CG

$$\boxed{\underline{s} = C_0 \underline{a} + C_1 \underline{b}}$$

Linear Combination.

$$\underline{a} \cdot \underline{b} = |a| \, |b| \cos\theta$$

$$\Rightarrow \cos\theta = \frac{\underline{a}}{|a|} \cdot \frac{\underline{b}}{|b|}$$

$$= \hat{a} \cdot \hat{b}$$

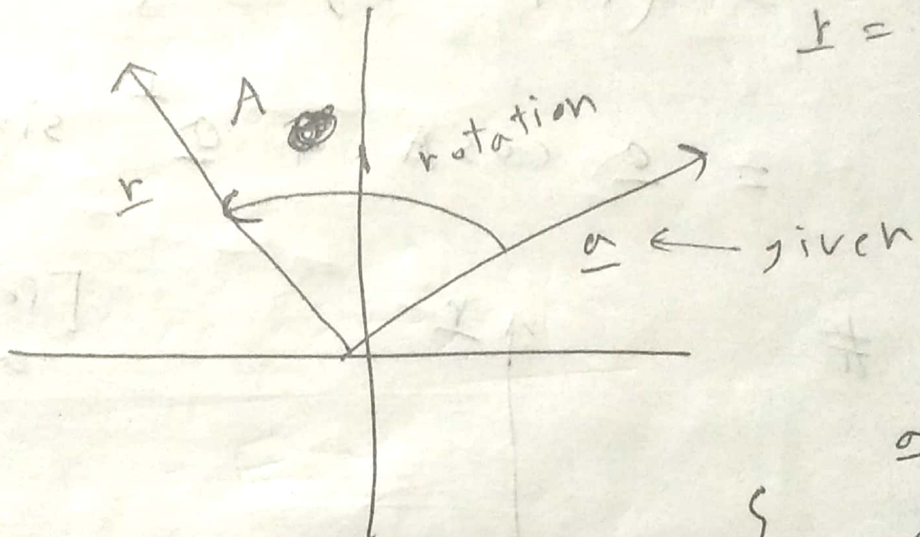
$\underline{a}$ given. We want $\underline{a}^{\perp}$ on 2D.

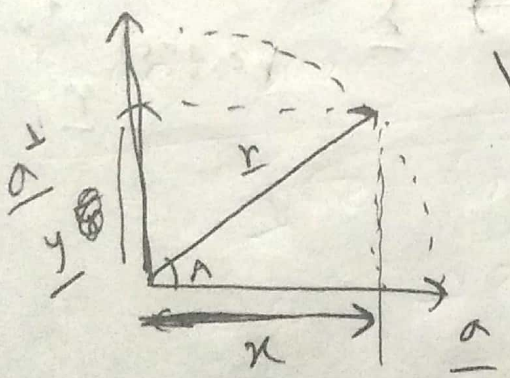
$\underline{a} \equiv (a_x, a_y)$

$\underline{a}^{\perp} \equiv (-a_y, a_x)$. or $(a_y, -a_x)$

#

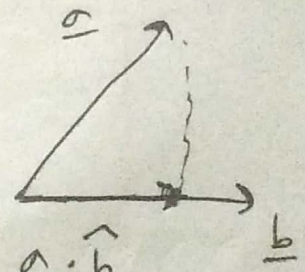
$\underline{r} = ?$





$|r| = |a|$

$\underline{r} = \underline{x} + \underline{y}$



$\underline{a} \cdot \hat{b}$ → projection

$(\underline{a} \cdot \hat{b})\,\hat{b}$ → component

$|x| = \underline{r} \cdot \hat{a}$

$\underline{r} = (\underline{r} \cdot \hat{a})\,\hat{a} + (\underline{r} \cdot \hat{a}^{\perp})\,\hat{a}^{\perp}$

