L-2

Concepts of probability theory

Bayes Rule:

$$P(E_1 | E_2) = \frac{P(E_2|E_1)P(E_1)}{P(E_2|E_1)P(E_1) + P(E_2|E_1')P(E_1')}$$, where E_1 and E_1' are mutually exclusive and collectively exhaustive

Example:

Let us consider data transmission over a lossy wireless channel. The probability of loss is as follows: prob of loss while transmitting 0 is 0·05 and while transmitting 1 is 0·1. Besides, probability of transmitting 1 is 0·6 and transmitting 0 is 0·4. Now, we have received an 1. What is the probability that the actual sent bit was 0.

Soln:

$E_1 \rightarrow$ Event of transmitting a 0 $\Rightarrow P(E_1) = 0{\cdot}4$

$E_1' \rightarrow$ " " " an 1 $\Rightarrow P(E_1') = 0{\cdot}6$

$E_2 \rightarrow$ " " receiving an 1 $\Rightarrow P(E_2) \rightarrow$ Not known

$P(E_2 | E_1) = 0{\cdot}05$; $P(E_2|E_1') = 1 - P(E_2'|E_1') = 1 - 0{\cdot}1 = 0{\cdot}9$

Now, $$P(E_1|E_2) = \frac{P(E_2|E_1)P(E_1)}{P(E_2|E_1)P(E_1) + P(E_2|E_1')P(E_1')}$$

$$= \frac{0{\cdot}05 * 0{\cdot}4}{0{\cdot}05 * 0{\cdot}4 + \underbrace{\cancel{0{\cdot}05}}_{0{\cdot}9} * 0{\cdot}6} = \frac{1}{28}$$

Bernoulli trials: N independent repetitions of a random experiment, which has two possible outcomes

- Let P(success) = p, P(failure) = q

  Then, P(K successes in n trials) $= \binom{n}{k} p^k q^{n-k} = \binom{n}{k} p^k (1-p)^{n-k}$

- Example problem: TMR reliability computatn → (Exercise, do yourself!)

* Discrete random variable: Takes values from a set of discrete numbers ← 2 different types of defn

* Contn " " : Assigns a real number to an uncountable event space's events ←

* Probability Mass Function (pmf): Gives probability that the value of a discrete random variable X is equal to x. It is denoted by $f_X(x)$.

$$f_X(x) = P(X = x)$$

$$\sum_{x \in A} f_X(x) = 1$$

* Cumulative Distribution Function (CDF): Gives probability that the value of a random variable remains within a certain value. It is denoted by $F_X(x)$.

$$F_X(x) = P(X \leq x)$$

$$P(a < X \leq b) = F_X(b) - F_X(a)$$ [Prob of remaining within a range]

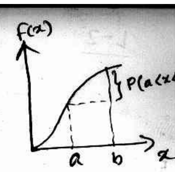
$$\lim_{x \to -\infty} F_X(x) = 0 \; ; \; \lim_{x \to +\infty} F_X(x) = 1,$$

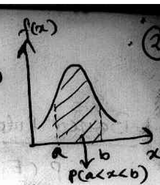
$$F_X(x) = \sum_{t \in A, t \leq x} f_X(t)$$ [for discrete var]

$$F_X(x) = \int_{-\infty}^{x} \underbrace{f(t)}_{\text{pmf}} dt$$ [for contn var]

* Probability ~~Mass~~ Density Function (pdf): Gives relative likelyhood of a random contn variable of taking a given value.

$$f_X(x) = \frac{P(X \le x+dx) - P(X \le x)}{dx}$$
$$= \frac{F(x+dx) - F(x)}{dx} = \frac{dF(x)}{dx}$$





Some simple random variables

($<, \le$ same for conts) $\Rightarrow P(a < x \le b) = \int_a^b f(t)\,dt$ [using pdf]
$= \int_{-\infty}^b f(t)dt - \int_{-\infty}^a f(t)dt = F(b) - F(a)$ [using cdf]

Discrete:

① Binomial: the # of successes in n independent trials (Bernoulli trial)

pmf $\rightarrow \binom{n}{k} p^k (1-p)^{n-k} = P(k)$ [cdf → not needed]

$\left[\sum_{k=0}^{n} P(k) = 1\right] \Rightarrow$ as $(p + (1-p))^n$

Example:

Vendor 1 → ~~ships~~ produces 1000 units where P(defective) = ·01

Vendor 2 → ~~ships~~ produces 2000 units where P(defective) = ·01

Let both ship 100 units each. Which shipment has the higher probability of having 5 defectives?

