Rasterization

Foley, Ch: 3

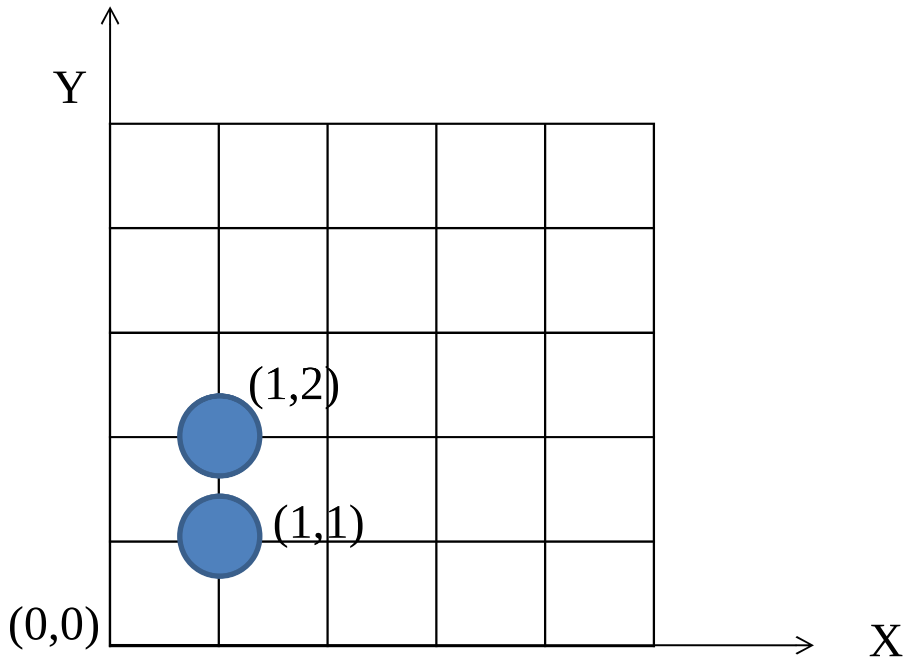
Upto 3.1, 3.2 (upto 3.2.3), 3.3, 3.6 (upto 3.6.3)

Overview

- Scan conversion : mapping objects/shapes to pixels
- Fast image generation (e.g. in computer games)
- Simple Raster Graphics Package (SRGP)
- Scan converting lines
- Scan converting circles
- Scan converting polygons

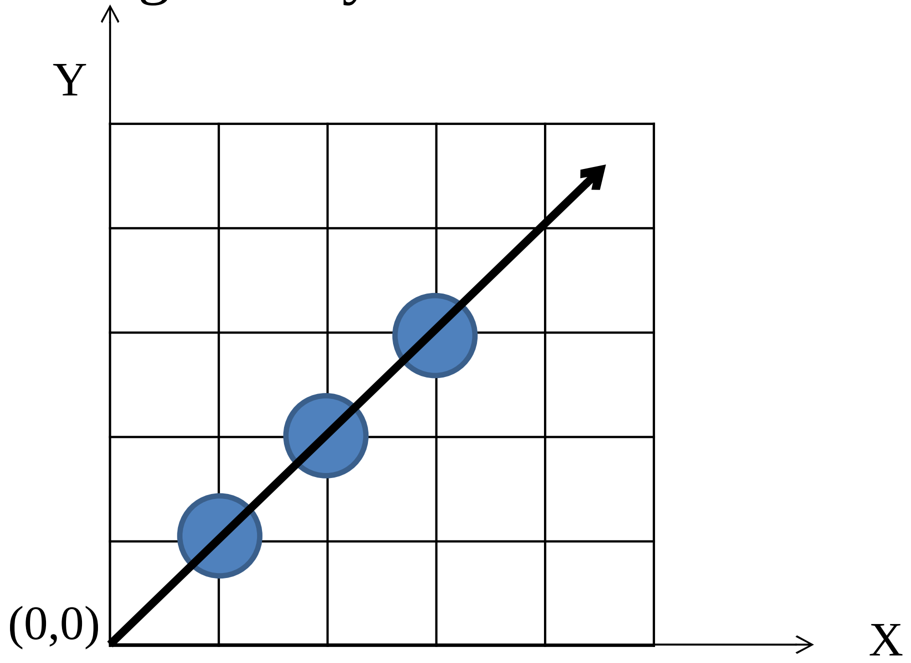
Pixels in SRGP

- Pixels are represented as circles centered on uniform grid
- Each (x,y) of the grid has a pixel



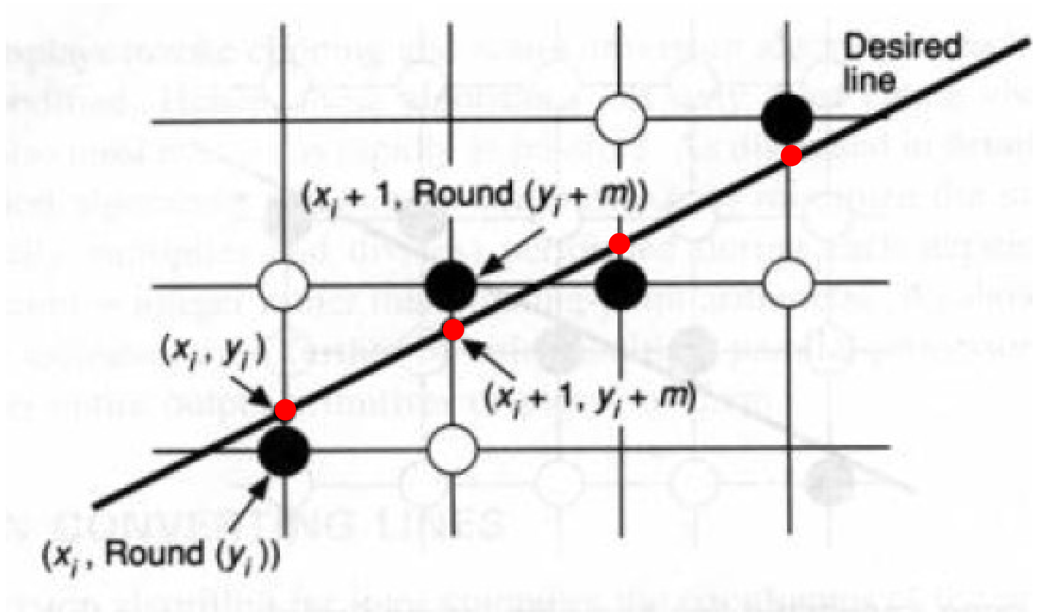
Scan Converting Lines

- If the line has slope, m = 1
- Incremental along x and y axis is 1



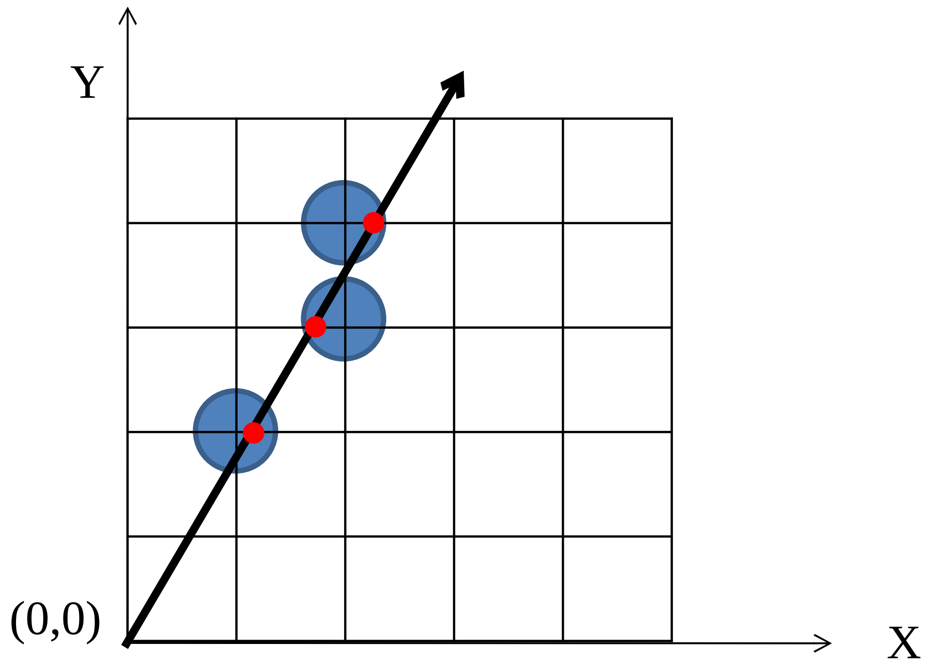
Scan Converting Lines

- If the line has slope < 1
- Increment along x is 1, increment along y is fractional
  - Round the value along y axis


Desired line
$(x_i + 1, \text{Round}(y_i + m))$
(x_i, y_i)
$(x_i + 1, y_i + m)$
$(x_i, \text{Round}(y_i))$

Scan Converting Lines

- If the line has slope > 1
- Increment along y is 1, increment along x is fractional
  - Round the value along y axis



Scan Converting Lines : Basic Incremental Algorithm

Two end points (x_o,y_o) and (x_1,y_1)

Calculate slope $m = (y_1-y_0) / (x_1-x_0)$

<u>If (m<1):</u>

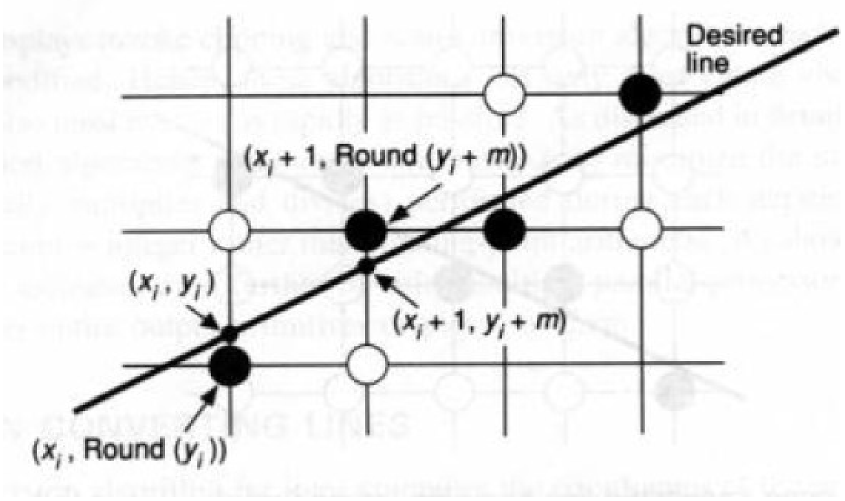
$y_{i+1} = mx_{i+1} + B$

$= m(x_i + \Delta x) + B$

$= (mx_i + B) + m\Delta x$

$= y_i + m\Delta x$

$= y_i + m$


Desired line
$(x_i + 1, \text{Round } (y_i + m))$
(x_i, y_i)
$(x_i + 1, y_i + m)$
$(x_i, \text{Round } (y_i))$

So next point to intensify is $(x_{i+1}, \text{round}(y_{i+1}))$ where $x_{i+1}= x_i+1$

Scan Converting Lines : Basic Incremental Algorithm

Two end points (x_o,y_o) and (x_1,y_1)

Calculate slope $m = (y_1-y_0) / (x_1-x_0)$

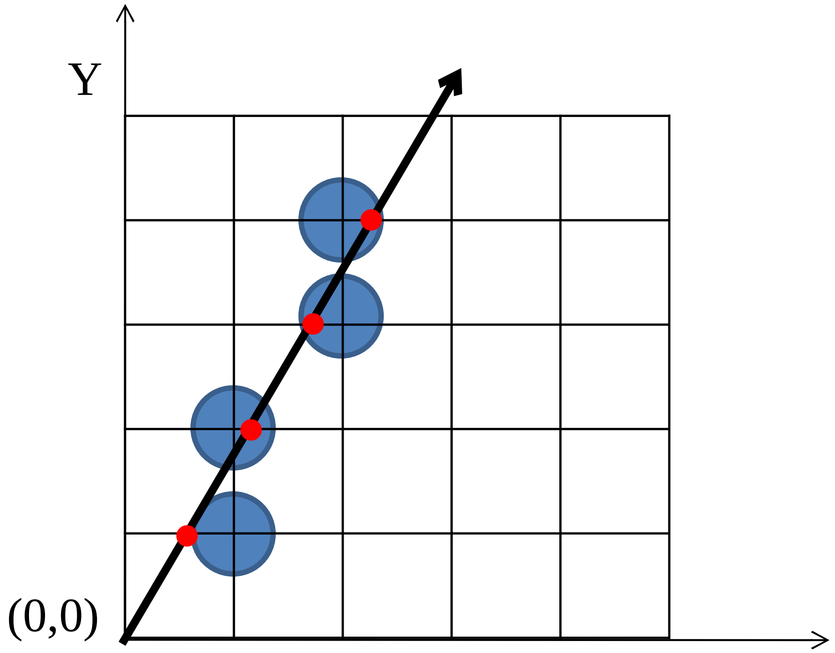
If(m>1) (Steeper):