$$\underline{r} = |r|\,\hat{r}$$
$$= |a|\,\hat{r}$$

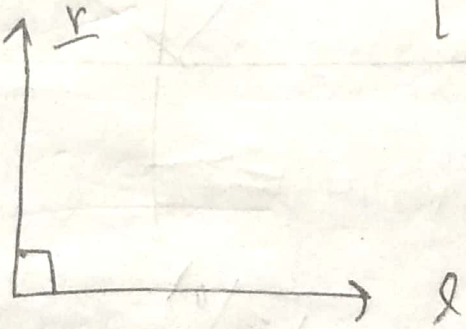
$$\therefore \underline{r} = (|a|\,\hat{r} \cdot \hat{a})\,\hat{a} + (|a|\,\hat{r} \cdot \hat{a}^{\perp})\,\hat{a}^{\perp}$$

$$= (|a| \cos A)\,\hat{a} + (|a| \cos(90^\circ - A))\,\hat{a}^{\perp}$$

$$= \underline{a} \cos A + \underline{a}^{\perp} \sin A$$

[Perpendicular axis rotation]

#



rotated with respect to $\underline{r}$

$$\underline{r} \xrightarrow{\text{Normalize}} \hat{r}$$

$$\underline{u} = \hat{r} \times \underline{l}$$

So, $\underline{u} \perp \underline{l}$ and $|u| = |l|$

Quiz → cube এর overlapping problem টা আসবে।

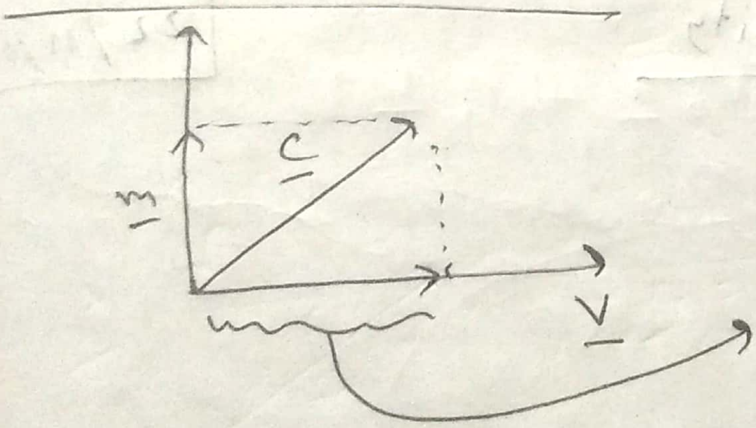
So, we got back to previous problem.

$$\underline{z} = \underline{l}\cos A + \underline{u}\sin A$$

$$= \underline{l}\cos A + (\hat{r} \times \underline{l})\sin A$$

for counter-clockwise

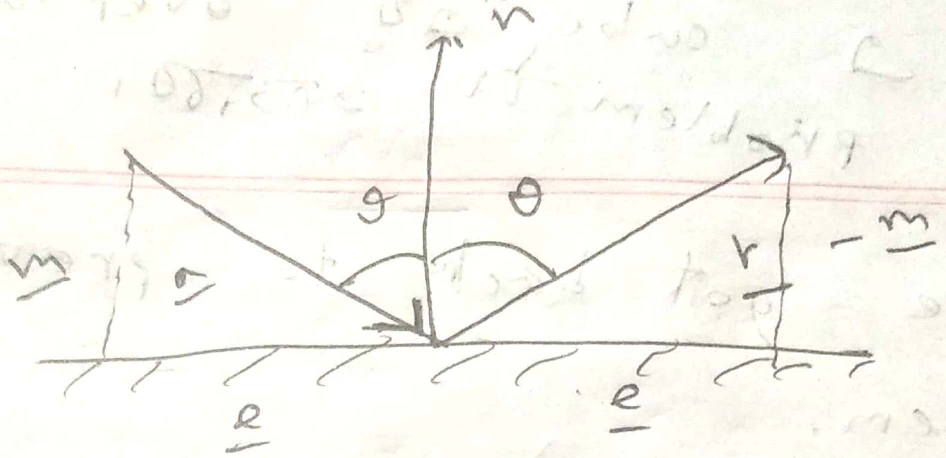
[For clockwise, use $(\underline{l} \times \hat{r})$]



