

Contents

1 Basic	1
1.1 .vimrc	1
1.2 Debug	1
1.3 Fast IO	1
1.4 Random	1
1.5 PBDS Tree	1
1.6 Pragma	1
1.7 SVG Writer	1
2 Data Structure	2
2.1 Heavy-Light Decomposition	2
2.2 Link Cut Tree	2
2.3 Treap	2
2.4 KD Tree	3
2.5 Leftist Tree	3
2.6 Convex 1D/1D	4
2.7 Dynamic Convex Hull	4
3 Flow & Matching	4
3.1 Dinic	4
3.2 Bounded Flow	4
3.3 MCMF	4
3.4 Min Cost Circulation	5
3.5 Gomory Hu	5
3.6 Stoer Wagner Algorithm	5
3.7 Bipartite Matching	5
3.8 Kuhn Munkres Algorithm	6
3.9 Max Simple Graph Matching	6
3.10 Flow Model	6
4 Geometry	7
4.1 Geometry Template	7
4.2 Convex Hull	7
4.3 Polar Angle Comparator	7
4.4 Minkowski Sum	8
4.5 Intersection of Circle and Convex Polygon	8
4.6 Intersection of Circles	8
4.7 Tangent Line of Circles	8
4.8 Intersection of Line and Convex Polygon	8
4.9 Intersection of Line and Circle	8
4.10 Point in Circle	8
4.11 Point in Convex	8
4.12 Half Plane Intersection	9
4.13 HPI General Line	9
4.14 Minimum Enclosing Circle	9
4.15 3D Point	9
4.16 ConvexHull3D	9
4.17 Delaunay Triangulation	10
4.18 Voronoi Diagram	10
4.19 Polygon Union	10
4.20 Tangent Point to Convex Hull	11
4.21 Heart	11
4.22 Rotating Sweep Line	11
4.23 Vector In Poly	11
4.24 Convex Hull DP	11
4.25 Calculate Points in Triangle	12
5 Graph	12
5.1 BCC	12
5.2 SCC	12
5.3 2-SAT	12
5.4 Dominator Tree	12
5.5 Virtual Tree	13
5.6 Fast DMST	13
5.7 Vizing	13
5.8 Maximum Clique	14
5.9 Number of Maximal Clique	14
5.10 Minimum Mean Cycle	14

1 Basic

Default code: Basic 9c8f02

Square: i+<esc>25A---+<esc>o|<esc>25A |<esc>ggVGyG35pGdd

1.1 .vimrc [9b4074]

```
sy on
se nu rnu bs=2 sw=4 ts=4 hls ls=2 si acd bo=all mouse=a
et
map <F9> :w<bar>!g++ "%" -o %:r -std=c++20 -Wall -
Wextra -Wshadow -O2 -Dzisk -g -fsanitize=address,
undefined<CR>
map <F8> :!./%:r<CR>
inoremap {<CR> {<CR>}<CR><ESC>ko
ca Hash w !cpp -dD -P -fpreprocessed \ tr -d '[:space
:]' \ md5sum \ cut -c-6
inoremap fj <ESC>
vnoremap fj <ESC>
" -D_GLIBCXX_ASSERTIONS, -D_GLIBCXX_DEBUG
```

5.11 Minimum Steiner Tree	14
5.12 Count Cycles	15
6 Math	15
6.1 Extended Euclidean Algo-	15
rithm	
6.2 Floor & Ceil	15
6.3 Legendre	15
6.4 Simplex	15
6.5 Simplex Construction	16
6.6 LP Dual	16
6.7 DiscreteLog	16
6.8 Miller Rabin & Pollard Rho	16
6.9 XOR Basis	16
6.10 Linear Equation	16
6.11 Chinese Remainder Theorem	17
6.12 Sqrt Decomposition	17
6.13 Floor Sum	17
6.14 Berlekamp-Massey	17
7 Polynomial	17
7.1 FWHT	17
7.2 FFT	18
7.3 NTT	18
7.4 Polynomial Operation	18
7.5 Generating Function	20
Ordinary Generating Func-	
tion	20
Exponential Generating	
Function	20
7.6 Bostan Mori	20
8 String	21
8.1 KMP Algorithm	21
8.2 Manacher Algorithm	21
8.3 Lyndon Factorization	21
8.4 Suffix Array	21
8.5 Suffix Automaton	21
8.6 Z-value Algorithm	22
8.7 Main Loretz	22
8.8 AC Automaton	22
8.9 Palindrome Automaton	23
8.10 Palindrome Partition	23
9 Misc	23
9.1 Cyclic Ternary Search	23
9.2 Matroid	23
9.3 Simulate Annealing	23
9.4 Binary Search On Fraction	23
9.5 Min Plus Convolution	23
9.6 SMAWK	24
9.7 Golden Ratio Search	24
9.8 Python Misc	24
10 Notes	24
10.1 Geometry	24
Rotation Matrix	24
Triangles	24
Quadrilaterals	24
Spherical coordinates	24
Green's Theorem	24
Point-Line Duality	24
10.2 Trigonometry	24
10.3 Calculus	25
10.4 Sum & Series	25
10.5 Misc	25
10.6 Number	25

1.2 Debug [aaf3f9]

```
ostream& operator<<(ostream& o, pair<auto, auto> p)
{ return o << '(' << p.ff << ',' << p.ss << ')'; }
#ifdef zisk
void debug() { cerr << "\033[0m\n"; }
template<class T> void _d(T&& x) {
    if constexpr (ranges::range<T> &&
        !is_convertible_v<T, string_view>) {
        cerr << "{ "; for (auto&& i : x) _d(i);
        cerr << " ";
    } else cerr << x << " ";
} // ranges::subrange(L, r)
void debug(auto&&... a)
{ cerr << "\033[33m"; (_d(a), ...); debug(); }
#else
#define debug(...) void()
#endif
```

1.3 Fast IO [4f6f0e]

```
char readchar() {
    const int N = 1<<20;
    static char buf[N];
    static char *p = buf, *end = buf;
    if(p == end) {
        if((end = buf + fread(buf, 1, N, stdin)) == buf)
            return EOF;
        p = buf;
    }
    return *p++;
}

const int buf_size = 524288;
struct Writer {
    char buf[buf_size]; int size = 0, ret;
    void flush() { ret = write(1, buf, size); size = 0; }
    void _flush(int sz) { if (sz + size > buf_size) flush
        (); }
    void write_char(char c) { _flush(1); buf[size++] = c;
    }
    void write_int(int x) {
        const int len = 20;
        _flush(len); int ptr = 0;
        if (x < 0) buf[size++] = '-', x = -x;
        if (x == 0) buf[size + (ptr++)] = '0';
        else for (; x; x /= 10) buf[size + (ptr++)] = '0' +
            x % 10;
        reverse(buf + size, buf + size + ptr);
        size += ptr;
    }
}; // remember to call flush
```

1.4 Random [4cf9ed]

```
mt19937 rng(chrono::system_clock::now().
    time_since_epoch().count());
```

1.5 PBDS Tree [9e57e3]

```
#include <bits/extc++.h>
using namespace __gnu_pbds;
using Tree = tree<int, null_type, less<>, rb_tree_tag,
    tree_order_statistics_node_update>;
// .find_by_order(x)
// .order_of_key(x)
```

1.6 Pragma [6006f6]

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target("popcnt,abm,mmx,avx,arch=skylake")
__builtin_ia32_ldmxcsr(__builtin_ia32_stmxcsr()|0x8040)
```

1.7 SVG Writer [7adcc8]

```
class SVG {
    void p(string_view s) { o << s; }
    void p(string_view s, auto v, auto... vs) {
        auto i = s.find('$');
        o << s.substr(0, i) << v, p(s.substr(i + 1), vs...);
    }
    ostream o; string c = "red";
public: // SVG svg("test.svg", 0, 0, 100, 100)
    SVG(auto f, auto x1, auto y1, auto x2, auto y2) : o(f)
    { }
```

```

p("<svg xmlns='http://www.w3.org/2000/svg' "
  "viewBox='$ $ $ $'\n"
  "<style>{*stroke-width:0.5%;}</style>\n",
  x1, -y2, x2 - x1, y2 - y1); }
~SVG() { p("</svg>\n"); }
void color(string nc) { c = nc; }
void line(auto x1, auto y1, auto x2, auto y2) {
  p("<line x1='$' y1='$' x2='$' y2='$' stroke='$'/>\n"
    ,
    x1, -y1, x2, -y2, c); }
void circle(auto x, auto y, auto r) {
  p("<circle cx='$' cy='$' r='$' stroke='$' "
    "fill='none'/>\n", x, -y, r, c); }
void text(auto x, auto y, string s, int w = 12) {
  p("<text x='$' y='$' font-size='$px'>$</text>\n",
    x, -y, w, s); }
};

```

2 Data Structure

2.1 Heavy-Light Decomposition [f2dbca]

```

struct HLD{ // 1-based
  int n, ts = 0; // ord is 1-based
  vector<vector<int>> g;
  vector<int> par, top, down, ord, dpt, sub;
  explicit HLD(int _n): n(_n), g(n + 1),
    par(n + 1), top(n + 1), down(n + 1),
    ord(n + 1), dpt(n + 1), sub(n + 1) {}
  void add_edge(int u, int v){ g[u].pb(v); g[v].pb(u); }
  void dfs(int now, int p){
    par[now] = p; sub[now] = 1;
    for(int i : g[now]){
      if(i == p) continue;
      dpt[i] = dpt[now] + 1;
      dfs(i, now);
      sub[now] += sub[i];
      if(sub[i] > sub[down[now]]) down[now] = i;
    }
  }
  void cut(int now, int t){
    top[now] = t; ord[now] = ++ts;
    if(!down[now]) return;
    cut(down[now], t);
    for(int i : g[now]){
      if(i != par[now] && i != down[now])
        cut(i, i);
    }
  }
  void build(){ dfs(1, 1), cut(1, 1); }
  int query(int a, int b){
    int ta = top[a], tb = top[b];
    while(ta != tb){
      if(dpt[ta] > dpt[tb]) swap(ta, tb), swap(a, b);
      // ord[ta], ord[b]
      tb = top[b = par[tb]];
    }
    if(ord[a] > ord[b]) swap(a, b);
    // ord[a], ord[b]
    return a; // Lca
  }
};

```

2.2 Link Cut Tree [502ab1]

```

// 1-based
// == 43515a ==
template <typename Val, typename SVal> struct LCT {
  struct node {
    int pa, ch[2]; bool rev; int size;
    Val v, sum, rsum; SVal sv, sub, vir;
    node() : pa{0}, ch{0, 0}, rev{false}, size{1}, v{},
      sum{}, rsum{}, sv{}, sub{}, vir{} {}
  };
  #define cur o[u]
  #define lc cur.ch[0]
  #define rc cur.ch[1]
  vector<node> o;
  bool is_root(int u) const {
    return o[cur.pa].ch[0] != u && o[cur.pa].ch[1] != u; }
  bool is_rch(int u) const {
    return o[cur.pa].ch[1] == u && !is_root(u); }
  void down(int u) {

```

```

    for (int c : {lc, rc}) if (c) {
      if (cur.rev) set_rev(c);
    }
    cur.rev = false;
  }
  void up(int u) {
    cur.sum = o[lc].sum + cur.v + o[rc].sum;
    cur.rsum = o[rc].rsum + cur.v + o[lc].rsum;
    cur.sub = cur.vir + o[lc].sub + o[rc].sub + cur.sv;
    cur.size = o[lc].size + o[rc].size + 1;
  }
  void set_rev(int u) {
    swap(lc, rc), swap(cur.sum, cur.rsum);
    cur.rev ^= 1;
  }
  // == 3a186b ==
  void rotate(int u) {
    int f = cur.pa, g = o[f].pa, l = is_rch(u);
    if (cur.ch[l ^ 1]) o[cur.ch[l ^ 1]].pa = f;
    if (not is_root(f)) o[g].ch[is_rch(f)] = u;
    o[f].ch[l] = cur.ch[l ^ 1], cur.ch[l ^ 1] = f;
    cur.pa = g, o[f].pa = u; up(f);
  }
  vector<int> stk;
  void splay(int u) {
    stk.clear(); stk.pb(u);
    while (not is_root(stk.back()))
      stk.push_back(o[stk.back()].pa);
    while (not stk.empty())
      down(stk.back()), stk.pop_back();
    for (int f = cur.pa; not is_root(u); f = cur.pa) {
      if (!is_root(f))
        rotate(is_rch(u) == is_rch(f) ? f : u);
      rotate(u);
    }
    up(u);
  }
  void access(int x) {
    for (int u = x, last = 0; u; u = cur.pa) {
      splay(u);
      cur.vir = cur.vir + o[rc].sub - o[last].sub;
      rc = last; up(last = u);
    }
    splay(x);
  }
  int find_root(int u) {
    int la = 0;
    for (access(u); u; u = lc) down(la = u);
    return la;
  }
  void split(int x, int y) { chroot(x); access(y); }
  void chroot(int u) { access(u); set_rev(u); }
  // == a238c2 ==
  LCT(int n = 0) : o(n + 1) { o[0].size = 0; }
  void set_val(int u, const Val &v) {
    splay(u); cur.v = v; up(u); }
  void set_sval(int u, const SVal &v) {
    access(u); cur.sv = v; up(u); }
  Val query(int x, int y) {
    split(x, y); return o[y].sum; }
  SVal subtree(int p, int u) {
    chroot(p); access(u); return cur.vir + cur.sv; }
  bool connected(int u, int v) {
    return find_root(u) == find_root(v); }
  void link(int x, int y) {
    chroot(x); access(y);
    o[y].vir = o[y].vir + o[x].sub;
    up(o[x].pa = y);
  }
  void cut(int x, int y) {
    split(x, y); o[y].ch[0] = o[x].pa = 0; up(y); }
  #undef cur
  #undef lc
  #undef rc
};

```

2.3 Treap [2ac37e]

```

mt19937 rng(880301);
// == fb4359 ==
struct node {
  ll data; int sz;
  node *l, *r;

```

```

node(ll k = 0) : data(k), sz(1), l(0), r(0) {}
void up() {
    sz = 1;
    if (l) sz += l->sz;
    if (r) sz += r->sz;
}
void down() {}
};
node pool[1000010]; int pool_cnt = 0;
node *newnode(ll k){ return &(pool[pool_cnt++] = node(k)); }

int sz(node *a) { return a ? a->sz : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (int(rng() % (sz(a) + sz(b))) < sz(a))
        return a->down(), a->r = merge(a->r, b), a->up(), a;
    return b->down(), b->l = merge(a, b->l), b->up(), b;
}
// a: key <= k, b: key > k
void split(node *o, node *&a, node *&b, ll k) {
    if (!o) return a = b = 0, void();
    o->down();
    if (o->data <= k)
        a = o, split(o->r, a->r, b, k), a->up();
    else b = o, split(o->l, a, b->l, k), b->up();
}
// a: size k, b: size n - k
void split2(node *o, node *&a, node *&b, int k) {
    if (sz(o) <= k) return a = o, b = 0, void();
    o->down();
    if (sz(o->l) + 1 <= k)
        a = o, split2(o->r, a->r, b, k - sz(o->l) - 1);
    else b = o, split2(o->l, a, b->l, k);
    o->up();
}
// == e9f4d8 ==
node *kth(node *o, ll k) { // 1-based
    if (k <= sz(o->l)) return kth(o->l, k);
    if (k == sz(o->l) + 1) return o;
    return kth(o->r, k - sz(o->l) - 1);
}
int Rank(node *o, ll key) { // num of key < key
    if (!o) return 0;
    if (o->data < key)
        return sz(o->l) + 1 + Rank(o->r, key);
    else return Rank(o->l, key);
}
bool erase(node *&o, ll k) {
    if (!o) return 0;
    if (o->data == k) {
        node *t = o;
        o->down(), o = merge(o->l, o->r);
        return 1;
    }
    node *&t = k < o->data ? o->l : o->r;
    return erase(t, k) ? o->up(), 1 : 0;
}
void insert(node *&o, ll k) {
    node *a, *b;
    split(o, a, b, k),
    o = merge(a, merge(new node(k), b));
}
tuple<node*, node*, node*> interval(node *&o, int l,
    int r) { // 1-based
    node *a, *b, *c; // b: [l, r]
    split2(o, a, b, l - 1), split2(b, b, c, r - l + 1);
    return make_tuple(a, b, c);
}

```

2.4 KD Tree [375ca2]

```

namespace kdt {
    int root, lc[maxn], rc[maxn], xl[maxn], xr[maxn],
    yl[maxn], yr[maxn];
    point p[maxn];
    int build(int l, int r, int dep = 0) {
        if (l == r) return -1;
        function<bool(const point &, const point &)> f =
            [dep](const point &a, const point &b) {
                if (dep & 1) return a.x < b.x;
                else return a.y < b.y;
            };
    }
}

```

```

int m = (l + r) >> 1;
nth_element(p + l, p + m, p + r, f);
xl[m] = xr[m] = p[m].x;
yl[m] = yr[m] = p[m].y;
lc[m] = build(l, m, dep + 1);
if (~lc[m]) {
    xl[m] = min(xl[m], xl[lc[m]]);
    xr[m] = max(xr[m], xr[lc[m]]);
    yl[m] = min(yl[m], yl[lc[m]]);
    yr[m] = max(yr[m], yr[lc[m]]);
}
rc[m] = build(m + 1, r, dep + 1);
if (~rc[m]) {
    xl[m] = min(xl[m], xl[rc[m]]);
    xr[m] = max(xr[m], xr[rc[m]]);
    yl[m] = min(yl[m], yl[rc[m]]);
    yr[m] = max(yr[m], yr[rc[m]]);
}
return m;
}
bool bound(const point &q, int o, long long d) {
    double ds = sqrt(d + 1.0);
    if (q.x < xl[o] - ds || q.x > xr[o] + ds ||
        q.y < yl[o] - ds || q.y > yr[o] + ds)
        return false;
    return true;
}
long long dist(const point &a, const point &b) {
    return (a.x - b.x) * 1ll * (a.x - b.x) +
        (a.y - b.y) * 1ll * (a.y - b.y);
}
void dfs(
    const point &q, long long &d, int o, int dep = 0)
{
    if (!bound(q, o, d)) return;
    long long cd = dist(p[o], q);
    if (cd != 0) d = min(d, cd);
    if ((dep & 1) && q.x < p[o].x ||
        !(dep & 1) && q.y < p[o].y) {
        if (~lc[o]) dfs(q, d, lc[o], dep + 1);
        if (~rc[o]) dfs(q, d, rc[o], dep + 1);
    } else {
        if (~rc[o]) dfs(q, d, rc[o], dep + 1);
        if (~lc[o]) dfs(q, d, lc[o], dep + 1);
    }
}
void init(const vector<point> &v) {
    for (int i = 0; i < v.size(); ++i) p[i] = v[i];
    root = build(0, v.size());
}
long long nearest(const point &q) {
    long long res = 1e18;
    dfs(q, res, root);
    return res;
}
} // namespace kdt

```

2.5 Leftist Tree [e91538]

```

struct node {
    ll v, data, sz, sum;
    node *l, *r;
    node(ll k)
        : v(0), data(k), sz(1), l(0), r(0), sum(k) {}
};
ll sz(node *p) { return p ? p->sz : 0; }
ll V(node *p) { return p ? p->v : -1; }
ll sum(node *p) { return p ? p->sum : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (a->data < b->data) swap(a, b);
    a->r = merge(a->r, b);
    if (V(a->r) > V(a->l)) swap(a->r, a->l);
    a->v = V(a->r) + 1, a->sz = sz(a->l) + sz(a->r) + 1;
    a->sum = sum(a->l) + sum(a->r) + a->data;
    return a;
}
void pop(node *&o) {
    node *tmp = o;
    o = merge(o->l, o->r);
    delete tmp;
}

```

2.6 Convex 1D/1D [a449dd]

