[now].pb(e);
                low[now] = min(low[now], in[v]);
                continue;
            }
            dfs(dfs, v, now, e);
            low[now] = min(low[now], low[v]);
        }
        if((now != par && low[now] >= in[par]) || (now ==
            par && SZ(g[now]) == 0)){
            make_bcc();
            for(int v = stk.back(); v = stk.back()){
                stk.pop_back(), add_v(v);
                for(int e : up[v]) add_e(e);
                if(v == now) break;
            }
            if(now != par) add_v(par);
        }
    };
    for(int i = 0; i < n; i++){
        if(in[i] == -1) _dfs(_dfs, i, i, -1);
    }
};

```

5.2 SCC [2c9a01]

```

struct SCC{ // 0-based, output reversed topo order
    int n, cnt = 0;
    vector<vector<int>> g;
    vector<int> sccid;
    explicit SCC(int _n): n(_n), g(n), sccid(n, -1) {}
    void add_edge(int u, int v){
        g[u].pb(v);
    }
    void build(){
        vector<int> in(n, -1), low(n), stk;
        vector<bool> instk(n);
        int ts = 0;
        auto dfs1 = [&](auto dfs, int now) -> void{
            stk.pb(now); instk[now] = true;
            in[now] = low[now] = ts++;
            for(int i : g[now]){
                if(in[i] == -1)
                    dfs(dfs, i), low[now] = min(low[now], low[i]);
                else if(instk[i] && in[i] < in[now])
                    low[now] = min(low[now], in[i]);
            }
            if(low[now] == in[now]){
                for(; stk.back() != now; stk.pop_back())
                    sccid[stk.back()] = cnt, instk[stk.back()] = false;
                sccid[now] = cnt++, instk[now] = false, stk.
                    pop_back();
            }
        };
    };
};

```

```

for(int i = 0; i < n; i++){
    if(in[i] == -1) dfs1(dfs1, i);
}
};

```

5.3 2-SAT [0686a5]

```

struct SAT { // 0-based
    int n;
    vector<bool> istrue;
    SCC scc;
    SAT(int _n): n(_n), istrue(n + n), scc(n + n) {}
    int neg(int a) {
        return a >= n ? a - n : a + n;
    }
    void add_clause(int a, int b) {
        scc.add_edge(neg(a), b), scc.add_edge(neg(b), a);
    }
    bool solve() {
        scc.build();
        for (int i = 0; i < n; ++i) {
            if (scc.sccid[i] == scc.sccid[i + n]) return false;
            istrue[i] = scc.sccid[i] < scc.sccid[i + n];
            istrue[i + n] = !istrue[i];
        }
        return true;
    }
};

```

5.4 Dominator Tree [2da9bb]

```

struct Dominator {
    int n;
    vector<vector<int>> g, r, rdom; int tk;
    vector<int> dfn, rev, fa, sdom, dom, val, rp;
    Dominator(int _n) : n(_n), g(n), r(n), rdom(n), tk(0)
    {
        dfn = rev = fa = sdom = dom =
            val = rp = vector<int>(n, -1);
    }
    void add_edge(int x, int y) { g[x].push_back(y); }
    void dfs(int x) {
        rev[dfn[x] = tk] = x;
        fa[tk] = sdom[tk] = val[tk] = tk; tk++;
        for (int u : g[x]) {
            if (dfn[u] == -1) dfs(u), rp[dfn[u]] = dfn[x];
            r[dfn[u]].push_back(dfn[x]);
        }
    }
    void merge(int x, int y) { fa[x] = y; }
    int find(int x, int c = 0) {
        if (fa[x] == x) return c ? -1 : x;
        if (int p = find(fa[x], 1); p != -1) {
            if (sdom[val[x]] > sdom[val[fa[x]])
                val[x] = val[fa[x]];
            fa[x] = p;
            return c ? p : val[x];
        } else return c ? fa[x] : val[x];
    }
    vector<int> build(int s) {
        // return the father of each node in dominator tree
        dfs(s); // p[i] = -2 if i is unreachable, par[s] = -1
        for (int i = tk - 1; i >= 0; --i) {
            for (int u : r[i])
                sdom[i] = min(sdom[i], sdom[find(u)]);
            if (i) rdom[sdom[i]].push_back(i);
            for (int u : rdom[i]) {
                int p = find(u);
                dom[u] = (sdom[p] == i ? i : p);
            }
            if (i) merge(i, rp[i]);
        }
        vector<int> p(n, -2); p[s] = -1;
        for (int i = 1; i < tk; ++i)
            if (sdom[i] != dom[i]) dom[i] = dom[dom[i]];
        for (int i = 1; i < tk; ++i)
            p[rev[i]] = rev[dom[i]];
        return p;
    }
};

```

5.5 Virtual Tree [6abeb5]

```
vector<int> vG[N];
int top, st[N];
int vrt = -1;
void insert(int u) {
    if (top == -1) return st[++top] = vrt = u, void();
    int p = LCA(st[top], u);
    if (dep[vrt] > dep[p]) vrt = p;
    if (p == st[top]) return st[++top] = u, void();
    while (top >= 1 && dep[st[top - 1]] >= dep[p])
        vG[st[top - 1]].pb(st[top]), --top;
    if (st[top] != p)
        vG[p].pb(st[top]), --top, st[++top] = p;
    st[++top] = u;
}
void reset(int u) {
    for (int i : vG[u]) reset(i);
    vG[u].clear();
}
void solve(vector<int> &v) {
    top = -1;
    sort(iter(v),
        [&](int a, int b) { return dfn[a] < dfn[b]; });
    for (int i : v) insert(i);
    while (top > 0) vG[st[top - 1]].pb(st[top]), --top;
    // do something
    reset(vrt);
}
```

5.6 Fast DMST [7b274d]

```
struct E { int s, t; ll w; }; // 0-base
struct PQ {
    struct P {
        ll v; int i;
        bool operator>(const P &b) const { return v > b.v; }
    };
    priority_queue<P, vector<P>, greater<>> pq; ll tag;
    // min heap
    void push(P p) { p.v -= tag; pq.emplace(p); }
    P top() { P p = pq.top(); p.v += tag; return p; }
    void join(PQ &b) {
        if (pq.size() < b.pq.size())
            swap(pq, b.pq), swap(tag, b.tag);
        while (!b.pq.empty()) push(b.top()), b.pq.pop();
    }
}; // O(E log^2 V), use leftist tree for O(E log V)
vector<int> dmst(const vector<E> &e, int n, int root) {
    vector<PQ> h(n * 2);
    for (int i = 0; i < int(e.size()); ++i)
        h[e[i].t].push({e[i].w, i});
    vector<int> a(n * 2); iota(iter(a), 0);
    vector<int> v(n * 2, -1), pa(n * 2, -1), r(n * 2);
    auto o = [&](auto Y, int x) -> int {
        return x == a[x] ? x : a[x] = Y(Y, a[x]);
    };
    auto S = [&](int i) { return o(o, e[i].s); };
    int pc = v[root] = n;
    for (int i = 0; i < n; ++i) if (v[i] == -1)
        for (int p = i; v[p] < 0 || v[p] == i; p = S(r[p])) {
            if (v[p] == i)
                for (int q = pc++; p != q; p = S(r[p])) {
                    h[p].tag -= h[p].top().v; h[q].join(h[p]);
                    pa[p] = a[p] = q;
                }
            while (S(h[p].top().i) == p) h[p].pq.pop();
            v[p] = i; r[p] = h[p].top().i;
        }
    vector<int> ans;
    for (int i = pc - 1; i >= 0; i--) if (v[i] != n) {
        for (int f = e[r[i]].t; f != -1 && v[f] != n; f = pa[f])
            v[f] = n;
        ans.push_back(r[i]);
    }
    return ans; // default minimize, returns edgeid array
}
```

5.7 Vizing [58a6ca]

// find $D+1$ edge coloring of a graph with max deg D , $O(nm)$

```
struct Vizing { // returns maxdeg+1 edge coloring in
    adjacent matrix G
    int n; // 1-based for vertices and colors, simple
    graph
    vector<vector<int>> C, G;
    vector<int> X, vst;
    Vizing(int _n): n(_n),
        C(n + 1, vector<int>(n + 2)), G(n + 1, vector<int>(n
            + 1)),
        X(n + 1, 1), vst(n + 1) {}
    void solve(vector<pii> &E) {
        auto update = [&](int u)
            { for (X[u] = 1; C[u][X[u]]; ++X[u]); };
        auto color = [&](int u, int v, int c) {
            int p = G[u][v];
            G[u][v] = G[v][u] = c;
            C[u][c] = v, C[v][c] = u;
            C[u][p] = C[v][p] = 0;
            if (p) X[u] = X[v] = p;
            else update(u), update(v);
            return p;
        };
        auto flip = [&](int u, int c1, int c2) {
            int p = C[u][c1];
            swap(C[u][c1], C[u][c2]);
            if (p) G[u][p] = G[p][u] = c2;
            if (!C[u][c1]) X[u] = c1;
            if (!C[u][c2]) X[u] = c2;
            return p;
        };
        for (int t = 0; t < SZ(E); ++t) {
            int u = E[t].ff, v0 = E[t].ss, v = v0, c0 = X[u],
                c = c0, d;
            vector<pii> L;
            fill(iter(vst), 0);
            while (!G[u][v0]) {
                L.emplace_back(v, d = X[v]);
                if (!C[v][c]) for (int a = SZ(L) - 1; a >= 0;
                    --a) c = color(u, L[a].ff, c);
                else if (!C[u][d]) for (int a = SZ(L) - 1; a >=
                    0; --a) color(u, L[a].ff, L[a].ss);
                else if (vst[d]) break;
                else vst[d] = 1, v = C[u][d];
            }
            if (!G[u][v0]) {
                for (; v; v = flip(v, c, d), swap(c, d));
                if (int a; C[u][c0]) {
                    for (a = SZ(L) - 2; a >= 0 && L[a].ss != c;
                        --a);
                    for (; a >= 0; --a) color(u, L[a].ff, L[a].ss
                        );
                }
                else --t;
            }
        }
    }
};
```

5.8 Maximum Clique [1ad4b2]