$$(\underline{c} \cdot \hat{v})\hat{v}$$

$$(\underline{c} \cdot \hat{v})\hat{v} + \underline{m} = \underline{c}$$

$$\therefore \underline{m} = \underline{c} - (\underline{c} \cdot \hat{v})\hat{v}$$



$$\underline{m} = (\underline{a}\cdot\hat{n})\,\hat{n}$$

$$\underline{e} = \underline{a} - (\underline{a}\cdot\hat{n})\,\hat{n}$$

$$\underline{r} = \underline{e} - \underline{m}$$

$$= \underline{a} - 2(\underline{a}\cdot\hat{n})\,\hat{n}$$

CG Tutorial

$a \times b = - b \times a$

Parametric representation of a line:

$$P + V = Q$$

$$L = P + tV$$

$t \longrightarrow$ Parameter
(Line এর উপর point এ যাওয়া যাবে)

$L \downarrow L(t)$

$$\text{Plane} = P + c_0 \underline{a} + c_1 \underline{b}$$

$\downarrow$

$P(c_0, c_1)$

$\underline{a}$ and $\underline{b}$ cannot be parallel

$$L(t) = (0, 2) + t(2, 3)$$

Point টা fix না করলে $c_0 \underline{a} + c_1 \underline{b}$ এর parallel যেকোনো plane ~~[illegible]~~ হত, এজন্য Point fix করা হচ্ছে।

- normal fix হলে, point fix থাকলেও plane fix হয়।

$$\underset{A,\ C,\ B}{\text{Three point}} \underset{\underline{a} = A - C,\quad \underline{b} = B - C}{\xrightarrow{}} \underset{C,\ \underline{a},\ \underline{b}}{\text{Parametric}}$$

$$\underset{A,\ C,\ B}{\text{Three point}} \xleftarrow{\begin{array}{c} C + \underline{a} = A \\ C + \underline{b} = B \end{array}} \underset{C,\ \underline{a},\ \underline{b}}{\text{Parametric}}$$

$$\underset{C,\ \underline{a},\ \underline{b}}{\text{Parametric}} \xrightarrow{\underline{n} = \underline{a} \times \underline{b}} \underset{C,\ \underline{n}}{\text{Point Normal}}$$

$$\underset{C,\ \underline{a},\ \underline{b}}{\text{Parametric}} \xleftarrow{} \underset{C,\ \underline{n}}{\text{Point Normal}}$$

$$\underline{a} = \underbrace{(0,0,1)}_{\text{random vector (just cannot be parallel to } \underline{n})} \times \underline{n}$$

~~b = (another one) × a~~

$$\underline{b} = \underline{n} \times \underline{a}$$

$$\underset{A,\ C,\ B}{\text{Three point}} \xrightarrow{\underline{n} = (A-C) \times (B-C)} \underset{C,\ \underline{n}}{\text{Point Normal}}$$

Let, plane intersects z-axis at Z

$$(C - Z) \cdot \underline{n} = 0 \Rightarrow C \cdot \underline{n} - \underbrace{Z \cdot \underline{n}}_{(0,0,z) \cdot (n_x, n_y, n_z) = z n_z} = 0$$

$\therefore c.\underline{n} = z n_z$

$\therefore z = \dfrac{c.\underline{n}}{n_z}$

$\therefore A = \left(0, 0, \dfrac{c.\underline{n}}{n_z}\right)$

$B = \left(0, \dfrac{n.c}{n_y}, 0\right)$

যদি যেকোনো
এর axis এর
সমান হয়,
তবে z-axis
e.g. যেখানে
আছে
(0, 0, 5)।
তাহলে
যেকোনো
(a, b, 5) ই
plane এ
থাকবে।

$ax + by + c = 0 \longrightarrow$ Line

$ax + by + cz + d = 0 \longrightarrow$ plane

if, $\sqrt{a^2 + b^2 + c^2} = 1$, it is called normalized form and $|d|$ equals distance of the plane frome the origin.