$x_{i+1} = (y_{i+1} – B)/m$

$= ((y_i + \Delta y) – B)/m$

$= (y_i - B)/m + \Delta y/m$

$= x_i + 1/m$



So next point to intensify is $(\text{round}(x_{i+1}), y_{i+1})$ where $y_{i+1}= y_i+1$

Scan Converting Lines : Basic Incremental Algorithm

Basic Incremental Algorithm Problems:

1. Handle fractional values ✉ Rounding takes time
2. Working with real numbers (e.g. y, m)

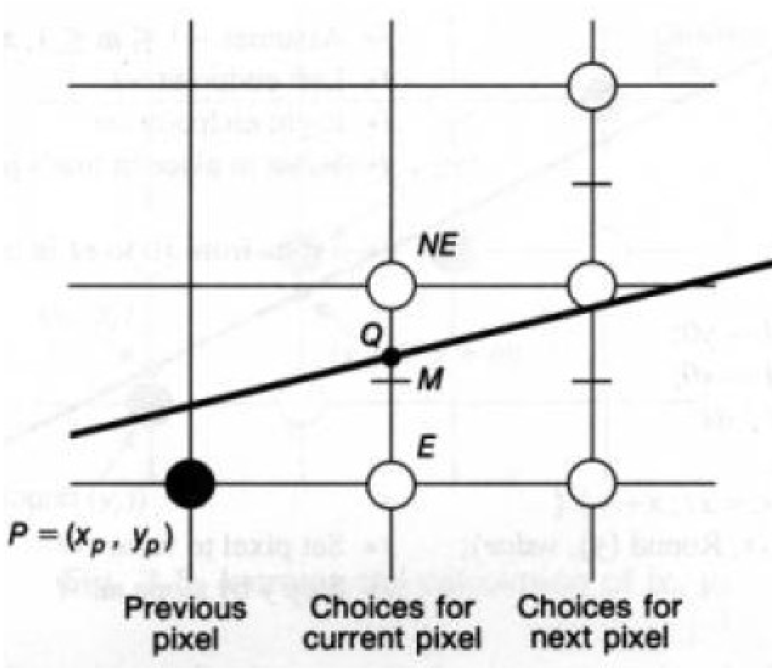
… etc.

Solution: **Midpoint Line Algorithm**

3. Only integer arithmetic
4. Avoid rounding

Scan Converting Lines : Midpoint Line Algorithm

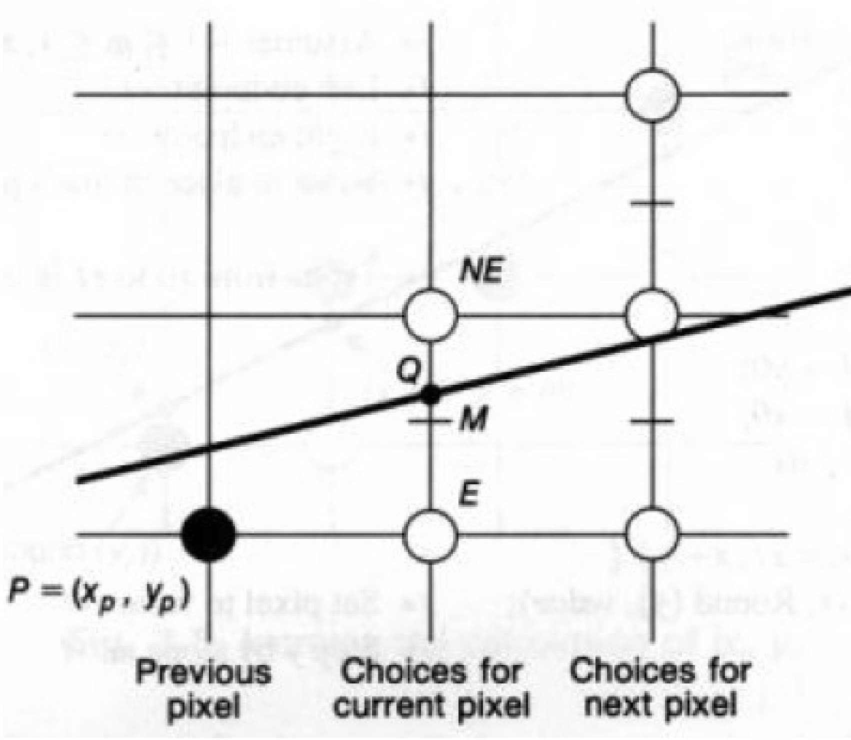
- Assumption : Slope is between 0 and 1
- Lower end point (x_o, y_o) and upper point (x_1, y_1)
- Initially we are at $P(x_p, y_p)$
- Choose between **NE** and **E**
- Midpoint **M** is $\mathbf{(x_p+1,\ y_p+1/2)}$
- **M lies on which side of the line?**
  - M is on the line ✉ any pixel NE / **E**
  - M is below the line ✉ choose NE
  - M is above the line ✉ choose E
- **Find the equation of the line f (x,y)**
  - $f(x,y) = ax + by + c$
    $a = dy$; $b = -dx$; $c = dx.B$ (B is the y-intercept in the slope-intercept form)
  - Let $d = f(M) = f(x_p+1, y_p+1/2)$
  - $d = 0$ ✉ M is on the line
    $d > 0$ ✉ M is below the line
    $d < 0$ ✉ M is above the line



Scan Converting Lines : Midpoint Line Algorithm

- Find the next value of the decision variable d?
  - Need to apply incremental approach

- Two cases:
  1. If E is chosen
  2. If NE is chosen



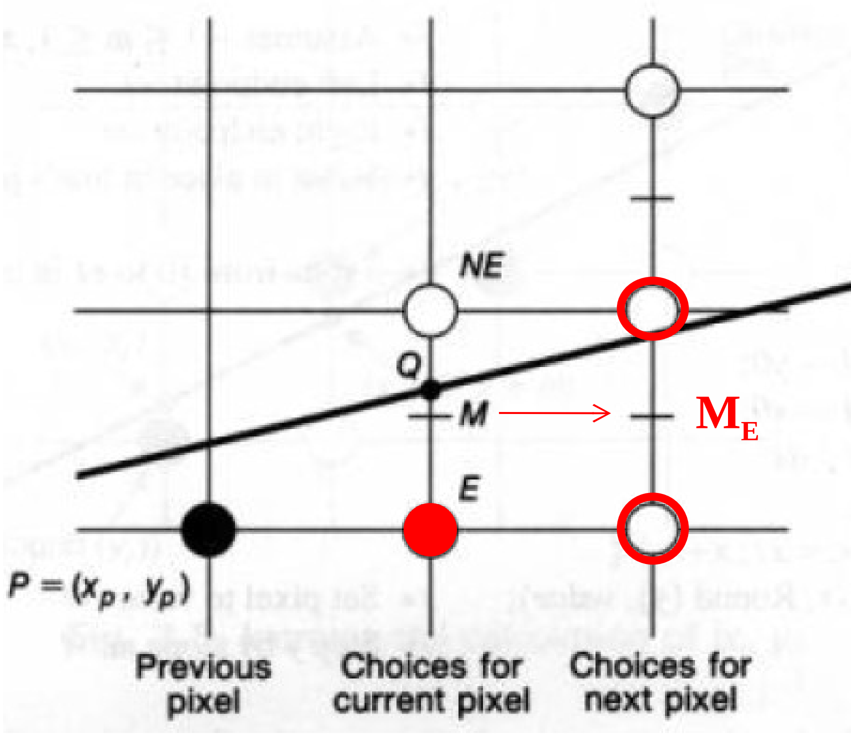
Scan Converting Lines : Midpoint Line Algorithm

- Find the next value of the decision variable d?
  - Need to apply incremental approach

If E is chosen :

- New Midpoint:
  M_E (x_p+2, y_p+ 1/2)
- New decision variable, d_{new}
  = f (M_E) = f (x_p+2, y_p+ 1/2)
  = f (x_p+1, y_p+ 1/2) + dy.1
  = d_{old}+ dy

So, d_E = dy



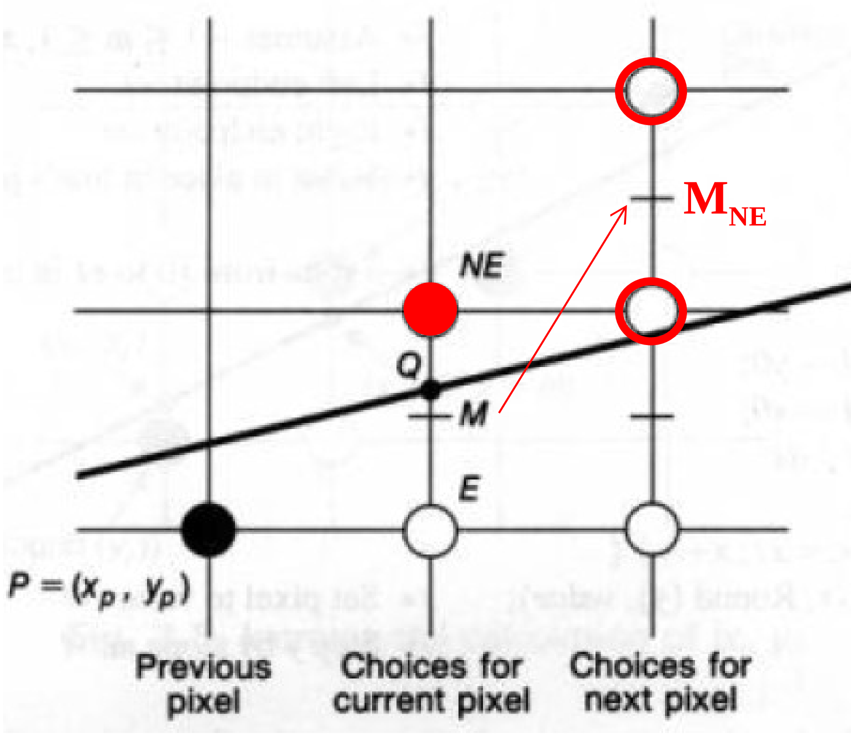
Scan Converting Lines : Midpoint Line Algorithm

- Find the next value of the decision variable d?
  - Need to apply incremental approach

If NE is chosen :

- New Midpoint:
  M_{NE} $(x_p+2, y_p+ 3/2)$
- New decision variable d_{new}
  $= f(M_{NE}) = f(x_p+2, y_p+ 3/2)$
  $= f(x_p+1, y_p+ 1/2) + dy.1- dx.1$
  $= d_{old} + dy – dx$

