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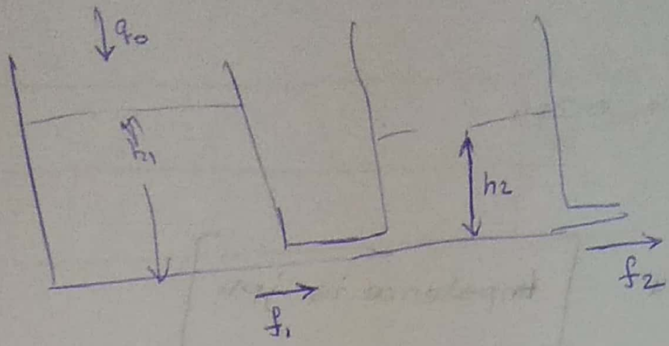
15EE35D028

(3)



(a)

$$f_1 = P_1 \sqrt{g(h_1 - h_2)}$$

$$f_2 = P_2 \sqrt{g h_2}$$

$$q_0 - f_1 = A \frac{dh_1}{dt}$$

$$f_1 - f_2 = A \frac{dh_2}{dt}$$

for equilibrium, $\frac{dh_1}{dt} = \frac{dh_2}{dt} = 0$

$$q_0 = f_1 = f_2$$

$$\Rightarrow \quad P_1 \sqrt{g(h_1 - h_2)} = P_2 \sqrt{g h_2}$$

$$P_1^2 (h_1 - h_2) = P_2^2 h_2$$

$$\frac{h_1}{h_2} = \left(\frac{P_2}{P_1}\right)^2 + 1$$

$$q_0 = P_2 \sqrt{g h_2}$$

$$\boxed{h_2 = \frac{q_0^2}{g P_2^2} \qquad h_1 = \frac{q_0^2 (P_1^2 + P_2^2)}{g P_1^2 P_2^2}}$$

Incremental state eqns

$\bar{h}_1 = h_{10} + h_1 \qquad \bar{h}_2 = h_{20} + h_2$

$\dot{\bar{h}}_1 = \dot{h}_1 \qquad \dot{\bar{h}}_2 = \dot{h}_2 \qquad q = q_0 + q_{10}$

$$A\dot{\bar{h}}_1 = q_0 - \rho_1\sqrt{\bar{h}_1 - \bar{h}_2}$$

$$= q_{10} - \rho_1\sqrt{(h_{10}-h_{20}) + h_1 - h_2} + q_0$$

$$A\dot{\bar{h}}_1 = q_{10} - \rho_1\sqrt{\frac{q_{10}^2}{\rho_1^2} + (h_1 - h_2)} + q_0$$

$$A\dot{\bar{h}}_1 = q_{10} - q_{10}\left(1 + \frac{(h_1-h_2)\rho_1^2}{q_{10}^2}\right)^{+1/2} + q_0$$

$$A\dot{\bar{h}}_1 = \frac{(h_2-h_1)\rho_1^2}{2q_{10}} + q_0$$

$$A\dot{\bar{h}}_2 = \rho_1\sqrt{\bar{h}_1 - \bar{h}_2} - \rho_2\sqrt{\bar{h}_2}$$

$$= q_{10} + \frac{(h_1-h_2)\rho_1^2}{2q_{10}} - \rho_2\sqrt{q_{10}^2\left(\frac{\cancel{\rho_2^2}}{\rho_2^2}\right) + h_2}$$

$$= q_{10} + \frac{(h_1-h_2)\rho_1^2}{2q_{10}} - q_{10}\left(1 + \frac{h_2\rho_2^2}{2q_{10}^2}\right)$$

$$= \cancel{\frac{h_2}{q_0}(\rho_2^2 - \rho_1^2)} - \cancel{\frac{h_1\rho_1^2}{q_0}}$$

$$A\dot{\bar{h}}_2 = \frac{h_1\rho_1^2}{2q_{10}} - \frac{h_2(\rho_1^2 + \rho_2^2)}{2q_{10}}$$

$$X = \begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \end{bmatrix} = \begin{bmatrix} \frac{-\rho_1^2}{2Aq_{10}} & \frac{\rho_1^2}{2Aq_{10}} \\ \frac{\rho_1^2}{2Aq_{10}} & -\frac{(\rho_1^2+\rho_2^2)}{2Aq_{10}} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} q_0$$

(c)

$$C = \frac{\text{change in liquid stored}}{\text{change in head}}$$

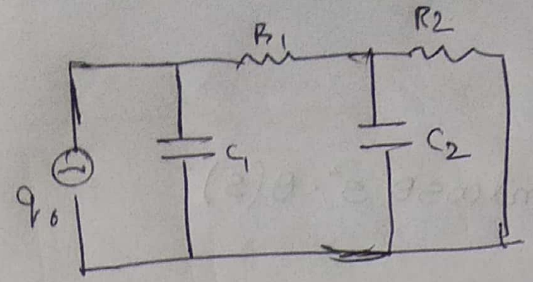
$$R = \frac{\text{change in level difference}}{\text{change in flow rate}}$$

