The Meaning of Slope and y-Intercept in the Context of Word Problems

In the equation of a straight line (strtlneq.htm) (when the equation is written as "$y = mx + b$"), the slope (slope.htm) is the number "m" that is multiplied on the x, and "b" is the y-intercept (intrcept.htm) (that is, the point where the line crosses the vertical y-axis). This useful form of the line equation is sensibly named the "slope-intercept form".

Graphing (slopgrph.htm) from this format can be quite straightforward, particularly if the values of "m" and "b" are relatively simple numbers — such as 2 or $–4.5$, rather than something messy like $\frac{17}{19}$ or 1.67385.

In this lesson, we are going to look at the "real world" meanings that the slope and the y-intercept of a line can have, in context. In other words, given a "word problem" modelling something in the real world, or an actual real-world linear model, what do the slope and intercept of the modelling equation stand for, in practical terms?

Back when we were first graphing straight lines, we saw that the slope of a given line measures how much the value of y changes for every so much that the value of x changes. For instance, consider this line:

$$y = \left(\frac{3}{5}\right)x - 2$$

The slope of the above line is the value $m = \frac{3}{5}$. This means that, starting at any point on this line, we can get to another point on the line by going up 3 units and then going to the right 5 units. But (and this is the useful thing) we could also view this slope as a fraction over 1; namely:

$$m = \frac{\left(\frac{3}{5}\right)}{1}$$

This tells us, in practical terms, that, for every one unit that the x-variable increases (that is, moves over to the right), the y-variable increases (that is, goes up) by three-fifths of a unit. While this doesn't necessarily graph as easily as "three up and five over", it can be a more useful way of viewing things when we're doing word problems or considering real-world models.

Slope: Very often, linear-equation word problems deal with changes over the course of time; the equations will deal with how much something (represented by the value on the vertical axis) changes as time (represented on the horizontal axis) passes.

An exercise might, say, talk about how the population grows, year on year, in a certain city, assuming that the population increases by a certain fixed amount every year. For every year that passes (that is, for every increase of 1 along the horizontal axis), the population would increase (that is, move up along the vertical axis) by that fixed amount.

Intercept: When $x = 0$, the corresponding y-value is the y-intercept. In the particular context of word problems, the y-intercept (that is, the point when $x = 0$) also refers to the starting value. For a time-based exercise, this will be the value when you started taking your reading or when you started tracking the time

and its related changes.

In the example from above, the y-intercept would be the population when the sociologists started keeping track of the population. If they started taking their measurements or doing their calculations from a "base" year of 1997, then "$x = 0$" would correspond to "the year 1997", and the y-intercept would correspond to "the population in 1997".

Advisory: "When you started keeping track" is *not* the same as "when (whatever it is that you're measuring) started". Using the example above, your population-growth model might be very accurate for the years 1997 through 2015, but the city whose population is being measured might have been founded way back in 1672. In this case, "$t = 0$" would stand for "when we started measuring, in 1997"; it would *not* stand for "the zero-year for the city, which was its founding back in 1672"; "$t = 2$" would stand for 1999, two years after you started counting; and so forth.

Pay careful attention to how the variables are defined!

The following are a few examples to help illustrate how this works.

- **The average lifespan of American women has been tracked, and the model for the data is $y = 0.2t + 73$, where $t = 0$ corresponds to 1960. Explain the meaning of the slope and the y-intercept.**

What is the slope? It is $m = 0.2$. This value tells me that, for every increase of 1 in my input variable t (that is, for every increase of one year), the value of my output variable y will increase by 0.2.

What is the meaning of the slope?

The slope tells me that, every year, the average lifespan of American women increased by 0.2 years, or about 2.4 months.

When $t = 0$, what is the value of y? Looking at the equation, I see that $y = 73$.

What is the meaning of this y-value?

The intercept value tells me that, in 1960 (when they started counting), the average lifespan of an American woman was 73 years.

- **The equation for the speed (not the height) of a ball which is thrown straight up in the air is given by $v = 128 – 32t$, where v is the velocity (in feet per second) and t is the number of seconds after the ball is thrown. With what initial velocity was the ball thrown? What is the meaning of the slope?**

What is the slope? It is $m = –32$. This value tells me that, for every increase by 1 in my input variable t, I get a decrease of 32 in my output variable v.

What is the meaning of the slope?

The slope tells me that, for every second that passes, the speed of the ball decreases by 32 feet per second.

(Also, by the way, the velocity will eventually become zero (when the ball reaches the peak of its arc), and will then become negative (when gravity takes over and pulls the ball back down to the ground).

When $t = 0$, what is the value of v? Looking at the equation, I see that $v = 128$. The exercise defines v as measuring the velocity of the ball.

What is the meaning of this v-value?

The intercept value tells me that, when the ball was released, it was launched upward at a speed of 128 feet per second.

- **Fisherman in the Finger Lakes Region have been recording the dead fish they encounter while fishing in the region. The Department of Environmental Conservation monitors the pollution index for the Finger Lakes Region. The model for the number of fish deaths "y" for a given pollution index "x" is $y = 9.607x + 111.958$. What is the meaning of the slope? What is the meaning of the y-intercept?**

What is the slope? It is $m = 9.607$. This value tells me that, for every increase by 1 in my input variable x, I get an increase of 9.607 in my output variable y.

What is the meaning of the slope?

The slope tells me that, fo every increase in the pollution index by one unit (say, from a pollution index of 6 to a pollution index of7), there are nine or ten more fish deaths during the year.

(Why "or"? Because the 0.607 portion of a fish makes no sense in practical terms. The number of fish found will be a whole number; in this case, that whole number is expected to be either nine or else ten.

When $x = 0$, what is the value of y? Looking at the equation, I see that $y = 111.958$.

What is the meaning of this y-value?

The intercept value tells me that, even if the index were zero (that is, even if the water were utterly pure), there would still be about 112 fish deaths a year anyway.

Word problems with linear equations (that is, with straight-line models) almost always work this way: the slope is the rate of change, and the y-intercept is the starting value. (I can't, off the top of my head, think of any instance in which this would not be the case.)

The main difficulty is usually in interpreting the horizontal variable, especially when that variable is keyed to a certain year. Always make sure that you're clear on the definitions of the variables, and you should be fine.