```
struct MaxClique { // fast when N <= 100
    bitset<N> G[N], cs[N];
    int ans, sol[N], q, cur[N], d[N], n;
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) G[i].reset();
    }
    void add_edge(int u, int v) {
        G[u][v] = G[v][u] = 1;
    }
    void pre_dfs(vector<int> &r, int l, bitset<N> mask) {
        if (l < 4) {
            for (int i : r) d[i] = (G[i] & mask).count();
            sort(iter(r), [&](int x, int y) { return d[x] > d
                [y]; });
        }
        vector<int> c(SZ(r));
        int lft = max(ans - q + 1, 1), rgt = 1, tp = 0;
        cs[1].reset(), cs[2].reset();
        for (int p : r) {
            int k = 1;
            while ((cs[k] & G[p]).any()) ++k;
            if (k > rgt) cs[++rgt + 1].reset();
        }
    }
};
```



```

    cs[k][p] = 1;
    if (k < lft) r[tp++] = p;
}
for (int k = lft; k <= rgt; ++k)
    for (int p = cs[k]._Find_first(); p < N; p = cs[k]._Find_next(p))
        r[tp] = p, c[tp] = k, ++tp;
dfs(r, c, l + 1, mask);
}
void dfs(vector<int> &r, vector<int> &c, int l,
        bitset<N> mask) {
    while (!r.empty()) {
        int p = r.back();
        r.pop_back(), mask[p] = 0;
        if (q + c.back() <= ans) return;
        cur[q++] = p;
        vector<int> nr;
        for (int i : r) if (G[p][i]) nr.pb(i);
        if (!nr.empty()) pre_dfs(nr, l, mask & G[p]);
        else if (q > ans) ans = q, copy_n(cur, q, sol);
        c.pop_back(), --q;
    }
}
int solve() {
    vector<int> r(n);
    ans = q = 0, iota(iter(r), 0);
    pre_dfs(r, 0, bitset<N>(string(n, '1')));
    return ans;
}
};

```

5.9 Number of Maximal Clique [11fa26]

```

struct BronKerbosch { // 1-base
    int n, a[N], g[N][N];
    int S, all[N][N], some[N][N], none[N][N];
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            for (int j = 1; j <= n; ++j) g[i][j] = 0;
    }
    void add_edge(int u, int v) {
        g[u][v] = g[v][u] = 1;
    }
    void dfs(int d, int an, int sn, int nn) {
        if (S > 1000) return; // pruning
        if (sn == 0 && nn == 0) ++S;
        int u = some[d][0];
        for (int i = 0; i < sn; ++i) {
            int v = some[d][i];
            if (g[u][v]) continue;
            int tsu = 0, tnn = 0;
            copy_n(all[d], an, all[d + 1]);
            all[d + 1][an] = v;
            for (int j = 0; j < sn; ++j)
                if (g[v][some[d][j]])
                    some[d + 1][tsu++] = some[d][j];
            for (int j = 0; j < nn; ++j)
                if (g[v][none[d][j]])
                    none[d + 1][tnn++] = none[d][j];
            dfs(d + 1, an + 1, tsu, tnn);
            some[d][i] = 0, none[d][nn++] = v;
        }
    }
    int solve() {
        iota(some[0], some[0] + n, 1);
        S = 0, dfs(0, 0, n, 0);
        return S;
    }
};

```

5.10 Minimum Mean Cycle [3e5d2b]

```

ll road[N][N]; // input here
struct MinimumMeanCycle {
    ll dp[N + 5][N], n;
    pll solve() {
        ll a = -1, b = -1, L = n + 1;
        for (int i = 2; i <= L; ++i)
            for (int k = 0; k < n; ++k)
                for (int j = 0; j < n; ++j)
                    dp[i][j] =
                        min(dp[i - 1][k] + road[k][j], dp[i][j]);
    }
};

```

```

for (int i = 0; i < n; ++i) {
    if (dp[L][i] >= INF) continue;
    ll ta = 0, tb = 1;
    for (int j = 1; j < n; ++j)
        if (dp[j][i] < INF &&
            ta * (L - j) < (dp[L][i] - dp[j][i]) * tb)
            ta = dp[L][i] - dp[j][i], tb = L - j;
    if (ta == 0) continue;
    if (a == -1 || a * tb > ta * b) a = ta, b = tb;
}
if (a != -1) {
    ll g = __gcd(a, b);
    return pll(a / g, b / g);
}
return pll(-1LL, -1LL);
}
void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j) dp[i + 2][j] = INF;
}
};

```

5.11 Minimum Steiner Tree [21acea]

```

// O(V 3^T + V^2 2^T)
struct SteinerTree { // 0-base
    static const int T = 10, N = 105, INF = 1e9;
    int n, dst[N][N], dp[1 << T][N], tdst[N];
    int vcost[N]; // the cost of vertex
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) {
            for (int j = 0; j < n; ++j) dst[i][j] = INF;
            dst[i][i] = vcost[i] = 0;
        }
    }
    void add_edge(int ui, int vi, int wi) {
        dst[ui][vi] = min(dst[ui][vi], wi);
    }
    void shortest_path() {
        for (int k = 0; k < n; ++k)
            for (int i = 0; i < n; ++i)
                for (int j = 0; j < n; ++j)
                    dst[i][j] =
                        min(dst[i][j], dst[i][k] + dst[k][j]);
    }
    int solve(const vector<int> &ter) {
        shortest_path();
        int t = SZ(ter);
        for (int i = 0; i < (1 << t); ++i)
            for (int j = 0; j < n; ++j) dp[i][j] = INF;
        for (int i = 0; i < n; ++i) dp[0][i] = vcost[i];
        for (int msk = 1; msk < (1 << t); ++msk) {
            if (!(msk & (msk - 1))) {
                int who = __lg(msk);
                for (int i = 0; i < n; ++i)
                    dp[msk][i] =
                        vcost[ter[who]] + dst[ter[who]][i];
            }
            for (int i = 0; i < n; ++i)
                for (int submsk = (msk - 1) & msk; submsk;
                    submsk = (submsk - 1) & msk)
                    dp[msk][i] = min(dp[msk][i],
                        dp[submsk][i] + dp[msk ^ submsk][i] -
                        vcost[i]);
            for (int i = 0; i < n; ++i) {
                tdst[i] = INF;
                for (int j = 0; j < n; ++j)
                    tdst[i] =
                        min(tdst[i], dp[msk][j] + dst[j][i]);
            }
            for (int i = 0; i < n; ++i) dp[msk][i] = tdst[i];
        }
        int ans = INF;
        for (int i = 0; i < n; ++i)
            ans = min(ans, dp[(1 << t) - 1][i]);
        return ans;
    }
};

```

5.12 Count Cycles [c7e8f2]

```
// ord = sort by deg decreasing, rk[ord[i]] = i
// D[i] = edge point from rk small to rk big
for (int x : ord) { // c3
    for (int y : D[x]) vis[y] = 1;
    for (int y : D[x]) for (int z : D[y]) c3 += vis[z];
    for (int y : D[x]) vis[y] = 0;
}
for (int x : ord) { // c4
    for (int y : D[x]) for (int z : adj[y])
        if (rk[z] > rk[x]) c4 += vis[z]++;
    for (int y : D[x]) for (int z : adj[y])
        if (rk[z] > rk[x]) --vis[z];
} // both are O(M*sqrt(M))
```

6 Math

6.1 Extended Euclidean Algorithm [c51ae9]

```
// ax+ny = 1, ax+ny == ax == 1 (mod n)
void extgcd(ll x, ll y, ll &g, ll &a, ll &b) {
    if (y == 0) g = x, a = 1, b = 0;
    else extgcd(y, x % y, g, b, a), b -= (x / y) * a;
}
```

6.2 Floor & Ceil [134881]

```
ll ifloor(ll a, ll b){
    return a / b - (a % b && (a < 0) ^ (b < 0));
}
ll iceil(ll a, ll b){
    return a / b + (a % b && (a < 0) ^ (b > 0));
}
```

6.3 Legendre [4e4b23]

```
// the Jacobi symbol is a generalization of the
// Legendre symbol,
// such that the bottom doesn't need to be prime.
// (n|p) -> same as Legendre
// (n|ab) = (n|a)(n|b)
// work with Long Long
int Jacobi(int a, int m) {
    int s = 1;
    for (; m > 1; ) {
        a %= m;
        if (a == 0) return 0;
        const int r = __builtin_ctz(a);
        if ((r & 1) && ((m + 2) & 4)) s = -s;
        a >>= r;
        if (a & m & 2) s = -s;
        swap(a, m);
    }
    return s;
}

// 0: a == 0
// -1: a isn't a quad res of p
// else: return X with X^2 % p == a
// doesn't work with Long Long
int QuadraticResidue(int a, int p) {
    if (p == 2) return a & 1;
    if (int jc = Jacobi(a, p); jc <= 0) return jc;
    int b, d;
    for (; ; ) {
        b = rand() % p;
        d = (1LL * b * b + p - a) % p;
        if (Jacobi(d, p) == -1) break;
    }
    int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (1LL + p) >> 1; e; e >>= 1) {
        if (e & 1) {
            tmp = (1LL * g0 * f0 + 1LL * d * (1LL * g1 * f1 % p)) % p;
            g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
            g0 = tmp;
        }
        tmp = (1LL * f0 * f0 + 1LL * d * (1LL * f1 * f1 % p)) % p;
        f1 = (2LL * f0 * f1) % p;
        f0 = tmp;
    }
    return g0;
}
```

6.4 Simplex [aa7741]

```
// maximize c^T x
// subject to Ax <= b, x >= 0
// and stores the solution;
typedef long double T; // Long double, Rational, double
// + mod<P>...
typedef vector<T> vd;
typedef vector<vd> vvd;

const T eps = 1e-9, inf = 1/.0;
#define ltj(X) if(s == -1 || mp(X[j],N[j]) < mp(X[s],N[s])) s=j
#define rep(i, 1, n) for(int i = 1; i < n; i++)

struct LPSolver {
    int m, n;
    vector<int> N, B;
    vvd D;

