Game Theoretic Analysis of PDNO and TNO

Reminder: Due to page limitations, the Appendix is provided online and will be attached to the paper later.

In this appendix, we show that the PDNO and TNO are in a non-cooperative competitive relationship, and the equilibrium can be interpreted from a game-theoretic perspective.

First, from a microeconomics viewpoint, the optimal power flow problem (11)-(17) is a consumption-minimization problem, which is strictly convex and satisfies strong duality, a unique competitive equilibrium exists for the PDNO. Each generator is assumed to be a price-taker, there exist individual generator's generation-minimization decisions that are consistent with the consumption-minimization solutions. Moreover, the market cleaning prices are specified as the LMPs, which alone can encourage generators to determine the optimal active power generation [1].

In the transportation network, EV users compete with other travelers for cost-minimizing routes while consuming electrical energy to power their vehicles. We model EV travelers as a non-atomic population, i.e., the traveler count is large, and each individual constitutes a negligible share of the traffic [2]. As electricity consumers, each EV driver seeks to minimize their own costs given the prevailing electricity price comprised of energy price and carbon price.

Obviously, EV travelers choose the optimal travel route in response to the electricity prices to minimize consumption. Alternatively, the generators minimize their generation consumption by providing energy at the energy price and carbon price. The strategies and payoffs of the PDNO and TNO are interconnected, which motivates a potential game-theoretic perspective with given incentives. The game can be formulated as follows:

Definition 1: The potential game can be defined by a cellular $\Psi = \{\{D_{\mathrm{E}} \cup D_{\mathrm{G}}\}, \{K_{\mathrm{E}}, K_{\mathrm{G}}\},\{C_{\mathrm{E}}, C_{\mathrm{G}}\}\}$, where the components are defined in the following:

1) **Players** $\{Q^{\mathrm{rs}} \cup \aleph^{\mathrm{MG}}\}$:
   In TNO, the player type corresponds to travel demand with different OD pairs. The continuum of players of type $rs \in \Gamma$ can be represented by the bound $[0, Q_{\mathrm{e}}^{rs} + Q_{\mathrm{g}}^{rs}]$.
   In PDNO, we have generators $\aleph^{\mathrm{MG}} = \{1, 2, ..., k\}$ as players.
2) **Strategies** $\{\Gamma, \{P_j^{\mathrm{g}}\}_{j \in \aleph^{\mathrm{MG}}}\}$:
   In TNO, strategy sets are set of routes connecting the OD pairs, i.e., Γ.
   In PDNO, the generators find an optimal active power generation strgtegy $p_j^{\mathrm{g}} \in P_j^{\mathrm{g}}$.
3) **Payoffs** $\{(C^{rs})_{rs \in \Gamma}, (C_j)_{j \in \aleph^{\mathrm{MG}}}\}$:
   For TNO, travelers minimize their consumption $C^{rs} := \sum_{a \in K_{\mathrm{e}}^{rs}} (\omega t_a + \lambda_a E_{\mathrm{B}}) + \sum_{a \in K_{\mathrm{g}}^{rs}} (\omega t_a + \phi \varepsilon_a l_a / 10^6), \forall rs \in \Gamma$.
   For PDNO, the generators minimize their consumption $C_j(p_j^{\mathrm{g}}) = a_j (p_j^{\mathrm{g}})^2 + b_j p_j^{\mathrm{g}} - \lambda^j p_j^{\mathrm{g}}$.

At equilibrium, given the electricity price, no traveler can unilaterally decrease their costs by switching routes (i.e., the Wardrop equilibrium [3], [4]), and no generator can decrease its consumption by unilaterally deviating from the optimal generation schedule, either. Furthermore, some properties of this game can be derived, which are given as follows:

Proposition 1. The game Ψ is a potential game with potential function $\Xi = C_{\mathrm{users}} + C_{\mathrm{PDNO}} - \sum\limits_{a \in A_{\mathrm{C}}} \lambda_a x_a E_{\mathrm{B}}$.

Proof. To prove the equivalency, we first form the partial Lagrangian equation with respect to constraints (3) and (11) as follows:

$$
\begin{aligned}
L = & C_{\mathrm{users}} + C_{\mathrm{PDNO}} - \sum_{a \in A_{\mathrm{C}}} \lambda_a x_a E_{\mathrm{B}} \\
& + \sum_{rs} \mu_{rs} \left(Q_{\mathrm{e}}^{rs} - \sum_{ke \in K_{\mathrm{e}}^{rs}} f_{ke}^{rs} \right) \\
& + \sum_{rs} \mu_{rs} \left(Q_{\mathrm{g}}^{rs} - \sum_{kg \in K_{\mathrm{g}}^{rs}} f_{kg}^{rs} \right) \\
& + \sum_{j=1}^{B} \zeta_j \left(\begin{array}{c} -P_{ij} - p_j^{\mathrm{g}} + r_{ij} I_{ij} + \\ \sum_{k \in \pi(j)} P_{jk} + p_j^{\mathrm{d}} + \Omega_{a,j} x_a^{\mathrm{e}} E_{\mathrm{B}} \end{array} \right)
\end{aligned} \tag{1}
$$

where μ, ζ are the Lagrange multiplier associated with (3) and (11), respectively (it should be noted that a given vehicle cannot simultaneously be a GV and an EV, hence, the multipliers μ appearing in the second and third lines cannot both be present.), B is the number of buses in the PDN.