$$C_1 = \frac{(q_0 - f_1)dt}{dh_1}$$

$$C_2 = \frac{(f_1 - f_2)dt}{dh_2}$$

$$R_1 = \frac{2h_1}{q_0}$$

$$R_2 = \frac{2h_2}{q_0}$$



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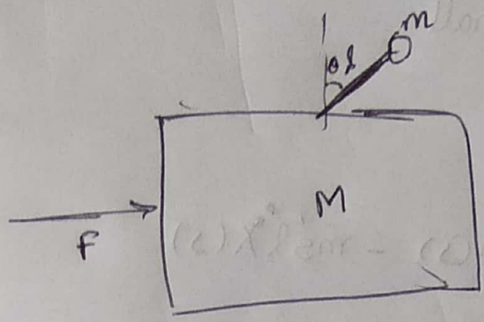
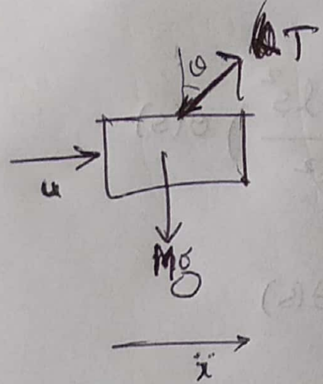
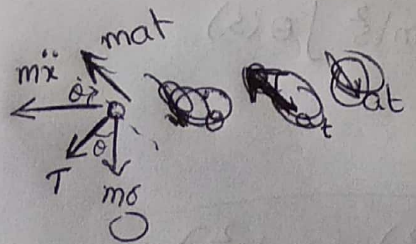




$$T\cos\theta = Mg$$

$$\boxed{u + T\sin\theta = M\ddot{x}} \quad — (1)$$



$$\cancel{T + mg\cos\theta + m\ddot{x}\sin\theta = ml\dot{\theta}^2}$$

$$l\,\cancel{(mg\sin\theta - m\ddot{x}\cos\theta) = ml^2\ddot{\theta}}$$

$$a_t = l\alpha$$

$$m\ddot{x} + m a_t\cos\theta + T\sin\theta = 0$$

$$\boxed{(M+m)\ddot{x} + m a_t\cos\theta = u}$$

$$\Rightarrow U(s) = (M+m)s^2 X(s) + ml\cos\theta \cdot s^2 \cdot \theta(s)$$

$$ml^2 s^2 \theta(s) = mgl\theta(s) - ms^2 X(s)\, l\cos\theta$$

since θ is very small

$\sin\theta \approx \theta$

$\cos\theta \approx 1$

$$\Rightarrow ml^2 s^2 \theta(s) = mgl\,\theta(s) - ms^2 l X(s)$$

$$\Rightarrow X(s) = \left(\frac{gl - l^2 s^2}{s^2 l}\right)\theta(s)$$

$$\Rightarrow X(s) = \left(\frac{g - ls^2}{s^2}\right)\theta(s)$$

$$U(s) = (M+m)s^2 X(s) + ml\,s^2\theta(s)$$

$$= \left[(M+m)(g - ls^2) + mls^2\right]\theta(s)$$

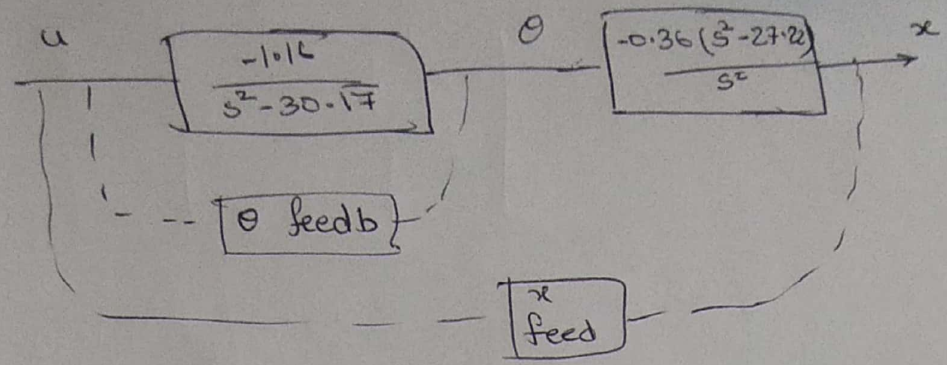
$$\Rightarrow \frac{\theta(s)}{U(s)} = \frac{1}{(M+m)(g-ls^2) + mls^2)}$$

$$\Rightarrow \frac{\theta(s)}{U(s)} = \frac{-1}{Mls^2 - (M+m)g}$$

$$\Rightarrow \boxed{\frac{\theta(s)}{U(s)} = \frac{-1.16}{s^2 - 30.17}}$$

$$\Rightarrow \frac{X(s)}{U(s)} = \frac{\left(\frac{s^2}{M} - \frac{g}{Ml}\right)}{s^2\left(s^2 - \frac{(M+m)g}{}\right)} = \frac{0.4166(s^2 - 27.22)}{s^2(s^2 - 30.171)}$$

(b)



→ for θ feed

$$\frac{\theta(s)}{U(s)} = \frac{-1.157}{(s+5.492)(s-5.492)}$$

+ve pole ⇒ unstable

— for x feed

$$\frac{X(s)}{U(s)} = \frac{ls^2-g}{s^2(Mls^2-(M+m)g)}$$

Since $M \gg m$ $\quad M+m \approx M$

$$\frac{X(s)}{U(s)} \approx \frac{ls^2-g}{s^2(Mls^2-Mg)} = \frac{1}{s^2M}$$

Cant be stable

→ with both θ, x can be stable.