🧩 Plan for the Template

We'll structure it in **sections** so you can easily scroll to what you need:

1. **Boilerplate & Fast I/O**
2. **Macros & Type Aliases**
3. **Debug Tools (optional in local mode)**
4. **Math Utilities**
    - GCD, LCM, ModPow, Inverse, nCr, Sieve, etc.
5. **Data Structures**
    - DSU, Segment Tree, Fenwick Tree, Sparse Table
6. **Graph Algorithms**
    - DFS, BFS, Dijkstra, Bellman-Ford, Floyd-Warshall, Toposort
7. **Trees**
    - LCA, Binary Lifting, Euler Tour
8. **Strings**
    - KMP, Z-Algorithm, Rolling Hash
9. **Miscellaneous**
    - Coordinate compression, custom comparators, etc.

```
#include <bits/stdc++.h>
using namespace std;
// ---------- FAST I/O ----------
#define FAST_IO ios::sync_with_stdio(false); cin.tie(nullptr)
// ---------- MACROS ----------
#define int long long
#define pb push_back
#define all(x) (x).begin(), (x).end()
#define rall(x) (x).rbegin(), (x).rend()
#define sz(x) (int)(x).size()
#define fi first
#define se second
// ---------- CONSTANTS ----------
const int MOD = 1e9 + 7;
const int INF = 1e18;
const double EPS = 1e-9;

// ---------- UTILITY ----------
template<class T> bool ckmin(T &a, const T &b){ if(b<a){a=b;return true;} return false; }
template<class T> bool ckmax(T &a, const T &b){ if(b>a){a=b;return true;} return false; }
int addmod(int a,int b,int m=MOD){ a%=m; b%=m; int r=a+b; if(r>=m) r-=m; return r; }
int submod(int a,int b,int m=MOD){ a%=m; b%=m; int r=a-b; if(r<0) r+=m; return r; }
int mulmod(long long a,long long b,int m=MOD){ return (int)((a*b)%m); }


// ---------- MATH UTILITIES ----------
int gcd(int a, int b) { return b == 0 ? a : gcd(b, a % b); }
int lcm(int a, int b) { return a / gcd(a, b) * b; }
int modpow(int a, int b, int mod = MOD) {
    int res = 1;
    while (b) {
        if (b & 1) res = res * a % mod;
        a = a * a % mod;
        b >>= 1;
    }
    return res;
}
int modinv(int a, int mod = MOD) { return modpow(a, mod - 2, mod); }
vector<int> fact(1, 1);
void init_fact(int n) {
    fact.resize(n+1);
    for (int i = 1; i <= n; i++) fact[i] = fact[i-1] * i % MOD;
}

int nCr(int n, int r) {
    if (r < 0 || r > n) return 0;
    return fact[n] * modinv(fact[r]) % MOD * modinv(fact[n - r]) % MOD;
}
```

```
// ---------- SIEVE ----------
vector<int> primes;
vector<bool> is_prime;
void sieve(int n) {
    is_prime.assign(n + 1, true);
    is_prime[0] = is_prime[1] = false;
    for (int i = 2; i * i <= n; i++)
        if (is_prime[i])
            for (int j = i * i; j <= n; j += i)
                is_prime[j] = false;
    for (int i = 2; i <= n; i++)
        if (is_prime[i]) primes.pb(i);
}

// ---------- DSU ----------
struct DSU {
    vector<int> p, sz;
    DSU(int n) { p.resize(n); sz.assign(n, 1); iota(all(p), 0); }
    int find(int x) { return p[x] == x ? x : p[x] = find(p[x]); }
    bool unite(int a, int b) {
        a = find(a), b = find(b);
        if (a == b) return false;
        if (sz[a] < sz[b]) swap(a, b);
        p[b] = a; sz[a] += sz[b];
        return true;
    }
};

// ---------- FENWICK TREE ----------
struct Fenwick {
    int n; vector<int> bit;
    Fenwick(int n) : n(n), bit(n+1) {}
    void update(int idx, int val) {
        for (; idx <= n; idx += idx & -idx)
            bit[idx] += val;
    }
    int query(int idx) {
        int res = 0;
        for (; idx > 0; idx -= idx & -idx)
            res += bit[idx];
        return res;
    }
};
```

```
// ---------- SEGMENT TREE ----------
struct SegTree {
    int n; vector<int> t;
    SegTree(int n) : n(n), t(4*n, 0) {}
    void build(vector<int> &a, int v, int tl, int tr) {
        if (tl == tr) t[v] = a[tl];
        else {
            int tm = (tl + tr) / 2;
            build(a, v*2, tl, tm);
            build(a, v*2+1, tm+1, tr);
            t[v] = t[v*2] + t[v*2+1];
        }
    }
    int query(int v, int tl, int tr, int l, int r) {
        if (l > r) return 0;
        if (l == tl && r == tr) return t[v];
        int tm = (tl + tr) / 2;
        return query(v*2, tl, tm, l, min(r, tm)) +
               query(v*2+1, tm+1, tr, max(l, tm+1), r);
    }
    void update(int v, int tl, int tr, int pos, int val) {
        if (tl == tr) t[v] = val;
        else {
            int tm = (tl + tr) / 2;
            if (pos <= tm) update(v*2, tl, tm, pos, val);
            else update(v*2+1, tm+1, tr, pos, val);
            t[v] = t[v*2] + t[v*2+1];
        }
    }
};
```

```
// ----------------- LCA (binary lifting) -----------------
struct LCA {
    int n, LOG;
    vector<int> depth;
    vector<vector<int>> up;
    vector<vector<int>> adj;
    LCA(int n=0){ init(n); }
    void init(int n_){
        n=n_;
        LOG = 1; while((1<<LOG) <= n) LOG++;
        depth.assign(n,0);
        up.assign(LOG, vector<int>(n, -1));
        adj.assign(n, {});
    }
    void addEdge(int u,int v){ adj[u].pb(v); adj[v].pb(u); }
    void dfs(int v,int p){
        up[0][v] = p==-1 ? v : p;
        for(int i=1;i<LOG;i++) up[i][v] = up[i-1][ up[i-1][v] ];
        for(int to: adj[v]){
            if(to==p) continue;
            depth[to] = depth[v]+1;
            dfs(to,v);
        }
    }
    void build(int root=0){
        dfs(root, -1);
    }
    int kthAncestor(int v, int k){
        for(int i=0;i<LOG;i++) if(k&(1<<i)) v = up[i][v];
        return v;
    }
    int lca(int a,int b){
        if(depth[a] < depth[b]) swap(a,b);
        int diff = depth[a]-depth[b];
        a = kthAncestor(a, diff);
        if(a==b) return a;
        for(int i=LOG-1;i>=0;i--) if(up[i][a] != up[i][b]) { a = up[i][a]; b = up[i][b]; }
        return up[0][a];
    }
};
```

```
// ---------- GRAPH ----------
vector<vector<int>> adj;
vector<int> dist, vis;

void bfs(int s) {
    queue<int> q; q.push(s);
    dist.assign(sz(adj), INF);
    dist[s] = 0;
    while (!q.empty()) {
        int v = q.front(); q.pop();
        for (int u : adj[v]) {
            if (dist[u] > dist[v] + 1) {
                dist[u] = dist[v] + 1;
                q.push(u);
            }
        }
    }
}

void dfs(int v) {
    vis[v] = 1;
    for (int u : adj[v])
        if (!vis[u]) dfs(u);
}

// ---------- DIJKSTRA ----------
vector<int> dijkstra(int n, int s, vector<vector<pair<int,int>>> &g) {
    vector<int> d(n, INF);
    d[s] = 0;
    priority_queue<pair<int,int>, vector<pair<int,int>>, greater<>> pq;
    pq.push({0, s});
    while (!pq.empty()) {
        auto [dist_u, u] = pq.top(); pq.pop();
        if (dist_u != d[u]) continue;
        for (auto [v, w] : g[u]) {
            if (d[v] > d[u] + w) {
                d[v] = d[u] + w;
                pq.push({d[v], v});
            }
        }
    }
    return d;
}
```

```
// ---------- STRINGS ----------
vector<int> prefix_function(string s) {
    int n = sz(s);
    vector<int> pi(n);
    for (int i = 1; i < n; i++) {
        int j = pi[i-1];
        while (j > 0 && s[i] != s[j]) j = pi[j-1];
        if (s[i] == s[j]) j++;
        pi[i] = j;
    }
    return pi;
}

vector<int> z_function(string s) {
    int n = sz(s);
    vector<int> z(n);
    for (int i = 1, l = 0, r = 0; i < n; i++) {
        if (i <= r) z[i] = min(r - i + 1, z[i - l]);
        while (i + z[i] < n && s[z[i]] == s[i + z[i]]) z[i]++;
        if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
    }
    return z;
}




// ----------------- MISC HELPERS -----------------
int bitcount(long long x){ return __builtin_popcountll((unsigned long long)x); }
int highest_bit(long long x){ return 63 - __builtin_clzll((unsigned long long)x); }



// ---------- MAIN ----------
void solve() {
    // your code here
}

int32_t main() {
    FAST_IO;
    int T = 1;
    // cin >> T;
    while (T--) solve();
}
```

```
import sys, math
from collections import deque, defaultdict, Counter
from bisect import bisect_left, bisect_right
from heapq import heappush, heappop
sys.setrecursionlimit(3 * 10**5)
input = sys.stdin.readline

# ---------------- FAST I/O ----------------
def inp(): return sys.stdin.readline().strip()
def inpi(): return int(inp())
def inpl(): return list(map(int, inp().split()))
def inps(): return inp().split()

# ---------------- CONSTANTS ----------------
INF = 10**18
MOD = 10**9 + 7

# ========================================================
# 🧱 MATH UTILITIES
# ========================================================
def gcd(a, b):
    """Greatest common divisor. O(log min(a,b))."""
    while b: a, b = b, a % b
    return a

def lcm(a, b):
    """Least common multiple using gcd."""
    return a * b // gcd(a, b)

def modpow(a, b, mod=MOD):
    """Fast exponentiation (a^b % mod). O(log b)."""
    res = 1
    a %= mod
    while b:
        if b & 1: res = res * a % mod
        a = a * a % mod
        b >>= 1
    return res

def modinv(a, mod=MOD):
    """Modular inverse using Fermat's theorem (mod must be prime)."""
    return modpow(a, mod - 2, mod)

# Example:
# print(gcd(10, 15)) → 5
# print(lcm(3, 5)) → 15
# print(modpow(2, 10)) → 1024
# print(modinv(3)) → 333333336 (since 3 * 333333336 % MOD = 1)
```

```
# ============================================================
# 🔢 COMBINATORICS
# ============================================================
MAXN = 10**6
fact = [1]*(MAXN+1)
invfact = [1]*(MAXN+1)