V1 → $\binom{100}{5} (\cdot 01)^5 (\cdot 99)^5$
V2 → $\binom{100}{5} (\cdot 01)^5 (\cdot 99)^5$
} Same!

② Geometric: the # of trials till the first success in Bernoulli trials

pmf → $P(X=k) = (1-p)^{k-1} p = P(k)$ | cdf → $1 - (1-p)^{k+1}$

P(k^{th} trial is the first success)

Example:

What is the prob of picking 10^{th} element in a shipment as its first defective?

$P(X=10) = (1-0{\cdot}01)^{10-1} \cdot (0{\cdot}01)^1$

~~③ Poisson~~ $\sum_1^\infty P(k) = \sum_1^\infty p(1-p)^{k-1} = p\sum_1^\infty (1-p)^{k-1} = p\sum_0^\infty (1-p)^k = p \cdot \frac{1}{1-(1-p)} = p \cdot \frac{1}{p} = 1$

E (# of trials till successes) $= \sum_1^\infty k\,p(k) = \sum_1^\infty k\,p(1-p)^{k-1}$

$= \sum_1^\infty k(1-q)q^{k-1} = \sum_1^\infty (kq^{k-1} - q^k k) = \sum_1^\infty kq^{k-1} - \sum_1^\infty kq^k$

$= \sum_0^\infty (t+1)q^t - \sum_1^\infty kq^k = \sum_0^\infty kq^k + \sum_0^\infty q^k - \sum_1^\infty kq^k = \sum_0^\infty q^k = \frac{1}{1-q} = 1/p$

③ Poisson: Gives the prob of k arrivals (or failures) (or events) in interval $(0, t]$

- Rate of arrival (constant) is λ. Here, a single parameter is $\alpha = \lambda t$.

pmf → $\frac{(\lambda t)^k}{k!} e^{-\lambda t}$ [k is the # of events] → k failures in time t

$R(t) = P(k = 0 \text{ at time } t) = e^{-\lambda t}$

Example: probability of k components failing in time λt

(4) Hypergeometric : Prob of k successes in n trials [success ⟷ defective]

- Gives probability of k defectives while m samples are drawn from a total of n items having d defectives

$$P(X=k) = \frac{\binom{d}{k}\binom{n-d}{m-k}}{\binom{n}{m}}$$

Prob of k successes from n trials with replacement ⟹ Binomial
" " k " " n " without " ⟹ Hypergeometric
as comes from independent trials

Cont^s :

(1) Exponential : ~~dis~~ Gives inter-arrival time where arrival follows a poisson process.

Example: Time to fail of a component, where failure occurs following a Poisson process.

$$X \sim Exp(\lambda)$$
$$f(x) = \lambda e^{-\lambda x}, \quad x > 0 \quad (pdf)$$
$$F(x) = 1 - e^{-\lambda x}, \quad x > 0$$

Memoryless property:

$$P(Y \le y \mid X > t) = P(X \le y), \text{ independent of } t$$

here, $Y = X - t$

So, $P(X \le y+t \mid X > t) = P(X \le y)$ [as $Y \le y \Rightarrow X - t \le y \Rightarrow X \le y+t$]

- Given a component has survives up to time t, what is the probability it will fail in $t+y$

Derivaⁿ:

$$P(X \le y+t \mid X > t) = \frac{P(t < X \le y+t)}{P(t < X)} = \frac{P(t < X \le y+t)}{1 - P(X \le t)}$$
$$= \frac{F(t+y) - F(t)}{1 - F(t)}$$
$$= \frac{(1 - e^{-\lambda(t+y)}) - (1 - e^{-\lambda t})}{1 - (1 - e^{-\lambda t})}$$
$$= \frac{e^{-\lambda t} - e^{-\lambda(t+y)}}{e^{-\lambda t}} = 1 - e^{-\lambda y} = P(X \le y)$$