So, $d_{NE} = dy - dx$

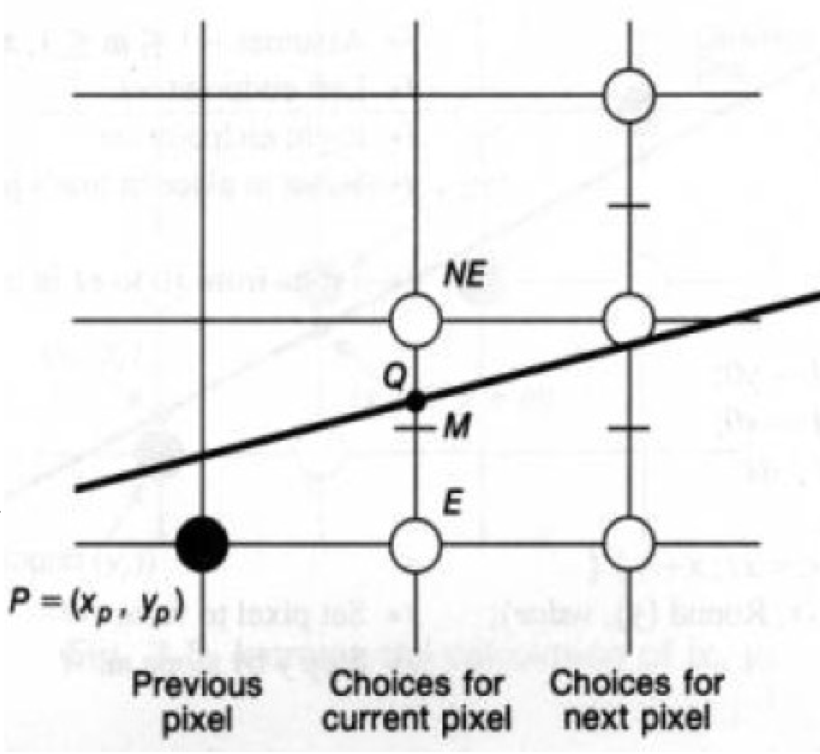


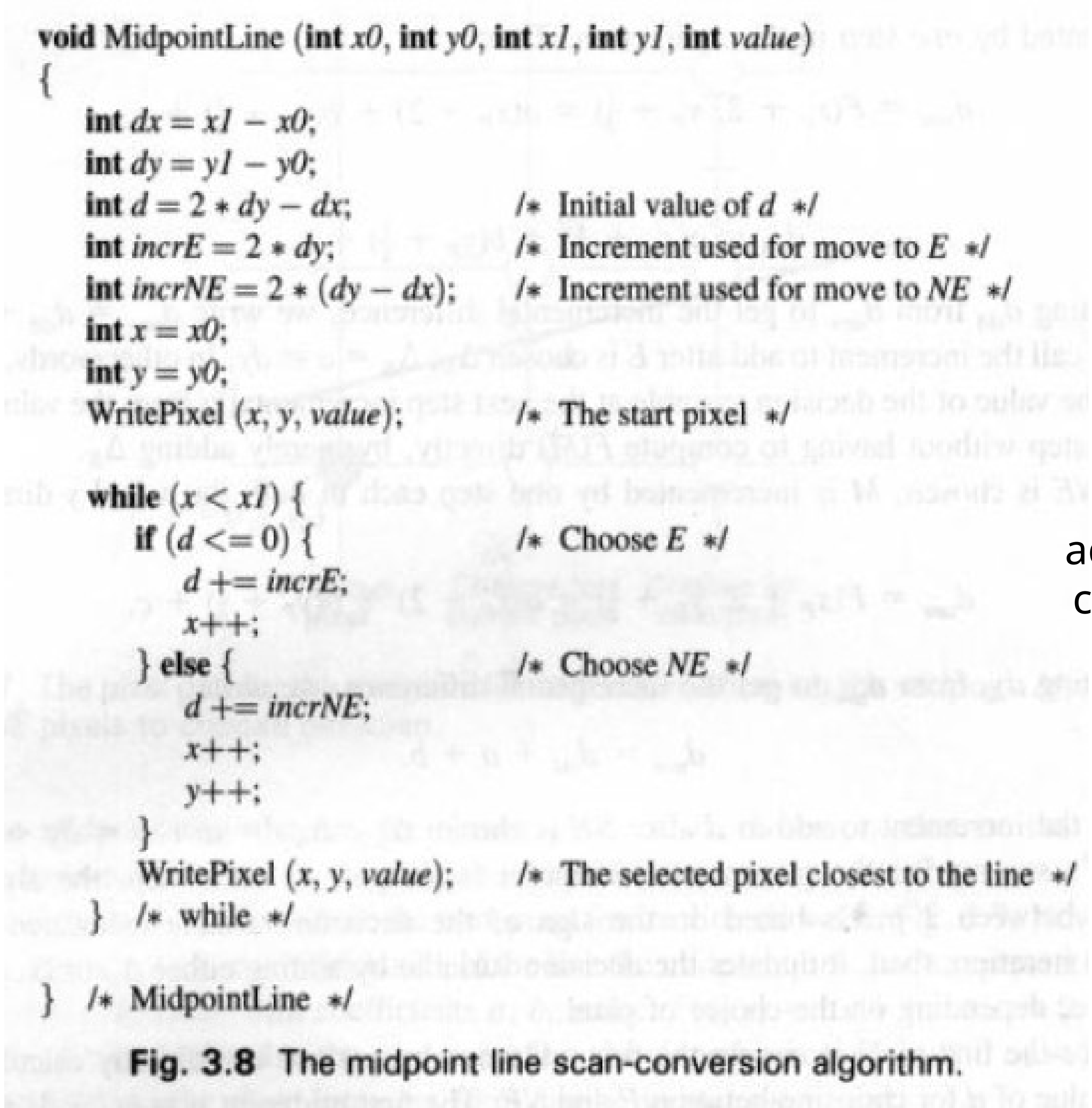
Scan Converting Lines : Midpoint Line Algorithm

- Initial value of decision variable $\mathbf{d_{start}}$
- Start point (x_o, y_o)
- $d_{start} = f(x_o+1,\ y_o + 1/2)$

$$= f(x_o, y_o) + dy - dx/2$$
$$= dy - dx/2$$

Here the fraction in **dx/2** can be avoided considering **f(x,y) = 2(ax+by+c)**





```
void MidpointLine (int x0, int y0, int x1, int y1, int value)
{
    int dx = x1 − x0;
    int dy = y1 − y0;
    int d = 2 * dy − dx;              /* Initial value of d */
    int incrE = 2 * dy;               /* Increment used for move to E */
    int incrNE = 2 * (dy − dx);       /* Increment used for move to NE */
    int x = x0;
    int y = y0;
    WritePixel (x, y, value);         /* The start pixel */

    while (x < x1) {
        if (d <= 0) {                 /* Choose E */
            d += incrE;
            x++;
        } else {                      /* Choose NE */
            d += incrNE;
            x++;
            y++;
        }
        WritePixel (x, y, value);     /* The selected pixel closest to the line */
    } /* while */

} /* MidpointLine */
```

Fig. 3.8 The midpoint line scan-conversion algorithm.

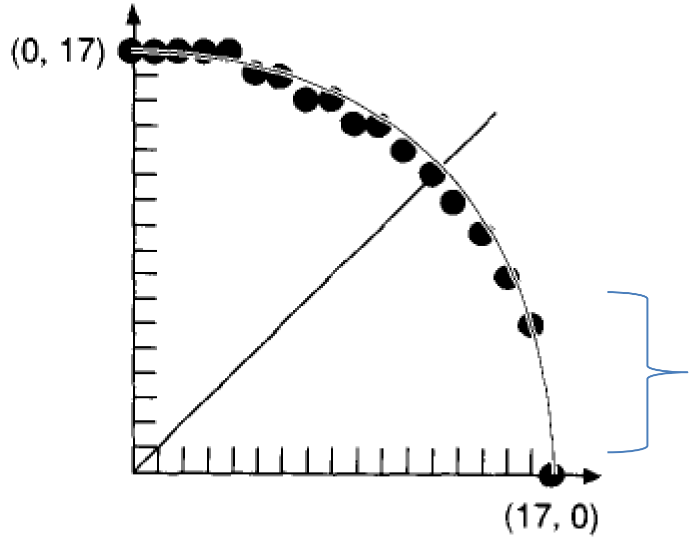
Simple addition and comparison

Scan Converting Lines : Midpoint Line Algorithm

- The algorithm works for $0<=m<=1$
- What about other slopes?
  - Think!

Scan Converting Circles

- Naive Approach (expensive computation)
  - For a circle centered at the origin, $y = \sqrt{(R^2 - x^2)}$
  - Draw a quarter by incrementing x from 0 to R in unit steps



Large gaps for values of x close to *R*

  - Circles not centered at the origin, can be translated for computation and pixels can be written with appropriate offsets

Scan Converting Circles

- Another Inefficient Method
  - For a circle centered at the origin, *y* = *RcosΘ*, *x* = *RsinΘ*
  - Vary *Θ* from 0 to 90 degree uniformly and plot *x*, *y*
  - Avoids large gaps but still computationally inefficient

- Eight Way Symmetry

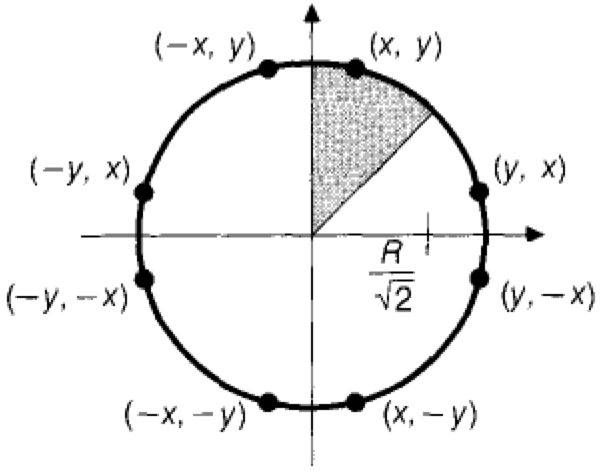
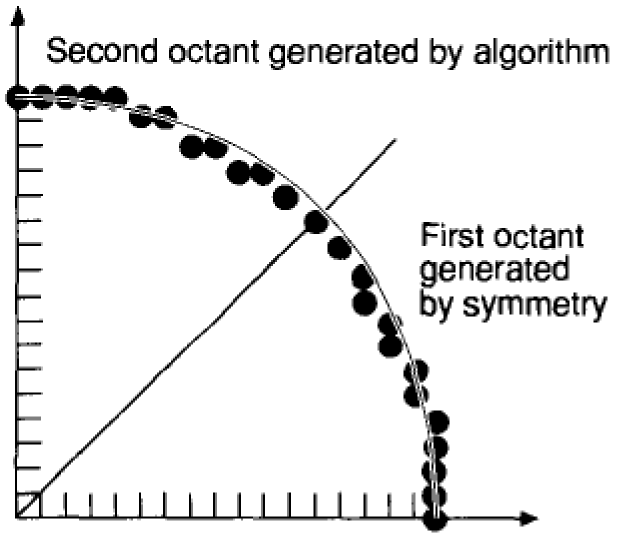
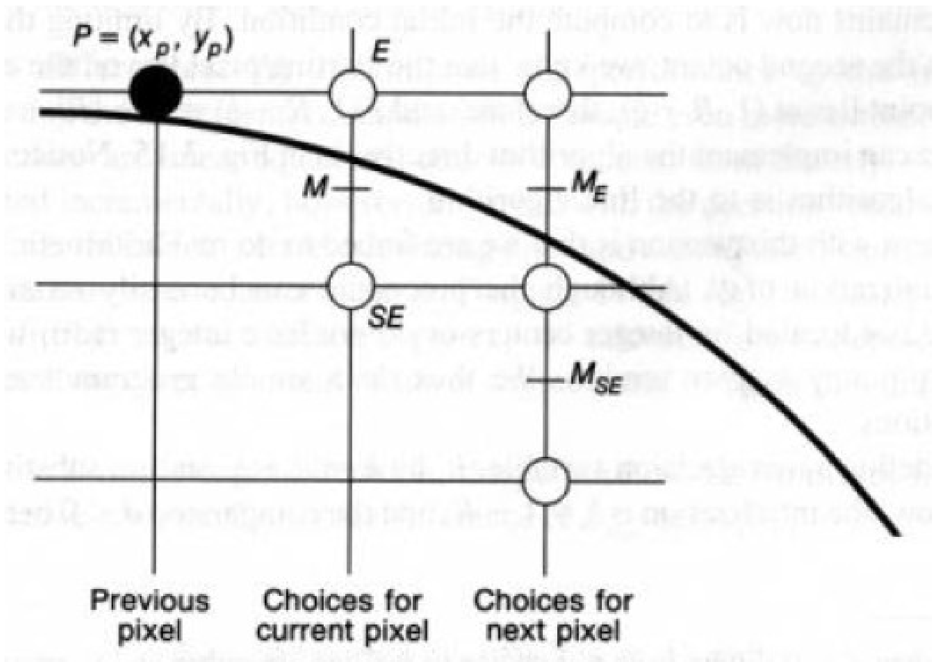


Fig. 3.13 Eight symmetrical points on a circle.