```
template<class T>
struct DynamicHull {
    struct seg { int x, l, r; };
    T f; int C; deque<seg> dq; // range: 1~C
    explicit DynamicHull(T _f, int _C): f(_f), C(_C) {}
    // max t s.t. f(x, t) >= f(y, t), x < y, maintain max
    int intersect(int x, int y) {
        int l = 0, r = C + 1;
        while (l + 1 < r) {
            int mid = (l + r) / 2;
            if (f(x, mid) >= f(y, mid)) l = mid;
            else r = mid;
        }
        return l;
    }
    void push_back(int x) {
        for (int i; !dq.empty() &&
            (i = dq.back().l, f(dq.back().x, i) < f(x, i));
            )
            dq.pop_back();
        if (dq.empty()) return dq.pb(seg({x, 1, C})), void();
        dq.back().r = intersect(dq.back().x, x);
        if (dq.back().r + 1 <= C) dq.pb(seg({x, dq.back().r
            + 1, C}));
    }
    int query(int x) {
        while (dq.front().r < x) dq.pop_front();
        return dq.front().x;
    }
};
```

2.7 Dynamic Convex Hull [7fcc55]

```
// only works for integer coordinates!! maintain max
struct Line {
    mutable ll a, b, p;
    bool operator<(const Line &rhs) const { return a <
        rhs.a; }
    bool operator<(ll x) const { return p < x; }
};
struct DynamicHull : multiset<Line, less<>> {
    static const ll kInf = 1e18;
    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = kInf; return 0; }
        if (x->a == y->a) x->p = x->b > y->b ? kInf : -kInf;
        else x->p = ifloor(y->b - x->b, x->a - y->a);
        return x->p >= y->p;
    }
    void addline(ll a, ll b) {
        auto z = insert({a, b, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y =
            erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    ll query(ll x) {
        auto l = *lower_bound(x);
        return l.a * x + l.b;
    }
};
```

3 Flow & Matching

3.1 Dinic [533718]

```
struct Dinic { // 0-based,  $O(V^2E)$ , unit flow:  $O(\min(V, E^{2/3}))$ , bipartite matching:  $O(\sqrt{V}E)$ 
    struct edge { ll to, cap, flow, rev; };
    int n, s, t;
    vector<vector<edge>> g; vector<int> dis, ind;
    explicit Dinic(int _n): n(_n), g(n) {}
    void reset() {
        for (int i = 0; i < n; ++i)
            for (auto &j : g[i]) j.flow = 0;
    }
    void add_edge(int u, int v, ll cap) {
        g[u].pb(edge{v, cap, 0, SZ(g[v])});
        g[v].pb(edge{u, 0, 0, SZ(g[u]) - 1});
        //change g[v] to cap for undirected graphs
    }
};
```

```
bool bfs() { // 115227
    dis.assign(n, -1);
    queue<int> q;
    q.push(s), dis[s] = 0;
    while (!q.empty()) {
        int cur = q.front(); q.pop();
        for (auto &e : g[cur]) {
            if (dis[e.to] == -1 && e.flow != e.cap) {
                q.push(e.to);
                dis[e.to] = dis[cur] + 1;
            }
        }
    }
    return dis[t] != -1;
}
ll dfs(int u, ll cap) { // 7b0fc6
    if (u == t || !cap) return cap;
    for (int &i = ind[u]; i < SZ(g[u]); ++i) {
        edge &e = g[u][i];
        if (dis[e.to] == dis[u] + 1 && e.flow != e.cap) {
            ll df = dfs(e.to, min(e.cap - e.flow, cap));
            if (df) {
                e.flow += df;
                g[e.to][e.rev].flow -= df;
                return df;
            }
        }
        dis[u] = -1;
        return 0;
    }
}
ll maxflow(int _s, int _t) { // 49f67d
    s = _s; t = _t;
    ll flow = 0, df;
    while (bfs()) {
        ind.assign(n, 0);
        while ((df = dfs(s, INF))) flow += df;
    }
    return flow;
};
```

3.2 Bounded Flow [345dfe]

```
struct BoundedFlow : Dinic {
    vector<ll> tot;
    BoundedFlow(int _n): Dinic(_n + 2), tot(n) {}
    void add_edge(int u, int v, ll lcap, ll rcap) {
        tot[u] -= lcap, tot[v] += lcap;
        g[u].pb(edge{v, rcap, lcap, SZ(g[v])});
        g[v].pb(edge{u, 0, 0, SZ(g[u]) - 1});
    }
    bool feasible() { // a263a8
        ll sum = 0;
        int vs = n - 2, vt = n - 1;
        for (int i = 0; i < n - 2; ++i)
            if (tot[i] > 0)
                add_edge(vs, i, 0, tot[i]), sum += tot[i];
            else if (tot[i] < 0) add_edge(i, vt, 0, -tot[i]);
        if (sum != maxflow(vs, vt)) sum = -1;
        for (int i = 0; i < n - 2; ++i)
            if (tot[i] > 0)
                g[vs].pop_back(), g[i].pop_back();
            else if (tot[i] < 0)
                g[i].pop_back(), g[vt].pop_back();
        return sum != -1;
    }
    ll boundedflow(int _s, int _t) {
        add_edge(_t, _s, 0, INF);
        if (!feasible()) return -1;
        ll x = g[_t].back().flow;
        g[_t].pop_back(), g[_s].pop_back();
        return x - maxflow(_t, _s); // min
        //return x + maxflow(_s, _t); // max
    }
};
```

3.3 MCMF [671e14]

```
struct MCMF { // 0-base
    struct Edge {
        ll from, to, cap, flow, cost, rev;
    };
    int n, s, t;
    vector<vector<Edge>> g;
    vector<Edge*> past;
```

```

vector<ll> dis, up, pot;
explicit MCMF(int _n): n(_n), g(n), past(n), dis(n),
    up(n), pot(n) {}
void add_edge(ll a, ll b, ll cap, ll cost) {
    g[a].pb(Edge{a, b, cap, 0, cost, SZ(g[b]))});
    g[b].pb(Edge{b, a, 0, 0, -cost, SZ(g[a]) - 1});
}
bool BellmanFord() {
    vector<bool> inq(n);
    fill(iter(dis), INF);
    queue<int> q;
    auto relax = [&](int u, ll d, ll cap, Edge *e) {
        if (cap > 0 && dis[u] > d) {
            dis[u] = d, up[u] = cap, past[u] = e;
            if (!inq[u]) inq[u] = 1, q.push(u);
        }
    };
    relax(s, 0, INF, 0);
    while (!q.empty()) {
        int u = q.front();
        q.pop(), inq[u] = 0;
        for (auto &e : g[u]) {
            ll d2 = dis[u] + e.cost + pot[u] - pot[e.to];
            relax(e.to, d2, min(up[u], e.cap - e.flow), &e);
        }
    }
    return dis[t] != INF;
}
pair<ll, ll> solve(int _s, int _t, bool neg = true) {
    s = _s, t = _t; ll flow = 0, cost = 0;
    if (neg) BellmanFord(), pot = dis;
    for (; BellmanFord(); pot = dis) {
        for (int i = 0; i < n; ++i)
            if (dis[i] != INF) dis[i] += pot[i] - pot[s];
        flow += up[t], cost += up[t] * dis[t];
        for (int i = t; past[i]; i = past[i]->from) {
            auto &e = *past[i];
            e.flow += up[t], g[e.to][e.rev].flow -= up[t];
        }
    }
    return {flow, cost};
}
};

```

3.4 Min Cost Circulation [47cf18]

```

struct MinCostCirculation { //  $\theta$ -based,  $O(VE * E \log C)$ 
    struct edge {
        ll from, to, cap, fcap, flow, cost, rev;
    };
    int n;
    vector<edge*> past;
    vector<vector<edge*>> g;
    vector<ll> dis;
    void BellmanFord(int s) {
        vector<int> inq(n);
        dis.assign(n, INF);
        queue<int> q;
        auto relax = [&](int u, ll d, edge *e) {
            if (dis[u] > d) {
                dis[u] = d, past[u] = e;
                if (!inq[u]) inq[u] = 1, q.push(u);
            }
        };
        relax(s, 0, 0);
        while (!q.empty()) {
            int u = q.front();
            q.pop(), inq[u] = 0;
            for (auto &e : g[u])
                if (e.cap > e.flow)
                    relax(e.to, dis[u] + e.cost, &e);
        }
    }
    void try_edge(edge &cur) {
        if (cur.cap > cur.flow) return ++cur.cap, void();
        BellmanFord(cur.to);
        if (dis[cur.from] + cur.cost < 0) {
            ++cur.flow, --g[cur.to][cur.rev].flow;
            for (int i = cur.from; past[i]; i = past[i]->from) {
                auto &e = *past[i];
                ++e.flow, --g[e.to][e.rev].flow;
            }
        }
    }
};

```

```

    }
}
++cur.cap;
}
void solve(int mxlg) { //  $mxlg \geq \log(\max cap)$ 
    for (int b = mxlg; b >= 0; --b) {
        for (int i = 0; i < n; ++i)
            for (auto &e : g[i])
                e.cap *= 2, e.flow *= 2;
        for (int i = 0; i < n; ++i)
            for (auto &e : g[i])
                if (e.fcap >> b & 1)
                    try_edge(e);
    }
}
void init(int _n) {
    n = _n;
    past.assign(n, nullptr);
    g.assign(n, vector<edge*>());
}
void add_edge(ll a, ll b, ll cap, ll cost) {
    g[a].pb(edge{a, b, 0, cap, 0, cost, SZ(g[b]) + (a
        == b)});
    g[b].pb(edge{b, a, 0, 0, 0, -cost, SZ(g[a]) - 1});
}
};

```

3.5 Gomory Hu [82d968]

```

void GomoryHu(Dinic &flow) { //  $\theta$ -based
    int n = flow.n;
    vector<int> par(n);
    for (int i = 1; i < n; ++i) {
        flow.reset();
        add_edge(i, par[i], flow.maxflow(i, par[i]));
        for (int j = i + 1; j < n; ++j)
            if (par[j] == par[i] && ~flow.dis[j])
                par[j] = i;
    }
}

```

3.6 Stoer Wagner Algorithm [a9917b]

```

struct StoerWagner { //  $\theta$ -based,  $O(V^3)$ 
    int n;
    vector<int> vis, del;
    vector<ll> wei;
    vector<vector<ll>> edge;
    void init(int _n) {
        n = _n;
        del.assign(n, 0);
        edge.assign(n, vector<ll>(n));
    }
    void add_edge(int u, int v, ll w) {
        edge[u][v] += w, edge[v][u] += w;
    }
    void search(int &s, int &t) {
        vis.assign(n, 0); wei.assign(n, 0);
        s = t = -1;
        while (1) {
            ll mx = -1, cur = 0;
            for (int i = 0; i < n; ++i)
                if (!del[i] && !vis[i] && mx < wei[i])
                    cur = i, mx = wei[i];
            if (mx == -1) break;
            vis[cur] = 1, s = t, t = cur;
            for (int i = 0; i < n; ++i)
                if (!vis[i] && !del[i]) wei[i] += edge[cur][i];
        }
    }
    ll solve() {
        ll ret = INF;
        for (int i = 0, x=0, y=0; i < n-1; ++i) {
            search(x, y), ret = min(ret, wei[y]), del[y] = 1;
            for (int j = 0; j < n; ++j)
                edge[x][j] = (edge[j][x] += edge[y][j]);
        }
        return ret;
    }
};

```

3.7 Bipartite Matching [5bb9be]

```

//  $O(E \sqrt{V})$ ,  $O(E \log V)$  for random sparse graphs
struct BipartiteMatching { //  $\theta$ -based

```



```

int nl, nr;
vector<int> mx, my, dis, cur;
vector<vector<int>> g;
bool dfs(int u) {
    for (int &i = cur[u]; i < SZ(g[u]); ++i) {
        int e = g[u][i];
        if (!~my[e] || (dis[my[e]] == dis[u] + 1 && dfs(
            my[e])))
            return mx[my[e] = u] = e, 1;
    }
    dis[u] = -1;
    return 0;
}
bool bfs() {
    int ret = 0;
    queue<int> q;
    dis.assign(nl, -1);
    for (int i = 0; i < nl; ++i)
        if (!~mx[i]) q.push(i), dis[i] = 0;
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        for (int e : g[u])
            if (!~my[e]) ret = 1;
            else if (!dis[my[e]]) {
                q.push(my[e]);
                dis[my[e]] = dis[u] + 1;
            }
    }
    return ret;
}
int matching() {
    int ret = 0;
    mx.assign(nl, -1); my.assign(nr, -1);
    while (bfs()) {
        cur.assign(nl, 0);
        for (int i = 0; i < nl; ++i)
            if (!~mx[i] && dfs(i)) ++ret;
    }
    return ret;
}
void add_edge(int s, int t) { g[s].pb(t); }
void init(int _nl, int _nr) {
    nl = _nl, nr = _nr;
    g.assign(nl, vector<int>());
}
};

```

3.8 Kuhn Munkres Algorithm [683e0a]

```

struct KM { // 0-based, maximum matching, O(V^3)
    int n, ql, qr;
    vector<vector<ll>> w;
    vector<ll> hl, hr, slk;
    vector<int> fl, fr, pre, qu, vl, vr;
    void init(int _n) {
        n = _n;
        // -INF for perfect matching
        w.assign(n, vector<ll>(n, 0));
        pre.assign(n, 0);
        qu.assign(n, 0);
    }
    void add_edge(int a, int b, ll wei) {
        w[a][b] = wei;
    }
    bool check(int x) {
        if (vl[x] = 1, ~fl[x])
            return (vr[qu[qr++]] = fl[x] = 1);
        while (~x) swap(x, fr[fl[x] = pre[x]]);
        return 0;
    }
    void bfs(int s) {
        slk.assign(n, INF); vl.assign(n, 0); vr.assign(n, 0);
        ql = qr = 0, qu[qr++] = s, vr[s] = 1;
        for (ll d;;) {
            while (ql < qr)
                for (int x = 0, y = qu[ql++]; x < n; ++x)
                    if (!vl[x] && slk[x] >= (d = hl[x] + hr[y] -
                        w[x][y])) {
                        if (pre[x] = y, d) slk[x] = d;
                        else if (!check(x)) return;
                    }
        }
    }
};

```

```

d = INF;
for (int x = 0; x < n; ++x)
    if (!vl[x] && d > slk[x]) d = slk[x];
for (int x = 0; x < n; ++x) {
    if (vl[x]) hl[x] += d;
    else slk[x] -= d;
    if (vr[x]) hr[x] -= d;
}
for (int x = 0; x < n; ++x)
    if (!vl[x] && !slk[x] && !check(x)) return;
}
}
ll solve() {
    fl.assign(n, -1); fr.assign(n, -1); hl.assign(n, 0)
        ; hr.assign(n, 0);
    for (int i = 0; i < n; ++i)
        hl[i] = *max_element(iter(w[i]));
    for (int i = 0; i < n; ++i) bfs(i);
    ll res = 0;
    for (int i = 0; i < n; ++i) res += w[i][fl[i]];
    return res;
}
};

```

3.9 Max Simple Graph Matching [907d7c]

```

struct Matching { // 0-based, O(V^3)
    queue<int> q; int n;
    vector<int> fa, s, vis, pre, match;
    vector<vector<int>> g;
    int Find(int u)
    { return u == fa[u] ? u : fa[u] = Find(fa[u]); }
    int LCA(int x, int y) {
        static int tk = 0; tk++; x = Find(x); y = Find(y);
        for (; swap(x, y)) if (x != n) {
            if (vis[x] == tk) return x;
            vis[x] = tk;
            x = Find(pre[match[x]]);
        }
    }
    void Blossom(int x, int y, int l) {
        for (; Find(x) != l; x = pre[y]) {
            pre[x] = y, y = match[x];
            if (s[y] == 1) q.push(y), s[y] = 0;
            for (int z: {x, y}) if (fa[z] == z) fa[z] = l;
        }
    }
    bool Bfs(int r) {
        iota(iter(fa), 0); fill(iter(s), -1);
        q = queue<int>(); q.push(r); s[r] = 0;
        for (; !q.empty(); q.pop()) {
            for (int x = q.front(); int u : g[x])
                if (s[u] == -1) {
                    if (pre[u] = x, s[u] = 1, match[u] == n) {
                        for (int a = u, b = x, last;
                            b != n; a = last, b = pre[a])
                            last = match[b], match[b] = a, match[a] =
                                b;
                        return true;
                    }
                    q.push(match[u]); s[match[u]] = 0;
                } else if (!s[u] && Find(u) != Find(x)) {
                    int l = LCA(u, x);
                    Blossom(x, u, l); Blossom(u, x, l);
                }
            }
        }
        return false;
    }
    Matching(int _n) : n(_n), fa(n + 1), s(n + 1), vis(n
        + 1), pre(n + 1, n), match(n + 1, n), g(n) {}
    void add_edge(int u, int v)
    { g[u].pb(v), g[v].pb(u); }
    int solve() {
        int ans = 0;
        for (int x = 0; x < n; ++x)
            if (match[x] == n) ans += Bfs(x);
        return ans;
    } // match[x] == n means not matched
};

```

3.10 Flow Model

- Maximum/Minimum flow with lower bound / Circulation problem
- 1. Construct super source S and sink T .

2. For each edge (x, y, l, u) , connect $x \rightarrow y$ with capacity $u - l$.
3. For each vertex v , denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
4. If $in(v) > 0$, connect $S \rightarrow v$ with capacity $in(v)$, otherwise, connect $v \rightarrow T$ with capacity $-in(v)$.
 - To maximize, connect $t \rightarrow s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T . If $f \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
 - To minimize, let f be the maximum flow from S to T . Connect $t \rightarrow s$ with capacity ∞ and let the flow from S to T be f' . If $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, f' is the answer.
5. The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
 1. Redirect every edge: $y \rightarrow x$ if $(x, y) \in M$, $x \rightarrow y$ otherwise.
 2. DFS from unmatched vertices in X .
 3. $x \in X$ is chosen iff x is unvisited.
 4. $y \in Y$ is chosen iff y is visited.
- Minimum cost cyclic flow
 1. Construct super source S and sink T
 2. For each edge (x, y, c) , connect $x \rightarrow y$ with $(cost, cap) = (c, 1)$ if $c > 0$, otherwise connect $y \rightarrow x$ with $(cost, cap) = (-c, 1)$
 3. For each edge with $c < 0$, sum these cost as K , then increase $d(y)$ by 1, decrease $d(x)$ by 1
 4. For each vertex v with $d(v) > 0$, connect $S \rightarrow v$ with $(cost, cap) = (0, d(v))$
 5. For each vertex v with $d(v) < 0$, connect $v \rightarrow T$ with $(cost, cap) = (0, -d(v))$
 6. Flow from S to T , the answer is the cost of the flow $C + K$
- Maximum density induced subgraph
 1. Binary search on answer, suppose we're checking answer T
 2. Construct a max flow model, let K be the sum of all weights
 3. Connect source $s \rightarrow v$, $v \in G$ with capacity K
 4. For each edge (u, v, w) in G , connect $u \rightarrow v$ and $v \rightarrow u$ with capacity w
 5. For $v \in G$, connect it with sink $v \rightarrow t$ with capacity $K + 2T - (\sum_{e \in E(v)} w_{2w}(v))$
 6. T is a valid answer if the maximum flow $f < K|V|$
- Minimum weight edge cover
 1. Let $w'(u, v) = w(u, v) - \mu(u) - \mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v .
 2. Find the minimum weight matching M with w' . The answer is $\sum \mu(v) + w'(M)$.
- Project selection problem
 1. If $p_v > 0$, create edge (s, v) with capacity p_v ; otherwise, create edge (v, t) with capacity $-p_v$.
 2. Create edge (u, v) with capacity w with w being the cost of choosing u without choosing v .
 3. The mincut is equivalent to the maximum profit of a subset of projects.
- Dual of minimum cost maximum flow
 1. Capacity c_{uv} , Flow f_{uv} , Cost w_{uv} , Required Flow difference for vertex b_u .
 2. If all w_{uv} are integers, then optimal solution can happen when all p_u are integers.

$$\min \sum_{uv} w_{uv} f_{uv} \quad \min \sum_u b_u p_u + \sum_{uv} c_{uv} \max(0, p_v - p_u - w_{uv})$$

$$-f_{uv} \geq -c_{uv} \Leftrightarrow p_u \geq 0$$

$$\sum_v f_{vu} - \sum_v f_{uv} = -b_u$$

4 Geometry

4.1 Geometry Template [dab500]

```
using ld = double;
using pdd = pair<ld, ld>;
#define X first
#define Y second
const ld eps = 1e-9;