25/11/23

CG

From point-normal to equation:

$P(x, y, z)$ any point on plane

$P_1(x_1, y_1, z_1)$ Given Point

$\bar{n} = (a, b, c)$

$\overrightarrow{PP_1} = P - P = (x - x_1, y - y_1, z - z_1)$

$\overrightarrow{PP_1} \cdot \bar{n} = 0$

$\Rightarrow (x - x_1, y - y_1, z - z_1) \cdot (a, b, c) = 0$

$\Rightarrow a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

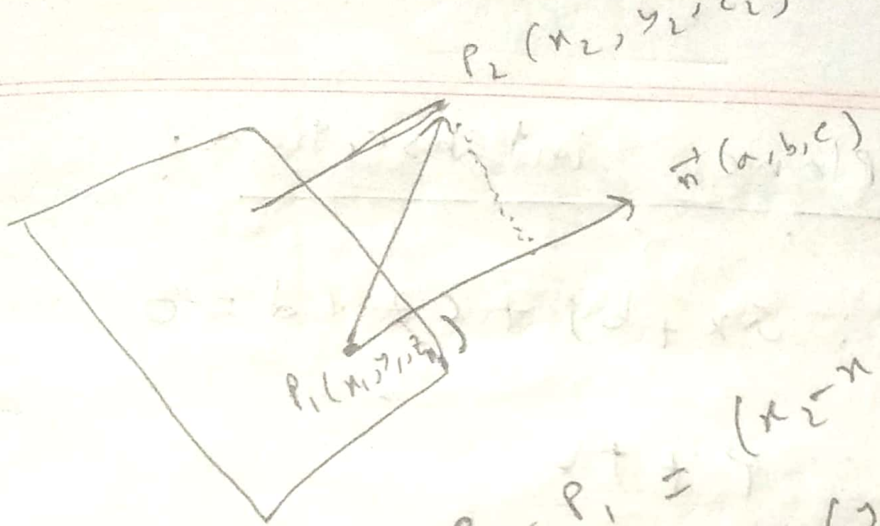
$\therefore ax + by + cz = ax_1 + by_1 + cz_1$

$= -d$ (Let)

$\therefore \underline{ax + by + cz} + \underline{d} = 0$

normal এর info contain করে

Point এর info contain করে।



$$P_2 - P_1 = (x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1)$$

$$\overrightarrow{P_2P_1} \cdot \hat{n} = \frac{a(x_2 - x_1) + b(y_2 - y_1) + c(z_2 - z_1)}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \cancel{a(x_2 + y_2 +}$$

$$= \frac{(ax_2 + by_2 + cz_2) - (ax_1 + by_1 + cz_1)}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{|ax_2 + by_2 + cz_2 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

just like distance of a line from a point.

$$= |n_x x_2 + n_y y_2 + n_z z_2 + d|$$

So, from origin, distance $= d$

Line-Plane intersection:

Plane: $ax + by + cz + d = 0$

Line: $P + tV$

#

$x + 2y + 3z + 4 = 0$

$(1, 2, 3) + t(3, 2, 1)$

These will be satisfied

$\therefore x = 1 + 3t$

$y = 2 + 2t$

$z = 3 + t$

$1 + 3t + 2(2 + 2t) + 3(3 + t) + 4 = 0$

$\Rightarrow 10t + 18 = 0$

$\therefore t = -1.8$

$P \equiv (-4.4, -1.6, 1.2)$

CamScanner

$\boxed{0 = 2}$ এই জাতীয় উত্তর answer আসলে parallel।

$\boxed{0 = 0}$ এই জাতীয় answer আসলে line টা plane এর উপরে।

Line - Line Intersection.

$$L_1 : P_1 + tV_1 = (x_1, y_1, z_1) + t(v_{11}, v_{12}, v_{13})$$

$$L_2 : P_2 + sV_2 = (x_2, y_2, z_2) + s(v_{21}, v_{22}, v_{23})$$

if parallel, $\overline{v}_1 = k\overline{v}_2$.

