UCI EECS
EECS241A
Fall 2017

HOMEWORK 2
(due 10/19/2017)

1. Consider a random process $Y(t)$ defined by

$$Y(t) = \int_0^t X(\tau) d\tau$$

where $X(t)$ is given by

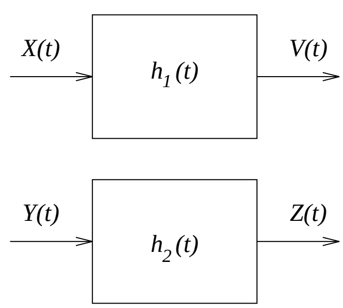
$$X(t) = A\cos(\omega t)$$

where ω is a constant and $A \sim \mathcal{N}(0, \sigma^2)$.

   (a) Determine the PDF of $Y(t)$ at $t = t_k$,
   (b) Is $Y(t)$ wide sense stationary?

2. Suppose that $X(t)$ is the input to a linear, time-invariant system with impulse response $h_1(t)$ and $Y(t)$ is the input to another linear, time-invariant system with impulse response $h_2(t)$ as shown in the figure below. The processes $X(t)$ and $Y(t)$ are jointly wide sense stationary, i.e., their means are constant with time and their cross-correlation function $R_{XY}(t+\tau, t) = E[X(t+\tau)Y(t)]$ depends only on τ, i.e., $E[X(t+\tau)Y(t)] = R_{XY}(\tau)$. Let $V(t)$ and $Z(t)$ denote the random process at the respective system outputs as shown in the figure below.

   (a) Express $V(t)$ and $Z(t)$ in terms of $X(t), Y(t), h_1(t)$, and $h_2(t)$.
   (b) Calculate $R_{VZ}(t+\tau, t) = E[V(t+\tau)Z(t)]$ in terms of $h_1(t), h_2(t)$, and $R_{XY}(\tau)$.
   (c) Are $V(t)$ and $Z(t)$ jointly stationary?
   (d) Calculate $S_{VZ}(f) = \int_{-\infty}^{\infty} R_{VZ}(\tau)e^{-j2\pi f\tau}d\tau$ in terms of $S_{XY}(f), H_1(f)$, and $H_2(f)$.



3. Two random processes $X(t)$ and $Y(t)$ are given by

$$X(t) = A\cos(2\pi ft + \Theta)$$

$$Y(t) = A\sin(2\pi ft + \Theta)$$

where A and f are constants and Θ is a uniform random variable over $[0, 2\pi]$. Find the following

(a) Mean $m_X(t) = E[X(t)]$

(b) Mean $m_Y(t) = E[Y(t)]$

(c) Autocorrelation $R_X(t, t+\tau) = E[X(t)X(t+\tau)]$

(d) Autocorrelation $R_Y(t, t+\tau) = E[Y(t)Y(t+\tau)]$

(e) Cross correlation $R_{XY}(t, t+\tau) = E[X(t)Y(t+\tau)]$

(f) Cross correlation $R_{YX}(t, t+\tau) = E[Y(t)X(t+\tau)]$

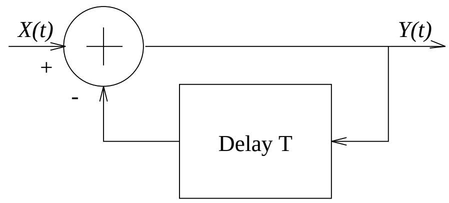
4. Consider a wide sense stationary process $X(t)$ with autocorrelation function $R_X(\tau) = E[X(t)X(t+\tau)]$ and power spectrum $S_X(f)$. Let $X'(t) = dX(t)/dt$. Show that

   (a) $R_{XX'}(\tau) = \dfrac{dR_X(\tau)}{d\tau}$

   (b) $R_{X'}(\tau) = -\dfrac{d^2R_X(\tau)}{d\tau^2}$

   (c) $S_{X'}(f) = 4\pi^2 f^2 S_X(f)$

5. Suppose that a wide sense stationary random process $X(t)$ with autocorrelation function $R_X(\tau) = e^{-\pi\tau^2}$ is the input to the filter shown in the figure below. What is the power spectral density $S_Y(f)$ of the output process $Y(t)$?



6. Consider the bandpass signal

$$s(t) = 2\sin(180\pi t) + 3\cos(220\pi t)$$

   (a) Calculate the frequency domain representation $S(f)$ of this signal

   (b) Calculate the frequency domain representation of the pre-envelope $S_+(f)$ of $s(t)$

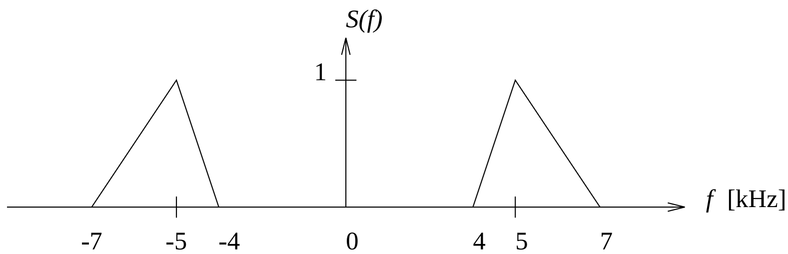
   (c) What is the pre-envelope $s_+(t)$ of $s(t)$?

   (d) Calculate the frequency domain representation $S_l(f)$ of the lowpass equivalent of the bandpass signal at $f_c = 100$ Hz

   (e) Calculate the time domain lowpass equivalent signal $s_l(t)$

   (f) What is the Hilbert transform $\hat{s}(t)$ of $s(t)$?

7. The frequency domain representation of a bandpass signal $s(t)$ is given in the figure below.



(a) Plot the frequency domain representation $\hat{S}(f)$ of the Hilbert transform $\hat{s}(t)$ of $s(t)$.

(b) Plot the frequency domain representation $S_+(f)$ of the pre-envelope (analytic signal) $s_+(t)$ of $s(t)$.

(c) Plot the frequency domain representation $S_l(f)$ of the low-pass equivalent $s_l(t)$ of $s(t)$ for $f_c = 5$ kHz.

(d) Note that $s_l(t) = s_I(t) + js_Q(t)$ is in general complex-valued, whereas its in-phase and quadrature parts ($s_I(t)$ and $s_Q(t)$, respectively) are real-valued (in time domain). Thus $s_I(t)$ and $s_Q(t)$ are the real and imaginary parts of $s_l(t)$. Using this fact, express $S_I(f)$ in terms of $S_l(f)$ (and its complex conjugate) and also $S_Q(f)$ in terms of $S_l(f)$ (and its complex conjugate).

(e) Using $S_I(f)$ and $S_Q(f)$ you found in part (e) above, reconstruct the frequency domain representations of

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$

and

$$\hat{s}(t) = s_I(t)\sin(2\pi f_c t) + s_Q(t)\cos(2\pi f_c t)$$