for i in range(1, MAXN+1): fact[i] = fact[i-1]*i%MOD
invfact[MAXN] = modinv(fact[MAXN])
for i in range(MAXN, 0, -1): invfact[i-1] = invfact[i]*i%MOD

def nCr(n, r):
    """n choose r mod MOD. O(1) after precompute."""
    if r<0 or r>n: return 0
    return fact[n]*invfact[r]%MOD*invfact[n-r]%MOD

# Example:
# print(nCr(5, 2)) → 10

# ============================================================
# 🔍 SIEVE OF ERATOSTHENES
# ============================================================
def sieve(n):
    """Returns all primes <= n. O(n log log n)."""
    isprime = [True]*(n+1)
    isprime[0]=isprime[1]=False
    for i in range(2, int(n**0.5)+1):
        if isprime[i]:
            for j in range(i*i, n+1, i):
                isprime[j]=False
    return [i for i in range(2,n+1) if isprime[i]]

# Example:
# print(sieve(10)) → [2,3,5,7]
```

```
# ========================================================
# 🌆 GRAPH ALGORITHMS
# ========================================================
def bfs(start, adj):
    """Shortest path in unweighted graph. O(V+E)."""
    dist=[-1]*len(adj)
    q=deque([start]); dist[start]=0
    while q:
        v=q.popleft()
        for u in adj[v]:
            if dist[u]==-1:
                dist[u]=dist[v]+1
                q.append(u)
    return dist

# Example:
# adj=[[1,2],[0,3],[0,3],[1,2]]
# print(bfs(0,adj)) → [0,1,1,2]

def dfs(v, adj, vis):
    """Depth-first search (connectivity, tree traversal)."""
    vis[v]=1
    for u in adj[v]:
        if not vis[u]:
            dfs(u,adj,vis)

# Example:
# vis=[0]*4; dfs(0,[[1,2],[0,3],[0,3],[1,2]],vis)
# print(vis) → [1,1,1,1]

def dijkstra(start, graph):
    """Shortest path with positive weights. O(E log V)."""
    n=len(graph)
    dist=[INF]*n; dist[start]=0
    pq=[(0,start)]
    while pq:
        d,u=heappop(pq)
        if d!=dist[u]: continue
        for v,w in graph[u]:
            if dist[v]>d+w:
                dist[v]=d+w
                heappush(pq,(dist[v],v))
    return dist

# Example:
# g=[[(1,2),(2,5)],[(0,2),(2,1)],[(0,5),(1,1)]]
# print(dijkstra(0,g)) → [0,2,3]
```

```
def bellman_ford(n, edges, src):
    """
    Shortest path allowing negative edges (no negative cycle).
    edges: list of (u,v,w)
    O(VE)
    """
    dist=[INF]*n
    dist[src]=0
    for _ in range(n-1):
        for u,v,w in edges:
            if dist[u]!=INF and dist[u]+w<dist[v]:
                dist[v]=dist[u]+w
    # detect negative cycle (optional)
    for u,v,w in edges:
        if dist[u]!=INF and dist[u]+w<dist[v]:
            return None  # negative cycle detected
    return dist

# Example:
# edges=[(0,1,1),(1,2,-1),(2,3,-1),(3,0,2)]
# print(bellman_ford(4,edges,0)) → [0,1,0,-1]

def topo_sort(n, adj):
    """Topological order (DAG). O(V+E)."""
    indeg=[0]*n
    for u in range(n):
        for v in adj[u]: indeg[v]+=1
    q=deque([i for i in range(n) if indeg[i]==0])
    topo=[]
    while q:
        u=q.popleft(); topo.append(u)
        for v in adj[u]:
            indeg[v]-=1
            if indeg[v]==0: q.append(v)
    return topo

# Example:
# adj=[[1,2],[],[1],[]]
# print(topo_sort(4,adj)) → [0,2,1,3] (one valid topo order)
```

```
# ======================================================
# ⚔️ DISJOINT SET UNION (DSU)
# ======================================================
class DSU:
    """Union-Find. O(α(N))."""
    def __init__(self,n):
        self.p=list(range(n))
        self.sz=[1]*n
    def find(self,x):
        if self.p[x]!=x:
            self.p[x]=self.find(self.p[x])
        return self.p[x]
    def unite(self,a,b):
        a=self.find(a); b=self.find(b)
        if a==b: return False
        if self.sz[a]<self.sz[b]: a,b=b,a
        self.p[b]=a; self.sz[a]+=self.sz[b]
        return True

# Example:
# d=DSU(4); d.unite(0,1); print(d.find(1)==d.find(0)) → True
```

```
# ========================================================
# 🌳 SEGMENT TREE
# ========================================================
class SegTree:
    """Range query (sum/min/max). O(log N)."""
    def __init__(self,arr,func=sum,default=0):
        self.n=len(arr)
        self.func=func
        self.default=default
        self.size=1
        while self.size<self.n: self.size<<=1
        self.data=[default]*(2*self.size)
        for i in range(self.n): self.data[self.size+i]=arr[i]
        for i in range(self.size-1,0,-1):
            self.data[i]=func([self.data[i*2],self.data[i*2+1]])
    def update(self,idx,val):
        idx+=self.size; self.data[idx]=val
        while idx>1:
            idx>>=1
            self.data[idx]=self.func([self.data[idx*2],self.data[idx*2+1]])
    def query(self,l,r):
        l+=self.size; r+=self.size
        resl,resr=self.default,self.default
        while l<=r:
            if l&1:
                resl=self.func([resl,self.data[l]]); l+=1
            if not (r&1):
                resr=self.func([self.data[r],resr]); r-=1
            l>>=1; r>>=1
        return self.func([resl,resr])

# Example:
# seg=SegTree([1,2,3,4])
# print(seg.query(1,3)) → 9
# seg.update(2,10); print(seg.query(0,2)) → 13
```

```
# ============================================================
# 🔤 STRING ALGORITHMS
# ============================================================
def prefix_function(s):
    """KMP preprocessing. Find longest prefix-suffix for each pos."""
    n=len(s); pi=[0]*n
    for i in range(1,n):
        j=pi[i-1]
        while j and s[i]!=s[j]: j=pi[j-1]
        if s[i]==s[j]: j+=1
        pi[i]=j
    return pi

# Example:
# print(prefix_function("ababc")) → [0,0,1,2,0]

def z_function(s):
    """Z[i] = length of longest prefix starting from i. O(N)."""
    n=len(s); z=[0]*n; l=r=0
    for i in range(1,n):
        if i<=r: z[i]=min(r-i+1,z[i-l])
        while i+z[i]<n and s[z[i]]==s[i+z[i]]: z[i]+=1
        if i+z[i]-1>r: l,r=i,i+z[i]-1
    return z

# Example:
# print(z_function("aaaa")) → [0,3,2,1]

# ============================================================
# 🔎 BINARY SEARCH
# ============================================================
def binary_search_first(lo,hi,cond):
    """Find smallest x s.t cond(x)=True."""
    ans=hi+1
    while lo<=hi:
        mid=(lo+hi)//2
        if cond(mid):
            ans=mid; hi=mid-1
        else: lo=mid+1
    return ans

# Example:
# print(binary_search_first(0,10,lambda x:x>=5)) → 5
```

```
# ============================================================
# 🧠 SOLVE
# ============================================================
def solve():
    """Example usage: Compute shortest path with negative edges."""
    n,m=4,5
    edges=[(0,1,4),(0,2,5),(1,2,-3),(2,3,4),(3,1,-10)]
    res=bellman_ford(n,edges,0)
    if res is None:
        print("Negative cycle detected")
    else:
        print("Shortest distances:",res)

# ============================================================
# 🚀 MAIN
# ============================================================
if __name__=="__main__":
    T=1
    # T=inpi()
    for _ in range(T):
        solve()
```

```
/* ---------- BITWISE OPERATIONS TEMPLATE ----------


#include <bits/stdc++.h>
using namespace std;

/* ---------- BASIC OPERATIONS ----------
   a & b  → bitwise AND
   a | b  → bitwise OR
   a ^ b  → bitwise XOR
   ~a     → bitwise NOT (flips bits)
   a << k → left shift (multiply by 2^k)
   a >> k → right shift (divide by 2^k)
*/
// Example values
void basicExamples() {
    int a = 13, b = 11;
    cout << "a & b = " << (a & b) << "\n";  // 9
    cout << "a | b = " << (a | b) << "\n";  // 15
    cout << "a ^ b = " << (a ^ b) << "\n";  // 6
    cout << "~a = " << (~a) << "\n";        // -14 (two's complement)
    cout << "a << 2 = " << (a << 2) << "\n"; // 52
    cout << "a >> 1 = " << (a >> 1) << "\n"; // 6
}
/* ---------- BITWISE UTILITIES ---------- */

bool isBitSet(int x, int k) {
    return (x >> k) & 1;         // Check if k-th bit is 1
}

int setBit(int x, int k) {
    return x | (1 << k);         // Set k-th bit
}

int clearBit(int x, int k) {
    return x & ~(1 << k);        // Clear k-th bit
}

int toggleBit(int x, int k) {
    return x ^ (1 << k);         // Flip k-th bit
}

int countBits(int x) {
    return __builtin_popcount(x); // Built-in function for counting 1s
}
bool isPowerOfTwo(int x) {
    return x > 0 && (x & (x - 1)) == 0;
}
```

```
/* ---------------- ADVANCED TRICKS ---------------- */

int rightmostSetBit(int x) {
    return x & -x;  // isolate lowest set bit
}

int turnOffRightmostBit(int x) {
    return x & (x - 1); // remove lowest set bit
}

/* ---------------- SUBSET ENUMERATION ----------------
   Commonly used in bitmask DP problems.
*/
void iterateSubsets(int n) {
    for (int mask = 0; mask < (1 << n); mask++) {
        cout << "Subset: ";
        for (int i = 0; i < n; i++) {
            if (mask & (1 << i))
                cout << i << " ";
        }
        cout << "\n";
    }
}

/* ---------------- XOR USE CASES ----------------
   1. Find unique element when every other element appears twice.
   2. Swap two numbers without temp variable.
*/
int findUnique(const vector<int> &a) {
    int x = 0;
    for (int v : a) x ^= v;
    return x;
}

void swapUsingXor(int &a, int &b) {
    if (a != b) {
        a ^= b;
        b ^= a;
        a ^= b;
    }
}
```