For intersecting point,

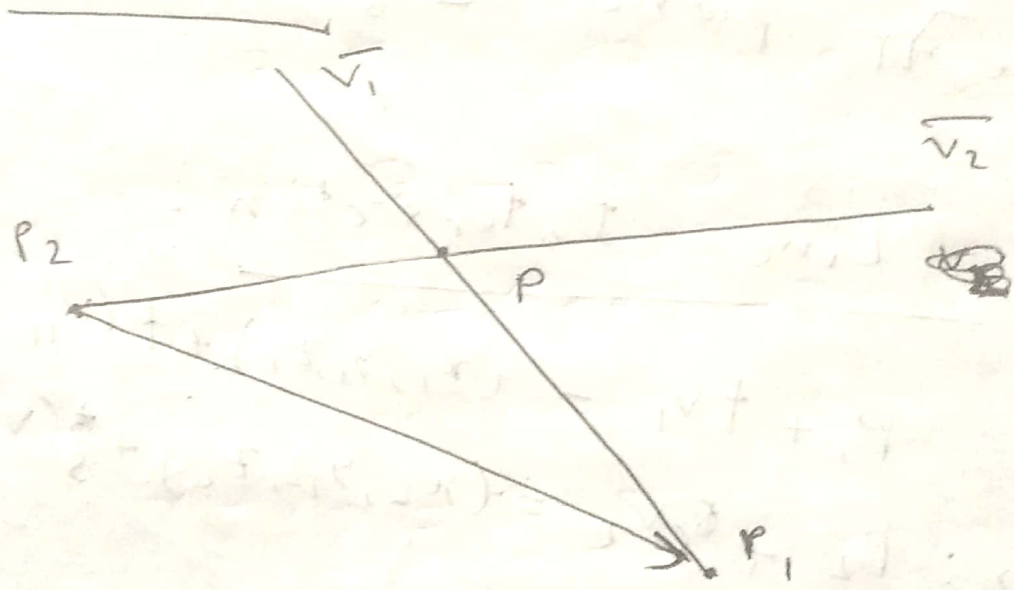
$$(x_1, y_1, z_1) + t(v_{11}, v_{12}, v_{13}) = (x_2, y_2, z_2) + s(v_{21}, v_{22}, v_{23})$$

$$x_1 + tv_{11} = x_2 + sv_{21} \quad \ldots (i)$$

$$y_1 + tv_{12} = y_2 + sv_{22} \quad \ldots (ii)$$

$$z_1 + tv_{13} = z_2 + sv_{23} \quad \ldots (iii)$$

i and ii solve and put in iii to check.
If all satisfied, lines intersect.

