ECE5720 Programming Assignment 5, due May 4 Spring 2017

1. General Remarks

This assignment is about hybrid MPI+OpenMP programming. MPI nodes in the cluster are

`en-openmpiXX.ece.cornell.edu,  XX =00, 02, 03, 04, 05, 06, 07`

(note that node `01` is not included).

2. Compile

Your codes should be compiled with `mpicc -fopenmp` (and any other flags that are required).

3. Assignment

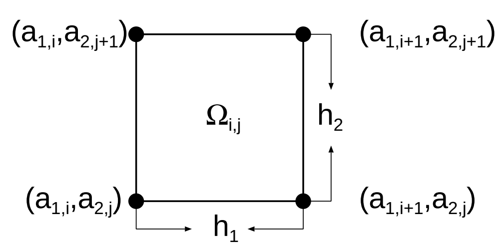
You are asked to approximate a 2D integral on a rectangular domain

$$\Omega = \{(x_1, x_2) \in R^2 \;:\; a_1 \leq x_1 \leq b_1,\; a_2 \leq x_2 \leq b_2\}$$

The theory of integrals tells us that if $f \;:\; \Omega \rightarrow R$ is continuous, then Reimann sums converge to the the integral of f.

In this assignment for Riemann sums we will use the four corner method. Specifically, the domain Ω is subdivided into smaller rectangles

$$\begin{aligned}
\Omega_{i,j} &= \{(x_1, x_2) \in R^2 \;:\; a_{1,i} \leq x_1 \leq b_{1,i},\; a_{2,j} \leq x_2 \leq b_{2,j}\} \\
& \quad a_{1,i+1} = b_{1,i},\; a_{2,j+1} = b_{2,j} \\
& \quad i = 0, ..., n-1, \quad j = 0, ..., m-1.
\end{aligned}$$



and the Riemann sum is

$$\begin{aligned}
F_{n,m} &= \sum_{i=0}^{n-1} \sum_{j=0}^{m-1} (b_{1,i} - a_{1,i})(b_{2,j} - a_{2,j}) c_{i,j} \\
c_{i,j} &= (f(a_{1,i}, a_{2,j}) + f(a_{1,i}, b_{2,j}) + f(b_{1,i}, a_{2,j}) + f(b_{1,i}, b_{2,j}))/4
\end{aligned}$$

Here $c_{i,j}$ is the average of f evaluated at the four corners of $\Omega_{i,j}$.

The four corners formula simplifies if we assume a uniform partition of intervals (a_1, b_1) and (a_2, b_2) with stepsizes h_1 and h_2, respectively. Then we can write

$$\begin{aligned}
F_{n,m} &= \sum_{i=0}^{n-1}\sum_{j=0}^{m-1} \frac{1}{4}(f(a_{1,i}, a_{2,j}) + f(a_{1,i}, a_{2,j+1}) + f(a_{1,i+1}, a_{2,j}) + f(a_{1,i+1}, a_{2,j+1})) \cdot h_1 \cdot h_2 \\
a_{1,i} &= a_1 + i \cdot h_1,\ h_1 = \frac{1}{n}(b_1 - a_1) \\
a_{2,j} &= a_2 + j \cdot h_2,\ h_2 = \frac{1}{m}(b_2 - a_2)
\end{aligned}$$

The specific function that we want to integrate is a hemisphere

$$f(x_1, x_2) = \begin{cases} (r^2 - x_1^2 - y_2^2)^{\frac{1}{2}} & \text{for } x_1^2 + y_2^2 \le r^2 \\ 0 & \text{otherwise} \end{cases}$$

The integral is the volume of the hemisphere and is given by

$$\begin{aligned}
V &= \int_\Omega f(x_1, x_2) dx_1 dx_2 = \frac{1}{3} 2\pi r^3 \\
\Omega &= [-r, r] \times [-r, r]
\end{aligned}$$

4. Format

- You are asked to write a hybrid MPI+OpenMP code for calculating the integral.
- Note that the domain is a square $[-r, r] \times [-r, r]$ but the function is nonzero on a disk of radius r. You may want to exploit this fact in your code.
- Choose $n = m = 2^k,\ k = 8, 12$ and possibly larger k.
- Divide the domain into several subdomains and assign a different MPI process to a different subdomain. Use OpenMP to calculate the integral in the subdomain. Finally merge values from subdomains.
- It is up to you how the work between MPI and OpenMP segments is divided.
- Investigate what a right balance between MPI processes and OpenMP threads is.

1. Your code must be saved in a file named `your_net_id_hw5.c`
2. Your codes must be described in a single file `your_net_id_hw5.pdf`. Please include your NAME and net ID on all pages of your write up. Please DO NOT submit `*.docx` files as I have difficulties with printing them.
3. All files need to be packed with the `tar` or `gzip` facilities. The packed file must have the name `your_net_id_hw_5.suffix` where `suffix` is either `tar` or `zip`.
4. If you relay on resources outside lecture notes but publically available, you need to cite sources in your write-up.