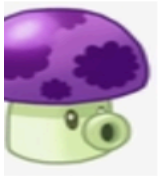
```
/* ---------------- DEMO ---------------- */
int main() {
    basicExamples();

    int x = 42; // 101010 (binary)
    cout << "Bit 1 set? " << isBitSet(x, 1) << "\n";
    cout << "Set bit 0: " << setBit(x, 0) << "\n";
    cout << "Clear bit 1: " << clearBit(x, 1) << "\n";
    cout << "Toggle bit 3: " << toggleBit(x, 3) << "\n";
    cout << "Count bits: " << countBits(x) << "\n";
    cout << "Rightmost set bit: " << rightmostSetBit(x) << "\n";
    cout << "After turning off rightmost bit: " << turnOffRightmostBit(x) << "\n";

    vector<int> nums = {5, 3, 5, 7, 3};
    cout << "Unique element: " << findUnique(nums) << "\n";

    int a = 10, b = 5;
    swapUsingXor(a, b);
    cout << "Swapped (a, b): " << a << " " << b << "\n";

    cout << "\nAll subsets of n=3:\n";
    iterateSubsets(3);
}
```

7 months ago, hide # | ☆

$1 \oplus 2 \oplus 3 \oplus \cdots \oplus (4n-1) = 0$ for all $n \in \mathbb{Z}^+$.

⟶ Reply

56



ew_Pew.

7 months ago, hide # ^ | ☆

A corollary:

If $n = 0 \ mod \ 4$ then XOR from 1 to n would be n

If $n = 1 \ mod \ 4$ then XOR from 1 to n would be 1

If $n = 2 \ mod \ 4$ then XOR from 1 to n would be $n + 1$

If $n = 3 \ mod \ 4$ then XOR from 1 to n would be 0

⟶ Reply

```
# ---------- Expected Value & Probability Template ----------
# Author: Krish
# Optimized for ICPC (PyPy3)
# Covers: modular arithmetic, rational probability, expectation DP
# -----------------------------------------------------------------

import sys, math
from fractions import Fraction
sys.setrecursionlimit(3 * 10**5)
input = sys.stdin.readline

MOD = 10**9 + 7
INF = float('inf')

# ---------- MODULAR MATH UTILS ----------
def modadd(a, b, m=MOD): return (a + b) % m
def modsub(a, b, m=MOD): return (a - b + m) % m
def modmul(a, b, m=MOD): return (a * b) % m

def modpow(a, b, m=MOD):
    """Fast modular exponentiation (a^b mod m)"""
    res = 1
    a %= m
    while b:
        if b & 1:
            res = (res * a) % m
        a = (a * a) % m
        b >>= 1
    return res

def modinv(a, m=MOD):
    """Modular inverse using Fermat's theorem (mod must be prime)."""
    return modpow(a, m - 2, m)

# Example:
# p = modmul(1, modinv(6)) → represents 1/6 mod 1e9+7

# ======================================================
# 🎲 Expected Value under Modulo (Discrete Random Events)
# ======================================================
def expected_mod(values, probs):
    """
    Expected value mod MOD.
    values: list of outcomes (int)
    probs: list of probabilities (each as modular fraction mod MOD)
    """
    assert len(values) == len(probs)
    E = 0
```

```
    for v, p in zip(values, probs):
        E = modadd(E, modmul(v, p))
    return E

# Example:
# Toss a biased coin: P(H)=1/3, P(T)=2/3, X=1 for head, 0 for tail
# values = [1, 0]
# probs = [modmul(1, modinv(3)), modmul(2, modinv(3))]
# print(expected_mod(values, probs)) → modular expectation = 333333336

# ======================================================
# 🎲 Expected Value with Fractions (Exact arithmetic)
# ======================================================
def expected_fraction(values, probs):
    """
    Expected value using Python Fractions for exact precision.
    values: [v1, v2, ...], probs: [p1, p2, ...] (Fractions)
    """
    E = Fraction(0, 1)
    for v, p in zip(values, probs):
        E += Fraction(v) * Fraction(p)
    return E

# Example:
# Expected outcome of dice roll (1..6)
# values = [1,2,3,4,5,6]
# probs = [Fraction(1,6)]*6
# print(expected_fraction(values, probs)) → 7/2

# ======================================================
# 🧩 DP on Expected Value (Common ICPC Pattern)
# ======================================================
"""
E[i] often represents expected steps/costs from state i.
Recurrence: E[i] = 1 + Σ(p(j) * E[next_state])
You solve by substitution or reverse iteration if transitions are acyclic.
"""

def expected_steps_dice(n):
    """
    Example: Expected rolls to reach sum >= n with dice rolls 1..6
    E[i] = 1 + (1/6)*Σ(E[min(i+k, n)]) for k=1..6
    """
    E = [0]*(n+7)
    for i in range(n-1, -1, -1):
        s = 0
        for k in range(1, 7):
            s += E[min(i+k, n)]
```

```
        E[i] = 1 + s/6
    return E[0]

# Example:
# print(expected_steps_dice(10)) → Expected rolls to reach 10

# ========================================================
# 📊 Conditional Probability Template
# ========================================================
def conditional_probability(p_a, p_b_given_a):
    """P(B) = P(B|A) * P(A)"""
    return p_b_given_a * p_a

def bayes_theorem(p_a, p_b_given_a, p_b):
    """P(A|B) = (P(B|A)*P(A)) / P(B)"""
    return (p_b_given_a * p_a) / p_b

# Example:
# P(A)=0.3, P(B|A)=0.7, P(B)=0.5
# print(bayes_theorem(0.3, 0.7, 0.5)) → 0.42

# ========================================================
# 🎯 Linearity of Expectation (Trick for ICPC)
# ========================================================
def linearity_of_expectation(expected_list):
    """Sum of expected values = expected sum (no independence needed)."""
    return sum(expected_list)

# Example:
# Two dice: E(X1)=3.5, E(X2)=3.5 → total E(X1+X2)=7
# print(linearity_of_expectation([3.5, 3.5])) → 7.0

# ========================================================
# 🧱 Expected Value for Continuous Distribution (approximation)
# ========================================================
def expected_continuous(func, a, b, steps=10000):
    """Numerical integration ∫ f(x)*x dx over [a,b]"""
    dx = (b - a) / steps
    s = 0
    for i in range(steps):
        x = a + i * dx
        s += func(x) * x * dx
    return s

# Example:
# Expectation of X when f(x)=2x for x in [0,1] (triangular distribution)
# print(round(expected_continuous(lambda x: 2*x, 0, 1), 3)) → 0.667
```

```
# ==========================================================
# 🧠 SOLVE (Sample usage combining everything)
# ==========================================================
def solve():
    # Example 1: Modular expected value (biased coin)
    values = [1, 0]
    probs = [modmul(1, modinv(3)), modmul(2, modinv(3))]
    print("Expected value mod:", expected_mod(values, probs))

    # Example 2: Fraction expected value (fair dice)
    vals = [1,2,3,4,5,6]
    probs = [Fraction(1,6)]*6
    print("Expected value fraction:", expected_fraction(vals, probs))

    # Example 3: Expected steps to reach n with dice rolls
    print("Expected rolls to reach 10:", round(expected_steps_dice(10), 3))

    # Example 4: Bayes theorem
    print("Bayes theorem example:", round(bayes_theorem(0.3, 0.7, 0.5), 3))

if __name__ == "__main__":
    solve()
```