Relⁿ betⁿ exp and Poisson:

Here, $P(\text{Component will fail in time } t) = F(t)$ following exp
$= 1 - P(\text{No failure in time } t) = 1 - P(N_t = 0 \text{ in poisson process})$
$= 1 - e^{-\lambda t} \cdot \frac{(\lambda t)^0}{0!} = 1 - e^{-\lambda t} \longrightarrow$ Exponential distrⁿ

Reliability: $R(t) = 1 - \text{Prob (Component fails in time } t)$

$$= 1 - F(t)$$

$$R'(t) = -f(t) \;; \text{ as } f(t) = F'(t)$$

Now,

P (A Component will not survive an additional time x | survived till time t)

$$= \frac{P(t < X < t+x)}{P(X > t)} = \frac{F(t+x) - F(t)}{R(t)} \quad \text{[similar to prev. derivaⁿ]}$$

Now, instantaneous ~~failure rate will be~~

$$h(t) = \lim_{x \to 0} \frac{F(t+x) - F(t)}{R(t)} \cdot \frac{1}{x}$$

$$= \frac{1}{R(t)} \lim_{x \to 0} \frac{F(t+x) - F(t)}{x} = \frac{1}{R(t)} \cdot \frac{d}{dx} F(t) = \frac{f(t)}{R(t)}$$

(*) For exponential distribuⁿ : $h(t) = \frac{f(t)}{R(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda$

(**) $h(t)\Delta t$: What is the prob of a component to ~~survives~~ fail in $(t, t+\Delta t]$, given that component has survived till t

$f(t)\Delta t$: What is the unconditional prob of a component to fail in $(t, t+\Delta t]$

Ques : $h(t) \; < / = / > \; f(t)$? [Hint: $h(t) = \lambda$, $f(t) = \lambda e^{-\lambda t}$]

Relationship betⁿ reliability and instantaneous failure rate

$$h(x) = \frac{f(x)}{R(x)} = -\frac{R'(x)}{R(x)}$$

$$\int_{-\infty}^{t} h(x)dx = \int_{-\infty}^{t} -\frac{1}{R(x)} \cdot \frac{dR(x)}{dx} \cdot dx = -\int_{-\infty}^{t} \frac{dR(x)}{R(x)} = -\left[\ln R(x)\right]_{-\infty}^{t}$$

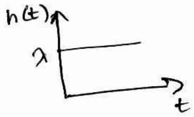
$$= -\left[\ln R(t) - \ln(R(-\infty))\right] = -\left[\ln R(t) - \ln 1\right] = -\ln R(t)$$

$$\ln R(t) = -\int_{-\infty}^{t} h(x)\,dx$$

$$\text{So, } R(t) = e^{-\int_{-\infty}^{t} h(x)dx}$$

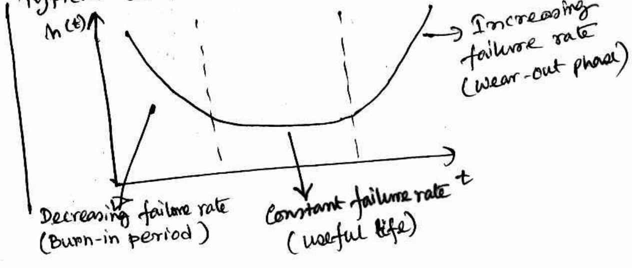
Failure Rate

With exponential case :



$$f(t) = \lambda e^{-\lambda t}$$
$$R(t) = e^{-\lambda t}$$
$$h(t) = \frac{f(t)}{R(t)} = \lambda$$

Typical computer failure rate distribuⁿ



(2) Weibull Distribution :

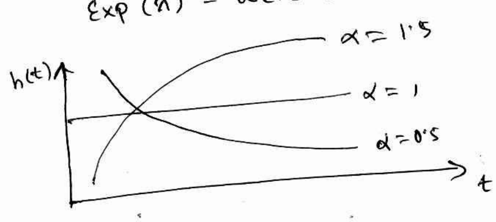
- The most widely used parametric family of failure distribu^{n}s