    LPSolver(const vvd& A, const vd& b, const vd& c) :
        m(SZ(b)), n(SZ(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
        rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];
        rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i]; }
        rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
        N[n] = -1; D[m+1][n] = 1;
    }

    void pivot(int r, int s) {
        T *a = D[r].data(), inv = 1 / a[s];
        rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
            T *b = D[i].data(), inv2 = b[s] * inv;
            rep(j,0,n+2) b[j] -= a[j] * inv2;
            b[s] = a[s] * inv2;
        }
        rep(j,0,n+2) if (j != s) D[r][j] *= inv;
        rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
        D[r][s] = inv;
        swap(B[r], N[s]);
    }

    bool simplex(int phase) {
        int x = m + phase - 1;
        for (; ) {
            int s = -1;
            rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
            if (D[x][s] >= -eps) return true;
            int r = -1;
            rep(i,0,m) {
                if (D[i][s] <= eps) continue;
                if (r == -1 || mp(D[i][n+1] / D[i][s], B[i]) < mp(D[r][n+1] / D[r][s], B[r])) r = i;
            }
            if (r == -1) return false;
            pivot(r, s);
        }
    }

    T solve(vd &x) {
        int r = 0;
        rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
        if (D[r][n+1] < -eps) {
            pivot(r, n);
            if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
            rep(i,0,m) if (B[i] == -1) {
                int s = 0;
                rep(j,1,n+1) ltj(D[i]);
                pivot(i, s);
            }
        }
        bool ok = simplex(1); x = vd(n);
        rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
        return ok ? D[m][n+1] : inf;
    }
};
```

6.5 Simplex Construction

Standard form: maximize $\sum_{1 \leq i \leq n} c_i x_i$ such that $\sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j$ for all $1 \leq j \leq m$ and $x_i \geq 0$ for all $1 \leq i \leq n$.

1. In case of minimization, let $c'_i = -c_i$

2. $\sum_{1 \leq i \leq n} A_{ji}x_i \geq b_j \rightarrow \sum_{1 \leq i \leq n} -A_{ji}x_i \leq -b_j$
3. $\sum_{1 \leq i \leq n} A_{ji}x_i = b_j \rightarrow \text{add} \leq \text{and} \geq$.
4. If x_i has no lower bound, replace x_i with $x_i - x'_i$

6.6 DiscreteLog [da27bf]

```
int DiscreteLog(int s, int x, int y, int m) {
    constexpr int kStep = 32000;
    unordered_map<int, int> p;
    int b = 1;
    for (int i = 0; i < kStep; ++i) {
        p[y] = i;
        y = 1LL * y * x % m;
        b = 1LL * b * x % m;
    }
    for (int i = 0; i < m + 10; i += kStep) {
        s = 1LL * s * b % m;
        if (p.find(s) != p.end()) return i + kStep - p[s];
    }
    return -1;
}

int DiscreteLog(int x, int y, int m) {
    if (m == 1) return 0;
    int s = 1;
    for (int i = 0; i < 100; ++i) {
        if (s == y) return i;
        s = 1LL * s * x % m;
    }
    if (s == y) return 100;
    int p = 100 + DiscreteLog(s, x, y, m);
    if (fpow(x, p, m) != y) return -1;
    return p; //returns: x^p = y (mod m)
}
```

6.7 Miller Rabin & Pollard Rho [d3ecd2]

```
// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633 4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383 6 : primes <= 13
// n < 2^64              7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
ll mul(ll a, ll b, ll n){
    return (__int128)a * b % n;
}

bool Miller_Rabin(ll a, ll n) { // 06308c
    if ((a = a % n) == 0) return 1;
    if (n % 2 == 0) return n == 2;
    ll tmp = (n - 1) / ((n - 1) & (1 - n));
    ll t = __lg(((n - 1) & (1 - n))), x = 1;
    for (; tmp; tmp >>= 1, a = mul(a, a, n))
        if (tmp & 1) x = mul(x, a, n);
    if (x == 1 || x == n - 1) return 1;
    while (--t)
        if ((x = mul(x, x, n)) == n - 1) return 1;
    return 0;
}

bool prime(ll n){ // 8859aa
    vector<ll> tmp = {2, 325, 9375, 28178, 450775,
        9780504, 1795265022};
    for(ll i : tmp)
        if(!Miller_Rabin(i, n)) return false;
    return true;
}

map<ll, int> cnt;
void PollardRho(ll n) { // 173531
    if (n == 1) return;
    if (prime(n)) return ++cnt[n], void();
    if (n % 2 == 0) return PollardRho(n / 2), ++cnt[2],
        void();
    ll x = 2, y = 2, d = 1, p = 1;
#define f(x, n, p) ((mul(x, x, n) + p) % n)
    while (true) {
        if (d != n && d != 1) {
            PollardRho(n / d);
            PollardRho(d);
            return;
        }
        if (d == n) ++p;
        x = f(x, n, p), y = f(f(y, n, p), n, p);
        d = gcd(abs(x - y), n);
    }
}
```

6.8 XOR Basis [006505]

```
const int digit = 60; // [0, 2^digit)
struct Basis{
    int total = 0, rank = 0;
    vector<ll> b;
    Basis(): b(digit) {}
    bool add(ll v){ // Gauss Jordan Elimination
        total++;
        for(int i = digit - 1; i >= 0; i--){
            if(!(1LL << i & v)) continue;
            if(b[i] != 0){
                v ^= b[i];
                continue;
            }
            for(int j = 0; j < i; j++){
                if(1LL << j & v) v ^= b[j];
            }
            for(int j = i + 1; j < digit; j++){
                if(1LL << i & b[j]) b[j] ^= v;
            }
            b[i] = v;
            rank++;
            return true;
        }
        return false;
    }
    ll getMax(ll x = 0){
        for(ll i : b) x = max(x, x ^ i);
        return x;
    }
    ll getMin(ll x = 0){
        for(ll i : b) x = min(x, x ^ i);
        return x;
    }
    bool can(ll x){
        return getMin(x) == 0;
    }
    ll kth(ll k){ // kth smallest, 0-indexed
        vector<ll> tmp;
        for(ll i : b) if(i) tmp.pb(i);
        ll ans = 0;
        for(int i = 0; i < SZ(tmp); i++){
            if(1LL << i & k) ans ^= tmp[i];
        }
        return ans;
    }
};
```

6.9 Linear Equation [056191]

```
vector<int> RREF(vector<vector<ll>> &mat) { // 9cd26b
    int N = SZ(mat), M = SZ(mat[0]);
    int rk = 0;
    vector<int> cols;
    for (int i = 0; i < M; i++) {
        int cnt = -1;
        for (int j = N - 1; j >= rk; j--)
            if(mat[j][i] != 0) cnt = j;
        if (cnt == -1) continue;
        swap(mat[rk], mat[cnt]);
        ll lead = mat[rk][i];
        for (int j = 0; j < M; j++) mat[rk][j] = mat[rk][j]
            * inv(lead) % MOD;
        for (int j = 0; j < N; j++) {
            if (j == rk) continue;
            ll tmp = mat[j][i];
            for (int k = 0; k < M; k++)
                mat[j][k] = (mat[j][k] - mat[rk][k] * tmp % MOD
                    + MOD) % MOD;
        }
        cols.pb(i);
        rk++;
    }
    return cols;
}

// sol = particular + linear combination of homogenous
struct LinearEquation { // 2702e2
    bool ok;
    vector<ll> par; //particular solution (Ax = b)
    vector<vector<ll>> homo; //homogenous (Ax = 0)
    vector<vector<ll>> rref;
    //first M columns are matrix A
    //last column of eq is vector b
    void solve(const vector<vector<ll>> &eq) {
        int M = SZ(eq[0]) - 1;
```

```

rref = eq;
auto piv = RREF(rref);
int rk = piv.size();
if(piv.size() && piv.back() == M)
    return ok = 0, void();
    return ok = 0, void();
ok = 1;
par.resize(M);
vector<bool> ispiv(M);
for (int i = 0; i < rk; i++) {
    par[piv[i]] = rref[i][M];
    ispiv[piv[i]] = 1;
}
for (int i = 0; i < M; i++) {
    if (ispiv[i]) continue;
    vector<ll> h(M);
    h[i] = 1;
    for (int j = 0; j < rk; j++)
        h[piv[j]] = rref[j][i] ? MOD - rref[j][i] : 0;
    homo.pb(h);
}
}
};

```

6.10 Chinese Remainder Theorem [6ef4a3]

```

pll solve_crt(ll x1, ll m1, ll x2, ll m2){
    ll g = gcd(m1, m2);
    if ((x2 - x1) % g) return {0, 0}; // no sol
    m1 /= g; m2 /= g;
    ll _, p, q;
    extgcd(m1, m2, _, p, q); // p <= C
    ll lcm = m1 * m2 * g;
    ll res = ((__int128)p * (x2 - x1) % lcm * m1 % lcm +
        x1) % lcm;
    // be careful with overflow, C^3
    return {(res + lcm) % lcm, lcm}; // (x, m)
}

```

6.11 Sqrt Decomposition [8d7bc0]