#define TEMP template<class T>
#define ptt pair<T, T>
TEMP ptt operator+(ptt a, ptt b)
{ return {a.X + b.X, a.Y + b.Y}; }
TEMP ptt operator-(ptt a, ptt b)
{ return {a.X - b.X, a.Y - b.Y}; }
TEMP ptt operator*(T i, ptt v)
{ return {i * v.X, i * v.Y}; }
TEMP ptt operator*(ptt v, T i)
```

```
{ return {i * v.X, i * v.Y}; }
TEMP pdd operator/(ptt v, ld i)
{ return {v.X / i, v.Y / i}; }
TEMP T dot(ptt a, ptt b)
{ return a.X * b.X + a.Y * b.Y; }
TEMP T cross(ptt a, ptt b)
{ return a.X * b.Y - a.Y * b.X; }
TEMP T abs2(ptt v)
{ return v.X * v.X + v.Y * v.Y; }
TEMP ld abs(ptt v)
{ return sqrt(abs2(v)); }
int sgn(ld v)
{ return v > 0 ? 1 : (v < 0 ? -1 : 0); }
int sgn(ld v){ return v > eps ? 1 : (v < -eps ? -1 : 0); }
TEMP int ori(ptt a, ptt b, ptt c)
{ return sgn(cross(b - a, c - a)); }
TEMP bool collinearity(ptt a, ptt b, ptt c)
{ return ori(a, b, c) == 0; }
TEMP bool btw(ptt p, ptt a, ptt b)
{ return collinearity(p, a, b) && sgn(dot(a - p, b - p)) <= 0; }

TEMP bool seg_intersect(ptt p1, ptt p2, ptt p3, ptt p4)
{
    if(btw(p1, p3, p4) || btw(p2, p3, p4) || btw(p3, p1, p2) || btw(p4, p1, p2))
        return true;
    return ori(p1, p2, p3) * ori(p1, p2, p4) < 0 && ori(p3, p4, p1) * ori(p3, p4, p2) < 0;
}
pdd intersect(pdd p1, pdd p2, pdd p3, pdd p4){
    ld a123 = cross(p2 - p1, p3 - p1);
    ld a124 = cross(p2 - p1, p4 - p1);
    return (p4 * a123 - p3 * a124) / (a123 - a124);
}
TEMP ptt perp(ptt p1)
{ return {-p1.Y, p1.X}; }
pdd projection(pdd p1, pdd p2, pdd p3)
{ return p1 + (p2 - p1) * dot(p3 - p1, p2 - p1) / abs2(p2 - p1); }
pdd reflection(pdd p1, pdd p2, pdd p3)
{ return p3 + perp(p2 - p1) * cross(p3 - p1, p2 - p1) / abs2(p2 - p1) * 2.0; }
pdd linearTransformation(pdd p0, pdd p1, pdd q0, pdd q1, pdd r) {
    pdd dp = p1 - p0, dq = q1 - q0, num(cross(dp, dq), dot(dp, dq));
    return q0 + pdd(cross(r - p0, num), dot(r - p0, num)) / abs2(dp);
} // from line p0--p1 to q0--q1, apply to r
```

4.2 Convex Hull [5a1ce0]

```
vector<int> getConvexHull(vector<pdd>& pts){
    vector<int> id(SZ(pts));
    iota(iter(id), 0);
    sort(iter(id), [&](int x, int y){ return pts[x] < pts[y]; });
    vector<int> hull;
    for(int tt = 0; tt < 2; tt++){
        int sz = SZ(hull);
        for(int j : id){
            pdd p = pts[j];
            while(SZ(hull) - sz >= 2 && ori(pts[hull.end()[-2]], pts[hull.back()], p) <= 0)
                hull.pop_back();
            hull.pb(j);
        }
        hull.pop_back();
        reverse(iter(id));
    }
    return hull;
}
```

4.3 Polar Angle Comparator [808e89]

```
// -1: a // b (if same), 0/1: a < b
int cmp(p11 a, p11 b, bool same = true){
#define is_neg(k) (sgn(k.Y) < 0 || (sgn(k.Y) == 0 && sgn(k.X) < 0))
    int A = is_neg(a), B = is_neg(b);
    if(A != B)
        return A < B;
```

```

if(sgn(cross(a, b)) == 0)
    return same ? abs2(a) < abs2(b) : -1;
return sgn(cross(a, b)) > 0;
}

```

4.4 Minkowski Sum [af1dac]

```

TEMP void reorder_poly(vector<ptt>& pts){
    rotate(pts.begin(), min_element(iter(pts),
        [&](ptt x, ptt y){
            return x.Y != y.Y ? x.Y < y.Y : x.X < y.X; })),
        pts.end());
}
TEMP vector<ptt> minkowski(vector<ptt> P, vector<ptt> Q
    ){
    reorder_poly(P); reorder_poly(Q);
    int psz = P.size(), qsz = Q.size();
    P.pb(P[0]); P.pb(P[1]); Q.pb(Q[0]); Q.pb(Q[1]);
    vector<ptt> ans; int i = 0, j = 0;
    while (i < psz || j < qsz) {
        ans.pb(P[i] + Q[j]);
        int t = sgn(cross(P[i + 1] - P[i], Q[j + 1] - Q[j]));
        if(t >= 0) i++; if(t <= 0) j++;
    }
    return ans;
}

```

4.5 Intersection of Circle and Convex Polygon

[63653d]

```

double _area(pdd pa, pdd pb, double r){
    if(abs(pa)<abs(pb)) swap(pa, pb);
    if(abs(pb)<eps) return 0;
    double S, h, theta;
    double a=abs(pb), b=abs(pa), c=abs(pb-pa);
    double cosB = dot(pb, pb-pa) / a / c, B = acos(cosB);
    double cosC = dot(pa, pb) / a / b, C = acos(cosC);
    if(a > r){
        S = (C/2)*r*r;
        h = a*b*sin(C)/c;
        if (h < r && B < PI/2) S -= (acos(h/r)*r*r - h*sqrt(
            (r*r-h*h)));
    }
    else if(b > r){
        theta = PI - B - asin(sin(B)/r*a);
        S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
    }
    else S = .5*sin(C)*a*b;
    return S;
}
double areaPolyCircle(const vector<pdd> poly, const pdd
    &O, const double r){
    double S=0;
    for(int i=0; i<SZ(poly); ++i)
        S += _area(poly[i]-O, poly[(i+1)%SZ(poly)]-O, r)*ori(0,
            poly[i], poly[(i+1)%SZ(poly)]);
    return fabs(S);
}

```

4.6 Intersection of Circles [f7a2fe]

```

bool CCinter(Cir &a, Cir &b, pdd &p1, pdd &p2) {
    pdd o1 = a.O, o2 = b.O;
    double r1 = a.R, r2 = b.R, d2 = abs2(o1 - o2), d =
        sqrt(d2);
    if(d < max(r1, r2) - min(r1, r2) || d > r1 + r2)
        return 0;
    pdd u = (o1 + o2) * 0.5 + (o1 - o2) * ((r2 * r2 - r1
        * r1) / (2 * d2));
    double A = sqrt((r1 + r2 + d) * (r1 - r2 + d) * (r1 +
        r2 - d) * (-r1 + r2 + d));
    pdd v = pdd(o1.Y - o2.Y, -o1.X + o2.X) * A / (2 * d2)
        ;
    p1 = u + v, p2 = u - v;
    return 1;
}

```

4.7 Tangent Line of Circles [c51d90]

```

vector<Line> CCTang(const Cir& c1, const Cir& c2,
    int sign1){
    vector<Line> ret;
    double d_sq = abs2(c1.O - c2.O);
    if(sgn(d_sq) == 0) return ret;
    double d = sqrt(d_sq);
    pdd v = (c2.O - c1.O) / d;

```

```

    double c = (c1.R - sign1 * c2.R) / d; // cos t
    if(c * c > 1) return ret;
    double h = sqrt(max(0.0, 1.0 - c * c)); // sin t
    for(int sign2 = 1; sign2 >= -1; sign2 -= 2) {
        pdd n = pdd(v.X * c - sign2 * h * v.Y,
            v.Y * c + sign2 * h * v.X);
        pdd p1 = c1.O + n * c1.R;
        pdd p2 = c2.O + n * (c2.R * sign1);
        if(sgn(p1.X - p2.X) == 0 and
            sgn(p1.Y - p2.Y) == 0)
            p2 = p1 + perp(c2.O - c1.O);
        ret.pb(Line(p1, p2));
    }
    return ret;
}

```

4.8 Intersection of Line and Convex Polygon

[157258]

```

int TangentDir(vector<p11> &C, p11 dir) {
    return cyc_tsearch(SZ(C), [&](int a, int b) {
        return cross(dir, C[a]) > cross(dir, C[b]);
    });
}
#define cml(i) sign(cross(C[i] - a, b - a))
pii lineHull(p11 a, p11 b, vector<p11> &C) {
    int A = TangentDir(C, a - b);
    int B = TangentDir(C, b - a);
    int n = SZ(C);
    if(cml(A) < 0 || cml(B) > 0)
        return pii(-1, -1); // no collision
    auto gao = [&](int l, int r) {
        for(int t = 1; (l + 1) % n != r; ) {
            int m = ((l + r + (l < r ? 0 : n)) / 2) % n;
            (cml(m) == cml(t) ? l : r) = m;
        }
        return (l + !cml(r)) % n;
    };
    pii res = pii(gao(B, A), gao(A, B)); // (i, j)
    if(res.X == res.Y) // touching the corner i
        return pii(res.X, -1);
    if(!cml(res.X) && !cml(res.Y)) // along side i, i
        +1
    switch((res.X - res.Y + n + 1) % n) {
        case 0: return pii(res.X, res.X);
        case 2: return pii(res.Y, res.Y);
    }
    // crossing sides (i, i+1) and (j, j+1)
    // crossing corner i is treated as side (i, i+1)
    // returned in the same order as the line hits the
    // convex */
    return res;
} // convex cut: (r, l]

```

4.9 Intersection of Line and Circle [9183db]

```

vector<pdd> circleLineIntersection(pdd c, double r, pdd
    a, pdd b) {
    pdd p = a + (b - a) * dot(c - a, b - a) / abs2(b - a)
        ;
    double s = cross(b - a, c - a), h2 = r * r - s * s /
        abs2(b - a);
    if(sgn(h2) < 0) return {};
    if(sgn(h2) == 0) return {p};
    pdd h = (b - a) / abs(b - a) * sqrt(h2);
    return {p - h, p + h};
}

```

4.10 Point in Circle [ecf954]

```

// return q's relation with circumcircle of tri(p[0], p
    [1], p[2])
bool in_cc(const array<p11, 3> &p, p11 q) {
    __int128 det = 0;
    for(int i = 0; i < 3; ++i)
        det += __int128(abs2(p[i]) - abs2(q)) * cross(p[(i
            + 1) % 3] - q, p[(i + 2) % 3] - q);
    return det > 0; // in: >0, on: =0, out: <0
}

```

4.11 Point in Convex [82b81e]

```

bool PointInConvex(const vector<p11> &C, p11 p, bool
    strict = true) {
    int a = 1, b = SZ(C) - 1, r = !strict;
    if(SZ(C) == 0) return false;

```



```

if (SZ(C) < 3) return r && btw(p, C[0], C.back());
if (ori(C[0], C[a], C[b]) > 0) swap(a, b);
if (ori(C[0], C[a], p) >= r || ori(C[0], C[b], p) <=
    -r)
    return false;
while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (ori(C[0], C[c], p) > 0 ? b : a) = c;
}
return ori(C[a], C[b], p) < r;
}

```

4.12 Half Plane Intersection [d34e39]

```

pll area_pair(Line a, Line b)
{ return pll(cross(a.Y - a.X, b.X - a.X), cross(a.Y - a
    .X, b.Y - a.X)); }
bool isin(Line l0, Line l1, Line l2) {
    // Check inter(l1, l2) strictly in l0
    auto [a02X, a02Y] = area_pair(l0, l2);
    auto [a12X, a12Y] = area_pair(l1, l2);
    if (a12X - a12Y < 0) a12X *= -1, a12Y *= -1;
    return ((__int128) a02Y * a12X - (__int128) a02X *
        a12Y > 0);
}
/* Having solution, check size > 2 */
/* ---^--- Line.X ---^--- Line.Y ---^--- */
vector<Line> halfPlaneInter(vector<Line> arr) {
    sort(iter(arr), [&](Line a, Line b) -> int {
        if (cmp(a.Y - a.X, b.Y - b.X, 0) != -1)
            return cmp(a.Y - a.X, b.Y - b.X, 0);
        return ori(a.X, a.Y, b.Y) < 0;
    });
    deque<Line> dq(1, arr[0]);
    auto pop_back = [&](int t, Line p) {
        while (SZ(dq) >= t && !isin(p, dq[SZ(dq) - 2], dq.
            back()))
            dq.pop_back();
    };
    auto pop_front = [&](int t, Line p) {
        while (SZ(dq) >= t && !isin(p, dq[0], dq[1]))
            dq.pop_front();
    };
    for (auto p : arr)
        if (cmp(dq.back().Y - dq.back().X, p.Y - p.X, 0) !=
            -1)
            pop_back(2, p), pop_front(2, p), dq.pb(p);
    pop_back(3, dq[0]), pop_front(3, dq.back());
    return vector<Line>(iter(dq));
}

```

4.13 HPI General Line [ba52e5]

```

// replace corresponding functions in vector HPI
using i128 = __int128;
struct LN {
    ll a, b, c; // ax + by + c <= 0
    pll dir() const { return pll(a, b); }
    LN(ll ta, ll tb, ll tc) : a(ta), b(tb), c(tc) {}
    LN(pll S, pll T) : a((T-S).Y), b(-(T-S).X), c(cross(T,
        S)) {}
};
pdd intersect(LN A, LN B) {
    double c = cross(A.dir(), B.dir());
    i128 a = i128(A.c) * B.a - i128(B.c) * A.a;
    i128 b = i128(A.c) * B.b - i128(B.c) * A.b;
    return pdd(-b / c, a / c);
}
bool isin(LN l, LN A, LN B) {
    i128 c = cross(A.dir(), B.dir());
    i128 a = i128(A.c) * B.a - i128(B.c) * A.a;
    i128 b = i128(A.c) * B.b - i128(B.c) * A.b;
    return sgn(a * l.b - b * l.a + c * l.c) * sgn(c) < 0;
}
int cmp(LN a, LN b) {
    if (int c = cmp(a.dir(), b.dir(), false); c != -1)
        return c;
    return i128(abs(b.a) + abs(b.b)) * a.c > i128(abs(a.a)
        + abs(a.b)) * b.c;
}

```

4.14 Minimum Enclosing Circle [5af6d5]

```

using ld = long double;
pair<pdd, ld> circumcenter(pdd a, pdd b, pdd c);

```

```

pair<pdd, ld> MinimumEnclosingCircle(vector<pdd> &pts){
    random_shuffle(iter(pts));
    pdd c = pts[0];
    ld r = 0;
    for(int i = 1; i < SZ(pts); i++){
        if(abs(pts[i] - c) <= r) continue;
        c = pts[i]; r = 0;
        for(int j = 0; j < i; j++){
            if(abs(pts[j] - c) <= r) continue;
            c = (pts[i] + pts[j]) / 2;
            r = abs(pts[i] - c);
            for(int k = 0; k < j; k++){
                if(abs(pts[k] - c) > r)
                    tie(c, r) = circumcenter(pts[i], pts[j], pts[
                        k]);
            }
        }
    }
    return {c, r};
}

```

4.15 3D Point [badbbd]

```

struct Point {
    double x, y, z;
    Point(double _x = 0, double _y = 0, double _z = 0) : x
        (_x), y(_y), z(_z){}
    Point(pdd p) { x = p.X, y = p.Y, z = abs2(p); }
};
Point operator-(Point p1, Point p2)
{ return Point(p1.x - p2.x, p1.y - p2.y, p1.z - p2.z); }
Point operator+(Point p1, Point p2)
{ return Point(p1.x + p2.x, p1.y + p2.y, p1.z + p2.z); }
Point operator*(Point p1, double v)
{ return Point(p1.x * v, p1.y * v, p1.z * v); }
Point operator/(Point p1, double v)
{ return Point(p1.x / v, p1.y / v, p1.z / v); }
Point cross(Point p1, Point p2)
{ return Point(p1.y * p2.z - p1.z * p2.y, p1.z * p2.x -
    p1.x * p2.z, p1.x * p2.y - p1.y * p2.x); }
double dot(Point p1, Point p2)
{ return p1.x * p2.x + p1.y * p2.y + p1.z * p2.z; }
double abs(Point a)
{ return sqrt(dot(a, a)); }
Point cross3(Point a, Point b, Point c)
{ return cross(b - a, c - a); }
double area(Point a, Point b, Point c)
{ return abs(cross3(a, b, c)); }
double volume(Point a, Point b, Point c, Point d)
{ return dot(cross3(a, b, c), d - a); }
//Azimuthal angle (Longitude) to x-axis in interval [-
    pi, pi]
double phi(Point p) { return atan2(p.y, p.x); }
//Zenith angle (Latitude) to the z-axis in interval [0,
    pi]
double theta(Point p) { return atan2(sqrt(p.x * p.x + p
    .y * p.y), p.z); }
Point masscenter(Point a, Point b, Point c, Point d)
{ return (a + b + c + d) / 4; }
pdd proj(Point a, Point b, Point c, Point u) {
    // proj. u to the plane of a, b, and c
    Point e1 = b - a;
    Point e2 = c - a;
    e1 = e1 / abs(e1);
    e2 = e2 - e1 * dot(e2, e1);
    e2 = e2 / abs(e2);
    Point p = u - a;
    return pdd(dot(p, e1), dot(p, e2));
}
Point rotate_around(Point p, double angle, Point axis)
{
    double s = sin(angle), c = cos(angle);
    Point u = axis / abs(axis);
    return u * dot(u, p) * (1 - c) + p * c + cross(u, p)
        * s;
}

```

4.16 ConvexHull3D [156311]

```

struct convex_hull_3D {
    struct Face {
        int a, b, c;
    };
};

```

```

Face(int ta, int tb, int tc): a(ta), b(tb), c(tc) {}
}; // return the faces with pt indexes
vector<Face> res;
vector<Point> P;
convex_hull_3D(const vector<Point> &_P): res(), P(_P) {
// all points coplanar case will WA, O(n^2)
int n = SZ(P);
if (n <= 2) return; // be careful about edge case
// ensure first 4 points are not coplanar
swap(P[1], *find_if(iter(P), [&](auto p) { return sgn
(abs2(P[0] - p)) != 0; }));
swap(P[2], *find_if(iter(P), [&](auto p) { return sgn
(abs2(cross3(p, P[0], P[1]))) != 0; }));
swap(P[3], *find_if(iter(P), [&](auto p) { return sgn
(volume(P[0], P[1], P[2], p)) != 0; }));
vector<vector<int>> flag(n, vector<int>(n));
res.emplace_back(0, 1, 2); res.emplace_back(2, 1, 0);
for (int i = 3; i < n; ++i) {
vector<Face> next;
for (auto f : res) {
int d = sgn(volume(P[f.a], P[f.b], P[f.c], P[i]));
if (d <= 0) next.pb(f);
int ff = (d > 0) - (d < 0);
flag[f.a][f.b] = flag[f.b][f.c] = flag[f.c][f.a] = ff;
}
for (auto f : res) {
auto F = [&](int x, int y) {
if (flag[x][y] > 0 && flag[y][x] <= 0)
next.emplace_back(x, y, i);
};
F(f.a, f.b); F(f.b, f.c); F(f.c, f.a);
}
res = next;
}
}
bool same(Face s, Face t) {
if (sgn(volume(P[s.a], P[s.b], P[s.c], P[t.a])) != 0)
return 0;
if (sgn(volume(P[s.a], P[s.b], P[s.c], P[t.b])) != 0)
return 0;
if (sgn(volume(P[s.a], P[s.b], P[s.c], P[t.c])) != 0)
return 0;
return 1;
}
int polygon_face_num() {
int ans = 0;
for (int i = 0; i < SZ(res); ++i)
ans += none_of(res.begin(), res.begin() + i, [&](
Face g) { return same(res[i], g); });
return ans;
}
double get_volume() {
double ans = 0;
for (auto f : res)
ans += volume(Point(0, 0, 0), P[f.a], P[f.b], P[f.c]);
return fabs(ans / 6);
}
double get_dis(Point p, Face f) {
Point p1 = P[f.a], p2 = P[f.b], p3 = P[f.c];
double a = (p2.y - p1.y) * (p3.z - p1.z) - (p2.z - p1.z) * (p3.y - p1.y);
double b = (p2.z - p1.z) * (p3.x - p1.x) - (p2.x - p1.x) * (p3.z - p1.z);
double c = (p2.x - p1.x) * (p3.y - p1.y) - (p2.y - p1.y) * (p3.x - p1.x);
double d = 0 - (a * p1.x + b * p1.y + c * p1.z);
return fabs(a * p.x + b * p.y + c * p.z + d) / sqrt(a * a + b * b + c * c);
}
};
// n^2 delaunay: facets with negative z normal of
// convexhull of (x, y, x^2 + y^2), use a pseudo-point
// (0, 0, inf) to avoid degenerate case

```

4.17 Delaunay Triangulation [6a9916]

/ Delaunay Triangulation:
Given a sets of points on 2D plane, find a
triangulation such that no points will strictly
inside circumcircle of any triangle. */*