🧱 C++ STL (Standard Template Library) – Complete ICPC Reference

Author: Krish
Prepared For: ICPC / Competitive Programming
Language: C++17 / C++20
Content: Detailed STL Documentation + Compact STL Cheat Sheet

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📘 Part 1 – Detailed STL Documentation

🧩 Overview

Category	Description
Containers	Store and organize data efficiently
Iterators	Provide a way to traverse container elements
Algorithms	Reusable functions for sorting, searching, counting, etc.
Utilities	Helper data structures like pairs, tuples, and bitsets
Strings	For handling text
Math & Numeric	Basic numerical operations and accumulation

1️⃣ Containers

🟦 `vector<T>`

Description:
Dynamic array with fast random access and amortized O(1) insertion at the end.
Header: `<vector>`

Common Operations:

Function	Description	Complexity
`push_back(x)`	Add to end	O(1) amortized

`pop_back()`	Remove last element	O(1)
`insert(pos, val)`	Insert at position	O(n)
`erase(pos)`	Remove at position	O(n)
`sort(all(v))`	Sort ascending	O(n log n)

Example:

```
vector<int> v = {1, 2, 3};
v.push_back(4);
v.erase(v.begin()+1);
sort(v.begin(), v.end());
for (int x : v) cout << x << " ";
```

🟩 `array<T, N>`

Description:
Fixed-size array with STL functions.
Header: `<array>`

```
array<int, 5> a = {1, 2, 3, 4, 5};
cout << a.size() << " " << a.front() << " " << a.back();
```

🟧 `deque<T>`

Description:
Double-ended queue. Fast insertion/removal from both ends.
Header: `<deque>`

```
deque<int> dq = {1, 2, 3};
dq.push_front(0);
dq.push_back(4);
dq.pop_front();
dq.pop_back();
```

🟨 list<T>

Description:
Doubly linked list allowing O(1) insert/erase given iterator.
Header: `<list>`

```
list<int> l = {1, 2, 3};
l.push_front(0);
l.push_back(4);
l.remove(2);
l.reverse();
```

🟪 stack<T>

Description:
LIFO (Last In First Out).
Header: `<stack>`

```
stack<int> s;
s.push(10);
s.push(20);
cout << s.top(); // 20
s.pop();
```

🟫 queue<T>

Description:
FIFO (First In First Out).
Header: `<queue>`

```
queue<int> q;
q.push(10);
q.push(20);
cout << q.front(); // 10
q.pop();
```

🟥 priority_queue<T>

Description:
Heap-based container.
Header: `<queue>`

```
priority_queue<int> pq; // Max-heap
pq.push(10);
pq.push(5);
cout << pq.top(); // 10

priority_queue<int, vector<int>, greater<int>> minpq; // Min-heap
```

🟩 set<T>

Description:
Stores **unique, sorted** elements.
Header: `<set>`

Operation	Time
insert/erase/find	O(log n)

```
set<int> s = {1, 3, 2};
s.insert(4);
s.erase(2);
for (int x : s) cout << x << " ";
```

🟧 multiset<T>

Description:
Sorted, allows duplicates.
Header: `<set>`

```
multiset<int> ms = {1, 1, 2, 3};
ms.insert(2);
ms.erase(ms.find(1)); // removes one instance
```

🟦 unordered_set<T>

Description:
Hash-based set. Fast O(1) average operations.
Header: `<unordered_set>`

```
unordered_set<int> us = {1, 2, 3};
us.insert(4);
us.erase(2);
```

🟥 map<K,V>

Description:
Sorted key-value pairs.
Header: `<map>`

```
map<string, int> mp;
mp["apple"] = 2;
mp["banana"] = 5;
for (auto &[k, v] : mp)
    cout << k << " " << v << "\n";
```

🟩 unordered_map<K,V>

Description:
Hash-based key-value pairs.
Header: `<unordered_map>`

```
unordered_map<string, int> ump;
ump["one"] = 1;
ump["two"] = 2;
ump.erase("two");
```

🟪 pair<T1, T2>

Description:
Stores two related values.
Header: `<utility>`

```
pair<int, string> p = {1, "apple"};
cout << p.first << " " << p.second;
```

🟨 `tuple<T1, T2, T3,...>`

Description:
Stores multiple related values.
Header: `<tuple>`

```
tuple<int, string, double> t = {1, "hi", 3.5};
cout << get<0>(t);
```

🟩 `bitset<N>`

Description:
Fixed-size binary array.
Header: `<bitset>`

```
bitset<8> b("1011");
b.count(); // number of 1s
b.flip(2);
b[0] = 0;
```

🟦 `string`

Description:
Dynamic character array.
Header: `<string>`

```
string s = "Krish";
s += " rocks";
cout << s.substr(1, 3);
reverse(s.begin(), s.end());
```

2️⃣ Iterators & Algorithms (`<algorithm>`)

Function	Description	Complexity
`sort(a,b)`	Sort ascending	O(n log n)

`reverse(a,b)`	Reverse range	O(n)
`count(a,b,x)`	Count occurrences	O(n)
`find(a,b,x)`	Iterator to element	O(n)
`min_element(a,b)`	Smallest element	O(n)
`max_element(a,b)`	Largest element	O(n)
`accumulate(a,b,init)`	Sum of range	O(n)
`unique(a,b)`	Remove duplicates	O(n)
`next_permutation(a,b)`	Next lexicographic order	O(n)
`lower_bound(a,b,x)`	First ≥ x	O(log n)
`upper_bound(a,b,x)`	First > x	O(log n)
`binary_search(a,b,x)`	Check existence	O(log n)

---

3 Numeric Utilities (`<numeric>`)

```
accumulate(v.begin(), v.end(), 0);              // sum
inner_product(a.begin(), a.end(), b.begin(), 0); // dot product
partial_sum(v.begin(), v.end(), pref.begin());
iota(v.begin(), v.end(), 1);                    // fill v = [1, 2, ...]
```

---

4 String Functions (`<string>`)

Function	Description
`stoi(s)`	Convert string to int
`stoll(s)`	Convert string to long long
`to_string(x)`	Convert number to string

`substr(l, len)`	Get substring
`find(sub)`	Find substring position
`erase(pos, len)`	Remove characters
`replace(l, len, s2)`	Replace substring

5️⃣ Math Utilities (`<cmath>`)

Function	Description
`abs(x)`	Absolute value
`pow(x,y)`	Power
`sqrt(x)`	Square root
`floor(x), ceil(x)`	Rounding
`log(x), log2(x), log10(x)`	Logarithms
`sin(x), cos(x), tan(x)`	Trig functions
`hypot(x,y)`	$\sqrt{(x^2+y^2)}$ safely
`__gcd(a,b)`	Built-in gcd

6️⃣ Example Program

```
#include <bits/stdc++.h>
using namespace std;

int main() {
    ios::sync_with_stdio(false);
    cin.tie(nullptr);

    vector<int> v = {3, 1, 4, 1, 5, 9};
    sort(v.begin(), v.end());
    v.erase(unique(v.begin(), v.end()), v.end());
```

```
    cout << "Sorted Unique Elements: ";
    for (int x : v) cout << x << " ";
    cout << "\nMax: " << *max_element(v.begin(), v.end());
    cout << "\nSum: " << accumulate(v.begin(), v.end(), 0);
}
```

For checking how many coprimes pairs are there

```
n = int(input())
a = list(map(int, input().split()))
MAX = 10**6

cnt = [0]*(MAX+1)
for x in a:
    cnt[x] += 1

f = [0]*(MAX+1)
for i in range(1, MAX+1):
    for j in range(i, MAX+1, i):
        f[i] += cnt[j]

mu = [1]*(MAX+1)
isprime = [True]*(MAX+1)
for i in range(2, MAX+1):
    if isprime[i]:
        for j in range(i, MAX+1, i):
            isprime[j] = False
            mu[j] *= -1
        j = i*i
        while j <= MAX:
            mu[j] = 0
            j += i*i

ans = 0
for i in range(1, MAX+1):
    if f[i] >= 2:
        ans += mu[i] * f[i]*(f[i]-1)//2
print(ans)
```

4️⃣ Solve ax + by = c (Extended Euclid)

```
def egcd(a,b):
    if b==0:return a,1,0
    g,x1,y1=egcd(b,a%b)
    return g,y1,x1-(a//b)*y1
def solve(a,b,c):
    g,x,y=egcd(a,b)
    if c%g:return None
    return x*(c//g),y*(c//g)
```