```

// for all i in [l, r], floor(n / i) = x
for(int l = 1, r; l <= n; l = r + 1){
    int x = ifloor(n, l);
    r = ifloor(n, x);
}
// for all i in [l, r], ceil(n / i) = x
for(int l = 1, r = n; r >= 1; r = l - 1){
    int x = iceil(n, r);
    l = iceil(n, x);
}

```

6.12 Floor Sum

- $m = \lfloor \frac{an+b}{c} \rfloor$
- Time complexity: $O(\log n)$

$$f(a, b, c, n) = \sum_{i=0}^n \left\lfloor \frac{ai+b}{c} \right\rfloor$$

$$= \begin{cases} \left\lfloor \frac{a}{c} \right\rfloor \cdot \frac{n(n+1)}{2} + \left\lfloor \frac{b}{c} \right\rfloor \cdot (n+1) \\ + f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm - f(c, c-b-1, a, m-1), & \text{otherwise} \end{cases}$$

$$g(a, b, c, n) = \sum_{i=0}^n i \left\lfloor \frac{ai+b}{c} \right\rfloor$$

$$= \begin{cases} \left\lfloor \frac{a}{c} \right\rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \left\lfloor \frac{b}{c} \right\rfloor \cdot \frac{n(n+1)}{2} \\ + g(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1)) \\ - h(c, c-b-1, a, m-1), & \text{otherwise} \end{cases}$$

$$h(a, b, c, n) = \sum_{i=0}^n \left\lfloor \frac{ai+b}{c} \right\rfloor^2$$

$$= \begin{cases} \left\lfloor \frac{a}{c} \right\rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \left\lfloor \frac{b}{c} \right\rfloor^2 \cdot (n+1) \\ + \left\lfloor \frac{a}{c} \right\rfloor \cdot \left\lfloor \frac{b}{c} \right\rfloor \cdot n(n+1) \\ + h(a \bmod c, b \bmod c, c, n) \\ + 2 \left\lfloor \frac{a}{c} \right\rfloor \cdot g(a \bmod c, b \bmod c, c, n) \\ + 2 \left\lfloor \frac{b}{c} \right\rfloor \cdot f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) \\ - 2f(c, c-b-1, a, m-1) - f(a, b, c, n), & \text{otherwise} \end{cases}$$

7 Polynomial

7.1 FWHT [c9cdb6]

```

/* x: a[j], y: a[j + (L >> 1)]
or: (y += x * op), and: (x += y * op)
xor: (x, y = (x + y) * op, (x - y) * op)
invop: or, and, xor = -1, -1, 1/2 */
void fwt(int *a, int n, int op) { //or
    for (int L = 2; L <= n; L <= 1)
        for (int i = 0; i < n; i += L)
            for (int j = i; j < i + (L >> 1); ++j)
                a[j + (L >> 1)] += a[j] * op;
}
const int N = 21;
int f[N][1 << N], g[N][1 << N], h[N][1 << N], ct[1 << N];
void subset_convolution(int *a, int *b, int *c, int L)
{
    // c_k = \sum_{i+j=k, i&j=0} a_i * b_j
    int n = 1 << L;
    for (int i = 1; i < n; ++i)
        ct[i] = ct[i & (i - 1)] + 1;
    for (int i = 0; i < n; ++i)
        f[ct[i]][i] = a[i], g[ct[i]][i] = b[i];
    for (int i = 0; i <= L; ++i)
        fwt(f[i], n, 1), fwt(g[i], n, 1);
    for (int i = 0; i <= L; ++i)
        for (int j = 0; j <= i; ++j)
            for (int x = 0; x < n; ++x)
                h[i][x] += f[j][x] * g[i - j][x];
    for (int i = 0; i <= L; ++i) fwt(h[i], n, -1);
    for (int i = 0; i < n; ++i) c[i] = h[ct[i]][i];
}

```

7.2 FFT [13ec2f]

```

// Errichto: FFT for double works when the result < 1
// e15, and < 1e18 with long double
using val_t = complex<double>;
template<int MAXN>
struct FFT {
    const double PI = acos(-1);
    val_t w[MAXN];
    FFT() {
        for (int i = 0; i < MAXN; ++i) {
            double arg = 2 * PI * i / MAXN;
            w[i] = val_t(cos(arg), sin(arg));
        }
    }
    void bitrev(vector<val_t> &a, int n) //same as NTT
    void trans(vector<val_t> &a, int n, bool inv = false)
    {
        bitrev(a, n);
        for (int L = 2; L <= n; L <= 1) {
            int dx = MAXN / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + dl; ++j, x += dx) {
                    val_t tmp = a[j + dl] * (inv ? conj(w[x]) : w[x]);
                    a[j + dl] = a[j] - tmp;
                    a[j] += tmp;
                }
            }
        }
        if (inv) {
            for (int i = 0; i < n; ++i) a[i] /= n;
        }
    }
    //multiplying two polynomials A * B:
    //fft.trans(A, siz, 0), fft.trans(B, siz, 0);
    //A[i] *= B[i], fft.trans(A, siz, 1);
};

```

7.3 NTT [bf683f]

```

//(2^16)+1, 65537, 3
//7*17*(2^23)+1, 998244353, 3
//1255*(2^20)+1, 1315962881, 3
//51*(2^25)+1, 1711276033, 29

```

```
// only works when sz(A) + sz(B) - 1 <= MAXN
template<int MAXN, ll P, ll RT> //MAXN must be 2^k
struct NTT {
    ll w[MAXN];
    ll mpow(ll a, ll n);
    ll minv(ll a) { return mpow(a, P - 2); }
    NTT() {
        ll dw = mpow(RT, (P - 1) / MAXN);
        w[0] = 1;
        for (int i = 1; i < MAXN; ++i) w[i] = w[i - 1] * dw
            % P;
    }
    void bitrev(vector<ll> &a, int n) {
        int i = 0;
        for (int j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (i ^ k) < k; k >>= 1);
            if (j < i) swap(a[i], a[j]);
        }
    }
    void operator()(vector<ll> &a, int n, bool inv =
        false) { //0 <= a[i] < P
        bitrev(a, n);
        for (int L = 2; L <= n; L <= 1) {
            int dx = MAXN / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + dl; ++j, x += dx) {
                    ll tmp = a[j + dl] * w[x] % P;
                    if ((a[j + dl] = a[j] - tmp) < 0) a[j + dl]
                        += P;
                    if ((a[j] += tmp) >= P) a[j] -= P;
                }
            }
        }
        if (inv) {
            reverse(a.begin() + 1, a.begin() + n);
            ll invn = minv(n);
            for (int i = 0; i < n; ++i) a[i] = a[i] * invn %
                P;
        }
    }
};
```

7.4 Polynomial Operation [77a8a8]

```
// == b4233a ==
#define fi(s, n) for (int i = (int)(s); i < (int)(n); ++i)
#define neg(x) (x ? P - x : 0)
#define V (*this)
template <int MAXN, ll P, ll RT> // MAXN = 2^k
struct Poly : vector<ll> { // coefficients in [0, P)
    using vector<ll>::vector;
    static inline NTT<MAXN, P, RT> ntt;
    int n() const { return (int)size(); } // n() >= 1
    Poly(const Poly &p, int m) : vector<ll>(m) { copy_n(p
        .data(), min(p.n(), m), data()); }
    Poly &irev() { return reverse(data(), data() + n()),
        V; }
    Poly &isz(int m) { return resize(m), V; }
    static ll minv(ll x) { return ntt.minv(x); }
    // == fb1867 ==
    Poly &iadd(const Poly &rhs) { // db5668
        fi(0, n()) if ((V[i] += rhs[i]) >= P) V[i] -= P;
        return V; // need n() == rhs.n()
    }
    Poly &imul(ll k) { // a8df26
        fi(0, n()) V[i] = V[i] * k % P;
        return V;
    }
    Poly Mul(const Poly &rhs) const { // 46caf3
        int m = 1;
        while (m < n() + rhs.n() - 1) m <= 1;
        assert(m <= MAXN);
        Poly X(V, m), Y(rhs, m);
        ntt(X, m), ntt(Y, m);
        fi(0, m) X[i] = X[i] * Y[i] % P;
        ntt(X, m, true);
        return X.isz(n() + rhs.n() - 1);
    }
    Poly Inv() const { // 796a37
        if (n() == 1) return {minv(V[0])};
        int m = 1; // need V[0] != 0, 2*sz<=MAXN
```

```
while (m < n() * 2) m <= 1;
assert(m <= MAXN);
Poly Xi = Poly(V, (n() + 1) / 2).Inv().isz(m);
Poly Y(V, m);
ntt(Xi, m), ntt(Y, m);
fi(0, m) {
    Xi[i] *= (2 - Xi[i] * Y[i]) % P;
    if ((Xi[i] %= P) < 0) Xi[i] += P;
}
ntt(Xi, m, true);
return Xi.isz(n());
}
Poly &shift_inplace(const ll &c) { // 0c04f6
    int n = V.n(); // 2 * sz <= MAXN
    vector<ll> fc(n), ifc(n);
    fc[0] = ifc[0] = 1;
    for (int i = 1; i < n; i++) {
        fc[i] = fc[i - 1] * i % P;
        ifc[i] = minv(fc[i]);
    }
    for (int i = 0; i < n; i++) V[i] = V[i] * fc[i] % P
        ;
    Poly g(n);
    ll cp = 1;
    for (int i = 0; i < n; i++) g[i] = cp * ifc[i] % P,
        cp = cp * c % P;
    V = V.irev().Mul(g).isz(n).irev();
    for (int i = 0; i < n; i++) V[i] = V[i] * ifc[i] %
        P;
    return V;
}
// == 7b2835 ==
Poly shift(const ll &c) const { return Poly(V).
    shift_inplace(c); }
Poly _Sqrt() const { // Jacobi(V[0], P) = 1
    if (n() == 1) return {QuadraticResidue(V[0], P)};
    Poly X = Poly(V, (n() + 1) / 2)._Sqrt().isz(n());
    return X.iadd(Mul(X.Inv()).isz(n())).imul(P / 2 +
        1);
}
// == b46641 ==
Poly Sqrt() const { // 1aa942
    Poly a; // 2 * sz <= MAXN
    bool has = 0;
    for (int i = 0; i < n(); i++) {
        if (V[i]) has = 1;
        if (has) a.push_back(V[i]);
    }
    if (!has) return V;
    if ((n() + a.n()) % 2 || Jacobi(a[0], P) != 1) {
        return Poly();
    }
    a = a.isz((n() + a.n()) / 2)._Sqrt();
    int sz = a.n();
    a.isz(n());
    rotate(a.begin(), a.begin() + sz, a.end());
    return a;
}
pair<Poly, Poly> DivMod(const Poly &rhs) const { // 5
    bd174
    if (n() < rhs.n()) return {{0}, V};
    const int m = n() - rhs.n() + 1;
    Poly X(rhs); // (rhs.)back() != 0
    X.irev().isz(m);
    Poly Y(V);
    Y.irev().isz(m);
    Poly Q = Y.Mul(X.Inv()).isz(m).irev();
    X = rhs.Mul(Q), Y = V;
    fi(0, n()) if ((Y[i] -= X[i]) < 0) Y[i] += P;
    return {Q, Y.isz(max(1, rhs.n() - 1))};
}
// == 76b1af ==
Poly Dx() const {
    Poly ret(n() - 1);
    fi(0, ret.n()) ret[i] = (i + 1) * V[i + 1] % P;
    return ret.isz(max(1, ret.n()));
}
Poly Sx() const {
    Poly ret(n() + 1);
    fi(0, n()) ret[i + 1] = minv(i + 1) * V[i] % P;
    return ret;
}
```