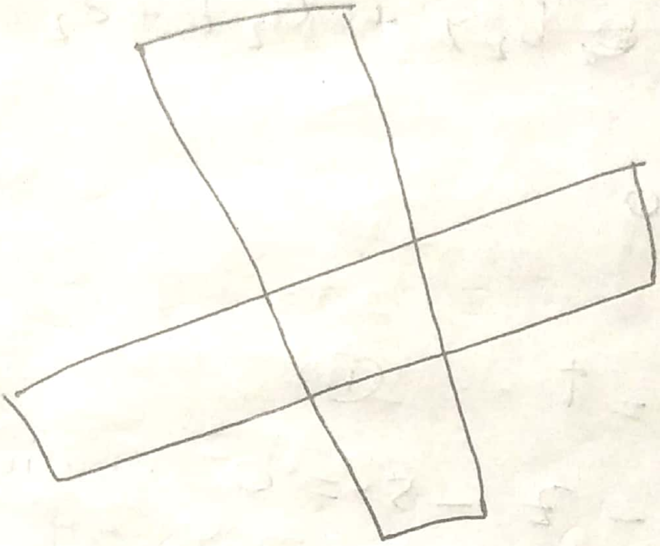


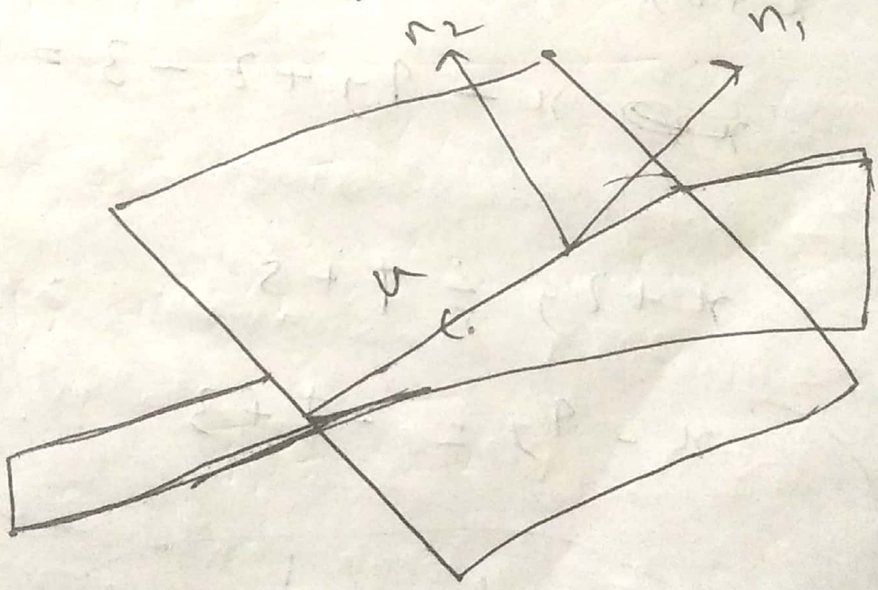
$$(P_2 - P_1) \cdot (v_1 \times v_2) = 0$$

Then, they intersects.

As, $P_2 - P_1$, $\vec{v_1}$, $\vec{v_2}$ makes a plane if they intersect.

Plane - Plane intersection:





$$\vec{u} = \vec{n}_1 \times \vec{n}_2$$

$P_1 : a_1 x + b_1 y + c_1 z + d_1$

$P_2 : a_2 x + \cancel{\;} b_2 y + c_2 z + d_2$

$P_3 : z = 0$

#

$z = t \quad \ldots \text{(i)}$

$x + 2y - z - 5 = 0 \quad \ldots \text{(ii)}$

$\cancel{x} \; x - 4y + z - 3 = 0 \quad \ldots \text{(iii)}$

$x + 2y = t + 5 \quad \ldots \text{(iv)}$

$x - 4y = -t + 3 \quad \ldots \text{(v)}$

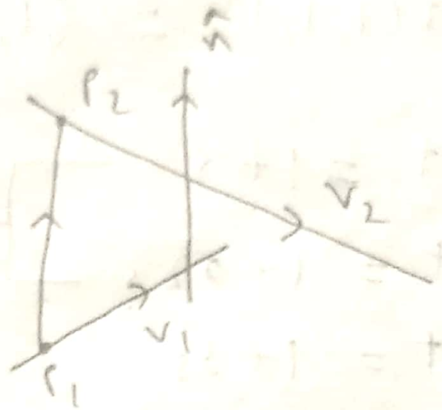
$y = \frac{1}{3} t + \frac{1}{3}$

$x = \frac{1}{13} (3t + 13)$

$L : (x, y, z) = \cancel{\tfrac{1}{13}} (1, \frac{1}{3}, 0) + t(\frac{3}{13}, \frac{1}{3}, 1)$

* $L_1: P_1 + t_1 V_1$, $L_2: P_2 + t_2 V_2$

L_1, L_2 is not parallel and do not intersect. Need to calculate shortest distance.