7️⃣ Count Coprime Pairs in Range

```
def phi_sieve(n):
    phi=list(range(n+1))
    for i in range(2,n+1):
        if phi[i]==i:
            for j in range(i,n+1,i):
                phi[j]-=phi[j]//i
    for i in range(2,n+1):phi[i]+=phi[i-1]
    return phi
```

`phi_sieve(N) gives prefix φ → number of coprime pairs ≤ n ≈ φ_prefix[n].`

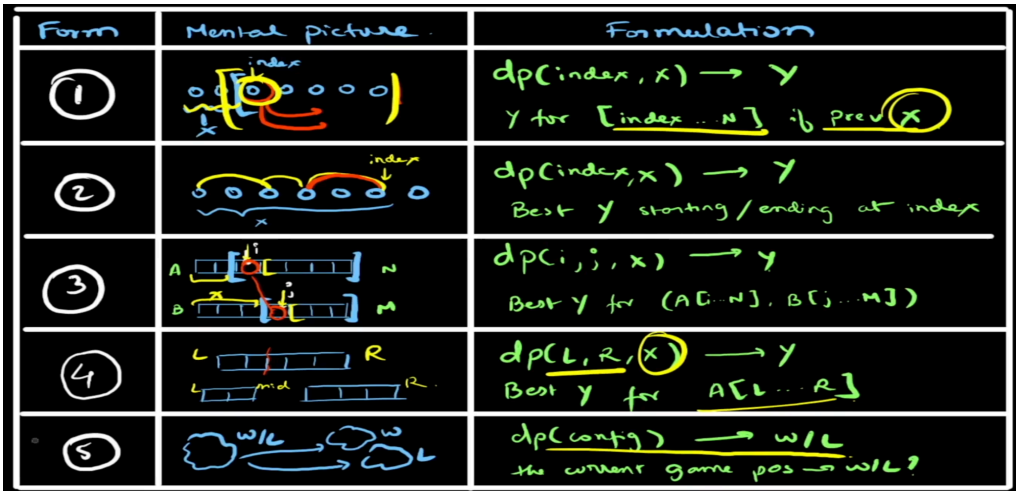
```
def sum_over_divisors(n):

    s=0

    i=1

    while i<=n:

        j=n//(n//i)

        s+=(n//i)*(j-i+1)

        i=j+1

    return s #  here s=summation of n//k for k in range 1 to n
```

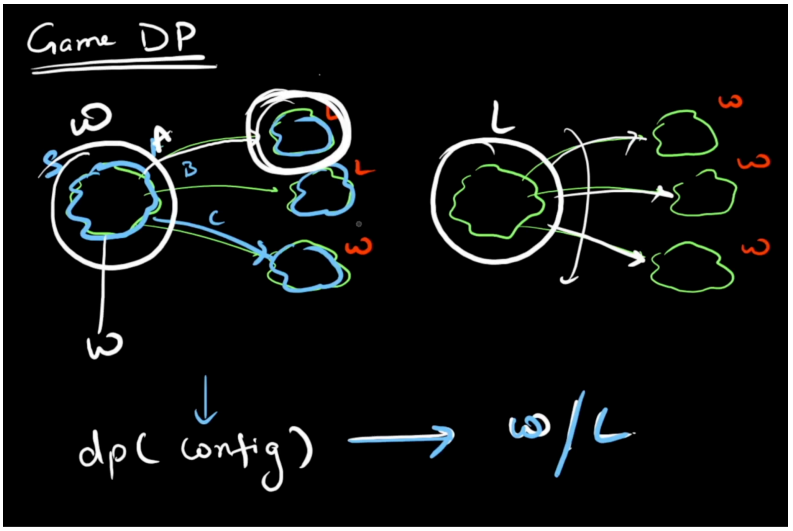
1. If s is a string of just a and b then if 1st and last element are equal then substring ‘ab’==‘ba’
2. If a number has odd divisors it means it is not a power of 2 .
3. If a number is prime it means in array if I want to find minimum effort to make product of an array divisible by number then I should only focus on making one no from number dib by that num.
4. For a length of len if we want to find no of subarrays of at least length k then ans=(len-k+1)*(len-k+2)//2
5. If I want to find two no a and b where a+b=n such that there gcd is maximum is n/2 and n/2 if n is even else n-x,x were x is highest multiple.
6. Always write n-m+1 not -m+1 in code it may give wrong solution.
7. int xor_1_to_n(int n) {
8. if (n % 4 == 0) return n;
9. if (n % 4 == 1) return 1;
10. if (n % 4 == 2) return n + 1;
11. return 0; // when n % 4 == 3
12. }
13. This is xor of n natural numbers.
14. If we want to check if a no is div by 2 do not use math.log(n,2) then int(x)==x it will give wa instead use n&(n-1)==0.

18.Once a Number is Not Divisible by 2^x, It's Never Divisible by Larger Powers Again.
19.a%m=b%m it means (a-b)is divisible by some m a>b.
If we want to find m such that a1%m==a2%m , a4%m==a3%m so on then we have to find gcd for all ai’s.

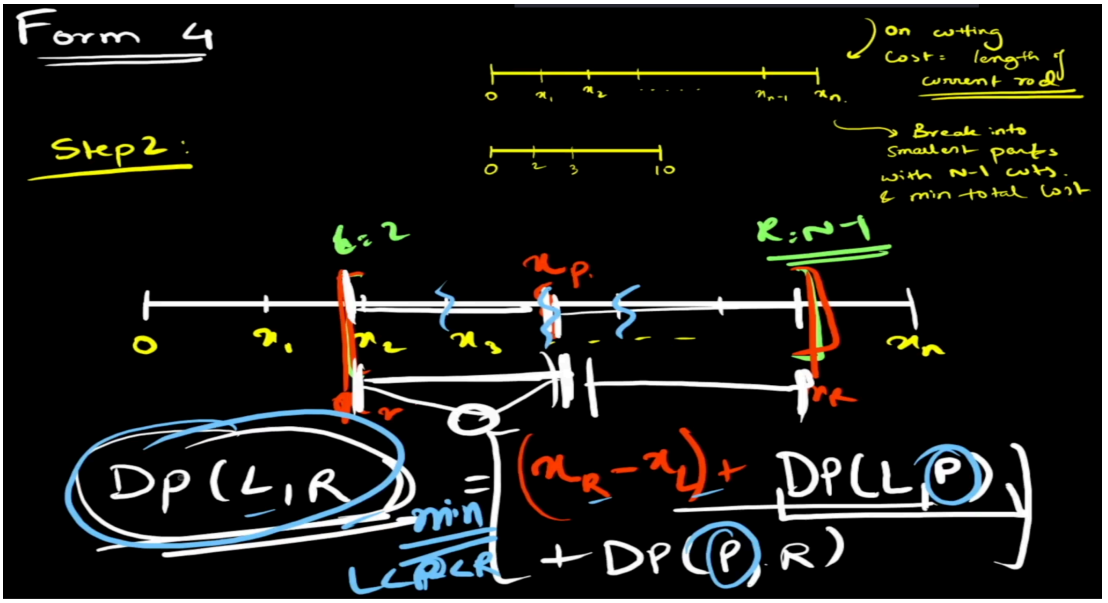
1. When in a question there is matrix then be careful of odd n.
2. Taking a[i] % 2^j is equivalent to: “look at the **last j bits** of a[I]”..

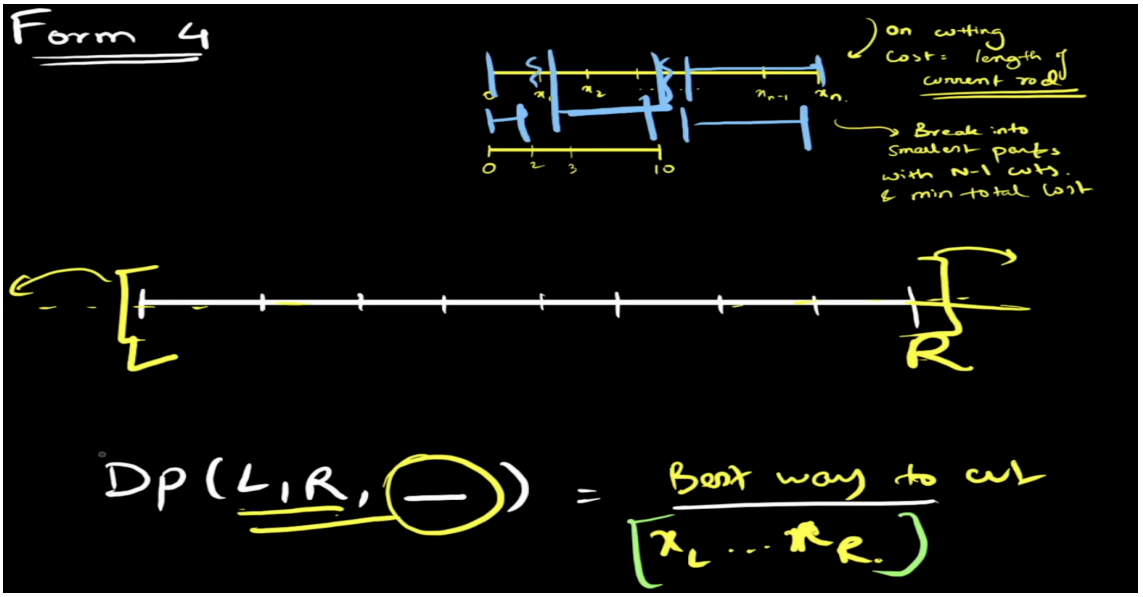
- 22. If I have to find k such that after taking module of every element of array the array should have 2 or 3 or some elements I have to check powers of two .