```

Poly _tmul(int nn, const Poly &rhs) const {
    Poly Y = Mul(rhs).isz(n() + nn - 1);
    return Poly(Y.data() + n() - 1, Y.data() + Y.n());
}
// == 3afa3f ==
vector<ll> _eval(const vector<ll> &x, const vector<
    Poly> &up) const { // fb6553
    const int m = (int)x.size();
    if (!m) return {};
    vector<Poly> down(m * 2);
    // down[1] = DivMod(up[1]).second;
    // fi(2, m * 2) down[i] = down[i / 2].DivMod(up[i])
    // .second;
    down[1] = Poly(up[1]).irev().isz(n()).Inv().irev().
        _tmul(m, V);
    fi(2, m * 2) down[i] = up[i ^ 1]._tmul(up[i].n() -
        1, down[i / 2]);
    vector<ll> y(m);
    fi(0, m) y[i] = down[m + i][0];
    return y;
}
static vector<Poly> _tree1(const vector<ll> &x) { //
    f5c433
    const int m = (int)x.size();
    vector<Poly> up(m * 2);
    fi(0, m) up[m + i] = {neg(x[i]), 1};
    for (int i = m - 1; i > 0; --i) up[i] = up[i * 2].
        Mul(up[i * 2 + 1]);
    return up;
}
vector<ll> Eval(const vector<ll> &x) const { // 1e5,
    1s
    auto up = _tree1(x);
    return _eval(x, up);
}
static Poly Interpolate(const vector<ll> &x, const
    vector<ll> &y) { // d7bae4
    const int m = (int)x.size(); // 1e5, 1.4s
    vector<Poly> up = _tree1(x), down(m * 2);
    vector<ll> z = up[1].Dx()._eval(x, up);
    fi(0, m) z[i] = y[i] * minv(z[i]) % P;
    fi(0, m) down[m + i] = {z[i]};
    for (int i = m - 1; i > 0; --i)
        down[i] = down[i * 2].Mul(up[i * 2 + 1]).iadd(
            down[i * 2 + 1].Mul(up[i * 2]));
    return down[1];
}
// == c066ab ==
Poly Ln() const { // V[0] == 1, 2*sz<=MAXN
    return Dx().Mul(Inv()).Sx().isz(n());
}
Poly Exp() const { // V[0] == 0, 2*sz<=MAXN
    if (n() == 1) return {1};
    Poly X = Poly(V, (n() + 1) / 2).Exp().isz(n());
    Poly Y = X.Ln();
    Y[0] = P - 1;
    fi(0, n()) if ((Y[i] = V[i] - Y[i]) < 0) Y[i] += P;
    return X.Mul(Y).isz(n());
}
// == 3f1d86 ==
// M := P(P - 1). If k >= M, k := k % M + M.
Poly Pow(ll k) const { // 2*sz<=MAXN // d08261
    int nz = 0;
    while (nz < n()) && !V[nz] ++nz;
    if (nz * min(k, (ll)n()) >= n()) return Poly(n());
    if (!k) return Poly(Poly{1}, n());
    Poly X(data() + nz, data() + nz + n() - nz * k);
    const ll c = ntt.mpow(X[0], k % (P - 1));
    return X.Ln().imul(k % P).Exp().imul(c).irev().isz(
        n()).irev();
}
// sum_j w_j [x^j] f(x^i) for i \in [0, m]
Poly power_projection(Poly wt, int m) { // 277119
    assert(n() == wt.n()); // 4*sz <= MAXN!
    if (!n()) {
        return Poly(m + 1, 0);
    }
    if (V[0] != 0) {
        ll c = V[0];
        V[0] = 0;
        Poly A = V.power_projection(wt, m);

```

```

        fi(0, m + 1) A[i] = A[i] * fac[i] % P; //
        factorial
    }
    Poly B(m + 1);
    ll pow = 1;
    fi(0, m + 1) B[i] = pow * ifac[i] % P, pow = pow
        * c % P; // inv. of fac
    A = A.Mul(B).isz(m + 1);
    fi(0, m + 1) A[i] = A[i] * fac[i] % P;
    return A;
}

int n = 1;
while (n < V.n()) n *= 2;
isz(n), wt.isz(n).irev();
int k = 1;
Poly p(wt, 2 * n), q(V, 2 * n);
q.imul(P - 1);

while (n > 1) {
    Poly r(2 * n * k);
    fi(0, 2 * n * k) r[i] = (i % 2 == 0 ? q[i] : neg(
        q[i]));
    Poly pq = p.Mul(r).isz(4 * n * k);
    Poly qq = q.Mul(r).isz(4 * n * k);
    fi(0, 2 * n * k) {
        pq[2 * n * k + i] += p[i];
        qq[2 * n * k + i] += q[i] + r[i];
        pq[2 * n * k + i] %= P;
        qq[2 * n * k + i] %= P;
    }
    fill(p.begin(), p.end(), 0);
    fill(q.begin(), q.end(), 0);
    for(int j = 0; j < 2 * k; j++) fi(0, n / 2) {
        p[n * j + i] = pq[(2 * n) * j + (2 * i + 1)];
        q[n * j + i] = qq[(2 * n) * j + (2 * i + 0)];
    }
    n /= 2, k *= 2;
}
Poly ans(k);
fi(0, k) ans[i] = p[2 * i];
return ans.irev().isz(m + 1);
}
Poly FPSinv() { // 2c54b4
    const int n = V.n() - 1;
    if (n == -1) return {};
    assert(V[0] == 0);
    if (n == 0) return V;
    assert(V[1] != 0);
    ll c = V[1], ic = minv(c);
    imul(ic);
    Poly wt(n + 1);
    wt[n] = 1;

    Poly A = V.power_projection(wt, n);
    Poly g(n);
    fi(1, n + 1) g[n - i] = n * A[i] % P * minv(i) % P;
    g = g.Pow(neg(minv(n)));
    g.insert(g.begin(), 0);

    ll pow = 1;
    fi(0, g.n()) g[i] = g[i] * pow % P, pow = pow * ic
        % P;
    return g;
}
Poly TMul(const Poly &rhs) const { // this[i] - rhs[j]
    j = k; // 7b552c
    return Poly(*this).irev().Mul(rhs).isz(n()).irev();
}
Poly FPScomp(Poly g) { // solves V(g(x)) // 332bb2
    auto rec = [&](auto &rec, int n, int k, Poly Q) ->
        Poly {
        if (n == 1) {
            Poly p(2 * k);
            irev();
            fi(0, k) p[2 * i] = V[i];
            return p;
        }
        Poly R(2 * n * k);
        fi(0, 2 * n * k) R[i] = (i % 2 == 0 ? Q[i] : neg(
            Q[i]));
        Poly QQ = Q.Mul(R).isz(4 * n * k);
        fi(0, 2 * n * k) {

```