$\Rightarrow$ $f(t) = \lambda \alpha t^{\alpha-1} e^{-\lambda t^{\alpha}}$

$F(t) = 1 - e^{-\lambda t^{\alpha}}$ [Here, the only difference is involvement of α]

$h(t) = \dfrac{f(t)}{1-F(t)} = \dfrac{\lambda \alpha t^{\alpha-1} e^{-\lambda t^{\alpha}}}{e^{-\lambda t^{\alpha}}} = \lambda \alpha t^{\alpha-1}$

$Exp(\lambda) = weib(\lambda, 1)$



$\alpha \Rightarrow$ shape parameter
$\alpha < 1$, Infant mortality
$\alpha = 1$, constant hazard rate (exponential
$1 < \alpha < 4$, rising hazard (fatigue) } wear out
$\alpha > 4$, rising hazard (rapid wearout) }

- Proper choice of ~~a~~ the shape parameter can make it IFR, DFR, or CFR

~~Another two~~
Two more concepts

(1) Variance of X : $\sigma_x^2 = \int_{-\infty}^{+\infty} (x - E_x)^2 f(x) dx$
$= \sum_k (x_k - E_x)^2 f(x_k)$

$\rightarrow$ measure of spreading out of a set of ~~xx~~ numbers
$\hookrightarrow$ How far a number lie from the mean

(2) Covariance of X and Y : $\Psi_{X,Y} = E[(X - E_X)(Y - E_Y)] = E[XY] - E_X E_Y$

$\rightarrow$ measure of how much two RVs vary together
Both ↑ and ↓ in a same way $\rightarrow$ Cov is (+) ve
Both ↑ and ↓ " " reverse way $\rightarrow$ " " (−) ve
$\Rightarrow$ sign shows the tendency of linear relationship bet^n the variables
[$\rightarrow$ magnitude not easy to interpret;
~~xx~~ normalized val of cov $\Rightarrow$ gives strength of linear rel^{n}ship]

$\int u(x) v(x) dx = u(x) \int v(x) dx - \int \frac{d}{dx} u(x) \int v(x) dx \, dx$

Mean Time to Failure (MTTF)

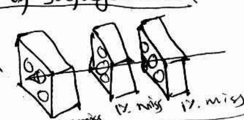
$E[T] = MTTF = \int_0^{\infty} R(t) dt = \int_0^{\infty} t f(t) dt$, as $E[x] = \int_{-\infty}^{\infty} x \underbrace{f(x)}_{pdf} dx$

Example : Exponential $\rightarrow$ MTTF = $1/\lambda$
Weibull $\rightarrow$ MTTF = $(1/\lambda)\,\Gamma(1 + 1/\alpha)$

Derivat^n
$MTTF = \int_0^{\infty} t f(t) dt$
$= \int_0^{\infty} t \frac{d}{dt} F(t) dt = -\int_0^{\infty} t \frac{d}{dt} R(t) dt$
$= [-tR(t) + \int R(t) dt]_0^{\infty} = \int_0^{\infty} R(t) dt$
0 as $R(t) \to 0$ if $t \to \infty$

Why are we ~~doing~~ going so deep? Why not simply use layers of safeguards?

$\rightarrow$ With multiple layers, a system fails only if warning symptoms and compensating actions are missed at every layer, which is quite unlikely.



10^{-6} miss probability

(*)(*) Multi~~pl~~layers increase the reliability significantly only if the "holes" are fairly randomly and uniformly distributed, so that the probability of their being aligned is ~~negl~~ negligible

Concepts related to failures & repairs

* Mean Time To Failure (MTTF): Expected time till the first failure
* Mean Time To Repair (MTTR): Expected time required to repair a failed module
* Mean Time Between Failure (MTBF): " " betn two consecutive failures
* Relnship: MTBF = MTTF + MTTR , where, $MTTF = 1/\lambda$ for exp

$MTTR = \frac{1}{\mu}$, where μ = repair rate

Now,

$$\text{availability} = \frac{MTTF}{MTTF+MTTR} = \frac{1/\lambda}{1/\lambda + \frac{1}{\mu}} = \frac{1}{1+\lambda/\mu} \text{ [if repair happens]?} = \frac{\text{\# of repairs}}{\text{time period for all repairs}}$$