Second octant generated by algorithm
First octant generated by symmetry

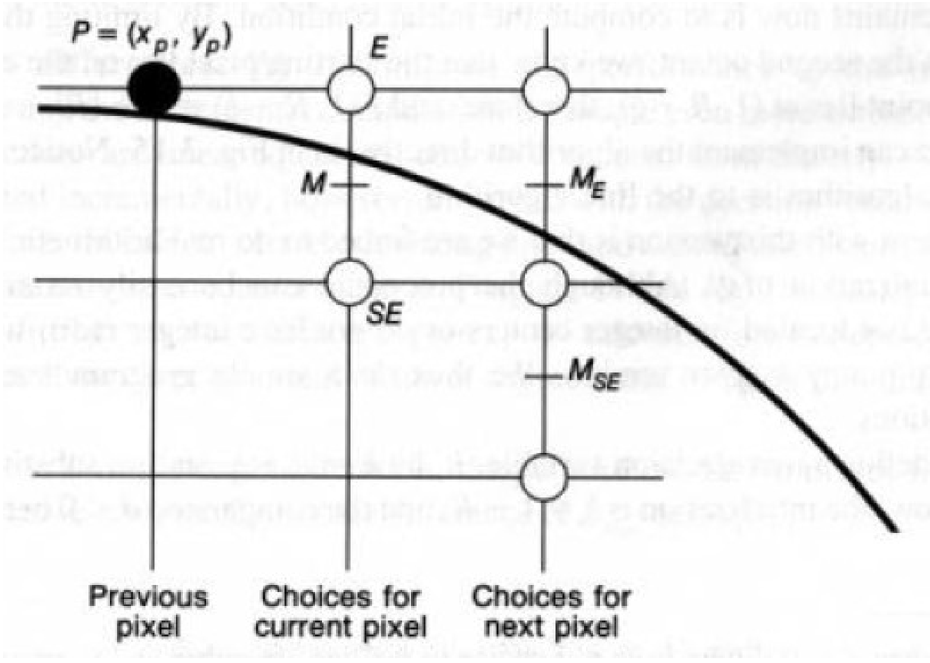
Scan Converting Circles : Midpoint Circle Algorithm

- Compute for the second octant only i.e. 45^0 of the circle, from $x = 0$ to $x = R/\sqrt{2}$
- Say, currently we are at $P(x_p, y_p)$.
- Choose between E or SE based on midpoint M$\mathbf{(x_p+1, y_p - 1/2)}$
- **M lies inside or outside the circle?**
  - M is outside the circle
    - ✉ choose SE
  - M is inside the circle
    - ✉ choose E
  - M is on the circle
    - ✉ any pixel SE /E



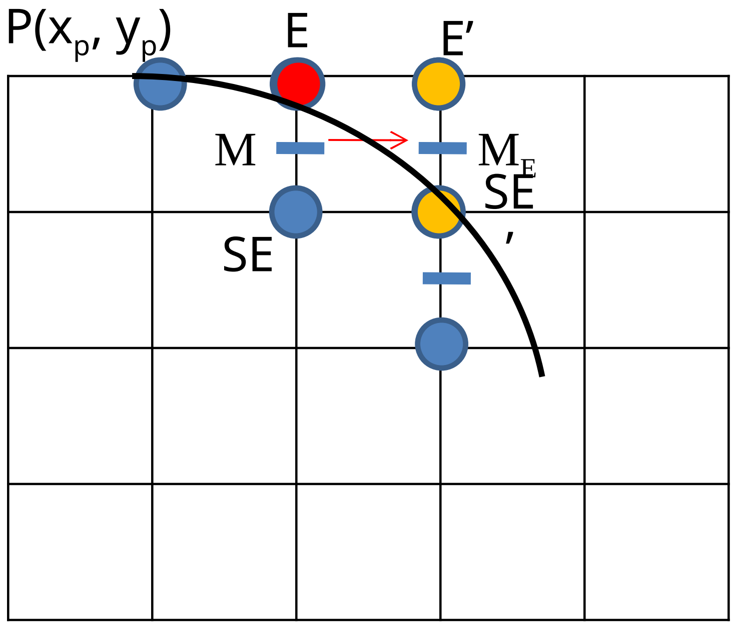
Scan Converting Circles : Midpoint Circle Algorithm

- Use $\mathbf{f(x,y) = x^2 + y^2 - R^2}$
- d = f(M) = f ($\mathbf{x_p + 1, y_p - 1/2}$)
- d = 0 : M is on the circle
  - any pixel SE/E
- d > 0 : M is outside the circle
  - choose SE
- d < 0 : M is inside the circle
  - choose E



Scan Converting Circles : Midpoint Circle Algorithm

- After we reach E or SE, what is the new value of d?
- <u>**Case 1-We are at E:**</u>
- New midpoint $M_E(\mathbf{x_p+2, y_p - 1/2})$
- $d_{new} = f(M_E) = f(\mathbf{x_p+2, y_p - 1/2})$
  $= d_{old} + (\mathbf{2x_p + 3})$
  $= d_{old} + \mathbf{\Delta E}$
- $d_{new} > 0$ ✉ M is outside the circle
  ✉ Choose SE′
- $d_{new} < 0$ ✉ M is inside the circle
  ✉ Choose E′
- $d_{new} = 0$ ✉ M is on the circle
  ✉ Choose E′ / SE′



Scan Converting Circles : Midpoint Circle Algorithm

- After we reach E or SE, what is the new value of d?
- **<u>Case 2-We are at SE:</u>**
- New midpoint M_{SE}(**x_p+2, y_p – 3/2**)
- $d_{new} = f(M_{SE}) = f(\mathbf{x_p+2, y_p - 3/2})$

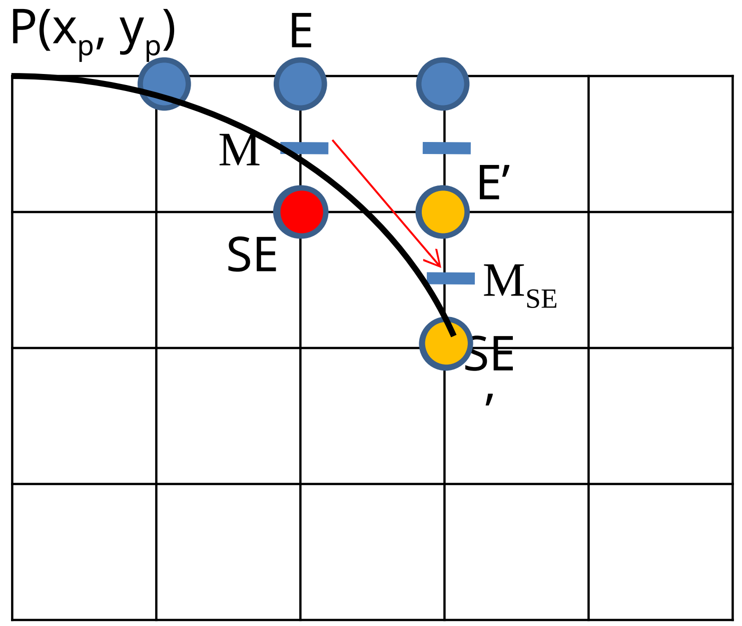
  $= d_{old} + (\mathbf{2x_p - 2y_p + 5})$

  $= d_{old} + \mathbf{\Delta SE}$
- $d_{new} > 0$ ✉ M is outside the circle

  ✉ Choose SE′
- $d_{new} < 0$ ✉ M is inside the circle

  ✉ Choose E′
- $d_{new} = 0$ ✉ M is on the circle

  ✉ Choose E′ / SE′



Scan Converting Circles : Midpoint Circle Algorithm

- Initial Value of d:

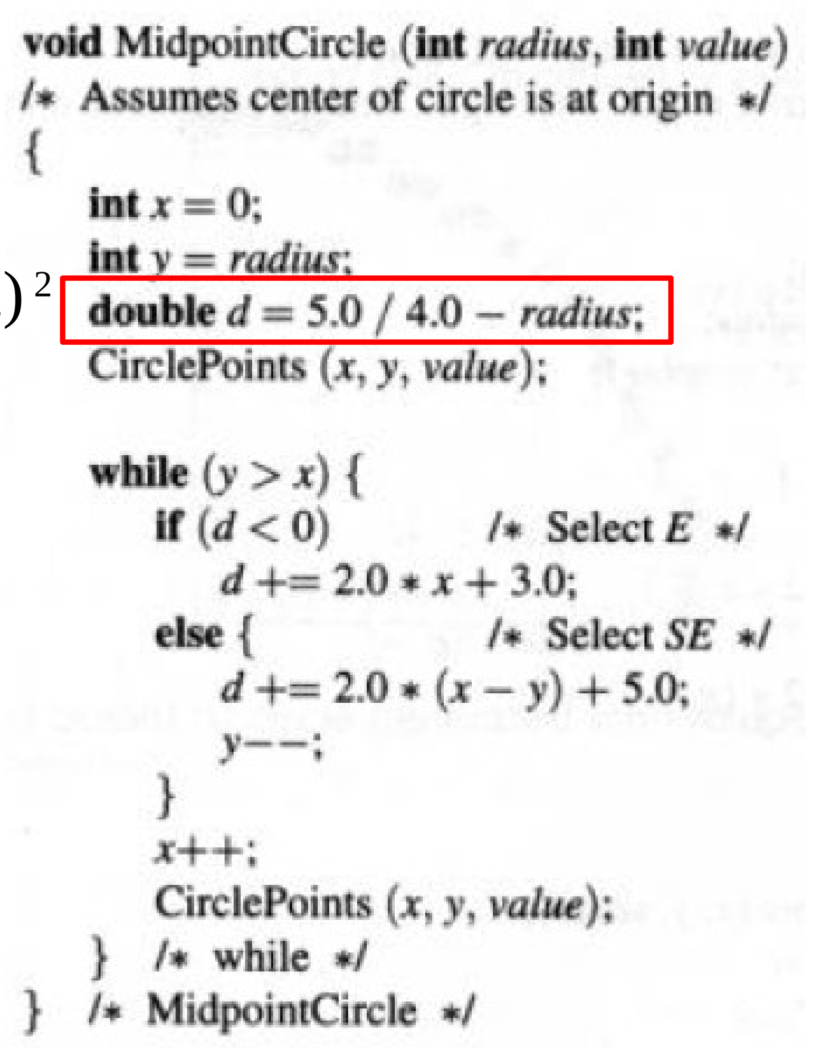
  $d_{start} = f(1, R-½)$

  $= 1^2 + (R-½)^2 - R^2$

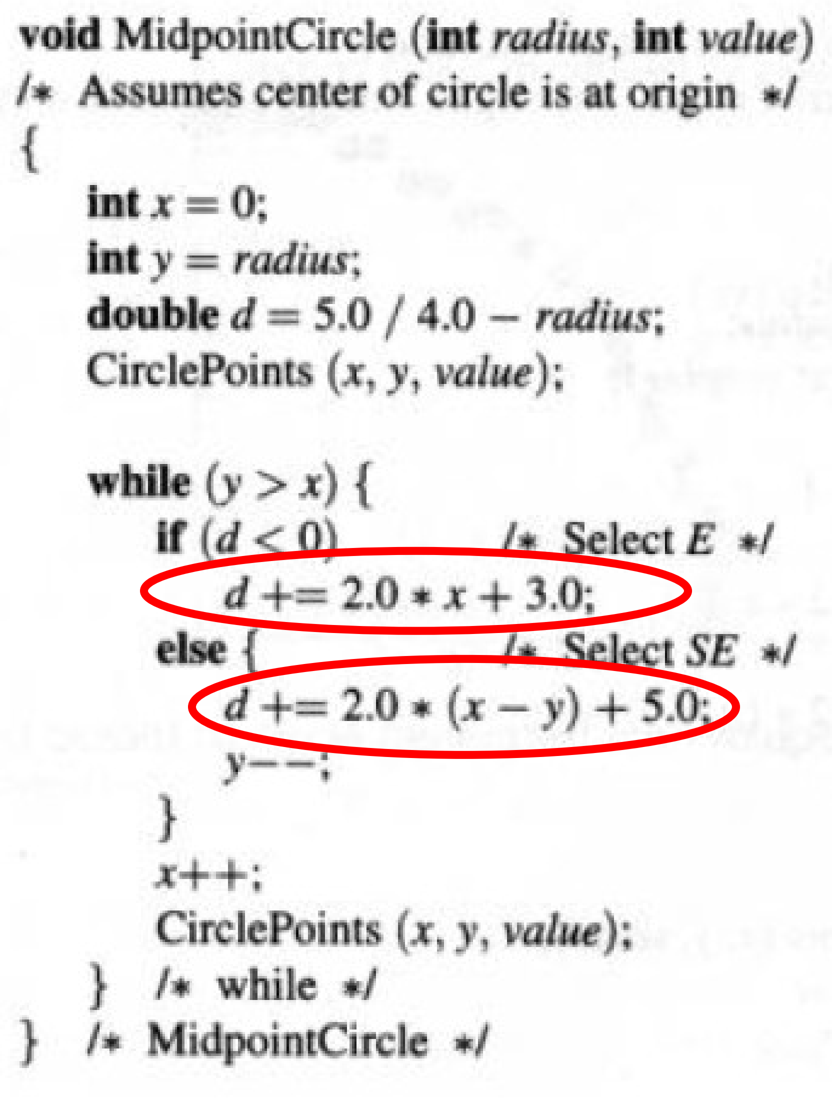
  $= 1 + R^2 – 2.R. ½ + (½)^2$
  $– R^2$