```

    QQ[2 * n * k + i] += Q[i] + R[i];
    QQ[2 * n * k + i] %= P;
}
Poly nxt_Q(2 * n * k);
for(int j = 0; j < 2 * k; j++) fi(0, n / 2) {
    nxt_Q[n * j + i] = QQ[(2 * n) * j + (2 * i + 0)];
}
Poly nxt_p = rec(rec, n / 2, k * 2, nxt_Q);
Poly pq(4 * n * k);
for(int j = 0; j < 2 * k; j++) fi(0, n / 2) {
    pq[(2 * n) * j + (2 * i + 1)] += nxt_p[n * j + i];
    pq[(2 * n) * j + (2 * i + 1)] %= P;
}
Poly p(2 * n * k);
fi(0, 2 * n * k) p[i] = (p[i] + pq[2 * n * k + i]) % P;
pq.pop_back();
Poly x = pq.TMul(R);
fi(0, 2 * n * k) p[i] = (p[i] + x[i]) % P;
return p;
};
int sz = 1;
while(sz < n() || sz < g.n()) sz <= 1;
return isz(sz), rec(rec, sz, 1, g.imul(P-1).isz(2 * sz)).isz(sz).irev();
}
};
#undef fi
#undef V
#undef neg
using Poly_t = Poly<1 << 19, 998244353, 3>;

```

7.5 Generating Function

Ordinary Generating Function

- $C(x) = A(rx)$: $c_n = r^n a_n$ 的一般生成函數。
- $C(x) = A(x) + B(x)$: $c_n = a_n + b_n$ 的一般生成函數。
- $C(x) = A(x)B(x)$: $c_n = \sum_{i=0}^n a_i b_{n-i}$ 的一般生成函數。
- $C(x) = A(x)^k$: $c_n = \sum_{i_1+i_2+\dots+i_k=n} a_{i_1} a_{i_2} \dots a_{i_k}$ 的一般生成函數。
- $C(x) = xA(x)'$: $c_n = na_n$ 的一般生成函數。
- $C(x) = \frac{A(x)}{1-x}$: $c_n = \sum_{i=0}^n a_i$ 的一般生成函數。
- $C(x) = A(1) + x \frac{A(1)-A(x)}{1-x}$: $c_n = \sum_{i=n}^{\infty} a_i$ 的一般生成函數。

常用展開式

- $\frac{1}{1-x} = 1 + x + x^2 + \dots + x^n + \dots$
- $(1+x)^a = \sum_{n=0}^{\infty} \binom{a}{n} x^n, \binom{a}{n} = \frac{a(a-1)(a-2)\dots(a-n+1)}{n!}$

常見生函

- 卡特蘭數: $f(x) = \frac{1-\sqrt{1-4x}}{2x}$

Exponential Generating Function

a_0, a_1, \dots 的指數生成函數:

$$\hat{A}(x) = \sum_{i=0}^{\infty} \frac{a_i}{i!} = a_0 + a_1 x + \frac{a_2}{2!} x^2 + \frac{a_3}{3!} x^3 + \dots$$

- $\hat{C}(x) = \hat{A}(x) + \hat{B}(x)$: $c_n = a_n + b_n$ 的指數生成函數
- $\hat{C}(x) = \hat{A}^{(k)}(x)$: $c_n = a_{n+k}$ 的指數生成函數
- $\hat{C}(x) = x\hat{A}(x)$: $c_n = na_n$ 的指數生成函數
- $\hat{C}(x) = \hat{A}(x)\hat{B}(x)$: $c_n = \sum_{k=0}^n \binom{n}{k} a_k b_{n-k}$ 的指數生成函數
- $\hat{C}(x) = \hat{A}(x)^k$: $\sum_{i_1+i_2+\dots+i_k=n} \binom{n}{i_1, i_2, \dots, i_k} a_{i_1} a_{i_2} \dots a_{i_k}$ 的指數生成函數
- $\hat{C}(x) = \exp(\hat{A}(x))$: 假設 $A(x)$ 是一個分量 (component) 的生成函數, 那 $\hat{C}(x)$ 是將 n 個有編號的東西分成若干個分量的指數生成函數

Lagrange's Inversion Formula

如果 F 跟 G 互反, 則有 $F(0), G(0) = 0, F'(0), G'(0) \neq 0$ 。若 H 為任意 FPS, 則

$$n[x^n]G(x) = [x^{n-1}] \frac{1}{(F(x)/x)^n}$$

$$n[x^n]H(G(x)) = [x^{n-1}]H'(x) \frac{1}{(F(x)/x)^n}$$

7.6 Bostan Mori [41c3bc]

```

const ll mod = 998244353;
NTT<262144, mod, 3> ntt;

```

```

// Finds the k-th coefficient of P / Q in O(d Log d Log k)
// size of NTT has to > 2 * d
ll BostanMori(vector<ll> P, vector<ll> Q, long long k)
{
    int d = max((int)P.size(), (int)Q.size() - 1);
    vector M = {P, Q};
    M[0].resize(d, 0);
    M[1].resize(d + 1, 0);
    int sz = (2 * d + 1 == 1 ? 2 : (1 << (__lg(2 * d) + 1)));
    vector<ll> Qn(sz);
    vector N(2, vector<ll>(sz));
    while(k) {
        fill(iter(Qn), 0);
        for(int i = 0; i < d + 1; i++){
            Qn[i] = M[1][i] * ((i & 1) ? -1 : 1);
            if(Qn[i] < 0) Qn[i] += mod;
        }
        ntt(Qn, sz, false);

        ll t[2] = {k & 1, 0};
        for(int i = 0; i < 2; i++){
            fill(iter(N[i]), 0);
            copy(iter(M[i]), N[i].begin());
            ntt(N[i], sz, false);
            for(int j = 0; j < sz; j++)
                N[i][j] = N[i][j] * Qn[j] % mod;
            ntt(N[i], sz, true);
            for(int j = t[i]; j < 2 * siz(M[i]); j += 2){
                M[i][j >> 1] = N[i][j];
            }
            k >>= 1;
        }
        return M[0][0] * ntt.minv(M[1][0]) % mod;
    }
}
ll LinearRecursion(vector<ll> a, vector<ll> c, ll k) {
    // a_n = \sum_{j=1}^d c_j a_{n-j}
    int d = siz(a);
    int sz = (2 * d + 1 == 1 ? 2 : (1 << (__lg(2 * d) + 1)));

    c[0] = mod - 1;
    for(ll &i : c) i = i ? mod - i : 0;

    auto A = a; A.resize(sz);
    auto C = c; C.resize(sz);
    ntt(A, sz, false), ntt(C, sz, false);
    for(int i = 0; i < sz; i++) A[i] = A[i] * C[i] % mod;
    ntt(A, sz, true);
    A.resize(d);

    return BostanMori(A, c, k);
}

```

8 String

8.1 KMP Algorithm [c8b75f]

```

// 0-based
// fail[i] = max k < i s.t. s[0..k] = s[i-k..i]
vector<int> kmp_build_fail(const string &s){
    int n = SZ(s);
    vector<int> fail(n, -1);
    int cur = -1;
    for(int i = 1; i < n; i++){
        while(cur != -1 && s[cur + 1] != s[i])
            cur = fail[cur];
        if(s[cur + 1] == s[i])
            cur++;
        fail[i] = cur;
    }
    return fail;
}

```

```

void kmp_match(const string &s, const vector<int> &fail, const string &t){
    int cur = -1;
    int n = SZ(s), m = SZ(t);
    for(int i = 0; i < m; i++){
        while(cur != -1 && (cur + 1 == n || s[cur + 1] != t[i]))

```

```

    cur = fail[cur];
    if (cur + 1 < n && s[cur + 1] == t[i])
        cur++;
    // cur = max k s.t. s[0..k] = t[i-k..i]
}
}

```

8.2 Manacher Algorithm [caf0f4]

```

/* center i: radius z[i * 2 + 1] / 2
   center i, i + 1: radius z[i * 2 + 2] / 2
   both aba, abba have radius 2 */
vector<int> manacher(const string &tmp) { // 0-based
    string s = "%";
    int l = 0, r = 0;
    for (char c : tmp) s += c, s += '%';
    vector<int> z(SZ(s));
    for (int i = 0; i < SZ(s); i++) {
        z[i] = r > i ? min(z[2 * l - i], r - i) : 1;
        while (i - z[i] >= 0 && i + z[i] < SZ(s)
            && s[i + z[i]] == s[i - z[i]])
            ++z[i];
        if (z[i] + i > r) r = z[i] + i, l = i;
    }
    return z;
}

```

8.3 Lyndon Factorization [7c612b]

```

// partition s = w[0] + w[1] + ... + w[k-1],
// w[0] >= w[1] >= ... >= w[k-1]
// each w[i] strictly smaller than all its suffix
void duval(const string &s, vector<pii> &w) {
    for (int n = (int)s.size(), i = 0, j, k; i < n; ) {
        for (j = i + 1, k = i; j < n && s[k] <= s[j]; j++)
            k = (s[k] < s[j] ? i : k + 1);
        // if (i < n / 2 && j >= n / 2) {
        //     for min cyclic shift, call duval(s + s)
        //     then here s.substr(i, n / 2) is min cyclic shift
        // }
        for (; i <= k; i += j - k)
            w.pb(pii(i, j - k)); // s.substr(l, len)
    }
}

```

8.4 Suffix Array [cd67ea]

```

struct SuffixArray {
    vector<int> sa, lcp, rank; // lcp[i] is lcp of sa[i]
    // and sa[i-1]
    // sa[0] = s.size()
    // character should be 1-
    // based
    SuffixArray(string& s, int lim=256) { // or
        basic_string<int>
        int n = s.size() + 1, k = 0, a, b;
        vector<int> x(n, 0), y(n), ws(max(n, lim));
        rank.assign(n, 0);
        for (int i = 0; i < n - 1; i++) x[i] = s[i];
        sa = lcp = y, iota(sa.begin(), sa.end(), 0);
        for (int j = 0, p = 0; p < n; j = max(1, j * 2),
            lim = p) {
            p = j, iota(y.begin(), y.end(), n - j);
            for (int i = 0; i < n; i++)
                if (sa[i] >= j) y[p++] = sa[i] - j;
            for (int &i : ws) i = 0;
            for (int i = 0; i < n; i++) ws[x[i]]++;
            for (int i = 1; i < lim; i++) ws[i] += ws[i - 1];
            for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
            swap(x, y), p = 1, x[sa[0]] = 0;
            for (int i = 1; i < n; i++) {
                a = sa[i - 1], b = sa[i];
                x[b] = (y[a] == y[b] && y[a + j] == y[b + j]) ?
                    p - 1 : p++;
            }
        }
        for (int i = 1; i < n; i++) rank[sa[i]] = i;
        for (int i = 0, j; i < n - 1; lcp[rank[i++]] = k)
            for (k && k--, j = sa[rank[i] - 1];
                s[i + k] == s[j + k]; k++);
    }
};

```

8.5 Suffix Automaton [016373]

```

// == a14210 ==
struct exSAM {
    const int CNUM = 26;
    // len: maxlength, link: fail link
    // lenSorted: topo order, cnt: occur
    vector<int> len, link, lenSorted, cnt;
    vector<vector<int>> next;
    int total = 0;
    int newnode() {
        return total++;
    }
    void init(int n) { // total number of characters
        len.assign(2 * n, 0); link.assign(2 * n, 0);
        lenSorted.assign(2 * n, 0); cnt.assign(2 * n, 0);
        next.assign(2 * n, vector<int>(CNUM));
        newnode(), link[0] = -1;
    }
    // == c83c9c ==
    int insertSAM(int last, int c) { // 081739
        // not exSAM: cur = newnode(), p = last
        int cur = next[last][c];
        len[cur] = len[last] + 1;
        int p = link[last];
        while (p != -1 && !next[p][c])
            next[p][c] = cur, p = link[p];
        if (p == -1) return link[cur] = 0, cur;
        int q = next[p][c];
        if (len[p] + 1 == len[q]) return link[cur] = q, cur;
        int clone = newnode();
        for (int i = 0; i < CNUM; ++i)
            next[clone][i] = len[next[q][i]] ? next[q][i] :
                0;
        len[clone] = len[p] + 1;
        while (p != -1 && next[p][c] == q)
            next[p][c] = clone, p = link[p];
        link[link[cur] = clone] = link[q];
        link[q] = clone;
        return cur;
    }
    void insert(const string &s) { // e47d43
        int cur = 0;
        for (auto ch : s) {
            int &nxt = next[cur][int(ch - 'a')];
            if (!nxt) nxt = newnode();
            cnt[cur = nxt] += 1;
        }
    }
    // == 0a715a ==
    void build() {
        queue<int> q;
        q.push(0);
        while (!q.empty()) {
            int cur = q.front();
            q.pop();
            for (int i = 0; i < CNUM; ++i)
                if (next[cur][i])
                    q.push(insertSAM(cur, i));
        }
        vector<int> lc(total);
        for (int i = 1; i < total; ++i) ++lc[len[i]];
        partial_sum(iter(lc), lc.begin());
        for (int i = 1; i < total; ++i) lenSorted[--lc[len[i]]] = i;
    }
    void solve() {
        for (int i = total - 2; i >= 0; --i)
            cnt[link[lenSorted[i]]] += cnt[lenSorted[i]];
    }
};

```

8.6 Z-value Algorithm [488d87]