?(availability = reliability [if no repair happens])

In case of a system either non-repairable, i.e., MTTR=0, or negligible repair time, i.e., MTTR << MTTF,

$$MTBF \approx MTTF = \int_0^\infty R(t)dt$$

Relnships between pdf, cdf, reliability, and hazard funcn

Expression in terms of	$f(t)$	$F(t)$	$R(t)$	$z(t)$
$f(t) =$	—	$\frac{dF(t)}{dt}$	$-\frac{dR(t)}{dt}$	$z(t)\, e^{-\int_0^t z(t)dt}$
$F(t) =$	$\int_0^t f(t)\,dt$	—	$1-R(t)$	$1-e^{-\int_0^t z(t)dt}$
$R(t) =$	$\int_t^\infty f(t)dt$	$1-F(t)$	—	$e^{-\int_0^t z(t)dt}$
$z(t) =$	$\frac{f(t)}{\int_t^\infty f(t)\,dt}$	$\frac{dF(t)/dt}{1-F(t)}$	$-\frac{d}{dt}\ln R(t)$	—

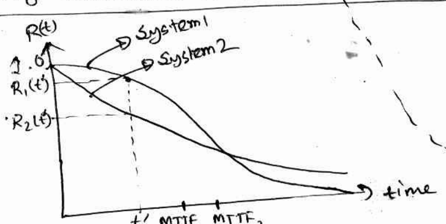
Comparing Reliabilities

(1) Reliability difference: $R_2 - R_1$ (2) Reliability gain: R_2/R_1 (3) Reliability improvement factor, $RIF_{2/1} = \frac{1-R_1(t_m)}{1-R_2(t_m)}$

Ex: $R_1(t_m) = 0.9$; $R_2(t_m) = 0.99$; $RIF_{2/1} = \frac{1-0.9}{1-0.99} = 10$

(4) Rel. impv. index: $RII = \frac{\log R_1(t_m)}{\log R_2(t_m)}$ [5] Mission time extension: $MTE_{2/1}(R_G) = T_2(R_G) - T_1(R_G)$ [6] Mission time imprv. factor $MTIF_{2/1}(R_G) = T_2(R_G)/T_1(R_G)$

Does comparing reliabilities indicate in the same way of comparing MTTFs?



$MTTF_2 > MTTF_1$

However, at $t=t'$, $R_1(t') > R_1(t')$

So, reliability can be much more imp. than MTTF ~~is~~ if mission time is known apriori [Ex: if mission time $\leq t'$, then System 1 is the better choice even though it has a lower MTTF]

Analog of Amdahl's law for Reliability:

Amdahl's law: If in a unit-time computan, a fracn f does not change and the remaining fraction 1-f is speeded up to run p times faster, then the overall speedup will be $S = \frac{1}{f + \frac{(1-f)}{p}}$

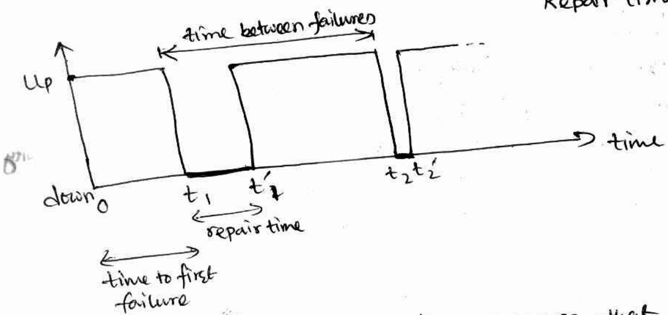
Reliability: Let a system has two parts having failure rates of ϕ and $\lambda - \phi$. ~~If~~ the second part is improved by a factor p to $\frac{\lambda-\phi}{p}$. So, $R_{org} = e^{-\lambda t}$, $R_{imp} = e^{-(\phi + \frac{\lambda-\phi}{p})t}$

$$RII = \frac{\log R_{org}}{\log R_{imp}} = \frac{\lambda}{\phi + (\lambda-\phi)/p} = \frac{1}{\phi/\lambda + (1-\phi/\lambda)/p} = \frac{1}{f+(1-f)/p} \text{ if } f = \frac{\phi}{\lambda}$$