```

struct Edge {
int id; // oidx[id]
list<Edge>::iterator twin;
Edge(int _id = 0): id(_id) {}
};
struct Delaunay { // 0-base
int n;
vector<int> oidx;
vector<list<Edge>> head; // result udir. graph
vector<p11> p;
Delaunay(int _n, vector<p11> _p): n(_n), oidx(n),
head(n), p(n) {
iota(iter(oidx), 0);
for (int i = 0; i < n; ++i) head[i].clear();
sort(iter(oidx), [&](int a, int b)
{ return _p[a] < _p[b]; });
for (int i = 0; i < n; ++i) p[i] = _p[oidx[i]];
divide(0, n - 1);
}
void addEdge(int u, int v) {
head[u].push_front(Edge(v));
head[v].push_front(Edge(u));
head[u].begin()->twin = head[v].begin();
head[v].begin()->twin = head[u].begin();
}
void divide(int l, int r) {
if (l == r) return;
if (l + 1 == r) return addEdge(l, l + 1);
int mid = (l + r) >> 1, nw[2] = {l, r};
divide(l, mid), divide(mid + 1, r);
auto gao = [&](int t) {
p11 pt[2] = {p[nw[0]], p[nw[1]]};
for (auto it : head[nw[t]]) {
int v = ori(pt[1], pt[0], p[it.id]);
if (v > 0 || (v == 0 && abs2(pt[t ^ 1] - p[it.
id]) < abs2(pt[1] - pt[0])))
return nw[t] = it.id, true;
}
return false;
};
while (gao(0) || gao(1));
addEdge(nw[0], nw[1]); // add tangent
while (true) {
p11 pt[2] = {p[nw[0]], p[nw[1]]};
int ch = -1, sd = 0;
for (int t = 0; t < 2; ++t)
for (auto it : head[nw[t]])
if (ori(pt[0], pt[1], p[it.id]) > 0 && (ch ==
-1 || in_cc({pt[0], pt[1], p[ch]}, p[it.
id])))
ch = it.id, sd = t;
if (ch == -1) break; // upper common tangent
for (auto it = head[nw[sd]].begin(); it != head[
nw[sd]].end(); )
if (seg_strict_intersect(pt[sd], p[it->id], pt[
sd ^ 1], p[ch]))
head[it->id].erase(it->twin), head[nw[sd]].
erase(it++);
else ++it;
nw[sd] = ch, addEdge(nw[0], nw[1]);
}
}
};

```

4.18 Voronoi Diagram [e4f408]

*// all coord. is even, you may want to call
halfPlaneInter after then*

```

vector<vector<Line>> vec;
void build_voronoi_line(int n, vector<p11> &pts) {
Delaunay tool(n, pts); // Delaunay
vec.clear(), vec.resize(n);
for (int i = 0; i < n; ++i)
for (auto e : tool.head[i]) {
int u = tool.oidx[i], v = tool.oidx[e.id];
p11 m = (pts[v] + pts[u]) / 2LL, d = perp(pts[v] -
pts[u]);
vec[u].pb(Line(m, m + d));
}
}

```

4.19 Polygon Union [9fbf66]

```
ld rat(p11 a, p11 b) {
```

```

    return sgn(b.X) ? (ld)a.X / b.X : (ld)a.Y / b.Y;
} // all poly. should be ccw
ld polyUnion(vector<vector<p11>> &poly) {
    ld res = 0;
    for (auto &p : poly)
        for (int a = 0; a < SZ(p); ++a) {
            p11 A = p[a], B = p[(a + 1) % SZ(p)];
            vector<pair<ld, int>> segs = {{0, 0}, {1, 0}};
            for (auto &q : poly) {
                if (&p == &q) continue;
                for (int b = 0; b < SZ(q); ++b) {
                    p11 C = q[b], D = q[(b + 1) % SZ(q)];
                    int sc = ori(A, B, C), sd = ori(A, B, D);
                    if (sc != sd && min(sc, sd) < 0) {
                        ld sa = cross(D - C, A - C), sb = cross(D - C, B - C);
                        segs.pb(sa / (sa - sb), sgn(sc - sd));
                    }
                    if (!sc && !sd && &q < &p && sgn(dot(B - A, D - C)) > 0) {
                        segs.pb(rat(C - A, B - A), 1);
                        segs.pb(rat(D - A, B - A), -1);
                    }
                }
            }
            sort(iter(segs));
            for (auto &s : segs) s.X = clamp(s.X, 0.0, 1.0);
            ld sum = 0;
            int cnt = segs[0].second;
            for (int j = 1; j < SZ(segs); ++j) {
                if (!cnt) sum += segs[j].X - segs[j - 1].X;
                cnt += segs[j].Y;
            }
            res += cross(A, B) * sum;
        }
    return res / 2;
}

```

4.20 Tangent Point to Convex Hull [523bc1]

```

/* The point should be strictly out of hull
   return arbitrary point on the tangent line */
pii get_tangent(vector<p11> &C, p11 p) {
    auto gao = [&](int s) {
        return cyc_tsearch(SZ(C), [&](int x, int y)
            { return ori(p, C[x], C[y]) == s; });
    };
    return pii(gao(1), gao(-1));
} // return (a, b), ori(p, C[a], C[b]) >= 0

```

4.21 Heart [082d19]

```

pdd circenter(pdd p0, pdd p1, pdd p2) { // 156d1f
    p1 = p1 - p0, p2 = p2 - p0; // radius = abs(center)
    double x1 = p1.X, y1 = p1.Y, x2 = p2.X, y2 = p2.Y;
    double m = 2. * (x1 * y2 - y1 * x2);
    pdd center;
    center.X = (x1 * x1 * y2 - x2 * x2 * y1 + y1 * y2 * (x1 - y2)) / m;
    center.Y = (x1 * x2 * (x2 - x1) - y1 * y1 * x2 + x1 * y2 * y2) / m;
    return center + p0;
}
pdd incenter(pdd p1, pdd p2, pdd p3) { // radius = area / s * 2
    double a = abs(p2 - p3), b = abs(p1 - p3), c = abs(p1 - p2);
    double s = a + b + c;
    return (a * p1 + b * p2 + c * p3) / s;
}
pdd masscenter(pdd p1, pdd p2, pdd p3) {
    return (p1 + p2 + p3) / 3;
}
pdd orthcenter(pdd p1, pdd p2, pdd p3) {
    return masscenter(p1, p2, p3) * 3 - circenter(p1, p2, p3) * 2;
}

```

4.22 Rotating Sweep Line [f5f689]

```

struct Event {
    p11 d; int u, v;
    bool operator<(const Event &b) const {
        int ret = cmp(d, b.d, false);
        return ret == -1 ? false : ret; // no tie-break
    };
    void rotatingSweepLine(const vector<p11> &p) {

```

```

        const int n = SZ(p);
        vector<Event> e; e.reserve(n * (n - 1));
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++) // pos[i] < pos[j] when
                the event occurs
                if (i != j) e.pb(p[j] - p[i], i, j);
        sort(iter(e));
        vector<int> ord(n), pos(n);
        iota(iter(ord), 0);
        sort(iter(ord), [&](int i, int j) { // initial order
            return p[i].Y != p[j].Y ? p[i].Y < p[j].Y : p[i].X < p[j].X; });
        for (int i = 0; i < n; i++) pos[ord[i]] = i;
        // initialize
        for (int i = 0, j = 0; i < SZ(e); i = j) {
            // do something
            vector<pii> tmp;
            for (; j < SZ(e) && !(e[i] < e[j]); j++)
                tmp.pb(pii(e[j].u, e[j].v));
            sort(iter(tmp), [&](pii x, pii y) {
                return pii(pos[x.ff], pos[x.ss]) < pii(pos[y.ff], pos[y.ss]); });
            for (auto [x, y] : tmp) // pos[x] + 1 == pos[y]
                tie(ord[pos[x]], ord[pos[y]], pos[x], pos[y]) =
                    make_tuple(ord[pos[y]], ord[pos[x]], pos[y], pos[x]);
        }
    }
}

```

4.23 Vector In Poly [c6d0fa]

```

// ori(a, b, c) >= 0, valid: "strict" angle from a-b to a-c
bool btwangle(p11 a, p11 b, p11 c, p11 p, int strict) {
    return ori(a, b, p) >= strict && ori(a, p, c) >= strict;
}
// whether vector{cur, p} in counter-clockwise order
prv, cur, nxt
bool inside(p11 prv, p11 cur, p11 nxt, p11 p, int strict) {
    if (ori(cur, nxt, prv) >= 0)
        return btwangle(cur, nxt, prv, p, strict);
    return !btwangle(cur, prv, nxt, p, !strict);
}

```

4.24 Convex Hull DP [6dc001]

```

sort(iter(pts), [&](p11 x, p11 y) {
    return x.Y != y.Y ? x.Y < y.Y : x.X < y.X;
});
auto getvec = [&](pii x) { return pts[x.ss] - pts[x.ff] };
vector<pii> trans;
for (int j = 0; j < n; j++)
    for (int k = 0; k < n; k++)
        if (j != k) trans.pb(pii(j, k));
sort(iter(trans), [&](pii x, pii y) -> bool {
    int tmp = cmp(getvec(x), getvec(y), false);
    if (tmp != -1) return tmp;
    p11 v = getvec(x);
    return dot(v, pts[x.ff]) > dot(v, pts[y.ff]);
});
// DP for convex hull vertices (no points on edges)
auto solve = [&](int bottom) { // O(n^3)
    // vector<ll> dp(n);
    for (int j = bottom + 1; j < n; j++) {
        // check whether bottom -> j is legal
        // init trans -> j
    }
    for (auto [i, j] : trans) {
        if (i <= bottom || j <= bottom ||
            ori(pts[bottom], pts[i], pts[j]) <= 0) continue;
        // check whether i -> j is legal
        // normal trans i -> j
    }
    for (int j = bottom + 1; j < n; j++) {
        // check whether j -> bottom is legal
        // end trans j ->
    }
};
for (int i = 0; i < n; i++) solve(i);

```

4.25 Calculate Points in Triangle [bf746f]

```
// all points are distinct
// cnt[i][j] = # of point k s.t. strictly above ij, and
//           i < k < j
// cnt2[i][j] = # of points k s.t. strictly in ij
// preprocess space: O(n^2), time: O(n^3), query time:
//               O(1)
vector cnt(n, vector<int>(n)), cnt2(n, vector<int>(n));
for (int i = 0; i < n; i++)
    for (int j = 0; j < n; j++) {
        if (pts[i] >= pts[j]) continue;
        for (int k = 0; k < n; k++) {
            if (pts[i] < pts[k] && pts[k] < pts[j]) {
                int tmp = ori(pts[i], pts[j], pts[k]);
                if (tmp > 0) cnt[i][j]++; // only for i < j
                else if (tmp == 0) cnt2[i][j]++, cnt2[j][i]++;
            }
        }
    }
auto calc_tri = [&](array<int, 3> arr) { // strictly
    inside
    sort(iter(arr), [&](int x, int y){ return pts[x] <
        pts[y]; });
    auto [x, y, z] = arr;
    int tmp = ori(pts[x], pts[y], pts[z]);
    if (tmp == 0) return 0;
    else if (tmp < 0)
        return cnt[x][z] - cnt[x][y] - cnt[y][z] - cnt2[x][
            y] - cnt2[y][z] - 1;
    else return cnt[x][y] + cnt[y][z] - cnt[x][z] - cnt2[
        x][z];
};
```

5 Graph

5.1 BCC [d04ebe]

```
struct BCC { // 0-based, allow multi edges but not allow
    loops
    int n, m, cnt = 0;
    // n:|V|, m:|E|, cnt:#bcc
    // bcc i : vertices bcc_v[i] and edges bcc_e[i]
    vector<vector<int>> bcc_v, bcc_e;
    vector<vector<pii>> g; // original graph
    vector<pii> edges; // 0-based
    BCC(int _n, vector<pii> _edges):
        n(_n), m(SZ(_edges)), g(_n), edges(_edges){
        for(int i = 0; i < m; i++){
            auto [u, v] = edges[i];
            g[u].pb(pii(v, i)); g[v].pb(pii(u, i));
        }
    }
    void make_bcc(){ bcc_v.pb(); bcc_e.pb(); cnt++; }
    // modify these if you need more information
    void add_v(int v){ bcc_v.back().pb(v); }
    void add_e(int e){ bcc_e.back().pb(e); }
    void build(){
        vector<int> in(n, -1), low(n, -1), stk;
        vector<vector<int>> up(n);
        int ts = 0;
        auto _dfs = [&](auto dfs, int now, int par, int pe)
            -> void{
            if(pe != -1) up[now].pb(pe);
            in[now] = low[now] = ts++;
            stk.pb(now);
            for(auto [v, e] : g[now]){
                if(e == pe) continue;
                if(in[v] != -1){
                    if(in[v] < in[now]) up[now].pb(e);
                    low[now] = min(low[now], in[v]);
                    continue;
                }
                dfs(dfs, v, now, e);
                low[now] = min(low[now], low[v]);
            }
            if((now != par && low[now] >= in[par]) || (now ==
                par && SZ(g[now]) == 0)){
                make_bcc();
                for(int v = stk.back(); v = stk.back()){
                    stk.pop_back(), add_v(v);
                    for(int e : up[v]) add_e(e);
                    if(v == now) break;
                }
            }
        };
        dfs(dfs, 0, 0, -1);
    }
};
```

```
        if(now != par) add_v(par);
    }
};
for(int i = 0; i < n; i++)
    if(in[i] == -1) _dfs(_dfs, i, i, -1);
};

5.2 SCC [2c9a01]
struct SCC { // 0-based, output reversed topo order
    int n, cnt = 0;
    vector<vector<int>> g;
    vector<int> sccid;
    explicit SCC(int _n): n(_n), g(_n), sccid(_n, -1) {}
    void add_edge(int u, int v){
        g[u].pb(v);
    }
    void build(){
        vector<int> in(n, -1), low(n), stk;
        vector<bool> instk(n);
        int ts = 0;
        auto dfs1 = [&](auto dfs, int now) -> void{
            stk.pb(now); instk[now] = true;
            in[now] = low[now] = ts++;
            for(int i : g[now]){
                if(in[i] == -1)
                    dfs(dfs, i), low[now] = min(low[now], low[i]);
                else if(instk[i] && in[i] < in[now])
                    low[now] = min(low[now], in[i]);
            }
            if(low[now] == in[now]){
                for(; stk.back() != now; stk.pop_back())
                    sccid[stk.back()] = cnt, instk[stk.back()] =
                        false;
                sccid[now] = cnt++, instk[now] = false, stk.
                    pop_back();
            }
        };
        for(int i = 0; i < n; i++)
            if(in[i] == -1) dfs1(dfs1, i);
    }
};
```

5.3 2-SAT [0686a5]

```
struct SAT { // 0-based
    int n;
    vector<bool> istrue;
    SCC scc;
    SAT(int _n): n(_n), istrue(n + n), scc(n + n) {}
    int neg(int a) {
        return a >= n ? a - n : a + n;
    }
    void add_clause(int a, int b) {
        scc.add_edge(neg(a), b), scc.add_edge(neg(b), a);
    }
    bool solve() {
        scc.build();
        for (int i = 0; i < n; ++i) {
            if (scc.sccid[i] == scc.sccid[i + n]) return
                false;
            istrue[i] = scc.sccid[i] < scc.sccid[i + n];
            istrue[i + n] = !istrue[i];
        }
        return true;
    }
};
```

5.4 Dominator Tree [2da9bb]

```
struct Dominator {
    int n;
    vector<vector<int>> g, r, rdom; int tk;
    vector<int> dfn, rev, fa, sdom, dom, val, rp;
    Dominator(int _n): n(_n), g(_n), r(_n), rdom(_n), tk(0)
        {
            dfn = rev = fa = sdom = dom =
                val = rp = vector<int>(n, -1);
        }
    void add_edge(int x, int y) { g[x].push_back(y); }
    void dfs(int x) {
        rev[dfn[x]] = tk; x = x;
        fa[tk] = sdom[tk] = val[tk] = tk; tk++;
        for (int u : g[x]) {
```

```

    if (dfn[u] == -1) dfs(u), rp[dfn[u]] = dfn[x];
    r[dfn[u]].push_back(dfn[x]);
}
}
void merge(int x, int y) { fa[x] = y; }
int find(int x, int c = 0) {
    if (fa[x] == x) return c ? -1 : x;
    if (int p = find(fa[x], 1); p != -1) {
        if (sdom[val[x]] > sdom[val[fa[x]]])
            val[x] = val[fa[x]];
        fa[x] = p;
        return c ? p : val[x];
    } else return c ? fa[x] : val[x];
}
vector<int> build(int s) {
    // return the father of each node in dominator tree
    dfs(s); // p[i] = -2 if i is unreachable, par[s] = -1
    for (int i = tk - 1; i >= 0; --i) {
        for (int u : r[i])
            sdom[i] = min(sdom[i], sdom[find(u)]);
        if (i) rdom[sdom[i]].push_back(i);
        for (int u : rdom[i]) {
            int p = find(u);
            dom[u] = (sdom[p] == i ? i : p);
        }
        if (i) merge(i, rp[i]);
    }
    vector<int> p(n, -2); p[s] = -1;
    for (int i = 1; i < tk; ++i)
        if (sdom[i] != dom[i]) dom[i] = dom[dom[i]];
    for (int i = 1; i < tk; ++i)
        p[rev[i]] = rev[dom[i]];
    return p;
}
};

```

5.5 Virtual Tree [6abeb5]

```

vector<int> vG[N];
int top, st[N];
int vrt = -1;
void insert(int u) {
    if (top == -1) return st[++top] = vrt = u, void();
    int p = LCA(st[top], u);
    if (dep[vrt] > dep[p]) vrt = p;
    if (p == st[top]) return st[++top] = u, void();
    while (top >= 1 && dep[st[top - 1]] >= dep[p])
        vG[st[top - 1]].pb(st[top]), --top;
    if (st[top] != p)
        vG[p].pb(st[top]), --top, st[++top] = p;
    st[++top] = u;
}
void reset(int u) {
    for (int i : vG[u]) reset(i);
    vG[u].clear();
}
void solve(vector<int> &v) {
    top = -1;
    sort(iter(v), [&](int a, int b) { return dfn[a] < dfn[b]; });
    for (int i : v) insert(i);
    while (top > 0) vG[st[top - 1]].pb(st[top]), --top;
    // do something
    reset(vrt);
}

```

5.6 Fast DMST [7b274d]

```

struct E { int s, t; ll w; }; // 0-base
struct PQ {
    struct P {
        ll v; int i;
        bool operator>(const P &b) const { return v > b.v; }
    };
    priority_queue<P, vector<P>, greater<>> pq; ll tag;
    // min heap
    void push(P p) { p.v -= tag; pq.emplace(p); }
    P top() { P p = pq.top(); p.v += tag; return p; }
    void join(PQ &b) {
        if (pq.size() < b.pq.size())
            swap(pq, b.pq), swap(tag, b.tag);
        while (!b.pq.empty()) push(b.top()), b.pq.pop();
    }
}

```