  $= 1 + 1/4 – R$

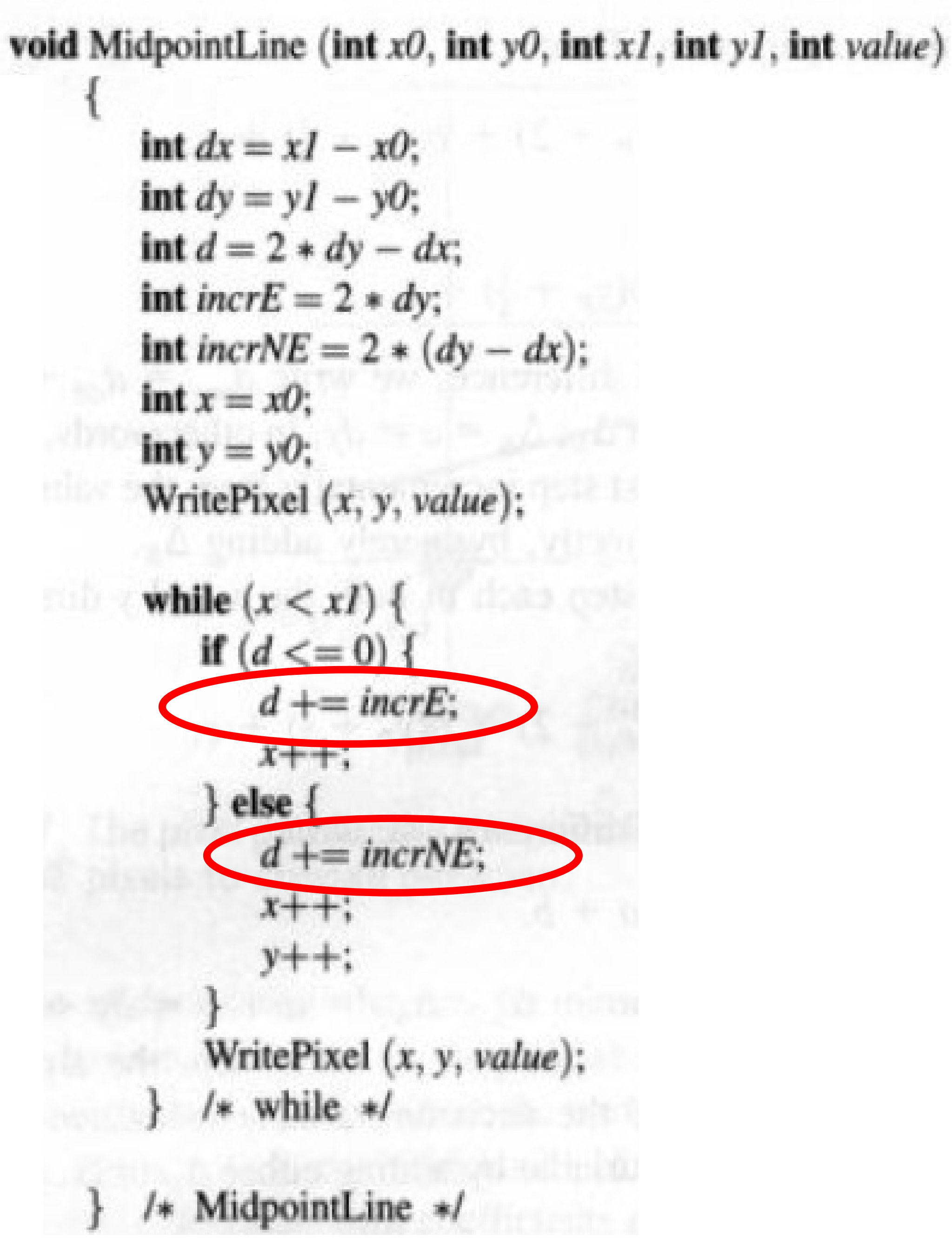
  $= 5/4 – R$



```
void MidpointCircle (int radius, int value)
/* Assumes center of circle is at origin */
{
    int x = 0;
    int y = radius;
    double d = 5.0 / 4.0 – radius;
    CirclePoints (x, y, value);

    while (y > x) {
        if (d < 0)              /* Select E */
            d += 2.0 * x + 3.0;
        else {                  /* Select SE */
            d += 2.0 * (x – y) + 5.0;
            y––;
        }
        x++;
        CirclePoints (x, y, value);
    }  /* while */
}  /* MidpointCircle */
```



```
void MidpointCircle (int radius, int value)
/* Assumes center of circle is at origin */
{
    int x = 0;
    int y = radius;
    double d = 5.0 / 4.0 - radius;
    CirclePoints (x, y, value);

    while (y > x) {
        if (d < 0)                 /* Select E */
            d += 2.0 * x + 3.0;
        else {                     /* Select SE */
            d += 2.0 * (x - y) + 5.0;
            y--;
        }
        x++;
        CirclePoints (x, y, value);
    } /* while */
} /* MidpointCircle */
```



```
void MidpointLine (int x0, int y0, int x1, int y1, int value)
{
    int dx = x1 - x0;
    int dy = y1 - y0;
    int d = 2 * dy - dx;
    int incrE = 2 * dy;
    int incrNE = 2 * (dy - dx);
    int x = x0;
    int y = y0;
    WritePixel (x, y, value);

    while (x < x1) {
        if (d <= 0) {
            d += incrE;
            x++;
        } else {
            d += incrNE;
            x++;
            y++;
        }
        WritePixel (x, y, value);
    } /* while */

} /* MidpointLine */
```

Scan Converting Circles : Midpoint Circle Algorithm

- Initial Value of d:
- $d_{start} = 5/4 - R$
- Say, $h = d - 1/4$
  $d = h + 1/4$
- $h + 1/4 = 5/4 - R$
  $h = 1 - R$
- $h = -1/4$ ✉ M is on the circle
  ✉ any pixel SE/E
  $h > -1/4$ ✉ M is outside the circle
  ✉ choose SE
  $h < -1/4$ ✉ M is inside the circle
  ✉ choose E

If h starts out with integer value and gets incremented by integer value (ΔE or ΔSE) then the comparisons,
$h = -1/4$, $h > -1/4$, $h < -1/4$
reduce to
$h=0$, $h>0$, and $h<0$

Scan Converting Circles : Midpoint Circle Algorithm

```
void MidpointCircle (int radius, int value)
/* Assumes center of circle is at origin */
{
    int x = 0;
    int y = radius;
    double d = 5.0 / 4.0 – radius;
    CirclePoints (x, y, value);

    while (y > x) {
        if (d < 0)              /* Select E */
            d += 2.0 * x + 3.0;
        else {                  /* Select SE */
            d += 2.0 * (x – y) + 5.0;
            y– –;
        }
        x++;
        CirclePoints (x, y, value);
    }  /* while */
}  /* MidpointCircle */
```

```
procedure MidpointCircle (radius, value: integer);
{Assumes center of circle is at origin. Integer arithmetic
var
    x, y, d : integer;
begin
    x := 0;                         {Initialization}
    y := radius;
    d := 1 – radius;
    CirclePoints (x, y, value);

    while y > x do
      begin
        if d < 0   then             {Select E}
          begin
            d := d + 2 * x + 3;
            x := x + 1
          end
        else
          begin                     {Select SE}
            d := d + 2 * (x – y) + 5;
            x := x + 1;
            y := y – 1
          end;
        CirclePoints (x, y, value)
      end   {while}
end;  {MidpointCircle}
```

Replacing h by d

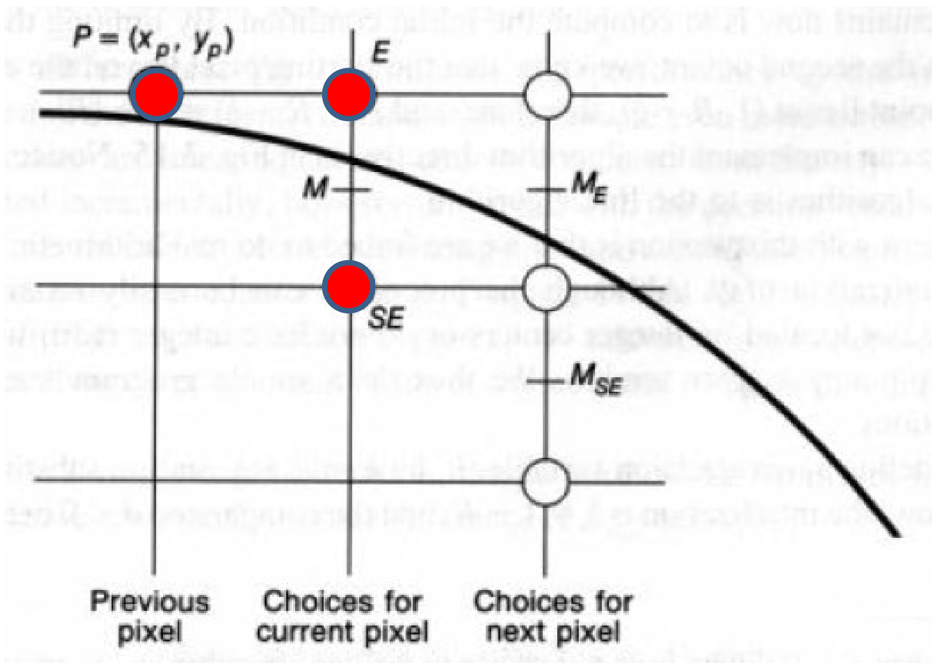
Scan Converting Circles : Midpoint Circle Algorithm

<u>Improvements by second order difference:</u>

- $\Delta E \quad = 2x_p + 3$
- $\Delta SE \quad = 2x_p - 2y_p + 5$

From the point of evaluation, $P(x_p, y_p)$

- E is chosen ✉ new point of evaluation is $E(x_p+1, y_p)$
- SE is chosen✉ new point of evaluation is $SE(x_p+1, y_p-1)$
- Find increments ΔE and ΔSE wrt new point of evaluation, $E(x_p+1, y_p)$, $SE(x_p+1, y_p-1)$

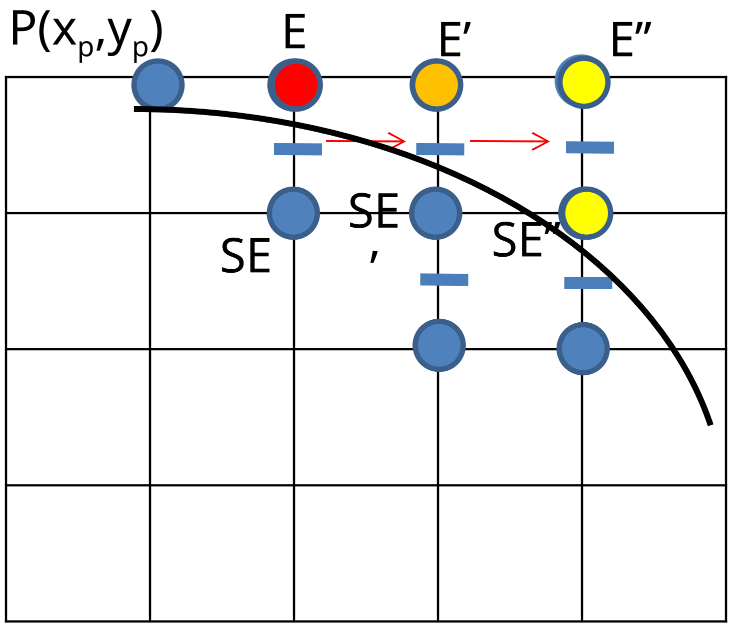


Second order difference wrt $E(x_p+1,y_p)$

- $P(x_p,y_p)$ $\rightarrow$ $\mathbf{\Delta E = 2x_p + 3 = \Delta E_{old}}$ Increment in **d** when E is chosen