[So, it is not ~~part~~ fraction of a system, rather ~~part~~ fraction of the failure rate of of a part]

Some other concepts

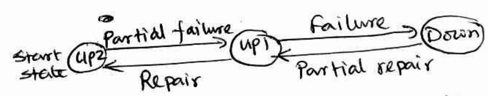
* Maintainability (serviceability) : Ability to maintain, which is related to repair time
Repair time ↓ : Good maintainability



Two-state repairable system

* Performability (P) : A composite measure that covers both performance and reliability

Example :
→ Worth of "Up2" is twice that of "Up1"
P_{up_i} = prob of being in state Up_i



Three-state dependable system

So, $P = 2 * P_{up_2} + P_{up_1}$ | $P_{up_2} = 0.92$, $P_{up_1} = 0.06$, and $P_{Down} = 0.02$

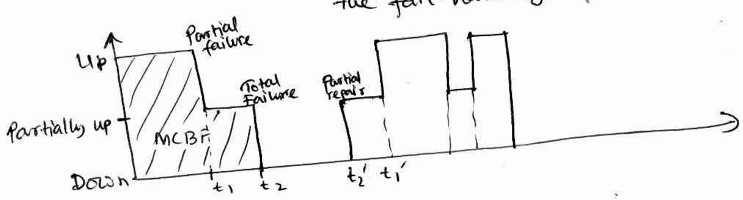
impact of performance
impact of reliability

= 1.90 ← system performance is equivalent to that of having 1.9 components on avg. [even though we are physically using 2 components]

⇒ Performability improvement factor of this system (similar to RIF) in comparison to a fail-hard system, which goes down when the system leaves up2 state.

$$PIF = (2 - \underbrace{2 \times 0.92}_{\text{performability of the fail-hard system}}) / (2 - 1.9) = 1.6$$

MCBF
(→ Mean Cycles Betn Failures)

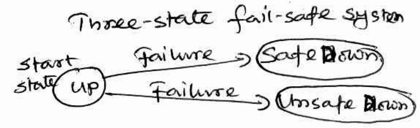


* Integrity and Safety : (Similar concepts)
→ Integrity is inward-looking : Capacity to protect system resources (e.g., data)
→ Safety is outward-looking : Consequences of incorrect actions to users
⇒ Safety is different from reliability : A fail-safe system may not be very reliable in the traditional sense

Basic safety assessment

→ Risk : Prob of being in "Unsafe down" state
⇒ Simple analysis : Merge "Up" and "Safe Down" states
⇒ More detailed " : Treat each state separately

Three-state fail-safe system



[There can be multiple unsafe states]

⇒ Quantifying safety:

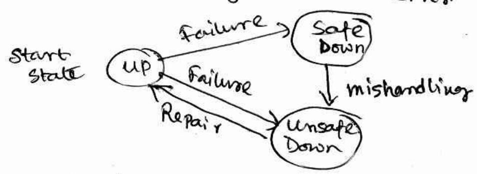
Risk (Consequence/Unit time) = Frequency (Events / Unit time) × Magnitude (Consequence/ ~~Unit time~~ Event)

Risk = Probability × Severity

→ Magnitude or severity is measured in suitable unit (ex: \$)

⇒ If there are multiple unsafe outcomes, the probability of each is multiplied by its severity (cost) and the results added up.

⇒ A safe failure can become unsafe or an unsafe failure can turn into a more severe safety problem due to mishandling or human error [can be modeled by adding appropriate transition]



* Privacy and Security: Human-related
  → Privacy: confidentiality and authorized access of data
  Security: Proper modificat^n of data

⇒ Security is distinct from reliability and safety: A secure system (automatically locks up when a breach is suspected) may not be very reliable or safe

⇒ Quantifying security:

Risk = Frequency × Magnitude

Risk = Probability × Severity

⇒ Security breaches are not generally suited for probabilistic treatment as such breaches are often not accidental.

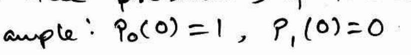
Markov modeling:

→ Markov Model: Analyzes systems having "Markovian Property", which indicates "memoryless" charateristic. The char can be described as:
- Future state of a system depends only on its current state.
- All transitions from one state to another occur at constant rates.