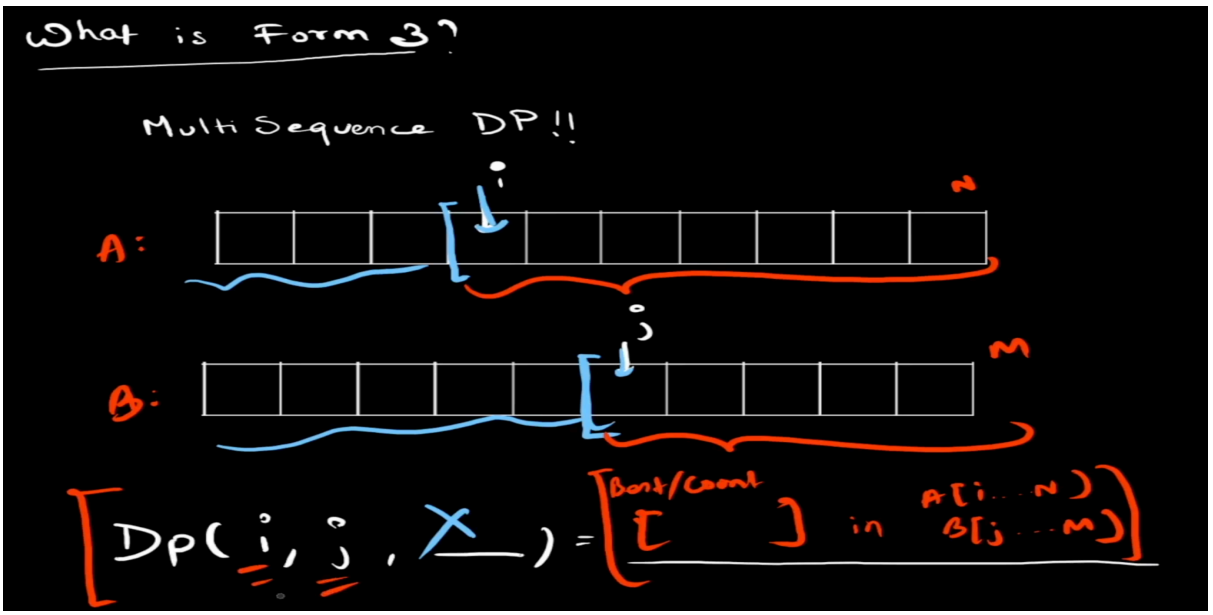



```
    // base case
    if(x==0){
        return 0;
    }
    //cache check
    if(dp[x]!=-1){
        return dp[x];
    }
    // compute
    int ans = 0;
    for(int m=0;(1<<m)<=x;m++){
        if(rec(x-(1<<m))==0){
            ans = 1;
            break;
        }
    }
    // save and return

    return dp[x] = ans;
}
```



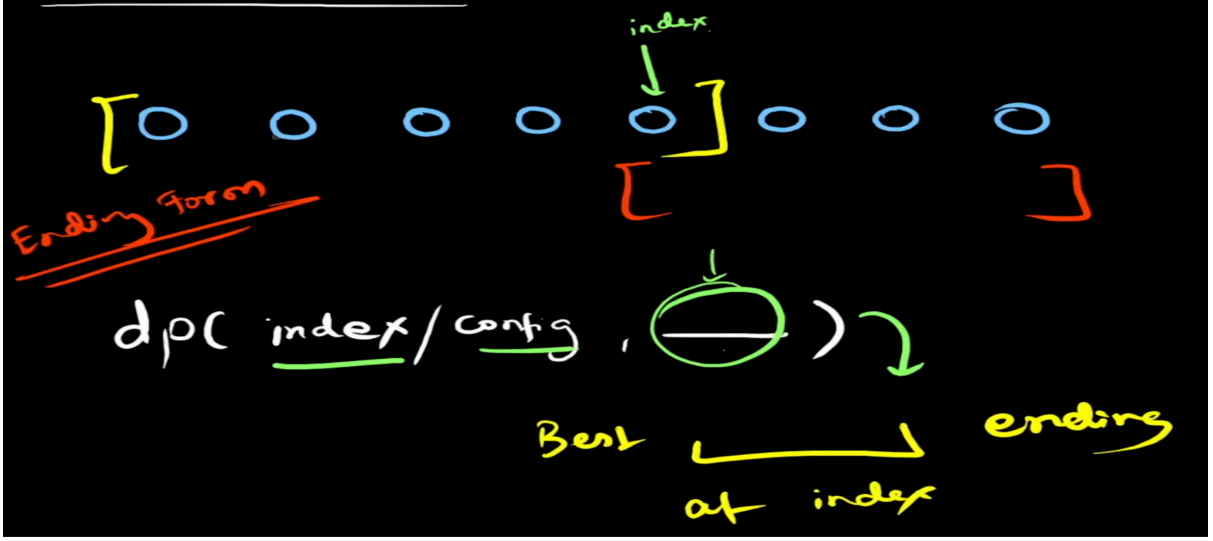



```cpp
int dp[1001][1001];

int rec(int l,int r){
    // pruning

    // basecase
    if(l+1==r){
        return 0;
    }
    // cache check
    if(dp[l][r]!=-1){
        return dp[l][r];
    }
    // compute
    int ans = 1e9;
    for(int p=l+1;p<=r-1;p++){
        ans = min(ans,x[r]-x[l]+rec(l,p)+rec(p,r));
    }
    // save and return
    return dp[l][r] = ans;
}
```