  $\rightarrow$ $\mathbf{\Delta SE = 2x_p - 2y_p + 5 = \Delta SE_{old}}$ Increment in **d** when E is chosen

- $E(x_p+1,y_p)$ $\longrightarrow$ $\mathbf{\Delta E = 2(x_p + 1) + 3 = (2x_p + 3) + 2 = \Delta E_{old} + 2}$



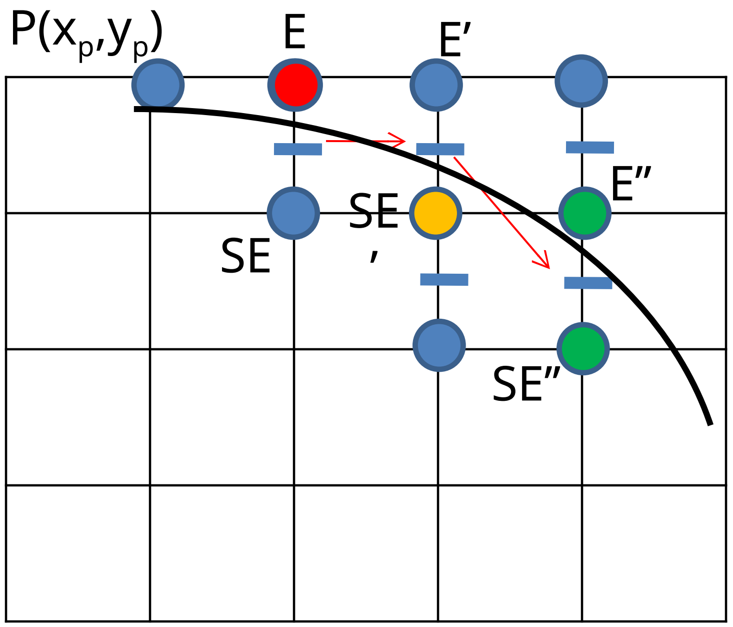
Second order difference wrt $E(x_p+1,y_p)$

- $$\begin{tikzcd}[row sep=small] & \mathbf{\Delta E= 2x_p + 3 = \Delta E_{old}} \\ P(x_p,y_p) \arrow[ur] \arrow[dr] & \\ & \mathbf{\Delta SE= 2x_p - 2y_p + 5= \Delta SE_{old}} \end{tikzcd}$$

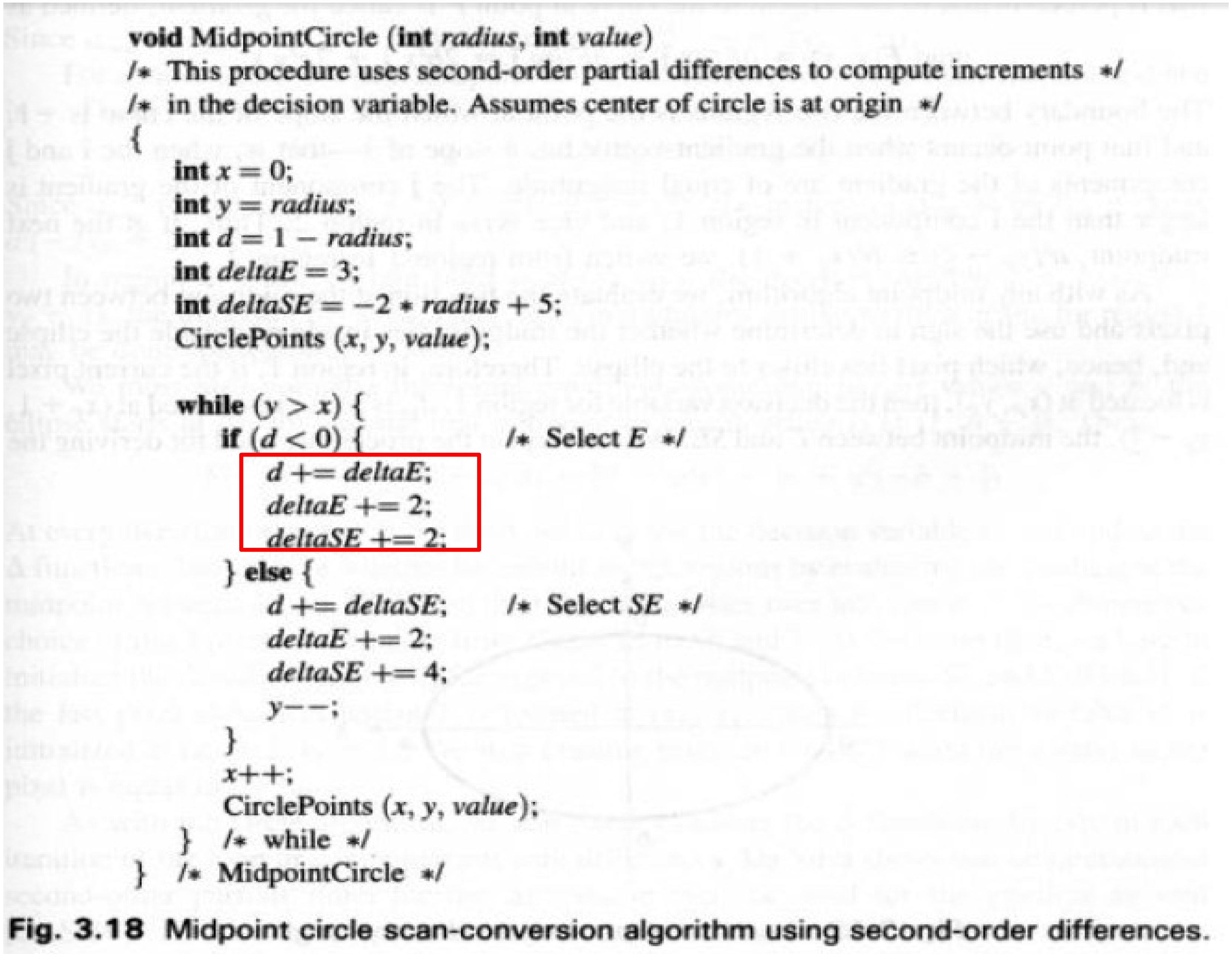
  Increment in **d** when E is chosen

  Increment in **d** when E is chosen

- $$\begin{tikzcd}[row sep=small] E(x_p+1,y_p) \arrow[r] \arrow[dr] & \mathbf{\Delta E= 2(x_p + 1) + 3 = (2x_p + 3) + 2 = \Delta E_{old} + 2} \\ & \mathbf{\Delta SE=2(x_p +1)- 2y_p + 5 = (2x_p - 2y_p + 5) + 2 = \Delta SE_{old} + 2} \end{tikzcd}$$



Second order difference wrt E(x_p+1,y_p)



```
void MidpointCircle (int radius, int value)
/* This procedure uses second-order partial differences to compute increments */
/* in the decision variable. Assumes center of circle is at origin */
{
    int x = 0;
    int y = radius;
    int d = 1 - radius;
    int deltaE = 3;
    int deltaSE = -2 * radius + 5;
    CirclePoints (x, y, value);

    while (y > x) {
        if (d < 0) {              /* Select E */
            d += deltaE;
            deltaE += 2;
            deltaSE += 2;
        } else {
            d += deltaSE;         /* Select SE */
            deltaE += 2;
            deltaSE += 4;
            y--;
        }
        x++;
        CirclePoints (x, y, value);
    } /* while */
} /* MidpointCircle */
```

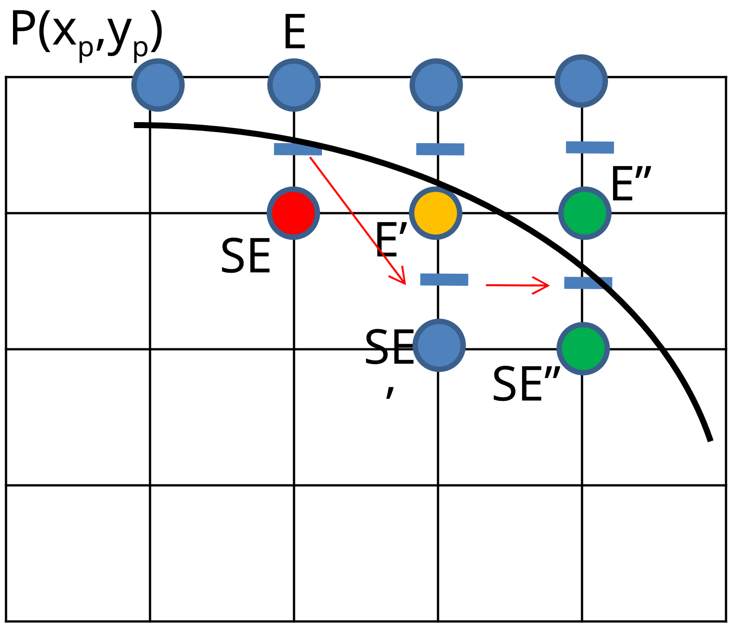
Fig. 3.18 Midpoint circle scan-conversion algorithm using second-order differences.

Second order difference wrt SE(x_p+1,y_p-1)

- P(x_p,y_p) $\rightarrow$ **$\Delta E = 2x_p + 3 = \Delta E_{old}$** Increment in **d** when E is chosen

  $\rightarrow$ **$\Delta SE = 2x_p - 2y_p + 5 = \Delta SE_{old}$** Increment in **d** when E is chosen

- SE(x_p+1,y_p-1) $\rightarrow$ **$\Delta E = 2(x_p + 1) + 3 = (2x_p + 3) + 2 = \Delta E_{old} + 2$**



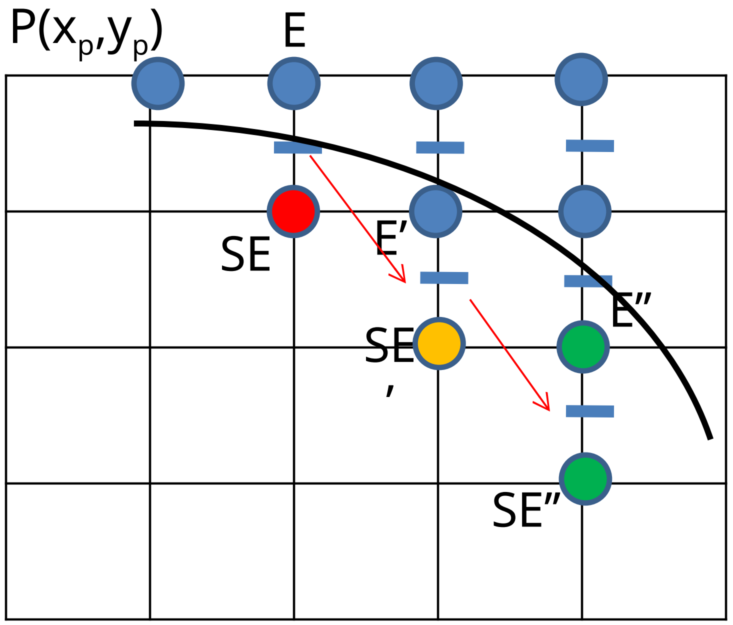
Second order difference wrt SE(x_p+1,y_p-1)

- P(x_p,y_p) $\rightarrow$ **$\Delta E = 2x_p + 3 = \Delta E_{old}$** Increment in **d** when E is chosen