make it normalized

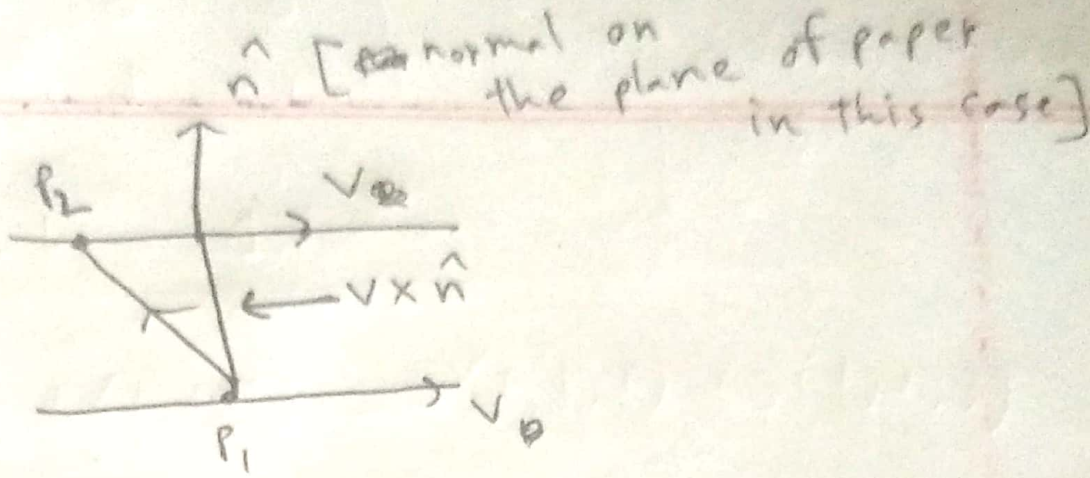
$\hat{n} = V_1 \times V_2$ [Perpendicular on both lines]

$\overrightarrow{P_1P_2} = P_2 - P_1$ [vector from L_1 to L_2]

$d = \overrightarrow{P_1P_2} \cdot \hat{n}$ [Projection of $\overrightarrow{P_1P_2}$ on normal]

* $L_1: P_1 + t_1 V$, $L_2: P_2 + t_2 V$

L_1 and L_2 parallel.



$\hat{n} = \overrightarrow{P_1P_2} \times V$ [normal on the plane]

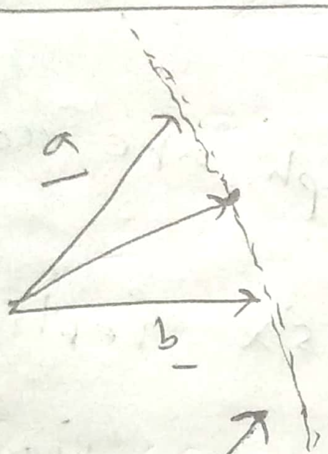
$\hat{l} = \hat{n} \times V$ [perpendicular on V and lies on the plane]

$d = \overrightarrow{P_1P_2} \cdot \hat{l}$ [Projection]

28/11/23

CG

Linear Combination of vector :



$$\underline{r} = c_0 \underline{a} + c_1 \underline{b}$$

if, $c_0 + c_1 = 1$, Affine combination

Affine combination এর হলে, তবে $\underline{r}$ একই line এর উপর পর্যন্ত থাকবে।

এইটাই

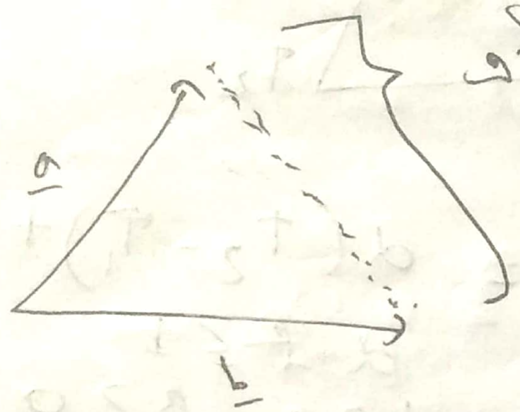
$$\underline{r} = (1 - c_1)\, \underline{a} + c_1 \underline{b}$$

$$= \underbrace{\underline{a} + c_1 (\underline{b} - \underline{a})}_{\text{Equation of a line}}$$