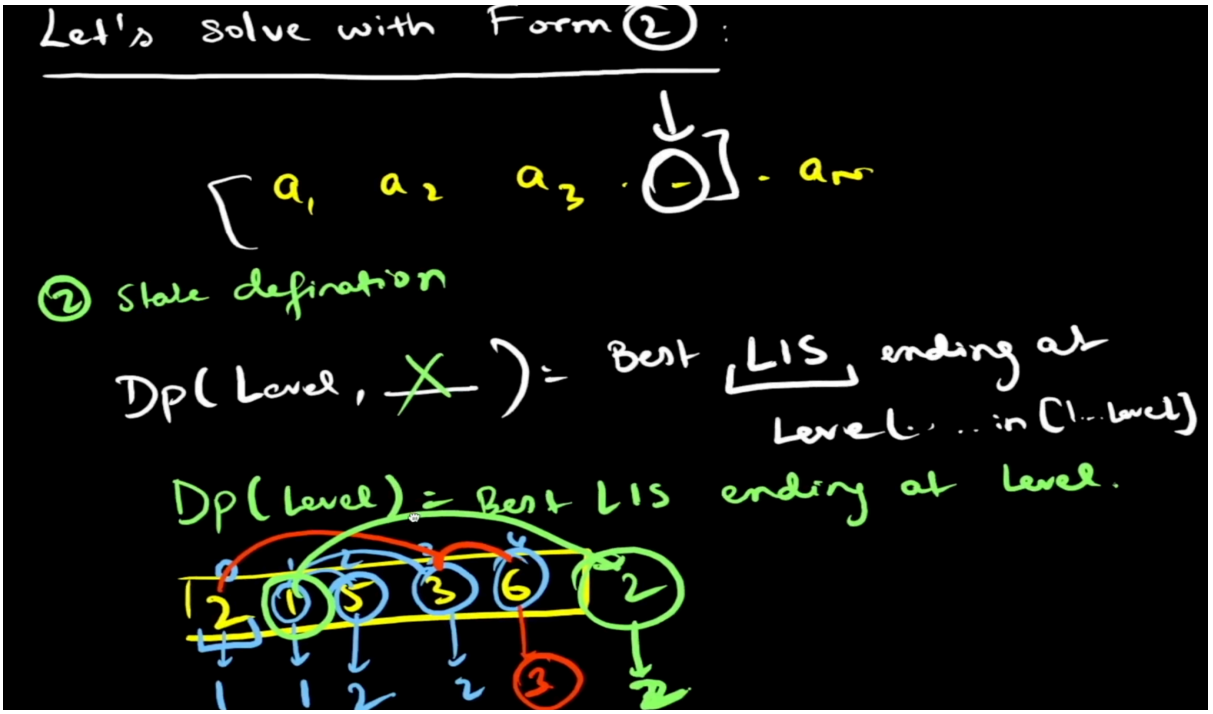


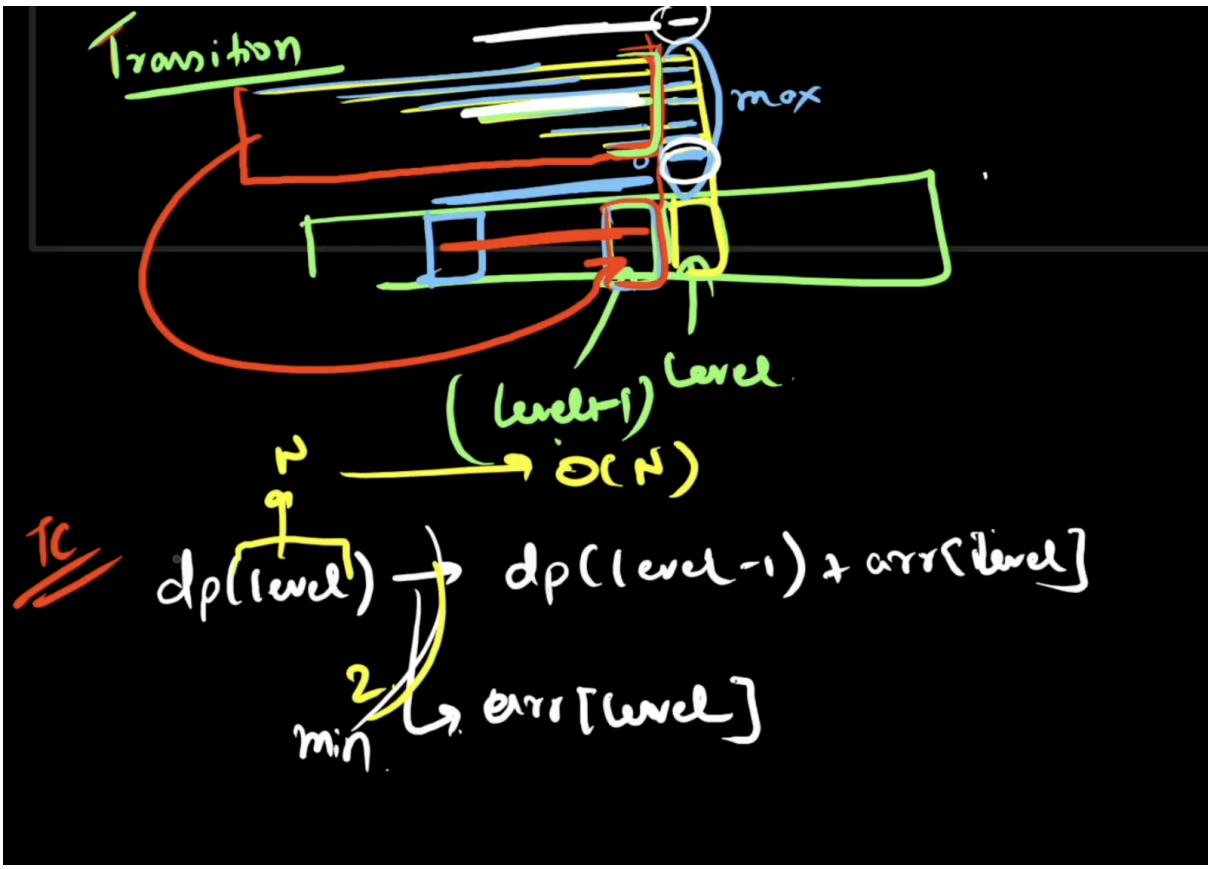

```
// return the LCS of a[i...n-1] and b[j...m-1].. c[k...x-
// pruning

// basecase
if(i>=n||j>=m||k>=x){
    return 0;
}
// save and compute
if(dp[i][j][k]!=-1){
    return dp[i][j][k];
}
// compute
int ans = 0;
ans = max(ans,rec(i+1,j,k));
ans = max(ans,rec(i,j+1,k));
ans = max(ans,rec(i,j,k+1));
if(a[i]==b[j]&&b[j]==c[k]){
    ans = max(ans,1+rec(i+1,j+1,k+1));
}
// save and return
return dp[i][j] = ans;
```

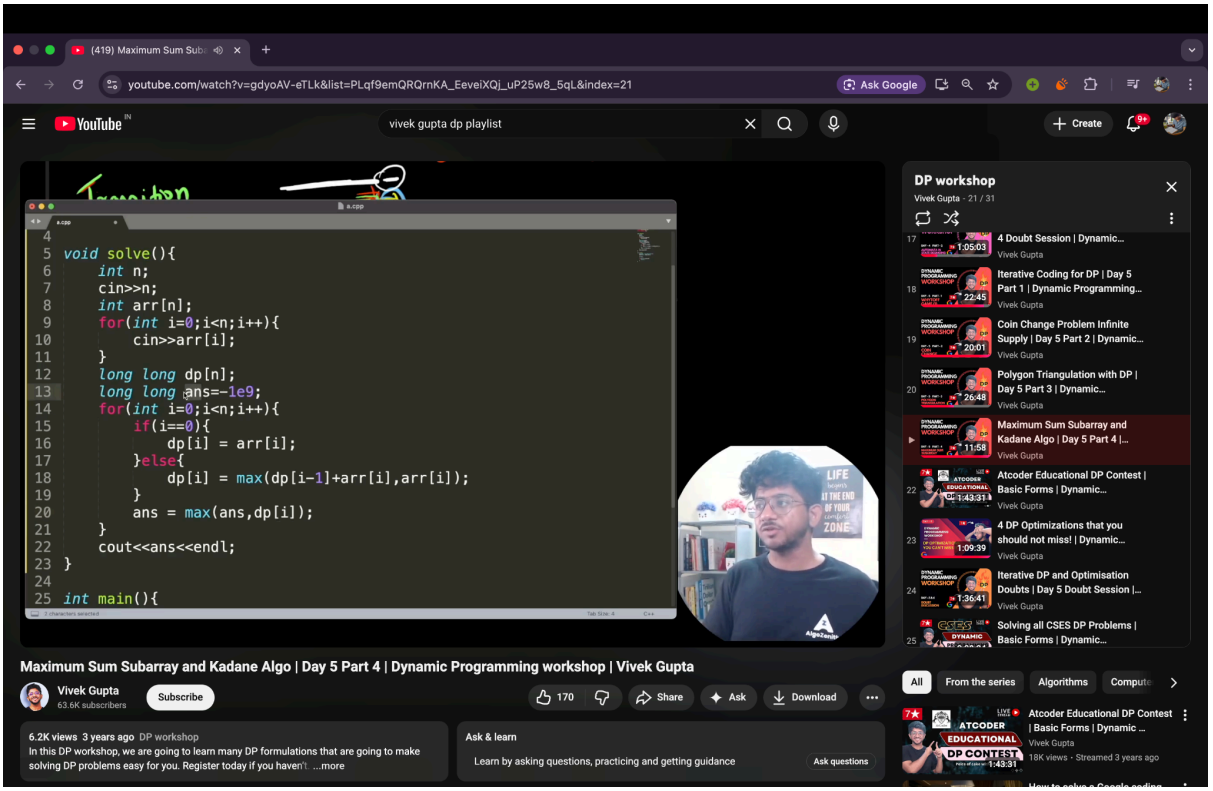
What is Form 2



Let's solve with Form (2):




Transition
max
(level-1)
level
N
O(N)
TC
dp(level)
dp(level-1) + arr[level]
2
min
arr[level]