```

}
}; // O(E Log^2 V), use Leftist tree for O(E Log V)
vector<int> dmst(const vector<E> &e, int n, int root) {
    vector<PQ> h(n * 2);
    for (int i = 0; i < int(e.size()); ++i)
        h[e[i].t].push({e[i].w, i});
    vector<int> a(n * 2); iota(iter(a), 0);
    vector<int> v(n * 2, -1), pa(n * 2, -1), r(n * 2);
    auto o = [&](auto Y, int x) -> int {
        return x == a[x] ? x : a[x] = Y(Y, a[x]);
    };
    auto S = [&](int i) { return o(o, e[i].s); };
    int pc = v[root] = n;
    for (int i = 0; i < n; ++i) if (v[i] == -1)
        for (int p = i; v[p] < 0 || v[p] == i; p = S(r[p])) {
            if (v[p] == i)
                for (int q = pc++; p != q; p = S(r[p])) {
                    h[p].tag -= h[p].top().v; h[q].join(h[p]);
                    pa[p] = a[p] = q;
                }
            while (S(h[p].top().i) == p) h[p].pq.pop();
            v[p] = i; r[p] = h[p].top().i;
        }
    vector<int> ans;
    for (int i = pc - 1; i >= 0; i--) if (v[i] != n) {
        for (int f = e[r[i]].t; f != -1 && v[f] != n; f = pa[f])
            v[f] = n;
        ans.push_back(r[i]);
    }
    return ans; // default minimize, returns edgeid array
}

```

5.7 Vizing [58a6ca]

```

// find D+1 edge coloring of a graph with max deg D, O(
nm)
struct Vizing { // returns maxdeg+1 edge coloring in
    adjacent matrix G
    int n; // 1-based for vertices and colors, simple
    graph
    vector<vector<int>> C, G;
    vector<int> X, vst;
    Vizing(int _n): n(_n),
        C(n + 1, vector<int>(n + 2)), G(n + 1, vector<int>(n
            + 1)),
        X(n + 1, 1), vst(n + 1) {}
    void solve(vector<pii> &E) {
        auto update = [&](int u) {
            for (X[u] = 1; C[u][X[u]]; ++X[u]);
        };
        auto color = [&](int u, int v, int c) {
            int p = G[u][v];
            G[u][v] = G[v][u] = c;
            C[u][c] = v, C[v][c] = u;
            C[u][p] = C[v][p] = 0;
            if (p) X[u] = X[v] = p;
            else update(u), update(v);
            return p;
        };
        auto flip = [&](int u, int c1, int c2) {
            int p = C[u][c1];
            swap(C[u][c1], C[u][c2]);
            if (p) G[u][p] = G[p][u] = c2;
            if (!C[u][c1]) X[u] = c1;
            if (!C[u][c2]) X[u] = c2;
            return p;
        };
        for (int t = 0; t < SZ(E); ++t) {
            int u = E[t].ff, v0 = E[t].ss, v = v0, c0 = X[u],
                c = c0, d;
            vector<pii> L;
            fill(iter(vst), 0);
            while (!G[u][v0]) {
                L.emplace_back(v, d = X[v]);
                if (!C[v][c]) for (int a = SZ(L) - 1; a >= 0;
                    --a) c = color(u, L[a].ff, c);
                else if (!C[u][d]) for (int a = SZ(L) - 1; a >=
                    0; --a) color(u, L[a].ff, L[a].ss);
                else if (vst[d]) break;
                else vst[d] = 1, v = C[u][d];
            }
            if (!G[u][v0]) {
                for (; v; v = flip(v, c, d), swap(c, d));
                if (int a; C[u][c0]) {

```



```

    for (a = SZ(L) - 2; a >= 0 && L[a].ss != c; --a);
    for (; a >= 0; --a) color(u, L[a].ff, L[a].ss);
}
else --t;
}
}
};

```

5.8 Maximum Clique [1ad4b2]

```

struct MaxClique { // fast when N <= 100
    bitset<N> G[N], cs[N];
    int ans, sol[N], q, cur[N], d[N], n;
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) G[i].reset();
    }
    void add_edge(int u, int v) {
        G[u][v] = G[v][u] = 1;
    }
    void pre_dfs(vector<int> &r, int l, bitset<N> mask) {
        if (l < 4) {
            for (int i : r) d[i] = (G[i] & mask).count();
            sort(iter(r), [&](int x, int y) { return d[x] > d[y]; });
        }
        vector<int> c(SZ(r));
        int lft = max(ans - q + 1, 1), rgt = 1, tp = 0;
        cs[1].reset(), cs[2].reset();
        for (int p : r) {
            int k = 1;
            while ((cs[k] & G[p]).any()) ++k;
            if (k > rgt) cs[++rgt + 1].reset();
            cs[k][p] = 1;
            if (k < lft) r[tp++] = p;
        }
        for (int k = lft; k <= rgt; ++k)
            for (int p = cs[k]._Find_first(); p < N; p = cs[k]._Find_next(p))
                r[tp] = p, c[tp] = k, ++tp;
        dfs(r, c, l + 1, mask);
    }
    void dfs(vector<int> &r, vector<int> &c, int l, bitset<N> mask) {
        while (!r.empty()) {
            int p = r.back();
            r.pop_back(), mask[p] = 0;
            if (q + c.back() <= ans) return;
            cur[q++] = p;
            vector<int> nr;
            for (int i : r) if (G[p][i]) nr.pb(i);
            if (!nr.empty()) pre_dfs(nr, l, mask & G[p]);
            else if (q > ans) ans = q, copy_n(cur, q, sol);
            c.pop_back(), --q;
        }
    }
    int solve() {
        vector<int> r(n);
        ans = q = 0, iota(iter(r), 0);
        pre_dfs(r, 0, bitset<N>(string(n, '1')));
        return ans;
    }
};

```

5.9 Number of Maximal Clique [11fa26]

```

struct BronKerbosch { // 1-base
    int n, a[N], g[N][N];
    int S, all[N][N], some[N][N], none[N][N];
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            for (int j = 1; j <= n; ++j) g[i][j] = 0;
    }
    void add_edge(int u, int v) {
        g[u][v] = g[v][u] = 1;
    }
    void dfs(int d, int an, int sn, int nn) {
        if (S > 1000) return; // pruning
        if (sn == 0 && nn == 0) ++S;
        int u = some[d][0];

```

```

        for (int i = 0; i < sn; ++i) {
            int v = some[d][i];
            if (g[u][v]) continue;
            int tsu = 0, tnn = 0;
            copy_n(all[d], an, all[d + 1]);
            all[d + 1][an] = v;
            for (int j = 0; j < sn; ++j)
                if (g[v][some[d][j]])
                    some[d + 1][tsu++] = some[d][j];
            for (int j = 0; j < nn; ++j)
                if (g[v][none[d][j]])
                    none[d + 1][tnn++] = none[d][j];
            dfs(d + 1, an + 1, tsu, tnn);
            some[d][i] = 0, none[d][nn++] = v;
        }
    }
    int solve() {
        iota(some[0], some[0] + n, 1);
        S = 0, dfs(0, 0, n, 0);
        return S;
    }
};

```

5.10 Minimum Mean Cycle [3e5d2b]

```

ll road[N][N]; // input here
struct MinimumMeanCycle {
    ll dp[N + 5][N], n;
    pll solve() {
        ll a = -1, b = -1, L = n + 1;
        for (int i = 2; i <= L; ++i)
            for (int k = 0; k < n; ++k)
                for (int j = 0; j < n; ++j)
                    dp[i][j] =
                        min(dp[i - 1][k] + road[k][j], dp[i][j]);
        for (int i = 0; i < n; ++i) {
            if (dp[L][i] >= INF) continue;
            ll ta = 0, tb = 1;
            for (int j = 1; j < n; ++j)
                if (dp[j][i] < INF &&
                    ta * (L - j) < (dp[L][i] - dp[j][i]) * tb)
                    ta = dp[L][i] - dp[j][i], tb = L - j;
            if (ta == 0) continue;
            if (a == -1 || a * tb > ta * b) a = ta, b = tb;
        }
        if (a != -1) {
            ll g = __gcd(a, b);
            return pll(a / g, b / g);
        }
        return pll(-1LL, -1LL);
    }
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j) dp[i + 2][j] = INF;
    }
};

```

5.11 Minimum Steiner Tree [21acea]

```

// O(V 3^T + V^2 2^T)
struct SteinerTree { // 0-base
    static const int T = 10, N = 105, INF = 1e9;
    int n, dst[N][N], dp[1 << T][N], tdst[N];
    int vcost[N]; // the cost of vertices
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) {
            for (int j = 0; j < n; ++j) dst[i][j] = INF;
            dst[i][i] = vcost[i] = 0;
        }
    }
    void add_edge(int ui, int vi, int wi) {
        dst[ui][vi] = min(dst[ui][vi], wi);
    }
    void shortest_path() {
        for (int k = 0; k < n; ++k)
            for (int i = 0; i < n; ++i)
                for (int j = 0; j < n; ++j)
                    dst[i][j] =
                        min(dst[i][j], dst[i][k] + dst[k][j]);
    }
    int solve(const vector<int> &ter) {
        shortest_path();

```

```

int t = SZ(ter);
for (int i = 0; i < (1 << t); ++i)
    for (int j = 0; j < n; ++j) dp[i][j] = INF;
for (int i = 0; i < n; ++i) dp[0][i] = vcost[i];
for (int msk = 1; msk < (1 << t); ++msk) {
    if (!(msk & (msk - 1))) {
        int who = __lg(msk);
        for (int i = 0; i < n; ++i)
            dp[msk][i] =
                vcost[ter[who]] + dst[ter[who]][i];
    }
    for (int i = 0; i < n; ++i)
        for (int submsk = (msk - 1) & msk; submsk;
             submsk = (submsk - 1) & msk)
            dp[msk][i] = min(dp[msk][i],
                             dp[submsk][i] + dp[msk ^ submsk][i] -
                             vcost[i]);
    for (int i = 0; i < n; ++i) {
        tdst[i] = INF;
        for (int j = 0; j < n; ++j)
            tdst[i] =
                min(tdst[i], dp[msk][j] + dst[j][i]);
    }
    for (int i = 0; i < n; ++i) dp[msk][i] = tdst[i];
}
int ans = INF;
for (int i = 0; i < n; ++i)
    ans = min(ans, dp[(1 << t) - 1][i]);
return ans;
}
};

```

5.12 Count Cycles [c7e8f2]

```

// ord = sort by deg decreasing, rk[ord[i]] = i
// D[i] = edge point from rk small to rk big
for (int x : ord) { // c3
    for (int y : D[x]) vis[y] = 1;
    for (int y : D[x]) for (int z : D[y]) c3 += vis[z];
    for (int y : D[x]) vis[y] = 0;
}
for (int x : ord) { // c4
    for (int y : D[x]) for (int z : adj[y])
        if (rk[z] > rk[x]) c4 += vis[z]++;
    for (int y : D[x]) for (int z : adj[y])
        if (rk[z] > rk[x]) --vis[z];
} // both are O(M*sqrt(M))

```

6 Math

6.1 Extended Euclidean Algorithm [c51ae9]

```

// ax+ny = 1, ax+ny == ax == 1 (mod n)
void extgcd(ll x, ll y, ll &g, ll &a, ll &b) {
    if (y == 0) g = x, a = 1, b = 0;
    else extgcd(y, x % y, g, b, a), b -= (x / y) * a;
}

```

6.2 Floor & Ceil [134881]

```

ll ifloor(ll a, ll b){
    return a / b - (a % b && (a < 0) ^ (b < 0));
}
ll iceil(ll a, ll b){
    return a / b + (a % b && (a < 0) ^ (b > 0));
}

```

6.3 Legendre [4e4b23]

```

// the Jacobi symbol is a generalization of the
// Legendre symbol,
// such that the bottom doesn't need to be prime.
// (n/p) -> same as Legendre
// (n/ab) = (n/a)(n/b)
// work with Long Long
int Jacobi(int a, int m) {
    int s = 1;
    for (; m > 1; ) {
        a %= m;
        if (a == 0) return 0;
        const int r = __builtin_ctz(a);
        if ((r & 1) && ((m + 2) & 4)) s = -s;
        a >>= r;
        if (a & m & 2) s = -s;
        swap(a, m);
    }
}

```

```

return s;
}

// 0: a == 0
// -1: a isn't a quad res of p
// else: return X with X^2 % p == a
// doesn't work with Long Long
int QuadraticResidue(int a, int p) {
    if (p == 2) return a & 1;
    if (int jc = Jacobi(a, p); jc <= 0) return jc;
    int b, d;
    for (; ; ) {
        b = rand() % p;
        d = (1LL * b * b + p - a) % p;
        if (Jacobi(d, p) == -1) break;
    }
    int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (1LL + p) >> 1; e; e >>= 1) {
        if (e & 1) {
            tmp = (1LL * g0 * f0 + 1LL * d * (1LL * g1 * f1 %
                p)) % p;
            g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
            g0 = tmp;
        }
        tmp = (1LL * f0 * f0 + 1LL * d * (1LL * f1 * f1 %
            p)) % p;
        f1 = (2LL * f0 * f1) % p;
        f0 = tmp;
    }
    return g0;
}

```

6.4 Simplex [aa7741]

```

// maximize c^T x
// subject to Ax <= b, x >= 0
// and stores the solution;
typedef long double T; // Long double, Rational, double
+ mod<P>...
typedef vector<T> vd;
typedef vector<vd> vvd;

const T eps = 1e-9, inf = 1/.0;
#define ltj(X) if(s == -1 || mp(X[j],N[j]) < mp(X[s],N[
s])) s=j
#define rep(i, l, n) for(int i = l; i < n; i++)

struct LPSolver {
    int m, n;
    vector<int> N, B;
    vvd D;

    LPSolver(const vvd& A, const vd& b, const vd& c) :
        m(SZ(b)), n(SZ(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
        rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];
        rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1]
            = b[i]; }
        rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
        N[n] = -1; D[m+1][n] = 1;
    }

    void pivot(int r, int s) {
        T *a = D[r].data(), inv = 1 / a[s];
        rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
            T *b = D[i].data(), inv2 = b[s] * inv;
            rep(j,0,n+2) b[j] -= a[j] * inv2;
            b[s] = a[s] * inv2;
        }
        rep(j,0,n+2) if (j != s) D[r][j] *= inv;
        rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
        D[r][s] = inv;
        swap(B[r], N[s]);
    }

    bool simplex(int phase) {
        int x = m + phase - 1;
        for (; ) {
            int s = -1;
            rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
            if (D[x][s] >= -eps) return true;
            int r = -1;
            rep(i,0,m) {
                if (D[i][s] <= eps) continue;

```

```

    if (r == -1 || mp(D[i][n+1] / D[i][s], B[i])
        < mp(D[r][n+1] / D[r][s], B[r])) r = i;
}
if (r == -1) return false;
pivot(r, s);
}
}

T solve(vd &x) {
    int r = 0;
    rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) {
        pivot(r, n);
        if (!simplex(2) || D[m+1][n+1] < -eps) return -
            inf;
        rep(i,0,m) if (B[i] == -1) {
            int s = 0;
            rep(j,1,n+1) ltj(D[i]);
            pivot(i, s);
        }
    }
    bool ok = simplex(1); x = vd(n);
    rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
}
};

```

6.5 Simplex Construction

Standard form: maximize $\sum_{1 \leq i \leq n} c_i x_i$ such that $\sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j$ for all $1 \leq j \leq m$ and $x_i \geq 0$ for all $1 \leq i \leq n$.

1. In case of minimization, let $c'_i = -c_i$
2. $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j \rightarrow \sum_{1 \leq i \leq n} -A_{ji} x_i \leq -b_j$
3. $\sum_{1 \leq i \leq n} A_{ji} x_i = b_j \rightarrow \text{add } \leq \text{ and } \geq$.
4. If x_i has no lower bound, replace x_i with $x_i - x'_i$

6.6 LP Dual

$$\max \sum_{i \in [n]} c_i x_i \Leftrightarrow \min \sum_{j \in [m]} b_j y_j$$

$$\sum_{i \in [n]} a_{ji} x_i \leq b_j \Leftrightarrow y_j \geq 0 \quad x_i \geq 0 \Leftrightarrow \sum_{j \in [m]} a_{ji} y_j \geq c_i$$

$$\sum_{i \in [n]} a_{ji} x_i \geq b_j \Leftrightarrow y_j \leq 0 \quad x_i \leq 0 \Leftrightarrow \sum_{j \in [m]} a_{ji} y_j \leq c_i$$

$$\sum_{i \in [n]} a_{ji} x_i = b_j \Leftrightarrow y_j \in \mathbb{R} \quad x_i \in \mathbb{R} \Leftrightarrow \sum_{j \in [m]} a_{ji} y_j = c_i$$

$\max c^T x$ subject to $Ax \leq b, x \geq 0 \Leftrightarrow \min b^T y$ subject to $A^T y \geq c, y \geq 0$
 $\sum_i c_i x_i \leq \sum_i (\sum_j a_{ji} y_j) x_i = \sum_j (\sum_i a_{ji} x_i) y_j \leq \sum_j b_j y_j$

Complementary slackness:

- Primal: $\forall i, x_i = 0 \vee c_i = \sum_j a_{ji} y_j$
- Dual: $\forall j, y_j = 0 \vee b_j = \sum_i a_{ji} x_i$

6.7 DiscreteLog [da27bf]

```

int DiscreteLog(int s, int x, int y, int m) {
    constexpr int kStep = 32000;
    unordered_map<int, int> p;
    int b = 1;
    for (int i = 0; i < kStep; ++i) {
        p[y] = i;
        y = 1LL * y * x % m;
        b = 1LL * b * x % m;
    }
    for (int i = 0; i < m + 10; i += kStep) {
        s = 1LL * s * b % m;
        if (p.find(s) != p.end()) return i + kStep - p[s];
    }
    return -1;
}

int DiscreteLog(int x, int y, int m) {
    if (m == 1) return 0;
    int s = 1;
    for (int i = 0; i < 100; ++i) {
        if (s == y) return i;
        s = 1LL * s * x % m;
    }
    if (s == y) return 100;
    int p = 100 + DiscreteLog(s, x, y, m);
    if (fpow(x, p, m) != y) return -1;
    return p; //returns: x^p = y (mod m)
}

```

6.8 Miller Rabin & Pollard Rho [d3ecd2]

```

// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633 4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383 6 : primes <= 13
// n < 2^64              7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
ll mul(ll a, ll b, ll n){
    return (__int128)a * b % n;
}

bool Miller_Rabin(ll a, ll n) { // 06308c
    if ((a = a % n) == 0) return 1;
    if (n % 2 == 0) return n == 2;
    ll tmp = (n - 1) / ((n - 1) & (1 - n));
    ll t = __lg(((n - 1) & (1 - n))), x = 1;
    for (; tmp; tmp >>= 1, a = mul(a, a, n))
        if (tmp & 1) x = mul(x, a, n);
    if (x == 1 || x == n - 1) return 1;
    while (--t)
        if ((x = mul(x, x, n)) == n - 1) return 1;
    return 0;
}

bool prime(ll n){ // 8859aa
    vector<ll> tmp = {2, 325, 9375, 28178, 450775,
        9780504, 1795265022};
    for(ll i : tmp)
        if(!Miller_Rabin(i, n)) return false;
    return true;
}

map<ll, int> cnt;
void PollardRho(ll n) { // 173531
    if (n == 1) return;
    if (prime(n)) return ++cnt[n], void();
    if (n % 2 == 0) return PollardRho(n / 2), ++cnt[2],
        void();
    ll x = 2, y = 2, d = 1, p = 1;
#define f(x, n, p) ((mul(x, x, n) + p) % n)
    while (true) {
        if (d != n && d != 1) {
            PollardRho(n / d);
            PollardRho(d);
            return;
        }
        if (d == n) ++p;
        x = f(x, n, p), y = f(f(y, n, p), n, p);
        d = gcd(abs(x - y), n);
    }
}

```

6.9 XOR Basis [cc5e62]