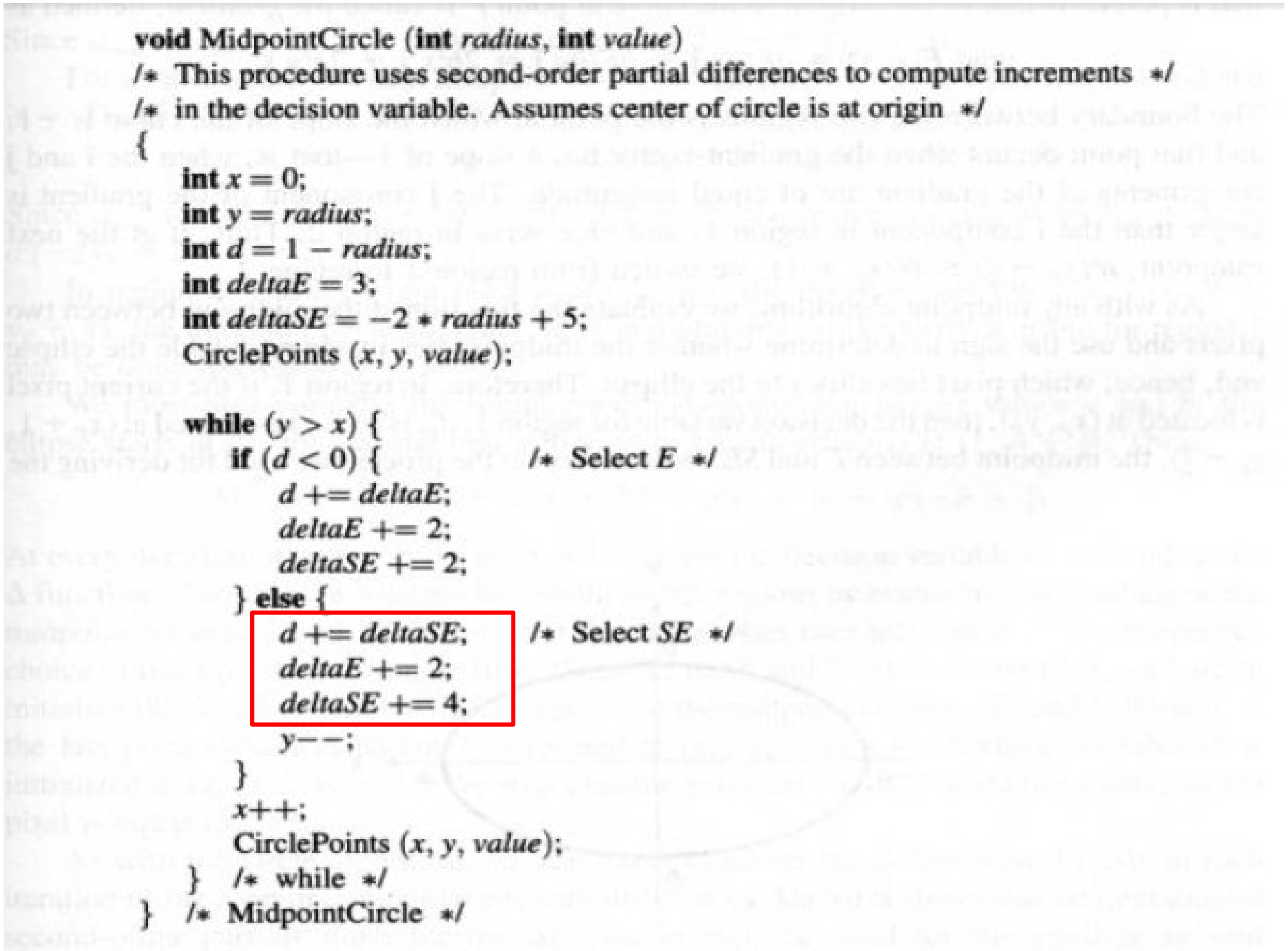
  $\rightarrow$ **$\Delta SE = 2x_p - 2y_p + 5 = \Delta SE_{old}$** Increment in **d** when E is chosen

- SE(x_p+1,y_p-1) $\rightarrow$ **$\Delta E = 2(x_p + 1) + 3 = (2x_p + 3) + 2 = \Delta E_{old} + 2$**

  $\rightarrow$ **$\Delta SE = 2(x_p+1) - 2(y_p-1)+5 = (2x_p - 2y_p+5)+2+2 = \Delta SE_{old} +4$**



Second order difference wrt SE(x_p+1,y_p-1)



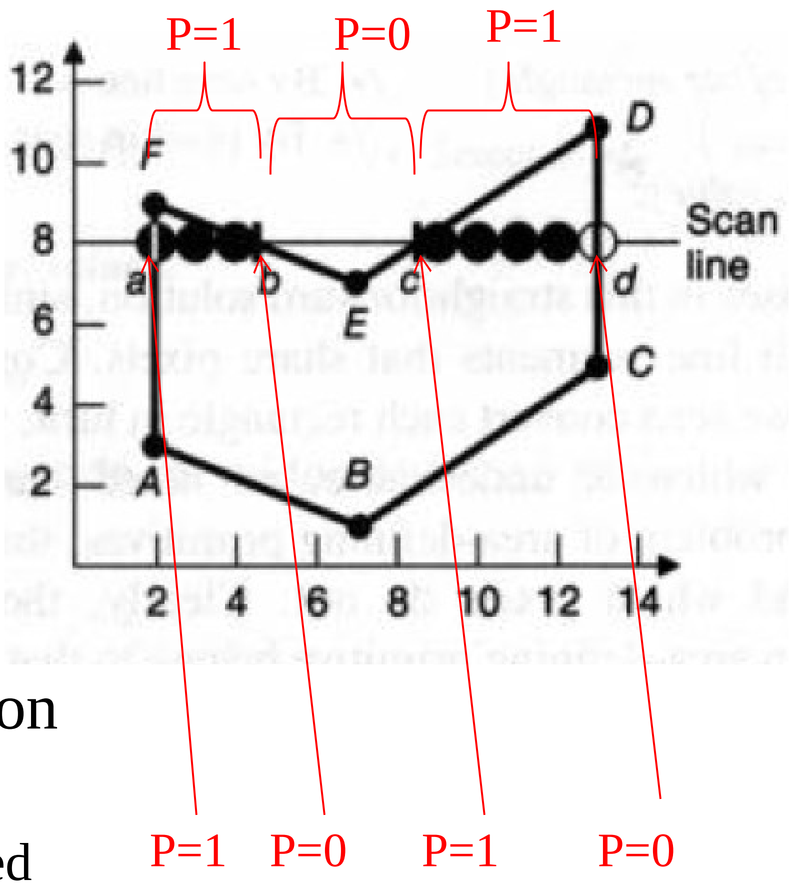
```
void MidpointCircle (int radius, int value)
/* This procedure uses second-order partial differences to compute increments */
/* in the decision variable. Assumes center of circle is at origin */
{
    int x = 0;
    int y = radius;
    int d = 1 - radius;
    int deltaE = 3;
    int deltaSE = -2 * radius + 5;
    CirclePoints (x, y, value);

    while (y > x) {
        if (d < 0) {                /* Select E */
            d += deltaE;
            deltaE += 2;
            deltaSE += 2;
        } else {
            d += deltaSE;           /* Select SE */
            deltaE += 2;
            deltaSE += 4;
            y--;
        }
        x++;
        CirclePoints (x, y, value);
    }   /* while */
}   /* MidpointCircle */
```

Fig. 3.18 Midpoint circle scan-conversion algorithm using second-order differences.

Polygon Scan Conversion

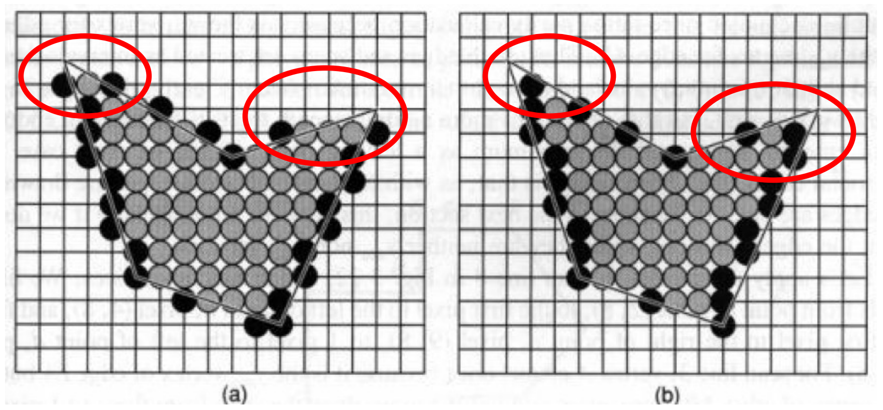
Steps:

1. Find the intersections of the scan line with the polygon edges
2. Sort the intersection points in increasing order of x coordinate
3. For each pair of intersections, use odd parity rule to draw the pixels that are interior to the polygon
   - Initially parity is even, P = 0
   - For each intersection, parity is changed
   - **Draw the pixels only when the parity is odd**



Polygon Scan Conversion

- Draw only those pixels which are strictly interior to the polygon



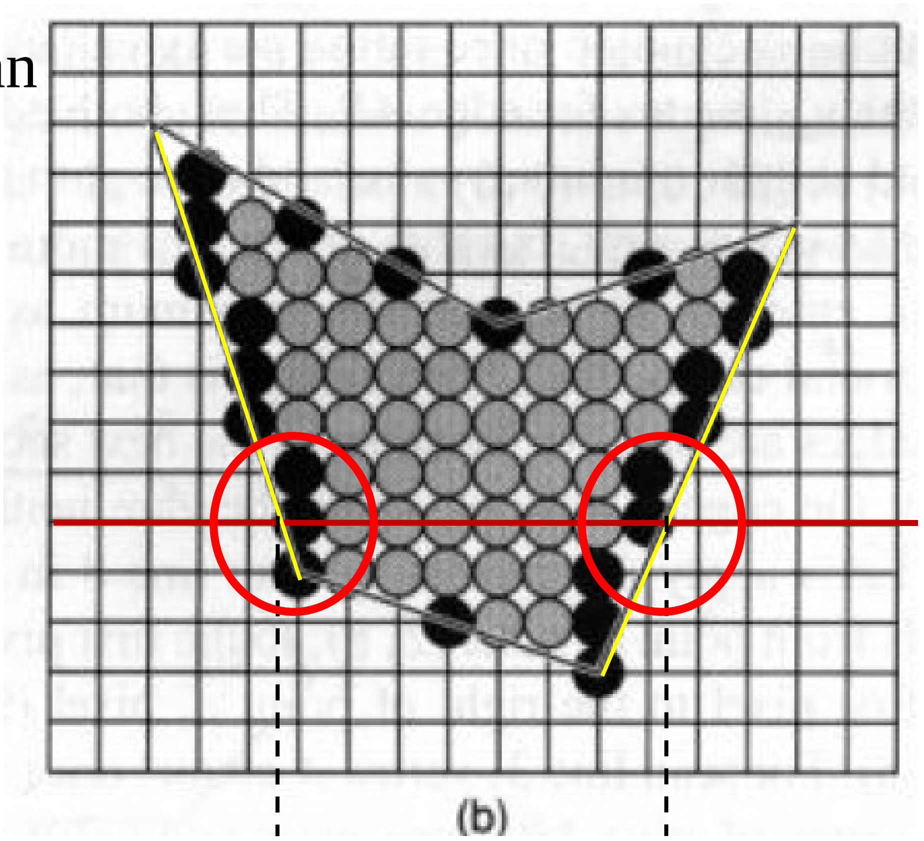
● Span extrema
● Other pixels in the span

- Cases to consider:
  1. Intersection point is fractional
  2. Intersection point is integer
  3. Integer intersection point that is a shared vertex
  4. Integer intersection points that define horizontal edge