→ Significance: A large class of real-world devices (such as electronic components) exhibit constant failure rates, and therefore be effectively represented and analyzed using Markov models.

(⇒ Sometimes ~~vari~~ Markov modeling also covers variable failure rates)

⇒ Modeling approach:
- Consists a set of possible states, possible transition paths, and rate parameters of those transitions. Example:



- $P_j(t)$ denotes the probability of the system of being in state j at time t. Example: $P_0(0)=1$, $P_1(0)=0$

Now, $\underbrace{dP_0}_{\text{incremental change in the prob of being at state 0}} = -\underbrace{(P_0)}_{\text{the prob of being at state 0 at the beginning of interval}}\underbrace{(\lambda dt)}_{\text{The probability of transition during the interval } dt}$

So, $\frac{dP_0}{dt} = -\lambda P_0$

Now, $P_0 + P_1 = 1$ gives $\frac{dP_0}{dt} + \frac{dP_1}{dt} = 0$

So, $\frac{dP_1}{dt} = \lambda P_0$

$\ln P_0 = -\lambda t$; So, $P_0(t) = e^{-\lambda t}$

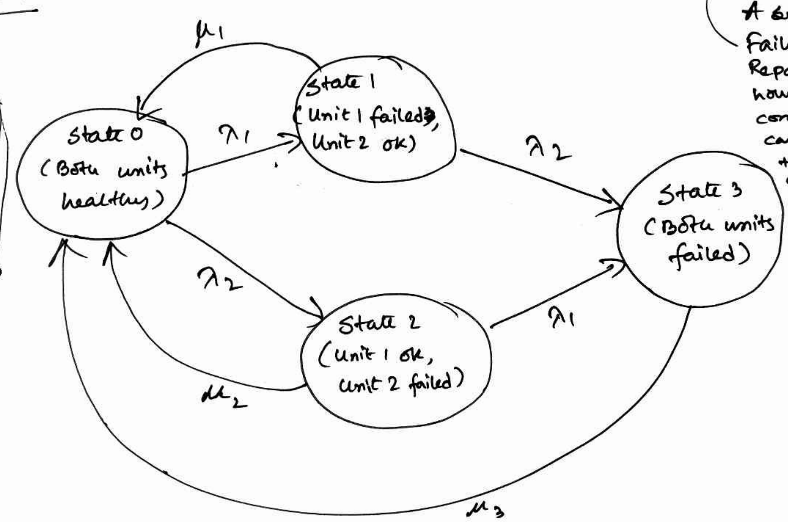
Therefore, $P_1(t) = 1 - e^{-\lambda t}$

Example :

never repaired
$\mu_3 = 0,\ P_3 > 0$

immediate repair
$\mu_3 = \infty,\ P_3 = 0$

in any case:
$\mu_3 P_3 = 0$



(Both in per hour)

A system having 2 components. Failure rates are '00005 and '00002. Repair time of unit 1 is 100 hours. System fails if both components fail. In that case an immediate repair takes place. In case of Unit 2 check occurs in each 1000 hours. What is the overall avg system failure rate?

periodic check; Here, unit 2 becomes operational at the end of the checking period.

$\lambda_1 = \cdot 00005$, $\lambda_2 = \cdot 00002$, $\mu_1 = 1/100$, $\underbrace{\mu_2 = 2/1000}$

as unit 2 has periodical checking, [assuming failure occurring in a uniform manner, i.e., same prob of failure occurring at the beginning & end of an interval]

time $= P(1+2+\cdots+999) = \frac{999 \cdot 1000}{2} \cdot 1/1000$

So, $\mu_2 = 1/\text{time} \approx 2/1000$

$\mu_3 = \cancel{0}\infty$, but $P_3 = 0$

Now,

$$(\lambda_1+\lambda_2)P_0 = \mu_1 P_1 + \mu_2 P_2 + \underbrace{\mu_3 P_3}_{\to 0 \text{ as } P_3 = 0} \quad \text{—— (1)}$$
$$\lambda_1 P_0 = (\lambda_2 + \mu_1) P_1 \quad \text{—— (2)}$$
$$\lambda_2 P_0 = (\lambda_1 + \mu_2) P_2 \quad \text{—— (3)}$$
$$\lambda_2 P_1 + \lambda_1 P_2 = \mu_3 P_3 \quad \text{—— (4)}$$

from (1), (4) $\Rightarrow (\lambda_1+\lambda_2)P_0 = (\mu_1+\lambda_2)P_1 + (\lambda_1+\mu_2)P_2$