```

// z[i] = max k s.t. s[0..k-1] = s[i..i+k-1]
// i.e. length of longest common prefix
// z[0] = 0
vector<int> z_function(const string &s) {
    int n = s.size();
    vector<int> z(n);
    for (int i = 1, l = 0, r = 0; i < n; i++) {
        if (i <= r) z[i] = min(r - i + 1, z[i - l]);
    }
}

```

```

while(i + z[i] < n && s[z[i]] == s[i + z[i]])
    z[i]++;
if(i + z[i] - 1 > r)
    l = i, r = i + z[i] - 1;
}
return z;
}

```

8.7 Main Lorentz [fcfb8f]

```

struct Rep{ int minl, maxl, len; };
vector<Rep> rep; // 0-base
// p \in [minl, maxl] => s[p, p + i) = s[p + i, p + 2i)
void main_lorentz(const string &s, int sft = 0) {
    const int n = s.size();
    if (n == 1) return;
    const int nu = n / 2, nv = n - nu;
    const string u = s.substr(0, nu), v = s.substr(nu,
        ru(u.rbegin(), u.rend()), rv(v.rbegin(), v.rend
        ());
    main_lorentz(u, sft), main_lorentz(v, sft + nu);
    const auto z1 = z_function(ru), z2 = z_function(v +
        '#' + u),
        z3 = z_function(ru + '#' + rv), z4 =
        z_function(v);
    auto get_z = [](const vector<int> &z, int i) {
        return (0 <= i and i < (int)z.size()) ? z[i] : 0;
    };
    auto add_rep = [&](bool left, int c, int l, int k1,
        int k2) {
        const int L = max(1, l - k2), R = min(l - left, k1)
        ;
        if (L > R) return;
        if (left) rep.emplace_back(Rep({sft + c - R, sft +
            c - L, l}));
        else rep.emplace_back(Rep({sft + c - R - l + 1, sft
            + c - L - l + 1, l}));
    };
    for (int cnt = 0; cnt < n; cnt++) {
        int l, k1, k2;
        if (cnt < nu) {
            l = nu - cnt;
            k1 = get_z(z1, nu - cnt);
            k2 = get_z(z2, nv + 1 + cnt);
        } else {
            l = cnt - nu + 1;
            k1 = get_z(z3, nu + 1 + nv - 1 - (cnt - nu));
            k2 = get_z(z4, (cnt - nu) + 1);
        }
        if (k1 + k2 >= 1)
            add_rep(cnt < nu, cnt, l, k1, k2);
    }
}

```

8.8 AC Automaton [f529e6]

```

const int SIGMA = 26;
struct AC_Automaton {
    // child: trie, next: automaton
    vector<vector<int>> child, next;
    vector<int> fail, cnt, ord;
    int total = 0;
    int newnode() {
        return total++;
    }
    void init(int len) { // Len >= 1 + total Len
        child.assign(len, vector<int>(26, -1));
        next.assign(len, vector<int>(26, -1));
        fail.assign(len, -1); cnt.assign(len, 0);
        ord.clear();
        newnode();
    }
    int input(string &s) {
        int cur = 0;
        for (char c : s) {
            if (child[cur][c - 'A'] == -1)
                child[cur][c - 'A'] = newnode();
            cur = child[cur][c - 'A'];
        }
        return cur; // return the end node of string
    }
    void make_fl() {
        queue<int> q;

```

```

q.push(0), fail[0] = -1;
while(!q.empty()) {
    int R = q.front();
    q.pop(); ord.pb(R);
    for (int i = 0; i < SIGMA; i++)
        if (child[R][i] != -1) {
            int X = next[R][i] = child[R][i], Z = fail[R
                ];
            while (Z != -1 && child[Z][i] == -1)
                Z = fail[Z];
            fail[X] = Z != -1 ? child[Z][i] : 0;
            q.push(X);
        }
        else next[R][i] = R ? next[fail[R]][i] : 0;
    }
}
void solve() {
    for (int i : ord | views::reverse)
        if (i) cnt[fail[i]] += cnt[i];
}
}

```

8.9 Palindrome Automaton [8a071b]

```

struct PalindromicTree {
    struct node {
        int nxt[26], fail, len; // num = depth of fail link
        int cnt, num; // cnt = occur, num = #pal_suffix of
        this node
        node(int l = 0) : nxt{}, fail(0), len(l), cnt(0), num
            (0) {}
    };
    vector<node> st; vector<int> s; int last, n;
    void init() {
        st.clear(); s.clear(); last = 1; n = 0;
        st.pb(0); st.pb(-1);
        st[0].fail = 1; s.pb(-1);
    }
    int getFail(int x) {
        while (s[n - st[x].len - 1] != s[n]) x = st[x].fail
        ;
        return x;
    }
    void add(int c) {
        s.pb(c == 'a'); ++n;
        int cur = getFail(last);
        if (!st[cur].nxt[c]) {
            int now = SZ(st);
            st.pb(st[cur].len + 2);
            st[now].fail = st[getFail(st[cur].fail)].nxt[c];
            st[cur].nxt[c] = now;
            st[now].num = st[st[now].fail].num + 1;
        }
        last = st[cur].nxt[c]; ++st[last].cnt;
    }
    void dpcnt() {
        for (int i = SZ(st) - 1; i >= 0; i--) {
            auto nd = st[i];
            st[nd.fail].cnt += nd.cnt;
        }
    }
    int size() { return (int)st.size() - 2; }
};

```

8.10 Palindrome Partition [c85c05]

```

// in PAM
/* node */ int dif = 0, slink = 0, g = 0;
vector<int> dp = {0};
// add
if (!st[cur].nxt[c]) {
    // ...
    st[now].dif = st[now].len - st[st[now].fail].len;
    if (st[now].dif == st[st[now].fail].dif)
        st[now].slink = st[st[now].fail].slink;
    else st[now].slink = st[now].fail;
}
dp.pb(0);
for (int x = last; x > 1; x = st[x].slink) {
    st[x].g = dp[n - st[st[x].slink].len - st[x].dif];
    if (st[x].dif == st[st[x].fail].dif)
        st[x].g = min(st[x].g, st[st[x].fail].g);
    dp[n] = min(dp[n], st[x].g + 1);
}
}

```

9 Misc

9.1 Cyclic Ternary Search [9017cc]

```
/* bool pred(int a, int b);
f(0) ~ f(n - 1) is a cyclic-shift U-function
return idx s.t. pred(x, idx) is false forall x*/
int cyc_tsearch(int n, auto pred) {
    if (n == 1) return 0;
    int l = 0, r = n; bool rv = pred(1, 0);
    while (r - l > 1) {
        int m = (l + r) / 2;
        if (pred(0, m) ? rv : pred(m, (m + 1) % n)) r = m;
        else l = m;
    }
    return pred(1, r % n) ? l : r % n;
}
```

9.2 Matroid

$M = (E, \mathcal{I})$, where $\mathcal{I} \subseteq 2^E$ is nonempty, is a matroid if:

- If $S \in \mathcal{I}$ and $S' \subseteq S$, then $S' \in \mathcal{I}$.
- For $S_1, S_2 \in \mathcal{I}$ s.t. $|S_1| < |S_2|$, there exists $e \in S_2 \setminus S_1$ s.t. $S_1 \cup \{e\} \in \mathcal{I}$.

Matroid intersection:

Start from $S = \emptyset$. In each iteration, let

- $Y_1 = \{x \notin S \mid S \cup \{x\} \in \mathcal{I}_1\}$
- $Y_2 = \{x \notin S \mid S \cup \{x\} \in \mathcal{I}_2\}$

If there exists $x \in Y_1 \cap Y_2$, insert x into S . Otherwise for each $x \in S, y \notin S$, create edges

- $x \rightarrow y$ if $S - \{x\} \cup \{y\} \in \mathcal{I}_1$.
- $y \rightarrow x$ if $S - \{x\} \cup \{y\} \in \mathcal{I}_2$.

Find a *shortest* path (with BFS) starting from a vertex in Y_1 and ending at a vertex in Y_2 which doesn't pass through any other vertices in Y_2 , and alternate the path. The size of S will be incremented by 1 in each iteration. For the weighted case, assign weight $w(x)$ to vertex x if $x \in S$ and $-w(x)$ if $x \notin S$. Find the path with the minimum number of edges among all minimum length paths and alternate it.

9.3 Simulate Annealing [ff826c]

```
ld anneal() {
    mt19937 rnd_engine(seed);
    uniform_real_distribution<ld> rnd(0, 1);
    const ld dT = 0.001;
    // Argument p
    ld S_cur = calc(p), S_best = S_cur;
    for (ld T = 2000; T > eps; T -= dT) {
        // Modify p to p_prime
        const ld S_prime = calc(p_prime);
        const ld delta_c = S_prime - S_cur;
        ld prob = min((ld)1, exp(-delta_c / T));
        if (rnd(rnd_engine) <= prob)
            S_cur = S_prime, p = p_prime;
        if (S_prime < S_best) // find min
            S_best = S_prime, p_best = p_prime;
    }
    return S_best;
}
```

9.4 Binary Search On Fraction [f6b9ec]