```

const int digit = 60; // [0, 2^digit)
struct Basis{
    int total = 0, rank = 0;
    vector<ll> b;
    Basis(): b(digit) {}
    bool add(ll v){ // Gauss Jordan Elimination
        total++;
        for(int i = digit - 1; i >= 0; i--){
            if(!(1LL << i & v)) continue;
            if(b[i] != 0){
                v ^= b[i];
                continue;
            }
            for(int j = 0; j < i; j++){
                if(1LL << j & v) v ^= b[j];
            }
            for(int j = i + 1; j < digit; j++){
                if(1LL << i & b[j]) b[j] ^= v;
            }
            b[i] = v;
            rank++;
            return true;
        }
        return false;
    }
};

```

6.10 Linear Equation [97a424]

```

struct GaussJordan { // bfa8b8
    int rk = 0; ll det = 1; // UB if n!=m
    vector<vector<ll>> rref;
    vector<int> cols;
    GaussJordan(const vector<vector<ll>> &_rref): rref(
        _rref) {
        if (rref.empty()) return;
    }
};

```

```

int N = SZ(rref), M = SZ(rref[0]);
auto swap_row = [&](int x, int y) {
    rref[x].swap(rref[y]); if (x != y) det = -det;
};
auto mul_row = [&](int x, ll mul) {
    for (auto &v : rref[x]) v = v * mul % MOD;
    det = det * inv(mul) % MOD;
};
auto minus_row = [&](int x, int y, ll mul) {
    for (int k = 0; k < M; k++)
        rref[x][k] = (rref[x][k] - rref[y][k] * mul %
            MOD + MOD) % MOD;
};
for (int i = 0; i < M; i++) {
    int cnt = -1;
    for (int j = N - 1; j >= rk; j--)
        if (rref[j][i] != 0) cnt = j;
    if (cnt == -1) continue;
    swap_row(rk, cnt);
    mul_row(rk, inv(rref[rk][i]));
    for (int j = 0; j < N; j++)
        if (j != rk) minus_row(j, rk, rref[j][i]);
    cols.pb(i); rk++;
}
if (rk < N) det = 0;
det = (det % MOD + MOD) % MOD;
}
};
// sol = particular + linear combination of homogenous
struct LinearEquation { // 8572ac
    bool ok;
    vector<ll> par; //Ax = b
    vector<vector<ll>> homo; //Ax = 0
    LinearEquation(int M, const GaussJordan& elim): par(M)
    {
        auto &piv = elim.cols;
        auto &rref = elim.rref;
        if (!piv.empty() && piv.back() == M)
            { ok = 0; return; }
        ok = 1;
        vector<bool> ispiv(M);
        for (int i = 0; i < elim.rk; i++) {
            par[piv[i]] = rref[i][M];
            ispiv[piv[i]] = 1;
        }
        for (int i = 0; i < M; i++) {
            if (ispiv[i]) continue;
            vector<ll> h(M);
            h[i] = 1;
            for (int j = 0; j < elim.rk; j++)
                h[piv[j]] = rref[j][i] ? MOD - rref[j][i] : 0;
            homo.pb(h);
        }
    }
};

```

6.11 Chinese Remainder Theorem [6ef4a3]

```

pll solve_crt(ll x1, ll m1, ll x2, ll m2){
    ll g = gcd(m1, m2);
    if ((x2 - x1) % g) return {0, 0}; // no sol
    m1 /= g; m2 /= g;
    ll _, p, q;
    extgcd(m1, m2, _, p, q); // p <= C
    ll lcm = m1 * m2 * g;
    ll res = ((__int128)p * (x2 - x1) % lcm * m1 % lcm +
        x1) % lcm;
    // be careful with overflow, C^3
    return {(res + lcm) % lcm, lcm}; // (x, m)
}

```

6.12 Sqrt Decomposition [8d7bc0]

```

// for all i in [L, r], floor(n / i) = x
for(int l = 1, r; l <= n; l = r + 1){
    int x = ifloor(n, l);
    r = ifloor(n, x);
}
// for all i in [L, r], ceil(n / i) = x
for(int l = 1, r = n; r >= 1; r = l - 1){
    int x = iceil(n, r);
    l = iceil(n, x);
}

```

6.13 Floor Sum

- $m = \lfloor \frac{an+b}{c} \rfloor$
- Time complexity: $O(\log n)$

$$f(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)}{2} + \lfloor \frac{b}{c} \rfloor \cdot (n+1) \\ + g(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm - f(c, c-b-1, a, m-1), & \text{otherwise} \end{cases}$$

$$g(a, b, c, n) = \sum_{i=0}^n i \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor \cdot \frac{n(n+1)}{2} \\ + g(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1) \\ - h(c, c-b-1, a, m-1)), & \text{otherwise} \end{cases}$$

$$h(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor^2$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor^2 \cdot (n+1) \\ + \lfloor \frac{a}{c} \rfloor \cdot \lfloor \frac{b}{c} \rfloor \cdot n(n+1) \\ + h(a \bmod c, b \bmod c, c, n) \\ + 2 \lfloor \frac{a}{c} \rfloor \cdot g(a \bmod c, b \bmod c, c, n) \\ + 2 \lfloor \frac{b}{c} \rfloor \cdot f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) \\ - 2f(c, c-b-1, a, m-1) - f(a, b, c, n), & \text{otherwise} \end{cases}$$

6.14 Berlekamp-Massey [a94d00]

```

// out[i] = me[0]*out[i-1] + me[1]*out[i-2]+...
template <typename T> // modify for mod
vector<T> BerlekampMassey(const vector<T> &output) {
    vector<T> d(output.size() + 1), me, he;
    for (size_t f = 0, i = 1; i <= output.size(); ++i) {
        for (size_t j = 0; j < me.size(); ++j)
            d[i] += output[i - j - 2] * me[j];
        if ((d[i] -= output[i - 1]) == 0) continue;
        if (me.empty()) {
            me.resize(f = i);
            continue;
        }
        vector<T> o(i - f - 1);
        T k = -d[i] / d[f]; o.push_back(-k);
        for (T x : he) o.push_back(x * k);
        if (o.size() < me.size()) o.resize(me.size());
        for (size_t j = 0; j < me.size(); ++j) o[j] += me[j];
        if (i - f + he.size() >= me.size()) he = me, f = i;
        me = o;
    }
    return me;
}

```

7 Polynomial

7.1 FWHT [c9cdb6]

```

/* x: a[j], y: a[j + (L >> 1)]
or: (y += x * op), and: (x += y * op)
xor: (x, y = (x + y) * op, (x - y) * op)
invop: or, and, xor = -1, -1, 1/2 */
void fwt(int *a, int n, int op) { //or
    for (int L = 2; L <= n; L <= 1)
        for (int i = 0; i < n; i += L)
            for (int j = i; j < i + (L >> 1); ++j)
                a[j + (L >> 1)] += a[j] * op;
}
const int N = 21;
int f[N][1 << N], g[N][1 << N], h[N][1 << N], ct[1 << N];
void subset_convolution(int *a, int *b, int *c, int L)
{
    // c_k = \sum_{i+j=k, i&j=0} a_i * b_j
    int n = 1 << L;
    for (int i = 1; i < n; ++i)
        ct[i] = ct[i & (i - 1)] + 1;
}

```

```

for (int i = 0; i < n; ++i)
    f[ct[i]][i] = a[i], g[ct[i]][i] = b[i];
for (int i = 0; i <= L; ++i)
    fwt(f[i], n, 1), fwt(g[i], n, 1);
for (int i = 0; i <= L; ++i)
    for (int j = 0; j <= i; ++j)
        for (int x = 0; x < n; ++x)
            h[i][x] += f[j][x] * g[i - j][x];
for (int i = 0; i <= L; ++i) fwt(h[i], n, -1);
for (int i = 0; i < n; ++i) c[i] = h[ct[i]][i];
}

```

7.2 FFT [13ec2f]

// Errichto: FFT for double works when the result < 1e15, and < 1e18 with long double

```

using val_t = complex<double>;
template<int MAXN>
struct FFT {
    const double PI = acos(-1);
    val_t w[MAXN];
    FFT() {
        for (int i = 0; i < MAXN; ++i) {
            double arg = 2 * PI * i / MAXN;
            w[i] = val_t(cos(arg), sin(arg));
        }
    }
    void bitrev(vector<val_t> &a, int n) //same as NTT
    void trans(vector<val_t> &a, int n, bool inv = false)
    {
        bitrev(a, n);
        for (int L = 2; L <= n; L <= 1) {
            int dx = MAXN / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + dl; ++j, x += dx) {
                    val_t tmp = a[j + dl] * (inv ? conj(w[x]) : w[x]);
                    a[j + dl] = a[j] - tmp;
                    a[j] += tmp;
                }
            }
        }
        if (inv) {
            for (int i = 0; i < n; ++i) a[i] /= n;
        }
    }
    //multiplying two polynomials A * B:
    //fft.trans(A, siz, 0), fft.trans(B, siz, 0);
    //A[i] *= B[i], fft.trans(A, siz, 1);
};

```

7.3 NTT [39f8b1]

```

//(2^16)+1, 65537, 3
//7*17*(2^23)+1, 998244353, 3
//1255*(2^20)+1, 1315962881, 3
//51*(2^25)+1, 1711276033, 29
// only works when sz(A) + sz(B) - 1 <= MAXN
// implement add, po, mul, inv first
template<int N, int RT>
struct NTT {
    int w[N];
    NTT() {
        int dw = po(RT, (mod-1) / N);
        w[0] = 1;
        for (int i = 1; i < N; ++i) w[i] = mul(w[i-1], dw);
    }
    void bitrev(vector<int> &a, int n) {
        int i = 0;
        for (int j = 1; j < n - 1; j++) {
            for (int k = n >> 1; (i ^ k) < k; k >>= 1);
            if (j < i) swap(a[i], a[j]);
        }
    }
    void operator()(vector<int> &a, int n, bool ix = false)
    {
        bitrev(a, n);
        for (int L = 2; L <= n; L <= 1) {
            int dx = N / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + dl; j++, x += dx) {
                    int tmp = mul(a[j + dl], w[x]);

```

```

                    a[j + dl] = a[j];
                    add(a[j + dl], -tmp);
                    add(a[j], tmp);
                }
            }
        }
        if (ix) {
            reverse(a.begin() + 1, a.begin() + n);
            int invn = inv(n);
            for (int i = 0; i < n; ++i) a[i] = mul(a[i], invn);
        }
    }
};

```

7.4 Polynomial Operation [2e2c93]

```

// maybe need fac ivf
// == ab8066 ==
#define fi(s, n) for (int i = (int)(s); i < (int)(n); i++)
#define neg(x) (x ? mod - x : 0)
#define V (*this)
template<int MAXN, int RT> // MAXN = 2^k
struct Poly : vector<int> { // coefficients in [0, P)
    using vector<int>::vector;
    static inline NTT<MAXN, RT> ntt;
    int n() const { return (int)size(); } // n() >= 1
    Poly(const Poly &p, int m) : vector<int>(m) { copy_n(p.data(), min(p.n(), m), data()); }
    Poly &rev() { return reverse(data(), data() + n()), V; }
    Poly &isz(int m) { return resize(m), V; }
    // == cd185f ==
    Poly &iadd(const Poly &rhs) { // 1c6277
        fi(0, n()) add(V[i], rhs[i]);
        return V; // need n() == rhs.n()
    }
    Poly &imul(int k) { // 7e5b36
        fi(0, n()) V[i] = mul(V[i], k);
        return V;
    }
    Poly mul_xk(int m) { // 13a612
        if (m < 0) {
            m = -m;
            fi(0, n() - m) V[i] = V[i + m];
            isz(n() - m);
        }
        else if (m > 0) {
            isz(n() + m);
            for (int i = n() - 1; i >= 0; i--) {
                if (i >= m) V[i] = V[i - m];
                else V[i] = 0;
            }
        }
        return V;
    }
}
Poly Mul(const Poly &rhs) const { // ecd03e
    int m = 1;
    while (m < n() + rhs.n() - 1) m <= 1;
    assert(m <= MAXN);
    Poly X(V, m), Y(rhs, m);
    ntt(X, m); ntt(Y, m);
    fi(0, m) X[i] = mul(X[i], Y[i]);
    ntt(X, m, true);
    return X.isz(n() + rhs.n() - 1);
}
Poly Inv() const { // 77f977
    if (n() == 1) return {inv(V[0])};
    int m = 1; // need V[0] != 0, 2*sz<=MAXN
    while (m < n() * 2) m <= 1;
    assert(m <= MAXN);
    Poly Xi = Poly(V, (n() + 1) / 2).Inv().isz(m);
    Poly Y(V, m);
    ntt(Xi, m); ntt(Y, m);
    fi(0, m) {
        Xi[i] = mul(Xi[i], (2 - mul(Xi[i], Y[i])));
        add(Xi[i], mod);
    }
    ntt(Xi, m, true);
    return Xi.isz(n());
}
// == 095701 ==

```



```

Poly &shift_inplace(const int &c) { // need fac[],
    ivf[]
    int n = V.n(); // 2 * sz <= MAXN
    for (int i = 0; i < n; i++) V[i] = mul(V[i], fac[i]);
    Poly g(n);
    int cp = 1;
    for (int i = 0; i < n; i++){
        g[i] = mul(cp, ivf[i]);
        cp = mul(cp, c);
    }
    V = V.irev().Mul(g).isz(n).irev();
    for (int i = 0; i < n; i++) V[i] = mul(V[i], ivf[i]);
    return V;
}
Poly Shift(const int &c) const { return Poly(V).
    shift_inplace(c); }
// == 0bd61d ==
Poly Dx() const {
    Poly ret(n() - 1);
    fi(0, ret.n()) ret[i] = mul(i+1, V[i+1]);
    return ret.isz(max(1, ret.n()));
}
Poly Sx() const {
    Poly ret(n() + 1);
    fi(0, n()) ret[i + 1] = mul(inv(i+1), V[i]);
    return ret;
}
// == 10e23d ==
Poly Ln() const { // V[0] == 1, 2*sz<=MAXN
    return Dx().Mul(Inv()).Sx().isz(n());
}
Poly Exp() const { // V[0] == 0, 2*sz<=MAXN
    if (n() == 1) return {1};
    Poly X = Poly(V, (n() + 1) / 2).Exp().isz(n());
    Poly Y = X.Ln();
    Y[0] = mod - 1;
    fi(0, n()){
        Y[i] = V[i] - Y[i];
        add(Y[i], mod);
    }
    return X.Mul(Y).isz(n());
}
// == eaf14c ==
// M := P(P - 1). If k >= M, k := k % M + M.
Poly Pow(ll k) const { // 2*sz<=MAXN
    int nz = 0;
    while (nz < n() && !V[nz]) nz++;
    if (nz * min(k, (ll)n()) >= n()) return Poly(n());
    if (!k) return Poly(Poly{1}, n());
    Poly X(data() + nz, data() + nz + n() - nz * k);
    const int c = po(X[0], k % (mod - 1));
    return X.Ln().imul(k % mod).Exp().imul(c).irev().
        isz(n()).irev();
}
// == cdf741 ==
Poly _tmul(int nn, const Poly &rhs) const {
    Poly Y = Mul(rhs).isz(n() + nn - 1);
    return Poly(Y.data() + n() - 1, Y.data() + Y.n());
}
vector<int> _eval(const vector<int> &x, const vector<
    Poly> &up) const { // 82d6be
    const int m = (int)x.size();
    if (!m) return {};
    vector<Poly> down(m * 2);
    down[1] = Poly(up[1]).irev().isz(n()).Inv().irev().
        _tmul(m, V);
    fi(2, m * 2) down[i] = up[i ^ 1]._tmul(up[i].n() -
        1, down[i / 2]);
    vector<int> y(m);
    fi(0, m) y[i] = down[m + i][0];
    return y;
}
static vector<Poly> _tree1(const vector<int> &x) { //
    2a0b6b
    const int m = (int)x.size();
    vector<Poly> up(m * 2);
    fi(0, m) up[m + i] = {neg(x[i]), 1};
    for (int i = m - 1; i > 0; --i) up[i] = up[i * 2].
        Mul(up[i * 2 + 1]);
    return up;
}

```

```

}
vector<int> Eval(const vector<int> &x) const {
    auto up = _tree1(x); // 2^17, 1.8s
    return _eval(x, up);
}
// == f9ecdcd ==
static Poly Interpolate(const vector<int> &x, const
    vector<int> &y) { // 8f2a08
    const int m = (int)x.size(); // 2^17, 2.3s
    vector<Poly> up = _tree1(x), down(m * 2);
    vector<int> z = up[1].Dx()._eval(x, up);
    fi(0, m) z[i] = mul(y[i], inv(z[i]));
    fi(0, m) down[m + i] = {z[i]};
    for (int i = m - 1; i > 0; --i)
        down[i] = down[i * 2].Mul(up[i * 2 + 1]).iadd(
            down[i * 2 + 1].Mul(up[i * 2]));
    return down[1];
}
pair<Poly, Poly> DivMod(const Poly &rhs) const { //
    a75170
    if (n() < rhs.n()) return {{0}, V}; // 5e5, 0.9s
    const int m = n() - rhs.n() + 1;
    Poly X(rhs); // (rhs).back() != 0
    X.irev().isz(m);
    Poly Y(V); Y.irev().isz(m);
    Poly Q = Y.Mul(X.Inv()).isz(m).irev();
    X = rhs.Mul(Q), Y = V;
    fi(0, n()) add(Y[i], -X[i]);
    return {Q, Y.isz(max(1, rhs.n() - 1))};
}
// == 7cd4c4 ==
Poly _Sqrt() const { // Jacobi(V[0], P) = 1
    if (n() == 1) return {QuadraticResidue(V[0], mod)};
    Poly X = Poly(V, (n() + 1) / 2)._Sqrt().isz(n());
    return X.iadd(Mul(X.Inv()).isz(n())).imul(mod / 2 +
        1);
}
Poly Sqrt() const { // 288427
    Poly a; // 2 * sz <= MAXN
    bool has = 0;
    for (int i = 0; i < n(); i++) {
        if (V[i]) has = 1;
        if (has) a.push_back(V[i]);
    }
    if (!has) return V;
    if ((n() + a.n()) % 2 || Jacobi(a[0], mod) != 1) {
        return Poly();
    }
    a = a.isz((n() + a.n()) / 2)._Sqrt();
    int sz = a.n();
    a.isz(n());
    rotate(a.begin(), a.begin() + sz, a.end());
    return a;
}
// == 0a54cf ==
Poly Shift_samples(int c, int m){
    // V = \sum f(n) x^n, return f(c), ..., f(c+m-1);
    2^19, 2s
    Poly A=V;
    Poly Q(n()+1), S(n());
    int nw=1;
    fi(0, n()+1) { Q[i] = mul(1-2*(i&1), nw); nw = mul(
        nw, n()-i, inv(i+1)); }
    nw=1;
    fi(0, n()) { S[i] = mul(1-2*(i&1), nw); nw = mul(nw,
        c-i, inv(i+1)); }
    S=S.Shift(1);
    fi(0,n()) if(i&1) {S[i] = mul(S[i],-1); add(S[i],
        mod);};
    Poly C=Mul(Q).mul_xk(-n());
    auto tmp=Q.isz(m).Inv();
    C=C.imul(-1).Mul(tmp).isz(m).mul_xk(n());
    A=A.isz(n()+m).iadd(C).irev().isz(n()+m);
    return A.Mul(S).mul_xk(-n()+1).isz(m+1).irev();
}
// == 224b20 ==
Poly power_projection(Poly wt, int m) { // 857237
    assert(n() == wt.n()); // 4*sz <= MAXN
    if (!n()) return Poly(m + 1, 0);
    if (V[0] != 0) {
        int c = V[0];
        V[0] = 0;
    }
}

```