(3) $\Rightarrow P_1 = \frac{\lambda_1 P_0}{\lambda_2 + \mu_1}$; (3) $\Rightarrow P_2 = \frac{\lambda_2 P_0}{\lambda_1 + \mu_2}$

Now, $P_3 = 0$

So, $P_0 + P_1 + P_2 = 1$

So, $P_0 + \frac{\lambda_1 P_0}{\lambda_2+\mu_1} + \frac{\lambda_2 P_0}{\lambda_1+\mu_2} = 1$

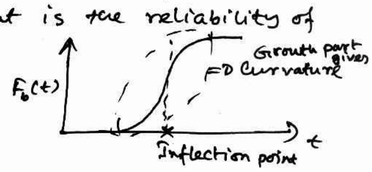
So, $P_0 = \frac{1}{1+\frac{\lambda_1}{\lambda_2+\mu_1}+\frac{\lambda_2}{\lambda_1+\mu_2}}$

Now, overall avg system failure rate will be

$$\lambda_{sys} = \lambda_2 P_1 + \lambda_1 P_2 = \frac{\lambda_2 \frac{\lambda_1}{\lambda_2+\mu_1} + \lambda_1 \frac{\lambda_2}{\lambda_1+\mu_2}}{1+\frac{\lambda_1}{\mu_1+\lambda_2}+\frac{\lambda_2}{\lambda_1+\mu_2}}$$

$= .000000579$ per hour

Example: A system consists of a device and a battery. pdf for the failure of the device is given by $f(t) = \lambda\alpha t^{\alpha-1} e^{-\lambda t^{\alpha}}$. ~~Reliability~~ CDF of the battery ~~follows~~ decaying follows a sigmoid curve having curvature a and inflection point b, i.e., $F_b(t) = \frac{e^{a(t-b)}}{1+e^{a(t-b)}}$. What is the reliability of the system.



Sol^n:

$X_d \sim$ Weibull (λ, α)

$X_b \sim$ Sigmoid (a, b)

Now $X_s \sim$ Min (X_d, X_b)

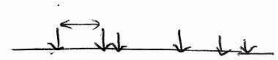
$R_s(t) = Pr\{X_s > t\} = Pr\{Min(X_d, X_b) > t\}$

$= Pr\{X_d > t\} \times Pr\{X_b > t\} = R_d(t) \times R_b(t)$

$= e^{-\lambda t^{\alpha}} \cdot \left(1 - \frac{e^{a(t-b)}}{1+e^{a(t-b)}}\right)$

$= \frac{1}{e^{\lambda t^{\alpha}} + e^{a(t-b)+\lambda t^{\alpha}}}$

Q: You are given ~~with~~ two independent computing systems. Let X_1 and X_2 are ~~denotes~~ two RVs that denote time lag between arrivals of two successive jobs in the two systems respectively. Here, the job arrivals follow two different Poisson distributions having rates of λ_1 and λ_2 respectively. Determine the probability of X_1 being less than X_2.



Ans: (Slide 14 in V20.pdf) [X_1, X_2 (interarrival time) → Exponential RVs as corresponding arrivals follow Poisson distributⁿ]

$P\{X_1 < X_2\}$

$= \int_0^\infty P\{X_1 < X_2 \mid X_1 = x\} f(x)\,dx = \int_0^\infty P\{X_1 < X_2 \mid X_1 = x\} \lambda_1 e^{-\lambda_1 x}\,dx$

$= \int_0^\infty P\{x < X_2\} \lambda_1 e^{-\lambda_1 x}\,dx = \int_0^\infty e^{-\lambda_2 x} \lambda_1 e^{-\lambda_1 x}\,dx = \int_0^\infty \lambda_1 e^{-(\lambda_1+\lambda_2)x}\,dx = \frac{\lambda_1}{\lambda_1+\lambda_2}$