(419) Maximum Sum Sub
youtube.com/watch?v=gdyoAV-eTLk&list=PLqf9emQRQrnKA_EeveiXQj_uP25w8_5qL&index=21
Ask Google
YouTube
vivek gupta dp playlist
Create
Transition
a.cpp
void solve(){
int n;
cin>>n;
int arr[n];
for(int i=0;i<n;i++){
cin>>arr[i];
}
long long dp[n];
long long ans=-1e9;
for(int i=0;i<n;i++){
if(i==0){
dp[i] = arr[i];
}else{
dp[i] = max(dp[i-1]+arr[i],arr[i]);
}
ans = max(ans,dp[i]);
}
cout<<ans<<endl;
}
int main(){
DP workshop
Vivek Gupta - 21 / 31
4 Doubt Session | Dynamic...
Iterative Coding for DP | Day 5 Part 1 | Dynamic Programming...
Coin Change Problem Infinite Supply | Day 5 Part 2 | Dynamic...
Polygon Triangulation with DP | Day 5 Part 3 | Dynamic...
Maximum Sum Subarray and Kadane Algo | Day 5 Part 4 |...
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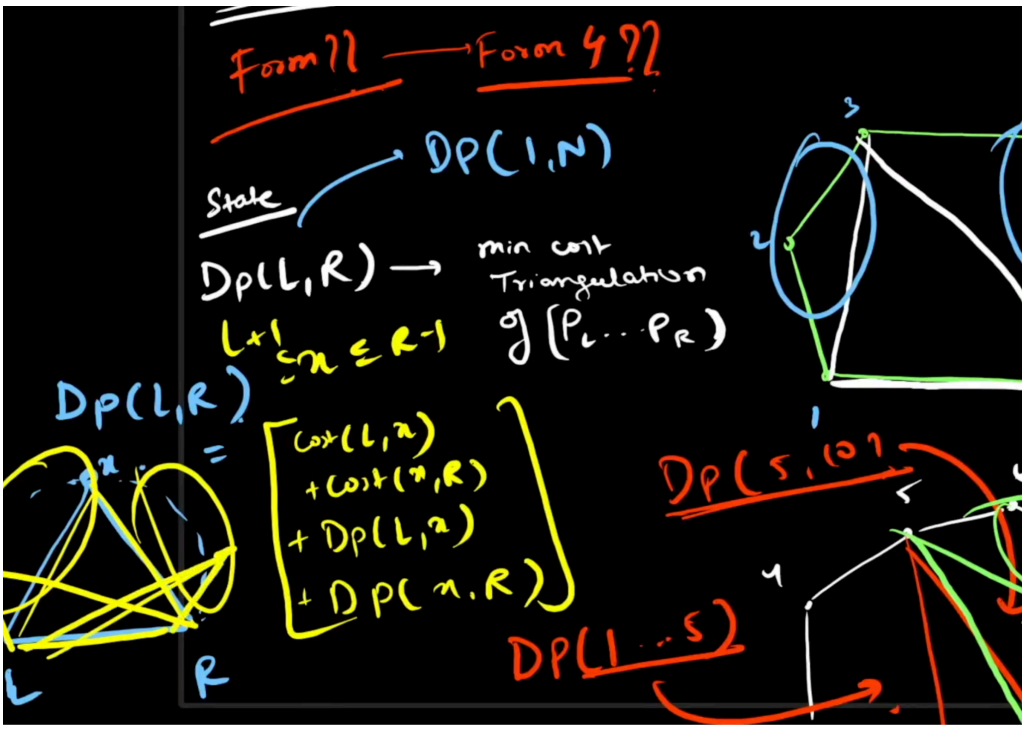
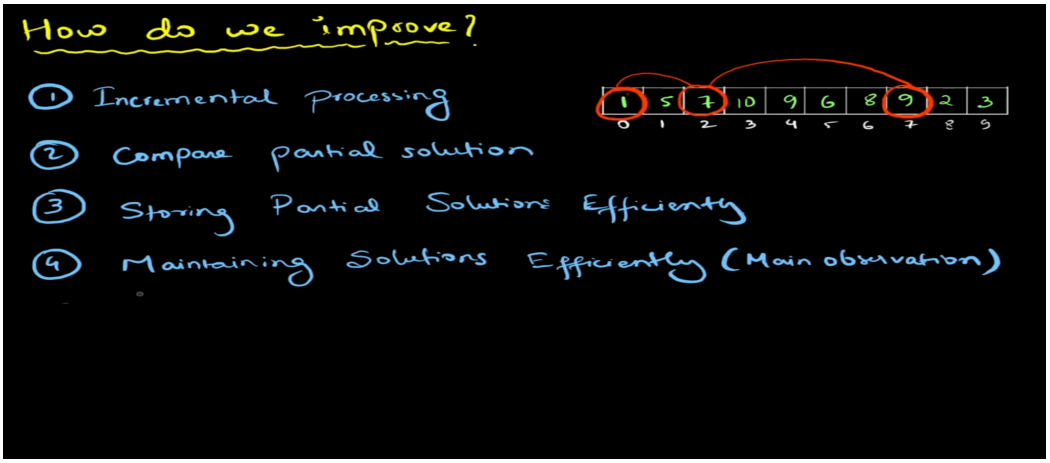
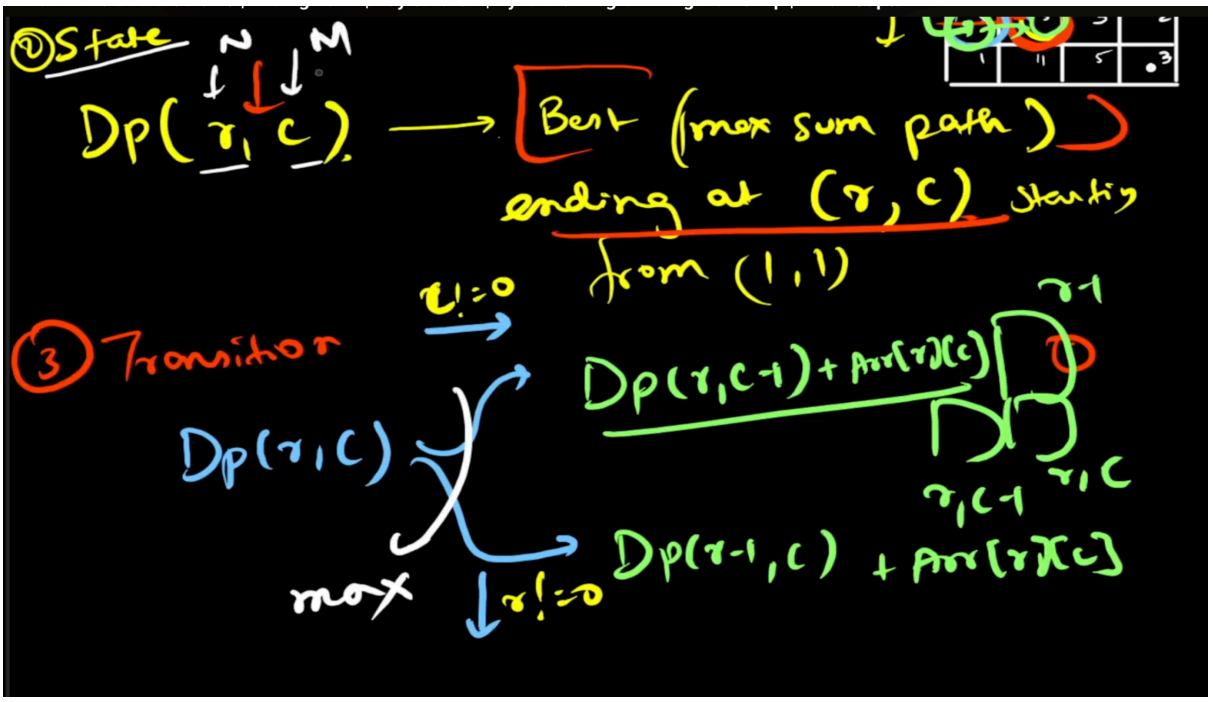
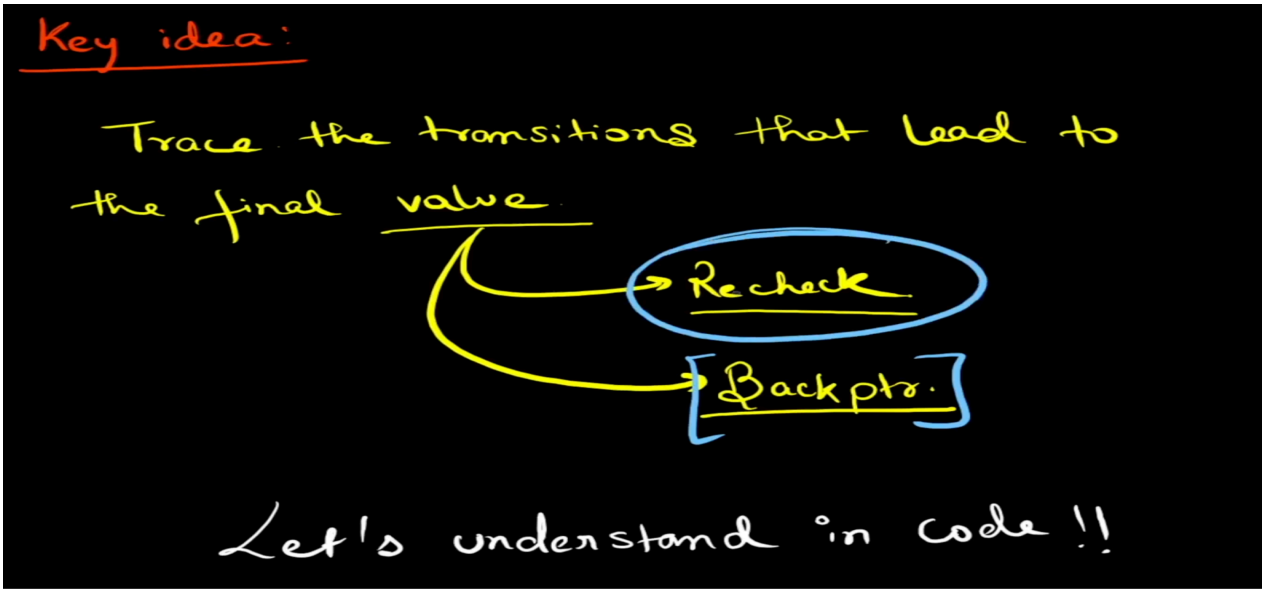




a.cpp

```
26      int ans = 0;
27      if(rec(level+1,left)==1){
28          ans = 1;
29      }else if(rec(rec+1,left-x[level])){
30          ans = 1;
31      }
32      // save and return
33      return dp[level][left]=ans;
34  }
35
36  void printset(int level,int left){
37      // base case
38      if(level==n+1){
39          return;
40      }
41      // find the correct transition
42      if(rec(level+1,left)==1){// don't take
43          printset(level+1,left);
44      }else if(rec(rec+1,left-x[level])){ //take
45          cout<<x[level]<<" ";
46          printset(rec+1,left-x[level])
47      }
48  }
49
```


①State
N
M
Dp(r, c)
Best (max sum path)
ending at (r, c) starting
from (1,1)
③ Transition
c!=0
Dp(r,c-1) + Arr[r][c]
Dp(r,c)
max
r!=0
Dp(r-1,c) + Arr[r][c]
r-1
r,c-1
r,c


Key idea:
Trace the transitions that lead to the final value
Recheck
Backptr.
Let's understand in code !!