CMPE320 Spring 2022
Homework #2 Assigned Feb. 9, 2022, Due Feb 16, 2022

1. Consider a 12-sided (dodecahedral)[1] unfair die. In rolling this die, the even numbered sides are twice as likely as the odd numbered sides.
   Define the events : $A = \{\text{odd numbered side}\},\ B = \{4,5,6,7,8\}$.
   $a)$ Find $\Pr[A]$,
   $b)$ Find $\Pr\left[B\right]$,
   $c)$ Find $\Pr\ [AB]$,
   $d)$ Are $A, B$ mutually exclusive?
   $e)$ Are $A, B$ independent
   $\Pr\left[\textit{any}\text{ single odd numbered side}\right] = p_o$
   $\Pr\left[\textit{any}\text{ single even numbered side}\right] = p_e = 2p_o$
   $\Pr\left[\mathcal{S}\right] = 1 = \sum_{n=1}^{12}\Pr\left[n\right] = \sum_{1}^{6}\Pr\left[odd\right] + \sum_{1}^{6}\Pr\left[even\right] = 6p_0 + 6p_e = 18p_0$
   Therefore, $p_0 = \frac{1}{18},\ p_e = 2p_0 = \frac{1}{9}$
   $a)$ Find $\Pr[A]$, $A$ is the union of simple events, which are mutually exclusive, so
   $\Pr\left[A\right] = \sum_{1}^{6}\Pr\left[odd\right] = 6\times\frac{1}{18} = \frac{1}{3}$
   $b)$ Find $\Pr\left[B\right], B$ is the union of simple events, which are mutually exclusive, so
   $\Pr\left[B\right] = 3\times\Pr\left[even\right] + 2\times\Pr\left[odd\right] = \frac{3}{9} + \frac{2}{18} = \frac{4}{9} = 0.4444$
   $c)$ Find $\Pr\ [AB]$,
   $AB = \{\text{odd}\}\cap\{4,5,6,7,9\} = \{5,7\}$
   $\Pr\left[AB\right] = \frac{2}{18} = \frac{1}{9}$, the simple events are mutually exclusive, but $A$ and $B$ are not.
   $d)$ Are $A, B$ mutually exclusive?
   No, $AB \neq \varnothing$
   $e)$ Are $A, B$ independent
   $\Pr\left[A\right]\times\Pr\left[B\right] = \frac{1}{3}\times\frac{4}{9} = \frac{4}{27} \neq \frac{1}{9} = \Pr\left[AB\right]$ So they are not independent.

2. Use the algebra of events to write expression for the desired events and the rules of probabilities for these event expressions.

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[1] Interesting side note, there are only five regular polyhedral with equal-sized sides. These are known as the *Platonic Solids.* They are the tetrahedron (4 sides), the cube (6 sides, our usual die), the octahedron (8 sides), the dodecahedron (12 sides) and the icosohedron (20 sides). There are no other solid shapes that can be used to construct a fair die.

a. A computer purchased from *Simon's Surplus* will experience hard drive failures with probability 0.3 and memory failures with probability 0.2 and will experience both types of failures simultaneously with probability 0.1.

b. What is the probability that there will be one type of failure, but not the other?

$$X = \{\text{one type of failure but not the other}\}$$
$$A = \{\text{disk fails}\}, B = \{\text{memory fails}\}$$
$$X = XOR(A,B), \text{ where } XOR \text{ is the exclusive OR.}$$
Short Way
$$A + B = X + AB \text{ put the intersection back in}$$
$$XAB = \varnothing \text{ by definition of } X$$
$$\Pr[A+B] = \Pr[X + AB]$$
$$\Pr[A] + \Pr[B] - \Pr[AB] = \Pr[X] + \Pr[AB]$$
$$\Pr[A] + \Pr[B] - 2\Pr[AB] = \Pr[X] \text{ and we're done}$$
$$0.3+0.2 \; -2 \times 0.1 = 0.3$$

c. What is the probability that there will be no failures of either kind.

$$b)\Pr[\text{no failures}]$$
$$Y = \{\text{no failures}\} = A^C B^C = (A+B)^C \text{ by DeMorgan's Laws}$$
$$\Pr[Y] = 1 - \Pr[A+B] = 1 - (\Pr[A] + \Pr[B] - \Pr[AB])$$
$$= 1 - (0.3 + 0.2 - 0.1) = 1 - 0.4 = 0.6$$

3. T&T 2.16 In a certain digital control system, the control command is represented by a set of four hexadecimal characters.
   a) What is the total number of control commands that are possible?
   b) If each control command must have a unique prefix, i.e., starting from left to right no hex character must be repeated, how many control commands are possible?

$a$) There are 4 characters, each can have 16 possibilities, therefore there are

$16 \times 16 \times 16 \times 16 = 16^4 = (2^4)^4 = 2^{16} = 65{,}536$ possibile command words.

$b$) If no character can be repeated, but the order of characters counts, then this

is a permutation, $P_4^{16} = \dfrac{16!}{(16-4)!} = 16 \times 15 \times 14 \times 13 = 43{,}680$ possible command

words with unique prefixes.

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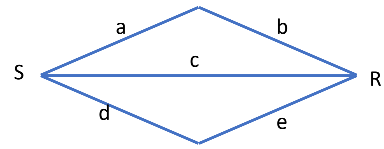
4. T&T Problem 2.20 The diagram below represents a communication network where the source S communicates with the receiver, R. Let A represent the event "link a fails", B "link b fails", etc. Write an expression in the algebra of events for the event F= "S fails to communicate with R"




The "positive method"

Let $C = \{\text{S can communicate with R}\}$

Then $F = C^C$.

$C = A^C B^C + C^C + D^C E^C$

$F = \left(A^C B^C + C^C + D^C E^C\right)^C$

$\Pr\left[F\right] = 1 - \Pr\left[A^C B^C + C^C + D^C E^C\right]$

The "negative method"

$F = (A+B)C(D+E) = ACD + ACE + BCD + BCE$

$\Pr\left[F\right] = \Pr\left[ACD + ACE + BCD + BCE\right]$, which is a mess.

5. Beetle Bailey has a date with Miss Buxley, but Beetle has an old jeep with will break down with probability 0.4. If his jeep breaks down, he will be late with a probability of 0.9. If it does not break down, he will be late with probability of 0.2. What is the probability he will be late for his date?

Let $A$ be "jeep breaks down", so that $A^C =$ "jeep doesn't break down"

Let $B$ be "Beetle is late for his date"

We are given $\Pr\left[B \mid A\right] = 0.9$, $\Pr\left[B \mid A^C\right] = 0.2$, $\Pr\left[A\right] = 0.4$, from which we can say $\Pr\left[A^C\right] = 0.6$.

From the Principle of Total Probability,

$\Pr\left[B\right] = \Pr\left[B \mid A\right]\Pr\left[A\right] + \Pr\left[B \mid A^C\right]\Pr\left[A^C\right]$

$= 0.9 \times 0.4 + 0.2 \times 0.6 = 0.48$