Polygon Scan Conversion

1. Intersection point is fractional

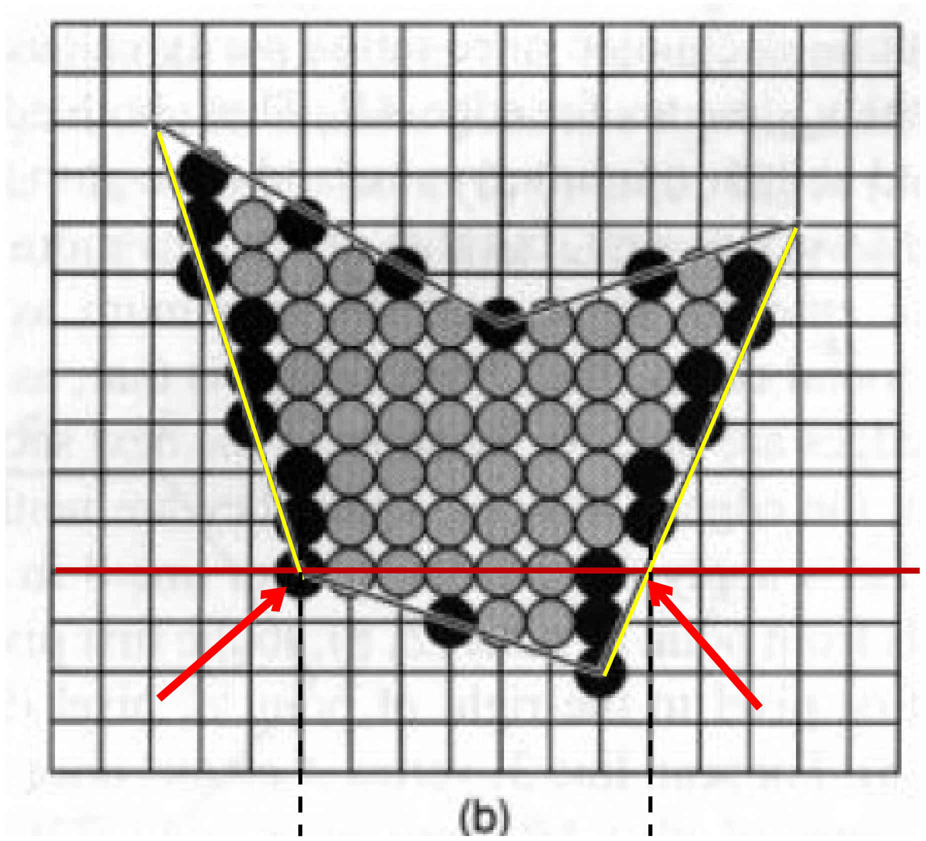
- If it is the left most of the span and before intersection we were outside the polygon, then round up
- If it is the right most of the span, and before intersection we were inside the polygon, then round down


(b)

Polygon Scan Conversion

2. Intersection point is integer

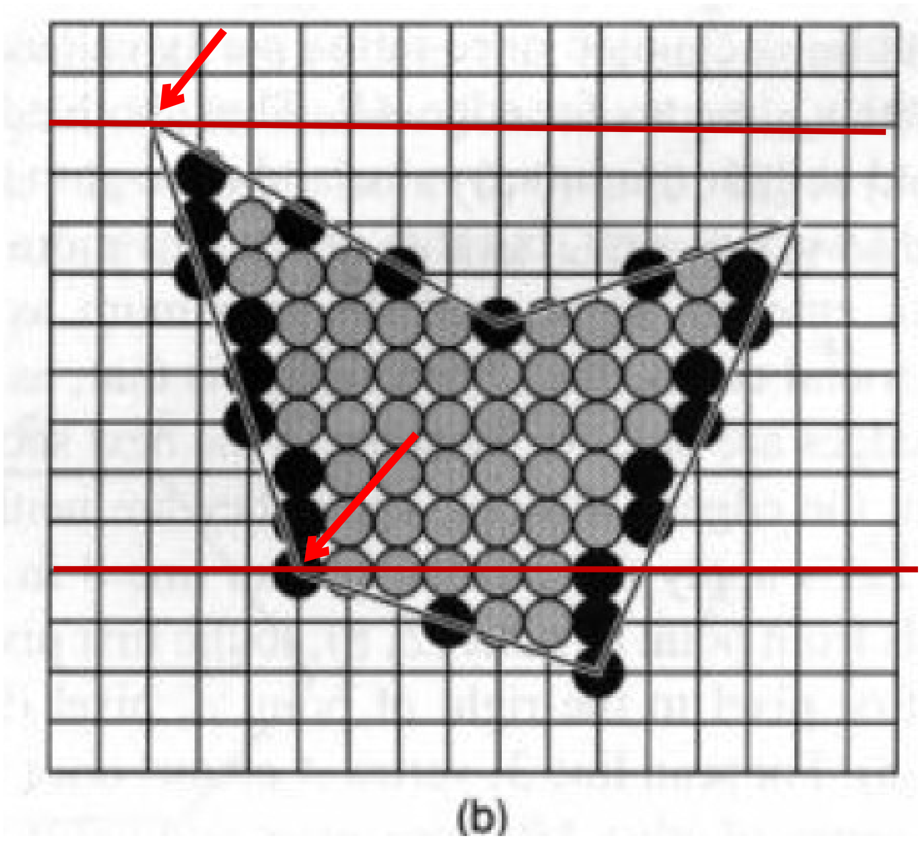
- If it is the left most of the span, draw it
- If it is the right most of the span, don't draw it



Polygon Scan Conversion

3. Integer intersection point that is a shared vertex

- Draw it only if it is y_{min} of an edge

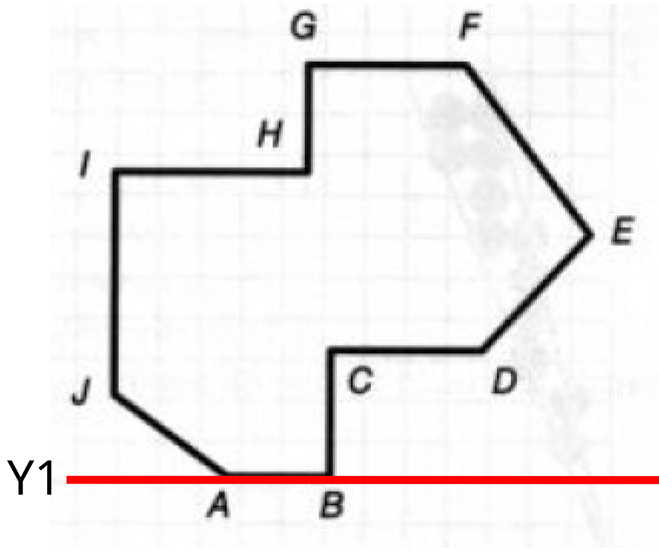


Polygon Scan Conversion

4. Integer intersection points that define horizontal edge

Scanline Y1:

- A is a Y_{min} of JA.
- A is drawn and parity becomes odd.
- Through out AB, parity remains odd, the span **AB is drawn.**
- B is Y_{min} of CB. Parity becomes even. Drawing stops.

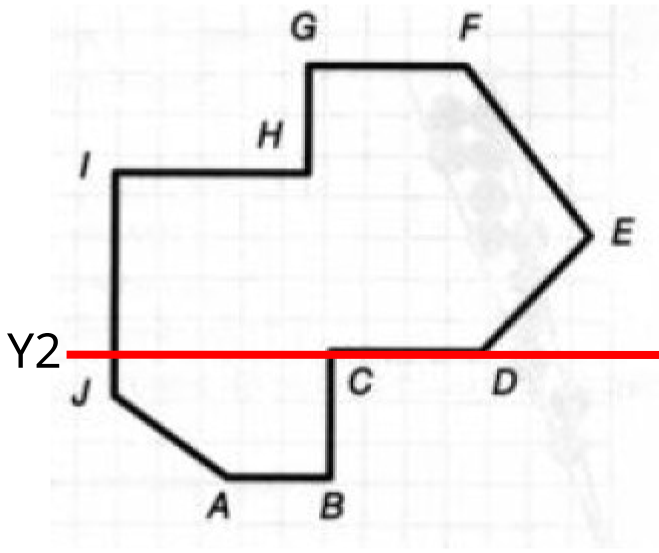


Polygon Scan Conversion

4. Integer intersection points that define horizontal edge

Scanline Y2:

- First Intersection point is drawn and parity becomes odd.
- Through out the span up to C, parity remains odd, the span is drawn.
- C is not Y_{min} of CD or BC. So C is not considered. Parity remains odd. Span **CD is drawn.**
- D is Y_{min} of DE. Parity becomes even. Drawing stops.

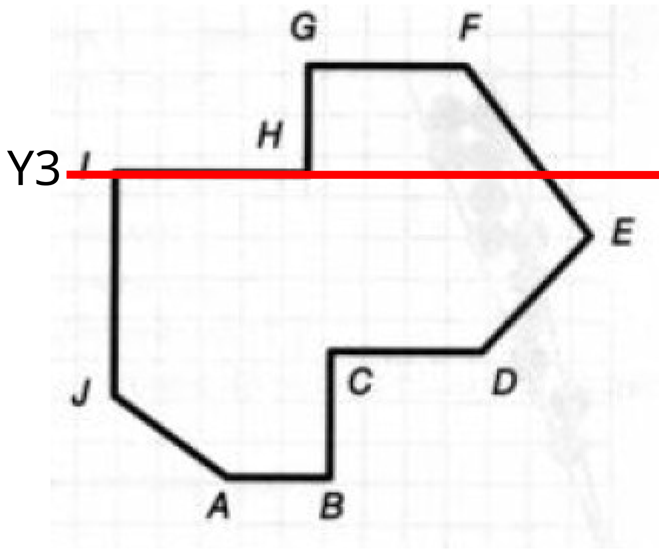


Polygon Scan Conversion

4. Integer intersection points that define horizontal edge

Scanline Y3:

- I is not Y_{min} of IH or IJ. So I is not considered. Parity remains even. Span **IH is not drawn.**
- H is Y_{min} of GH. So H is considered and Parity becomes odd. The span up to next intersection point is drawn.



Polygon Scan Conversion

4. Integer intersection points that define horizontal edge

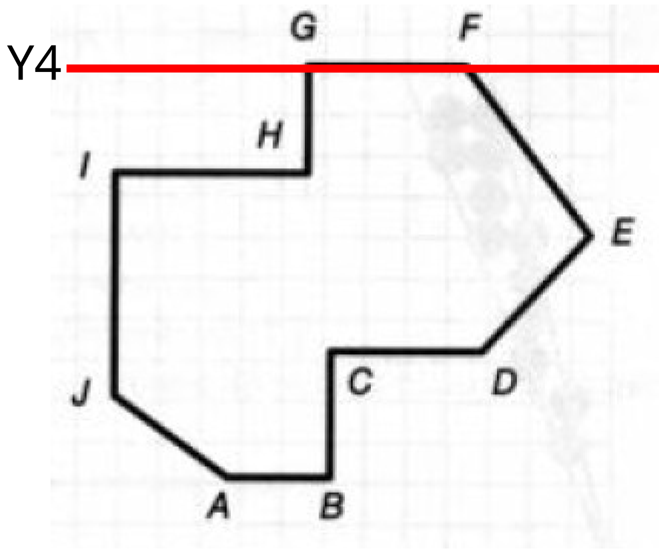
Scanline Y4:

- G is not Y_{min} of GF or GH. So G is not considered. Parity remains even. Span **GF is not drawn.**

Summary:

Top edges are not drawn

Bottom edges are drawn



Polygon Scan Conversion

- Silver : In case of thin polygon area, each scan line may not have a distinct span (e.g. a single pixel or no pixel)

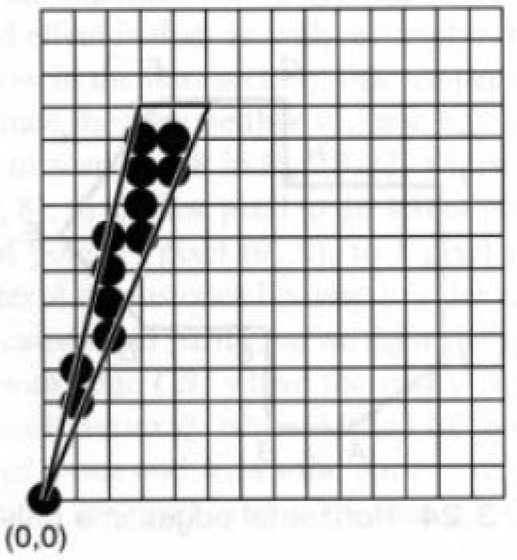


Fig. 3.25 Scan converting slivers of polygons.

Summary

- A Brief Idea of Rasterization
- Scan Conversion of Lines, Circles in Detail
  - Inefficient techniques and why they are inefficient
  - Efficient algorithms based on the position of midpoint calculation
- Scan Conversion of Filled Polygons
  - Challenges and some conventions

Thank you ⏪