```
struct Q {
    ll p, q;
    Q go(Q b, ll d) { return {p + b.p * d, q + b.q * d};
    };
    // returns smallest p/q in [lo, hi] such that
    // pred(p/q) is true, and 0 <= p, q <= N
    Q frac_bs(ll N, auto &&pred) {
        Q lo{0, 1}, hi{1, 0};
        if (pred(lo)) return lo;
        assert(pred(hi));
        bool dir = 1, L = 1, H = 1;
        for (; L || H; dir = !dir) {
            ll len = 0, step = 1;
            for (int t = 0; t < 2 && (t ? step /= 2 : step *= 2);)
                if (Q mid = hi.go(lo, len + step);
                    mid.p > N || mid.q > N || dir ^ pred(mid))
                    t++;
                else len += step;
            swap(lo, hi = hi.go(lo, len));
            (dir ? L : H) = !len;
        }
        return dir ? hi : lo;
    }
}
```

9.5 Min Plus Convolution [09b5c3]

```
// a is convex a[i+1]-a[i] <= a[i+2]-a[i+1]
vector<int> min_plus_convolution(vector<int> &a, vector<int> &b) {
    int n = SZ(a), m = SZ(b);
    vector<int> c(n + m - 1, INF);
    auto dc = [&](auto Y, int l, int r, int jl, int jr) {
        if (l > r) return;
        int mid = (l + r) / 2, from = -1, &best = c[mid];
        for (int j = jl; j <= jr; ++j)
            if (int i = mid - j; i >= 0 && i < n)
                if (best > a[i] + b[j])
                    best = a[i] + b[j], from = j;
        Y(Y, l, mid - 1, jl, from), Y(Y, mid + 1, r, from, jr);
    };
    return dc(dc, 0, n - 1 + m - 1, 0, m - 1), c;
}
```

9.6 SMAWK [a2a4ce]

```
// For all 2x2 submatrix:
// If M[1][0] < M[1][1], M[0][0] < M[0][1]
// If M[1][0] == M[1][1], M[0][0] <= M[0][1]
// M[i][ans_i] is the best value in the i-th row
// select(int r, int u, int v) return true if f(r, v)
// is better than f(r, u)
vector<int> smawk(int N, int M, auto &&select) {
    auto dc = [&](auto self, const vector<int> &r, const vector<int> &c) {
        if (r.empty()) return vector<int>{};
        const int n = SZ(r); vector<int> ans(n), nr, nc;
        for (int i : c) {
            while (!nc.empty() &&
                select(r[nc.size() - 1], nc.back(), i))
                nc.pop_back();
            if (int(nc.size()) < n) nc.push_back(i);
        }
        for (int i = 1; i < n; i += 2) nr.push_back(r[i]);
        const auto na = self(self, nr, nc);
        for (int i = 1; i < n; i += 2) ans[i] = na[i >> 1];
        for (int i = 0, j = 0; i < n; i += 2) {
            ans[i] = nc[j];
            const int end = i + 1 == n ? nc.back() : ans[i + 1];
            while (nc[j] != end)
                if (select(r[i], ans[i], nc[++j])) ans[i] = nc[j];
        }
        return ans;
    };
    vector<int> R(N), C(M); iota(iter(R), 0), iota(iter(C), 0);
    return dc(dc, R, C);
}
```

9.7 Golden Ratio Search [ce06a8]

```
ld goldenRatioSearch(ld a, ld b, auto &&f) {
    ld r = (sqrt(5)-1)/2, eps = 1e-7;
    ld x1 = b - r*(b-a), x2 = a + r*(b-a);
    ld f1 = f(x1), f2 = f(x2);
    while (b-a > eps)
        if (f1 < f2) { //change to > to find maximum
            b = x2; x2 = x1; f2 = f1;
            x1 = b - r*(b-a); f1 = f(x1);
        } else {
            a = x1; x1 = x2; f1 = f2;
            x2 = a + r*(b-a); f2 = f(x2);
        }
    return a;
}
```

10 Notes

10.1 Geometry

Rotation Matrix

$$\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

- rotate 90° : $(x, y) \rightarrow (-y, x)$
- rotate -90° : $(x, y) \rightarrow (y, -x)$

Triangles

Side lengths: a, b, c

Semiperimeter: $p = \frac{a+b+c}{2}$

Area: $A = \sqrt{p(p-a)(p-b)(p-c)}$

Circumradius: $R = \frac{abc}{4A}$

Inradius: $r = \frac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two): $s_a = \sqrt{bc\left(1 - \left(\frac{a}{b+c}\right)^2\right)}$

Law of sines: $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} = \frac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc \cos \alpha$

Law of tangents: $\frac{a+b}{a-b} = \frac{\tan \frac{\alpha+\beta}{2}}{\tan \frac{\alpha-\beta}{2}}$

Quadrilaterals

With side lengths a, b, c, d , diagonals e, f , diagonals angle θ , area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2 f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is 180° , $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

Spherical coordinates

$$\begin{aligned} x &= r \sin \theta \cos \phi & r &= \sqrt{x^2 + y^2 + z^2} \\ y &= r \sin \theta \sin \phi & \theta &= \arccos(z/\sqrt{x^2 + y^2 + z^2}) \\ z &= r \cos \theta & \phi &= \operatorname{atan2}(y, x) \end{aligned}$$

Green's Theorem

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy = \oint_{L^+} (P dx + Q dy)$$

$$\text{Area} = \frac{1}{2} \oint_L x dy - y dx$$

- Circular sector:

$$x = x_0 + r \cos \theta$$

$$y = y_0 + r \sin \theta$$

$$\begin{aligned} A &= r \int_{\alpha}^{\beta} (x_0 + \cos \theta) \cos \theta + (y_0 + \sin \theta) \sin \theta d\theta \\ &= r(r\theta + x_0 \sin \theta - y_0 \cos \theta)|_{\alpha}^{\beta} \end{aligned}$$

- Centroid:

$$\bar{x} = \frac{1}{2A} \int_C x^2 dy \quad \bar{y} = -\frac{1}{2A} \int_C y^2 dx$$

Point-Line Duality

$$p = (a, b) \leftrightarrow p^* : y = ax - b$$

- $p \in l \iff l^* \in p^*$
- p_1, p_2, p_3 are collinear $\iff p_1^*, p_2^*, p_3^*$ intersect at a point
- p lies above $l \iff l^*$ lies above p^*
- lower convex hull \leftrightarrow upper envelope

10.2 Trigonometry

$$\sinh x = \frac{1}{2}(e^x - e^{-x}) \quad \cosh x = \frac{1}{2}(e^x + e^{-x})$$

$$\sin n\pi = 0 \quad \cos n\pi = (-1)^n$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\sin(2\alpha) = 2 \cos \alpha \sin \alpha$$

$$\cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha = 2 \cos^2 \alpha - 1 = 1 - 2 \sin^2 \alpha$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha \sin \beta = \frac{1}{2}(\cos(\alpha - \beta) - \cos(\alpha + \beta))$$

$$\sin \alpha \cos \beta = \frac{1}{2}(\sin(\alpha + \beta) + \sin(\alpha - \beta))$$

$$\cos \alpha \sin \beta = \frac{1}{2}(\sin(\alpha + \beta) - \sin(\alpha - \beta))$$

$$\cos \alpha \cos \beta = \frac{1}{2}(\cos(\alpha - \beta) + \cos(\alpha + \beta))$$

$$(V + W) \tan(\alpha - \beta)/2 = (V - W) \tan(\alpha + \beta)/2$$

where V, W are lengths of sides opposite angles α, β .

$$a \cos x + b \sin x = r \cos(x - \phi)$$

$$a \sin x + b \cos x = r \sin(x + \phi)$$

where $r = \sqrt{a^2 + b^2}$, $\phi = \operatorname{atan2}(b, a)$.

10.3 Calculus

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \tan x = 1 + \tan^2 x$$

$$\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$$

$$\int \tan ax = -\frac{\ln |\cos ax|}{a}$$

$$\int x \sin ax = \frac{\sin ax - ax \cos ax}{a^2}$$

$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2} \operatorname{erf}(x)$$

$$\int x e^{ax} = \frac{e^{ax}}{a^2} (ax - 1)$$

$$\int \sin^2 x = \frac{x}{2} - \frac{1}{4} \sin 2x$$

$$\int \sin^3 x = \frac{1}{12} \cos 3x - \frac{3}{4} \cos x$$

$$\int \cos^2 x = \frac{x}{2} + \frac{1}{4} \sin 2x$$

$$\int \cos^3 x = \frac{1}{12} \sin 3x + \frac{3}{4} \sin x$$

$$\int x \sin x = \sin x - x \cos x$$

$$\int x \cos x = \cos x + x \sin x$$

$$\int x e^x = e^x (x - 1)$$

$$\int x^2 e^x = e^x (x^2 - 2x + 2)$$

$$\int x^2 \sin x = 2x \sin x - (x^2 - 2) \cos x$$

$$\int x^2 \cos x = 2x \cos x + (x^2 - 2) \sin x$$

$$\int e^x \sin x = \frac{1}{2} e^x (\sin x - \cos x)$$

$$\int e^x \cos x = \frac{1}{2} e^x (\sin x + \cos x)$$

$$\int x e^x \sin x = \frac{1}{2} e^x (x \sin x - x \cos x + \cos x)$$

$$\int x e^x \cos x = \frac{1}{2} e^x (x \sin x + x \cos x - \sin x)$$

10.4 Sum & Series

$$c^a + c^{a+1} + \dots + c^b = \frac{c^{b+1} - c^a}{c - 1}, c \neq 1$$

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(2n+1)(n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \dots, (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots, (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots, (-\infty < x < \infty)$$