The following optimality conditions need to hold for the route flows:

$$f_{ke}^{rs}\frac{\partial L}{\partial f_{ke}^{rs}} = 0 \text{ and } \frac{\partial L}{\partial f_{ke}^{rs}} >= 0, \forall ke \in K_{\mathrm{e}}, \forall rs \in \Gamma \tag{2}$$

$$f_{kg}^{rs}\frac{\partial L}{\partial f_{kg}^{rs}} = 0 \text{ and } \frac{\partial L}{\partial f_{kg}^{rs}} >= 0, \forall kg \in K_{\mathrm{g}}, \forall rs \in \Gamma \tag{3}$$

where

$$\begin{aligned}
\frac{\partial L}{\partial f_{ke}^{rs}} =& \frac{\partial C_{\text{users}}}{\partial f_{ke}^{rs}} + \frac{\partial C_{\text{PDNO}}}{\partial f_{ke}^{rs}} \\
& - \frac{\partial}{\partial f_{ke}^{rs}} \sum_{a \in A_{\mathrm{C}}} \lambda_a x_a E_{\mathrm{B}} - \mu_{rs} + \frac{\partial}{\partial f_{ke}^{rs}} \sum_{j=1}^{B} \zeta_j \Omega_{a,j} x_a E_{\mathrm{B}} \\
=& \sum_{a \in A_{\mathrm{R}}} \omega t_a \frac{\partial x_a}{\partial f_{ke}^{rs}} + \sum_{a \in A_{\mathrm{C}}} \omega t_a \frac{\partial x_a}{\partial f_{ke}^{rs}} + \sum_{a \in A_{\mathrm{C}}} \lambda_a E_{\mathrm{B}} \frac{\partial x_a}{\partial f_{ke}^{rs}} \\
& - \sum_{a \in A_{\mathrm{C}}} \lambda_a E_{\mathrm{B}} \frac{\partial x_a}{\partial f_{ke}^{rs}} - \mu_{rs} + \sum_{j=1}^{n} \zeta_j \Omega_{a,j} E_{\mathrm{B}} \frac{\partial x_a}{\partial f_{ke}^{rs}} \\
=& \sum_{a \in A_{\mathrm{R}}} \omega t_a \delta_{a,ke}^{rs} + \sum_{a \in A_{\mathrm{C}}} \omega t_a \delta_{a,ke}^{rs} - \mu_{rs} \\
& + \sum_{a \in A_{\mathrm{C}}} \lambda_a E_{\mathrm{B}} \delta_{a,ke}^{rs} \\
=& \sum_{a \in A_{\mathrm{R}}} \omega t_a \delta_{a,ke}^{rs} + \sum_{a \in A_{\mathrm{C}}} (\omega t_a + \lambda_a E_{\mathrm{B}}) \delta_{a,ke}^{rs} - \mu_{rs}
\end{aligned} \tag{4}$$

Therefore, (4) yields

$$\frac{\partial L}{\partial f_{ke}^{rs}} = \frac{\partial C_{\text{users}}}{\partial f_{ke}^{rs}} \tag{5}$$

which demonstrates that the potential function Ξ yields a Wardropian equilibrium traffic flow pattern for EVs. The same method can be applied to prove Ξ yields a Wardropian equilibrium traffic flow pattern for GVs. Hence, Ξ is a potential function for travelers.

For the generators, a necessary and sufficient condition for a a minimum $p^{\mathrm{g}} > 0$ is to satisfy:

$$\frac{\partial L}{\partial p_j^{\mathrm{g}}} = \frac{\partial L}{\partial C_{\text{PDNO}}} - \zeta_j \tag{6}$$

such that $\frac{\partial L}{\partial p_j^{\mathrm{g}}} - \frac{\partial C_j}{\partial p_j^{\mathrm{g}}} = \lambda_j - \zeta_j = \phi$ holds. Consequently, Ξ is a potential function for generators as well [5].

It's worth noting that the potential function Ξ can be viewed as a social welfare for the operation of coupled TN and PDN with given incentives, i.e., charging price λ_a, thus, the following corollaries can be derived from the properties of the potential games.

Corollary 1. An equilibrium state exists for the proposed game Ψ, namely, a strategies that minimize Ξ [6] **(Existence)**.

Corollary 2. The equilibrium state of game Ψ is unique if Ξ is a strict convex problem [6] **(Uniqueness)**.

Thus, the reformulation is conducted in our work to convert the original non-convex problem into a convex one, such that the existence and uniqueness of the equilibrium can be guaranteed.

References

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