Convex Combination

$$c_0 + c_1 = 1 \quad \dots \text{(i)}$$

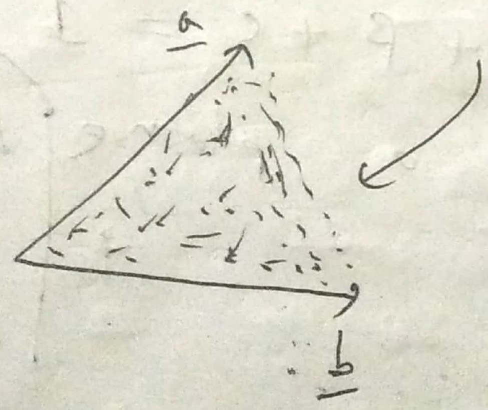
$$0 \le c_0 \text{ and } 0 \le c_1 \quad \dots \text{(ii)}$$



এইটুকুর এর
মধ্যেই থাকবে

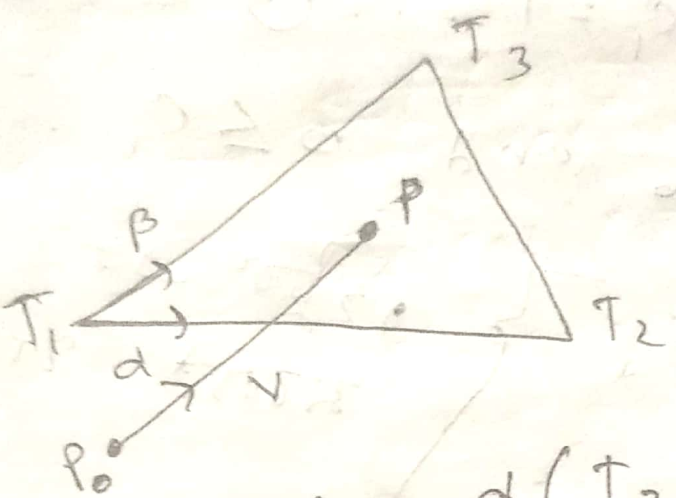
If, $c_0 + c_1 \le 1$

$0 \le c_0$ and $0 \le c_1$



এর মধ্যে
সব point

Ray - Triangle Intersection :



$$P = \alpha(T_2 - T_1) + \beta(T_3 - T_1)$$

$$\alpha + \beta \le 1$$

$$\alpha \ge 0, \quad \beta \ge 0$$

Barycentric Triangle :

$$P(\alpha, \beta, \gamma) = \alpha a + \beta b + \gamma c$$

with $\alpha + \beta + c = 1$

We will get a plane

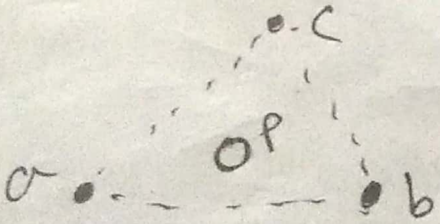
AND

$0 < \alpha < 1$

$0 < \beta < 1$

$0 < \gamma < 1$

$\underbrace{\hphantom{0 < \gamma < 1}}$ Inside the triangle



$$p(\beta, \gamma) = (1-\beta-\gamma)a + \beta b + \gamma c$$

$$= a + \beta(b-a) + \gamma(c-a)$$

$$p(t) = p(\beta, \gamma)$$

$$p_0 + t \cdot \underline{v} = a + \beta(b-a) + \gamma(c-a)$$

t, β, γ variables.

x, y, z coordinate -এ তিন. হবে solve হবে।

$$\boxed{\beta + \gamma < 1 \;\&\; \beta > 0 \;\&\; \gamma > 0}$$

এটা hold হলে intersect হবে।