```

Poly A = V.power_projection(wt, m);
fi(0, m + 1) A[i] = mul(A[i], fac[i]);
Poly B(m + 1);
int pow = 1;
fi(0, m + 1) {B[i] = mul(pow, ivf[i]); pow = mul(
    pow, c);} // inv. of fac
A = A.Mul(B).isz(m + 1);
fi(0, m + 1) A[i] = mul(A[i], fac[i]);
return A;
}
int n = 1;
while (n < V.n()) n *= 2;
isz(n), wt.isz(n).irev();
int k = 1;
Poly p(wt, 2 * n), q(V, 2 * n);
q.imul(mod - 1);
while (n > 1) {
    Poly r(2 * n * k);
    fi(0, 2 * n * k) r[i] = (i % 2 == 0 ? q[i] : neg(
        q[i]));
    Poly pq = p.Mul(r).isz(4 * n * k);
    Poly qq = q.Mul(r).isz(4 * n * k);
    fi(0, 2 * n * k) {
        add(pq[2 * n * k + i], p[i]);
        add(qq[2 * n * k + i], (q[i] + r[i]) % mod);
    }
    fill(p.begin(), p.end(), 0);
    fill(q.begin(), q.end(), 0);
    for(int j = 0; j < 2 * k; j++) fi(0, n / 2) {
        p[n * j + i] = pq[(2 * n) * j + (2 * i + 1)];
        q[n * j + i] = qq[(2 * n) * j + (2 * i + 0)];
    }
    n /= 2; k *= 2;
}
Poly ans(k);
fi(0, k) ans[i] = p[2 * i];
return ans.irev().isz(m + 1);
}
Poly FPSInv() { // 49e5ad
    const int n = V.n() - 1; // 2^17, 4s
    if (n == -1) return {};
    assert(V[0] == 0);
    if (n == 0) return V;
    assert(V[1] != 0);
    int c = V[1], ic = inv(c);
    imul(ic);
    Poly wt(n + 1);
    wt[n] = 1;
    Poly A = V.power_projection(wt, n);
    Poly g(n);
    fi(1, n + 1) g[n - i] = mul(n, A[i], inv(i));
    g = g.Pow(neg(inv(n)));
    g.insert(g.begin(), 0);
    int pow = 1;
    fi(0, g.n()) {g[i] = mul(g[i], pow); pow = mul(pow,
        ic);}
    return g;
}
// == 71ee33 ==
Poly TMul(const Poly &rhs) const { // this[i] - rhs[j]
    int j = k;
    return Poly(V).irev().Mul(rhs).isz(n()).irev();
}
Poly comp_rec(int n, int k, Poly Q){ // 06ca07
    if (n == 1) {
        Poly p(2 * k);
        irev();
        fi(0, k) p[2 * i] = V[i];
        return p;
    }
    Poly R(2 * n * k);
    fi(0, 2 * n * k) R[i] = (i % 2 == 0 ? Q[i] : neg(Q[
        i]));
    Poly QQ = Q.Mul(R).isz(4 * n * k);
    fi(0, 2 * n * k)
        add(QQ[2 * n * k + i], (Q[i] + R[i]) % mod);
    Poly nxt_Q(2 * n * k);
    for(int j = 0; j < 2 * k; j++) fi(0, n / 2)
        nxt_Q[n * j + i] = QQ[(2 * n) * j + (2 * i + 0)];
    Poly nxt_p = comp_rec(n / 2, k * 2, nxt_Q);
    Poly pq(4 * n * k);
    for(int j = 0; j < 2 * k; j++) fi(0, n / 2)

```

```

        add(pq[(2 * n) * j + (2 * i + 1)], nxt_p[n * j +
            i]);
    Poly p(2 * n * k);
    fi(0, 2 * n * k) add(p[i], pq[2 * n * k + i]);
    pq.pop_back();
    Poly x = pq.TMul(R);
    fi(0, 2 * n * k) add(p[i], x[i]);
    return p;
}
Poly FPScomp(Poly g) { // solves V(g(x))
    int sz = 1; // 2^17, 5s
    while(sz < n() || sz < g.n()) sz <= 1;
    return isz(sz), comp_rec(sz, 1, g.imul(mod-1).isz(2
        * sz)).isz(sz).irev();
}
// == 1d60ad ==
};
#undef fi
#undef V
#undef neg
using Poly_t = Poly<1 << 21, 3>;

```

7.5 Generating Function

Ordinary Generating Function

$$\begin{array}{c|c}
 \begin{array}{l}
 A(rx) \quad r^n a_n \\
 A(x)B(x) \quad \sum_{i=0}^n a_i b_{n-i} \\
 xA(x)' \quad na_n \\
 A(1) + x \frac{A(1)-A(x)}{1-x} \quad \sum_{i=n}^{\infty} a_i
 \end{array}
 &
 \begin{array}{l}
 A(x) + B(x) \quad a_n + b_n \\
 A(x)^k \quad \sum_{i_1+\dots+i_k=n} a_{i_1} \dots a_{i_k} \\
 \frac{A(x)}{1-x} \quad \sum_{i=0}^n a_i
 \end{array}
 \end{array}$$

- $\frac{1}{1-x} = 1 + x + x^2 + \dots + x^n + \dots$
- $(1+x)^a = \sum_{n=0}^{\infty} \binom{a}{n} x^n$, $\binom{a}{n} = \frac{a(a-1)(a-2)\dots(a-n+1)}{n!}$
- Catalan number: $f(x) = \frac{1-\sqrt{1-4x}}{2x}$

Exponential Generating Function

$$\begin{array}{c|c}
 \begin{array}{l}
 \hat{A}(x) + \hat{B}(x) \quad c_n = a_n + b_n \\
 \hat{A}^{(k)}(x) \quad c_n = a_{n+k} \\
 x\hat{A}(x) \quad c_n = na_n \\
 \hat{A}(x)\hat{B}(x) \quad c_n = \sum_{k=0}^n \binom{n}{k} a_k b_{n-k} \\
 \hat{A}(x)^k \quad \sum_{i_1+i_2+\dots+i_k=n} \binom{n}{i_1, i_2, \dots, i_k} a_{i_1} a_{i_2} \dots a_{i_k}
 \end{array}
 &
 \end{array}$$

$\exp(A(x))$: 假設 $A(x)$ 是一個分量 (component) 的生成函數, 那 $\hat{C}(x)$ 是將 n 個有編號的東西分成若干個分量的指數生成函數

Lagrange's Inversion Formula

如果 F 跟 G 互反, 則有 $F(0), G(0) = 0, F'(0), G'(0) \neq 0$ 。若 H 為任意 FPS, 則

$$\begin{aligned}
 n[x^n]G(x) &= [x^{n-1}] \frac{1}{(F(x)/x)^n} \\
 n[x^n]H(G(x)) &= [x^{n-1}] H'(x) \frac{1}{(F(x)/x)^n}
 \end{aligned}$$

Newton's iteration

如果有 $F(G(x)) = 0$ 已知 F 則可以用 $G_2N(x) = G_N(x) - F(G_N(x)) / \frac{\partial F}{\partial P}(G_N(x))$ 求出 G 的前 N 項, 其中 P 代表多項式。

7.6 Bostan Mori [dd6fb]

// Finds the k -th coefficient of P / Q in $O(d \log d \log k)$

// size of NTT has to $> 2 * d$

NTT<1<<19, 3> ntt;

int BostanMori(vector<int> P, vector<int> Q, long long k) {

int d = max((int)P.size(), (int)Q.size() - 1);

vector M = {P, Q};

M[0].resize(d, 0);

M[1].resize(d + 1, 0);

int sz = (2 * d + 1 == 1 ? 2 : (1 << (__lg(2 * d) + 1)));

vector<int> Qn(sz);

vector N(2, vector<int>(sz));

while(k) {

fill(iter(Qn), 0);

for(int i = 0; i < d + 1; i++){

Qn[i] = M[1][i] * ((i & 1) ? -1 : 1);

add(Qn[i], mod);

}

ntt(Qn, sz, false);

int t[2] = {(int)k & 1, 0};

for(int i = 0; i < 2; i++){

```

    fill(iter(N[i]), 0);
    copy(iter(M[i]), N[i].begin());
    ntt(N[i], sz, false);
    for(int j = 0; j < sz; j++) N[i][j] = mul(N[i][j]
        ], Qn[j]);
    ntt(N[i], sz, true);
    for(int j = t[i]; j < 2 * SZ(M[i]); j += 2){
        M[i][j >> 1] = N[i][j];
    }
    k >>= 1;
}
return mul(M[0][0], inv(M[1][0]));
}
int LinearRecursion(vector<int> a, vector<int> c, ll k)
{ // a_n = \sum_{j=1}^d c_j a_{n-j}, c_0 = 0
  int d = SZ(a), sz = 1; // 1e5, 5s
  while(sz <= 2*d) sz <<= 1;
  c[0] = mod - 1;
  for(auto &i : c) i = i % mod - i % 0;
  auto A = a; A.resize(sz);
  auto C = c; C.resize(sz);
  ntt(A, sz, false), ntt(C, sz, false);
  for(int i = 0; i < sz; i++) A[i] = mul(A[i], C[i]);
  ntt(A, sz, true);
  A.resize(d);
  return BostanMori(A, c, k);
}

```

8 String

8.1 KMP Algorithm [c8b75f]

```

// 0-based
// fail[i] = max k<i s.t. s[0..k] = s[i-k..i]
vector<int> kmp_build_fail(const string &s){
  int n = SZ(s);
  vector<int> fail(n, -1);
  int cur = -1;
  for(int i = 1; i < n; i++){
    while(cur != -1 && s[cur + 1] != s[i])
      cur = fail[cur];
    if(s[cur + 1] == s[i])
      cur++;
    fail[i] = cur;
  }
  return fail;
}

void kmp_match(const string &s, const vector<int> &fail
, const string &t){
  int cur = -1;
  int n = SZ(s), m = SZ(t);
  for(int i = 0; i < m; i++){
    while(cur != -1 && (cur + 1 == n || s[cur + 1] != t
        [i]))
      cur = fail[cur];
    if(cur + 1 < n && s[cur + 1] == t[i])
      cur++;
    // cur = max k s.t. s[0..k] = t[i-k..i]
  }
}

```

8.2 Manacher Algorithm [caf0f4]

```

/* center i: radius z[i * 2 + 1] / 2
   center i, i + 1: radius z[i * 2 + 2] / 2
   both aba, abba have radius 2 */
vector<int> manacher(const string &tmp){ // 0-based
  string s = "%";
  int l = 0, r = 0;
  for(char c : tmp) s += c, s += '%';
  vector<int> z(SZ(s));
  for(int i = 0; i < SZ(s); i++){
    z[i] = r > i ? min(z[2 * l - i], r - i) : 1;
    while(i - z[i] >= 0 && i + z[i] < SZ(s)
        && s[i + z[i]] == s[i - z[i]])
      ++z[i];
    if(z[i] + i > r) r = z[i] + i, l = i;
  }
  return z;
}

```

8.3 Lyndon Factorization [7c612b]

```

// partition s = w[0] + w[1] + ... + w[k-1],
// w[0] >= w[1] >= ... >= w[k-1]
// each w[i] strictly smaller than all its suffix
void duval(const string &s, vector<pii> &w) {
  for (int n = (int)s.size(), i = 0, j, k; i < n; ) {
    for (j = i + 1, k = i; j < n && s[k] <= s[j]; j++)
      k = (s[k] < s[j] ? i : k + 1);
    // if (i < n / 2 && j >= n / 2) {
    //   for min cyclic shift, call duval(s + s)
    //   then here s.substr(i, n / 2) is min cyclic shift
    // }
    for (; i <= k; i += j - k)
      w.pb(pii(i, j - k)); // s.substr(L, len)
  }
}

```

8.4 Suffix Array [36a3ce]

```

auto sais(const auto &s) { // 172fe4
  const int n = (int)s.size(), z = ranges::max(s) + 1;
  if (n == 1) return vector{0};
  vector<int> c(z); for (int x : s) ++c[x];
  partial_sum(iter(c), begin(c));
  vector<int> sa(n); auto I = views::iota(0, n);
  vector<bool> t(n); t[n - 1] = true;
  for (int i = n - 2; i >= 0; --i)
    t[i] = (s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
  auto is_lms = views::filter([&t](int x) {
    return x && t[x] && !t[x - 1]; });
  auto induce = [&] {
    for (auto x = c; int y : sa)
      if (y-- && !t[y]) sa[x[s[y] - 1]++] = y;
    for (auto x = c; int y : sa | views::reverse)
      if (y-- && t[y]) sa[--x[s[y]]] = y;
  };
  vector<int> lms, q(n); lms.reserve(n);
  for (auto x = c; int i : I | is_lms) {
    q[i] = int(lms.size());
    lms.push_back(sa[--x[s[i]]] = i);
  }
  induce(); vector<int> ns(lms.size());
  for (int j = -1, nz = 0; int i : sa | is_lms) {
    if (j >= 0) {
      int len = min({n - i, n - j, lms[q[i] + 1] - i});
      ns[q[i]] = nz += lexicographical_compare(
          begin(s) + j, begin(s) + j + len,
          begin(s) + i, begin(s) + i + len);
    }
    j = i;
  }
  ranges::fill(sa, 0); auto nsa = sais(ns);
  for (auto x = c; int y : nsa | views::reverse)
    y = lms[y], sa[--x[s[y]]] = y;
  return induce(), sa;
}

// hi[i]: LCP of suffix sa[i] and suffix sa[i - 1].
struct Suffix { // f35a00
  int n; vector<int> sa, hi, rev;
  Suffix(const auto &s) : n((int)s.size()),
    hi(n), rev(n) {
    vector<int> _s(n + 1); // _s[n] = 0;
    copy(iter(s), begin(_s)); // s shouldn't contain 0
    sa = sais(_s); sa.erase(sa.begin());
    for (int i = 0; i < n; ++i) rev[sa[i]] = i;
    for (int i = 0, h = 0; i < n; ++i) {
      if (!rev[i]) { h = 0; continue; }
      for (int j = sa[rev[i] - 1]; i + h < n && j + h <
          n
          && s[i + h] == s[j + h];) ++h;
      hi[rev[i]] = h ? h - 1 : 0;
    }
  }
};

```

8.5 Suffix Automaton [016373]

```

// == a14210 ==
struct exSAM {
  const int CNUM = 26;
  // Len: maxLength, Link: fail link
  // LenSorted: topo order, cnt: occur
  vector<int> len, link, lenSorted, cnt;
  vector<vector<int>> next;
};

```

```

int total = 0;
int newnode() {
    return total++;
}
void init(int n) { // total number of characters
    len.assign(2 * n, 0); link.assign(2 * n, 0);
    lenSorted.assign(2 * n, 0); cnt.assign(2 * n, 0);
    next.assign(2 * n, vector<int>(CNUM));
    newnode(), link[0] = -1;
}
// == c83c9c ==
int insertSAM(int last, int c) { // 081739
    // not exSAM: cur = newnode(), p = last
    int cur = next[last][c];
    len[cur] = len[last] + 1;
    int p = link[last];
    while (p != -1 && !next[p][c])
        next[p][c] = cur, p = link[p];
    if (p == -1) return link[cur] = 0, cur;
    int q = next[p][c];
    if (len[p] + 1 == len[q]) return link[cur] = q, cur;
    int clone = newnode();
    for (int i = 0; i < CNUM; ++i)
        next[clone][i] = len[next[q][i]] ? next[q][i] : 0;
    len[clone] = len[p] + 1;
    while (p != -1 && next[p][c] == q)
        next[p][c] = clone, p = link[p];
    link[link[cur] = clone] = link[q];
    link[q] = clone;
    return cur;
}
void insert(const string &s) { // e47d43
    int cur = 0;
    for (auto ch : s) {
        int &nxt = next[cur][int(ch - 'a')];
        if (!nxt) nxt = newnode();
        cnt[cur = nxt] += 1;
    }
}
// == 0a715a ==
void build() {
    queue<int> q;
    q.push(0);
    while (!q.empty()) {
        int cur = q.front();
        q.pop();
        for (int i = 0; i < CNUM; ++i)
            if (next[cur][i])
                q.push(insertSAM(cur, i));
    }
    vector<int> lc(total);
    for (int i = 1; i < total; ++i) ++lc[len[i]];
    partial_sum(iter(lc), lc.begin());
    for (int i = 1; i < total; ++i) lenSorted[--lc[len[i]]] = i;
}
void solve() {
    for (int i = total - 2; i >= 0; --i)
        cnt[link[lenSorted[i]]] += cnt[lenSorted[i]];
}
};

```

8.6 Z-value Algorithm [488d87]

// $z[i] = \max k \text{ s.t. } s[0..k-1] = s[i..i+k-1]$
 // i.e. Length of longest common prefix
 // $z[0] = 0$

```

vector<int> z_function(const string &s){
    int n = s.size();
    vector<int> z(n);
    for(int i = 1, l = 0, r = 0; i < n; i++){
        if(i <= r) z[i] = min(r - i + 1, z[i - l]);
        while(i + z[i] < n && s[z[i]] == s[i + z[i]])
            z[i]++;
        if(i + z[i] - 1 > r)
            l = i, r = i + z[i] - 1;
    }
    return z;
}

```

8.7 Main Lorentz [fcfb8f]

```

struct Rep{ int minl, maxl, len; };
vector<Rep> rep; // 0-base
//  $p \text{ \textit{in} } [minl, maxl] \Rightarrow s[p, p + i] = s[p + i, p + 2i]$ 
void main_lorentz(const string &s, int sft = 0) {
    const int n = s.size();
    if (n == 1) return;
    const int nu = n / 2, nv = n - nu;
    const string u = s.substr(0, nu), v = s.substr(nu,
        ru(u.rbegin(), u.rend()), rv(v.rbegin(), v.rend
        ());
    main_lorentz(u, sft), main_lorentz(v, sft + nu);
    const auto z1 = z_function(ru), z2 = z_function(v + '
        #' + u),
        z3 = z_function(ru + '#' + rv), z4 =
        z_function(v);
    auto get_z = [](const vector<int> &z, int i) {
        return (0 <= i and i < (int)z.size()) ? z[i] : 0;
    };
    auto add_rep = [&](bool left, int c, int l, int k1,
        int k2) {
        const int L = max(1, l - k2), R = min(l - left, k1)
        ;
        if (L > R) return;
        if (left) rep.emplace_back(Rep({sft + c - R, sft +
            c - L, l}));
        else rep.emplace_back(Rep({sft + c - R - l + 1, sft
            + c - L - l + 1, l}));
    };
    for (int cntr = 0; cntr < n; cntr++) {
        int l, k1, k2;
        if (cntr < nu) {
            l = nu - cntr;
            k1 = get_z(z1, nu - cntr);
            k2 = get_z(z2, nv + 1 + cntr);
        } else {
            l = cntr - nu + 1;
            k1 = get_z(z3, nu + 1 + nv - 1 - (cntr - nu));
            k2 = get_z(z4, (cntr - nu) + 1);
        }
        if (k1 + k2 >= 1)
            add_rep(cntr < nu, cntr, l, k1, k2);
    }
}

```

8.8 AC Automaton [f50309]

```

const int SIGMA = 26;
struct AC_Automaton {
    // child: trie, next: automaton
    vector<vector<int>> child, next;
    vector<int> fail, cnt, ord;
    int total = 0;
    int newnode() {
        return total++;
    }
    void init(int len) { // Len >= 1 + total Len
        child.assign(len, vector<int>(SIGMA, -1));
        next.assign(len, vector<int>(SIGMA, -1));
        fail.assign(len, -1); cnt.assign(len, 0);
        ord.clear();
        newnode();
    }
    int input(string &s) {
        int cur = 0;
        for (char c : s) {
            if (child[cur][c - 'A'] == -1)
                child[cur][c - 'A'] = newnode();
            cur = child[cur][c - 'A'];
        }
        return cur; // return the end node of string
    }
    void make_fl() {
        queue<int> q;
        q.push(0), fail[0] = -1;
        while(!q.empty()) {
            int R = q.front();
            q.pop(); ord.pb(R);
            for (int i = 0; i < SIGMA; ++i)
                if (child[R][i] != -1) {
                    int X = next[R][i] = child[R][i], Z = fail[R]
                    ;
                    while (Z != -1 && child[Z][i] == -1)
                        Z = fail[Z];
                }
        }
    }
}

```

```

        fail[X] = Z != -1 ? child[Z][i] : 0;
        q.push(X);
    }
    else next[R][i] = R ? next[fail[R]][i] : 0;
}
}
void solve() {
    for (int i : ord | views::reverse)
        if (i) cnt[fail[i]] += cnt[i];
}
};

```

8.9 Palindrome Automaton [8a071b]

```

struct PalindromicTree {
    struct node {
        int nxt[26], fail, len; // num = depth of fail link
        int cnt, num; // cnt = occur, num = #pal_suffix of this node
        node(int l = 0) : nxt{}, fail(0), len(l), cnt(0), num(0) {}
    };
    vector<node> st; vector<int> s; int last, n;
    void init() {
        st.clear(); s.clear(); last = 1; n = 0;
        st.pb(0); st.pb(-1);
        st[0].fail = 1; s.pb(-1);
    }
    int getFail(int x) {
        while (s[n - st[x].len - 1] != s[n]) x = st[x].fail;
        return x;
    }
    void add(int c) {
        s.pb(c - 'a'); ++n;
        int cur = getFail(last);
        if (!st[cur].nxt[c]) {
            int now = SZ(st);
            st.pb(st[cur].len + 2);
            st[now].fail = st[getFail(st[cur].fail)].nxt[c];
            st[cur].nxt[c] = now;
            st[now].num = st[st[now].fail].num + 1;
        }
        last = st[cur].nxt[c]; ++st[last].cnt;
    }
    void dpcnt() {
        for (int i = SZ(st) - 1; i >= 0; i--) {
            auto nd = st[i];
            st[nd.fail].cnt += nd.cnt;
        }
    }
    int size() { return (int)st.size() - 2; }
};

```

8.10 Palindrome Partition [c85c05]

```

// in PAM
/* node */ int dif = 0, slink = 0, g = 0;
vector<int> dp = {0};
// add
if (!st[cur].nxt[c]) {
    // ...
    st[now].dif = st[now].len - st[st[now].fail].len;
    if (st[now].dif == st[st[now].fail].dif)
        st[now].slink = st[st[now].fail].slink;
    else st[now].slink = st[now].fail;
}
dp.pb(0);
for (int x = last; x > 1; x = st[x].slink) {
    st[x].g = dp[n - st[st[x].slink].len - st[x].dif];
    if (st[x].dif == st[st[x].fail].dif)
        st[x].g = min(st[x].g, st[st[x].fail].g);
    dp[n] = min(dp[n], st[x].g + 1);
}

```

9 Misc

9.1 Cyclic Ternary Search [9017cc]

```

/* bool pred(int a, int b);
f(0) ~ f(n - 1) is a cyclic-shift U-function
return idx s.t. pred(x, idx) is false forall x*/
int cyc_tsearch(int n, auto pred) {
    if (n == 1) return 0;
    int l = 0, r = n; bool rv = pred(1, 0);

```

```

while (r - l > 1) {
    int m = (l + r) / 2;
    if (pred(0, m) ? rv : pred(m, (m + 1) % n)) r = m;
    else l = m;
}
return pred(1, r % n) ? l : r % n;
}

```

9.2 Matroid

$M = (E, \mathcal{I})$, where $\mathcal{I} \subseteq 2^E$ is nonempty, is a matroid if:

- If $S \in \mathcal{I}$ and $S' \subsetneq S$, then $S' \in \mathcal{I}$.
- For $S_1, S_2 \in \mathcal{I}$ s.t. $|S_1| < |S_2|$, there exists $e \in S_2 \setminus S_1$ s.t. $S_1 \cup \{e\} \in \mathcal{I}$.

Matroid intersection:

Start from $S = \emptyset$. In each iteration, let

- $Y_1 = \{x \notin S \mid S \cup \{x\} \in \mathcal{I}_1\}$
- $Y_2 = \{x \notin S \mid S \cup \{x\} \in \mathcal{I}_2\}$

If there exists $x \in Y_1 \cap Y_2$, insert x into S . Otherwise for each $x \in S, y \notin S$, create edges

- $x \rightarrow y$ if $S - \{x\} \cup \{y\} \in \mathcal{I}_1$.
- $y \rightarrow x$ if $S - \{x\} \cup \{y\} \in \mathcal{I}_2$.

Find a *shortest* path (with BFS) starting from a vertex in Y_1 and ending at a vertex in Y_2 which doesn't pass through any other vertices in Y_2 , and alternate the path. The size of S will be incremented by 1 in each iteration. For the weighted case, assign weight $w(x)$ to vertex x if $x \in S$ and $-w(x)$ if $x \notin S$. Find the path with the minimum number of edges among all minimum length paths and alternate it.

9.3 Simulate Annealing [ff826c]

```

ld anneal() {
    mt19937 rnd_engine(seed);
    uniform_real_distribution<ld> rnd(0, 1);
    const ld dT = 0.001;
    // Argument p
    ld S_cur = calc(p), S_best = S_cur;
    for (ld T = 2000; T > eps; T -= dT) {
        // Modify p to p_prime
        const ld S_prime = calc(p_prime);
        const ld delta_c = S_prime - S_cur;
        ld prob = min((ld)1, exp(-delta_c / T));
        if (rnd(rnd_engine) <= prob)
            S_cur = S_prime, p = p_prime;
        if (S_prime < S_best) // find min
            S_best = S_prime, p_best = p_prime;
    }
    return S_best;
}

```

9.4 Binary Search On Fraction [f6b9ec]

```

struct Q {
    ll p, q;
    Q go(Q b, ll d) { return {p + b.p * d, q + b.q * d};
    };
    // returns smallest p/q in [lo, hi] such that
    // pred(p/q) is true, and 0 <= p, q <= N
    Q frac_bs(ll N, auto &&pred) {
        Q lo{0, 1}, hi{1, 0};
        if (pred(lo)) return lo;
        assert(pred(hi));
        bool dir = 1, L = 1, H = 1;
        for (; L || H; dir = !dir) {
            ll len = 0, step = 1;
            for (int t = 0; t < 2 && (t ? step /= 2 : step *= 2);)
                if (Q mid = hi.go(lo, len + step);
                    mid.p > N || mid.q > N || dir ^ pred(mid))
                    t++;
                else len += step;
            swap(lo, hi = hi.go(lo, len));
            (dir ? L : H) = !len;
        }
        return dir ? hi : lo;
    }
};

```

9.5 Min Plus Convolution [09b5c3]

```

// a is convex a[i+1]-a[i] <= a[i+2]-a[i+1]
vector<int> min_plus_convolution(vector<int> &a, vector<int> &b) {
    int n = SZ(a), m = SZ(b);
    vector<int> c(n + m - 1, INF);
    auto dc = [&](auto Y, int l, int r, int j1, int jr) {
        if (l > r) return;
        int mid = (l + r) / 2, from = -1, &best = c[mid];

```



```

for (int j = j1; j <= jr; ++j)
    if (int i = mid - j; i >= 0 && i < n)
        if (best > a[i] + b[j])
            best = a[i] + b[j], from = j;
Y(Y, 1, mid - 1, j1, from), Y(Y, mid + 1, r, from, jr);
};
return dc(dc, 0, n - 1 + m - 1, 0, m - 1), c;
}

```

9.6 SMAWK [a2a4ce]

```

// For all 2x2 submatrix:
// If M[1][0] < M[1][1], M[0][0] < M[0][1]
// If M[1][0] == M[1][1], M[0][0] <= M[0][1]
// M[i][ans_i] is the best value in the i-th row
// select(int r, int u, int v) return true if f(r, v)
// is better than f(r, u)
vector<int> smawk(int N, int M, auto &&select) {
    auto dc = [&](auto self, const vector<int> &r, const
        vector<int> &c) {
        if (r.empty()) return vector<int>{};
        const int n = SZ(r); vector<int> ans(n), nr, nc;
        for (int i : c) {
            while (!nc.empty() &&
                select(r[nc.size() - 1], nc.back(), i))
                nc.pop_back();
            if (int(nc.size()) < n) nc.push_back(i);
        }
        for (int i = 1; i < n; i += 2) nr.push_back(r[i]);
        const auto na = self(self, nr, nc);
        for (int i = 1; i < n; i += 2) ans[i] = na[i >> 1];
        for (int i = 0, j = 0; i < n; i += 2) {
            ans[i] = nc[j];
            const int end = i + 1 == n ? nc.back() : ans[i + 1];
            while (nc[j] != end)
                if (select(r[i], ans[i], nc[++j])) ans[i] = nc[j];
        }
        return ans;
    };
    vector<int> R(N), C(M); iota(iter(R), 0), iota(iter(C), 0));
    return dc(dc, R, C);
}

```

9.7 Golden Ratio Search [ce06a8]

```

ld goldenRatioSearch(ld a, ld b, auto &&f) {
    ld r = (sqrt(5)-1)/2, eps = 1e-7;
    ld x1 = b - r*(b-a), x2 = a + r*(b-a);
    ld f1 = f(x1), f2 = f(x2);
    while (b-a > eps)
        if (f1 < f2) { //change to > to find maximum
            b = x2; x2 = x1; f2 = f1;
            x1 = b - r*(b-a); f1 = f(x1);
        } else {
            a = x1; x1 = x2; f1 = f2;
            x2 = a + r*(b-a); f2 = f(x2);
        }
    return a;
}

```

9.8 Python Misc [d41d8c]

```

from decimal import *
setcontext(Context(prec=MAX_PREC, Emax=MAX_EMAX,
    rounding=ROUND_FLOOR))
print(Decimal(input()) * Decimal(input()))
from fractions import Fraction
Fraction('3.14159').limit_denominator(10).numerator #
22
import string, random
print(string.ascii_letters + string.digits + string.
    punctuation)
# string.ascii_lowercase
print(random.choice('abc'), random.randint(1, 10),
    random.random())

```

10 Notes

10.1 Geometry

Rotation Matrix

- rotate 90°: $(x, y) \rightarrow (-y, x)$
- rotate -90°: $(x, y) \rightarrow (y, -x)$

$$\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

Triangles

Side lengths: a, b, c , Semiperimeter: $p = \frac{a+b+c}{2}$

Area: $A = \sqrt{p(p-a)(p-b)(p-c)}$

Circumradius: $R = \frac{abc}{4A}$, Inradius: $r = \frac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two): $s_a = \sqrt{bc \left(1 - \left(\frac{a}{b+c}\right)^2\right)}$

Law of sines: $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} = \frac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc \cos \alpha$

Law of tangents: $\frac{a+b}{a-b} = \frac{\tan \frac{\alpha+\beta}{2}}{\tan \frac{\alpha-\beta}{2}}$

Quadrilaterals

With side lengths a, b, c, d , diagonals e, f , diagonals angle θ , area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2 f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is 180° , $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

Spherical coordinates

$$\begin{aligned} x &= r \sin \theta \cos \phi & r &= \sqrt{x^2 + y^2 + z^2} \\ y &= r \sin \theta \sin \phi & \theta &= \arccos(z/\sqrt{x^2 + y^2 + z^2}) \\ z &= r \cos \theta & \phi &= \operatorname{atan2}(y, x) \end{aligned}$$

Green's Theorem

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy = \oint_{L^+} (P dx + Q dy)$$

$$\text{Area} = \frac{1}{2} \oint_L x dy - y dx$$

- Circular sector:

$$x = x_0 + r \cos \theta$$

$$y = y_0 + r \sin \theta$$

$$\begin{aligned} A &= r \int_{\alpha}^{\beta} (x_0 + \cos \theta) \cos \theta + (y_0 + \sin \theta) \sin \theta d\theta \\ &= r(r\theta + x_0 \sin \theta - y_0 \cos \theta) \Big|_{\alpha}^{\beta} \end{aligned}$$

- Centroid:

$$\bar{x} = \frac{1}{2A} \int_C x^2 dy, \quad \bar{y} = -\frac{1}{2A} \int_C y^2 dx$$

Point-Line Duality

$$p = (a, b) \leftrightarrow p^* : y = ax - b$$

- $p \in l \iff l^* \in p^*$
- p_1, p_2, p_3 are collinear $\iff p_1^*, p_2^*, p_3^*$ intersect at a point
- p lies above $l \iff l^*$ lies above p^*
- lower convex hull \leftrightarrow upper envelope

10.2 Trigonometry

$$\sinh x = \frac{1}{2}(e^x - e^{-x}), \cosh x = \frac{1}{2}(e^x + e^{-x}), \sin n\pi = 0, \cos n\pi = (-1)^n$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\sin(2\alpha) = 2 \cos \alpha \sin \alpha$$

$$\cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha = 2 \cos^2 \alpha - 1 = 1 - 2 \sin^2 \alpha$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}, \sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}, \sin \alpha \sin \beta = \frac{1}{2}(\cos(\alpha - \beta) - \cos(\alpha + \beta))$$

$$\sin \alpha \cos \beta = \frac{1}{2}(\sin(\alpha + \beta) + \sin(\alpha - \beta)), \cos \alpha \sin \beta = \frac{1}{2}(\sin(\alpha + \beta) - \sin(\alpha - \beta))$$

$$\cos \alpha \cos \beta = \frac{1}{2}(\cos(\alpha - \beta) + \cos(\alpha + \beta))$$

$$(V + W) \tan(\alpha - \beta)/2 = (V - W) \tan(\alpha + \beta)/2$$

where V, W are lengths of sides opposite angles α, β .

$$a \cos x + b \sin x = r \cos(x - \phi), \quad a \sin x + b \cos x = r \sin(x + \phi)$$

where $r = \sqrt{a^2 + b^2}, \phi = \operatorname{atan2}(b, a)$.

10.4 Sum & Series

$$\begin{array}{ll} \frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}} & \frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1-x^2}} \\ \frac{d}{dx} \tan x = 1 + \tan^2 x & \frac{d}{dx} \arctan x = \frac{1}{1+x^2} \\ \int \tan ax = -\frac{\ln |\cos ax|}{a} & \int x \sin ax = \frac{\sin ax - ax \cos ax}{a^2} \end{array}$$

- Vandermonde's Identity

$$C(n+m, k) = \sum_{i=0}^k C(n, i)C(m, k-i)$$

- **Kirchhoff's Theorem**
Denote L be a $n \times n$ matrix as the Laplacian matrix of graph G , where $L_{ii} = d(i)$, $L_{ij} = -c$ where c is the number of edge (i, j) in G .
 - The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
 - The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.
 - BEST theorem: the number of eulerian circuits in a directed graph is $|\det(\tilde{L}_{ww})| \cdot \prod_{v \in V} (\deg(v) - 1)!$.
- **Tutte's Matrix**
Let D be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ (x_{ij} is chosen uniformly at random) if $i < j$ and $(i, j) \in E$, otherwise $d_{ij} = -d_{ji} \cdot \frac{\text{rank}(D)}{2}$ is the maximum matching on G .
- **Cayley's Formula**
 - Given a degree sequence d_1, d_2, \dots, d_n for each *labeled* vertices, there are $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\dots(d_n-1)!}$ spanning trees.
 - Let $T_{n,k}$ be the number of *labeled* forests on n vertices with k components, such that vertex $1, 2, \dots, k$ belong to different components. Then $T_{n,k} = kn^{n-k-1}$.
- **Erdős-Gallai theorem**
A sequence of nonnegative integers $d_1 \geq \dots \geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1 + \dots + d_n$ is even and $\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$ holds for every $1 \leq k \leq n$.
- **Gale-Ryser theorem**
A pair of sequences of nonnegative integers $a_1 \geq \dots \geq a_n$ and b_1, \dots, b_n is bigraphic if and only if $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$ and $\sum_{i=1}^k a_i \leq \sum_{i=1}^n \min(b_i, k)$ holds for every $1 \leq k \leq n$.
- **Fulkerson-Chen-Anstee theorem**
A sequence $(a_1, b_1), \dots, (a_n, b_n)$ of nonnegative integer pairs with $a_1 \geq \dots \geq a_n$ is digraphic if and only if $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$ and $\sum_{i=1}^k a_i \leq \sum_{i=1}^k \min(b_i, a_i)$ holds for every $1 \leq k \leq n$.
- **Pick's theorem**
For simple polygon, when points are all integer, we have $A = \#\{\text{lattice points in the interior}\} + \frac{\#\{\text{lattice points on the boundary}\}}{2} - 1$.
- **Möbius inversion formula**
 - $\mu(d) = (-1)^k$ if n is the product of k distinct primes, 0 if $p^2 \mid n$
 - $f(n) = \sum_{d \mid n} g(d) \Leftrightarrow g(n) = \sum_{d \mid n} \mu(d) f(\frac{n}{d})$
 - $f(n) = \sum_{n \mid d} g(d) \Leftrightarrow g(n) = \sum_{n \mid d} \mu(\frac{d}{n}) f(d)$
- **Spherical cap**
 - A portion of a sphere cut off by a plane.

- r : sphere radius, a : radius of the base of the cap, h : height of the cap, θ : $\arcsin(a/r)$.
- Volume = $\pi h^2(3r - h)/3 = \pi h(3a^2 + h^2)/6 = \pi r^3(2 + \cos\theta)(1 - \cos\theta)^2/3$.
- Area = $2\pi rh = \pi(a^2 + h^2) = 2\pi r^2(1 - \cos\theta)$.
- Lagrange multiplier
 - Optimize $f(x_1, \dots, x_n)$ when k constraints $g_i(x_1, \dots, x_n) = 0$.
 - Lagrangian function

$$\mathcal{L}(x_1, \dots, x_n, \lambda_1, \dots, \lambda_k) = f(x_1, \dots, x_n) - \sum_{i=1}^k \lambda_i g_i(x_1, \dots, x_n).$$
 - The solution corresponding to the original constrained optimization is always a saddle point of the Lagrangian function.

- Nearest points of two skew lines

- Line 1 : $v_1 = p_1 + t_1 d_1$
- Line 2 : $v_2 = p_2 + t_2 d_2$
- $n = d_1 \times d_2$
- $n_1 = d_1 \times n$
- $n_2 = d_2 \times n$
- $c_1 = p_1 + \frac{(p_2 - p_1) \cdot n_2}{d_1 \cdot n_2} d_1$
- $c_2 = p_2 + \frac{(p_1 - p_2) \cdot n_1}{d_2 \cdot n_1} d_2$

- Bernoulli numbers

$$B_0 = 1, B_1^\pm = \pm \frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0$$

$$\sum_{j=0}^m \binom{m+1}{j} B_j = 0, \text{ EGF is } B(x) = \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}.$$

$$S_m(n) = \sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k^+ n^{m+1-k}$$

- Stirling numbers of the second kind Partitions of n distinct elements into exactly k groups.

$$S(n, k) = S(n - 1, k - 1) + kS(n - 1, k), S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{i=0}^k (-1)^{k-i} \binom{k}{i} i^n$$

$$x^n = \sum_{i=0}^n S(n, i)(x)_i$$

- Pentagonal number theorem

$$\prod_{n=1}^{\infty} (1 - x^n) = 1 + \sum_{k=1}^{\infty} (-1)^k \left(x^{k(3k+1)/2} + x^{k(3k-1)/2} \right)$$

- Catalan numbers

$$C_n^{(k)} = \frac{1}{(k-1)n+1} \binom{kn}{n}$$

$$C^{(k)}(x) = 1 + x[C^{(k)}(x)]^k$$

- Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j 's s.t. $\pi(j) > \pi(j+1)$, $k+1$ j 's s.t.

$$\pi(j) \geq j, k \text{ j:s s.t. } \pi(j) > j.$$

$$E(n, k) = (n - k)E(n - 1, k - 1) + (k + 1)E(n - 1, k)$$

$$E(n, 0) = E(n, n - 1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

10.6 Number

- Number of partitions of n :

n	2	3	4	5	6	7	8	9	20	30	40	50	100
$p(n)$	2	3	5	7	11	15	22	30	627	5604	4e4	2e5	2e8

- Maximum number of divisors:

n	100	1e3	1e6	1e9	1e12	1e15	1e18
$d(i)$	12	32	240	1344	6720	26880	103680

- | | | | | | | | | | | | | | | | |
|---------------------------|---|---|----|----|-----|-----|------|-------|-------|--------|-----|-----|-----|-----|-------|
| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| $\frac{1}{\binom{2n}{n}}$ | 2 | 6 | 20 | 70 | 252 | 924 | 3432 | 12870 | 48620 | 184756 | 7e5 | 2e6 | 1e7 | 4e7 | 1.5e8 |

- Number of ways to partition a set of n labeled elements:

n	2	3	4	5	6	7	8	9	10	11	12	13
B_n	2	5	15	52	203	877	4140	21147	115975	7e5	4e6	3e7

- Fibonacci numbers:

n	1	2	3	4	5	31	45	88
F_n	1	1	3	5	8	1346269	1e9	1e18

[illegible]