ECE 405: Communication Systems

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Outline

Introduction: Signals, Energy, Power, RMS

- A signal is a function of variables that we care about to study. the signal can be deterministic, or stochastic (random).
- The total energy of a signal $g(t)$ (real, or complex) is defined as

$$E_g = \int_{-\infty}^{+\infty} |g(t)|^2 dt$$

- The total average power of the signal is defined as

$$P_g = \lim_{t \to +\infty} \frac{1}{T} \int_{-T/2}^{+T/2} |g(t)|^2 dt$$

- A signal is called energy signal if $E_g < +\infty$.
- a signal is called power signal if $P_g < +\infty$

Introduction: Signals, Energy, Power, RMS (cont'd)

- Fact: For energy signals, $P_g = 0$.
- Example 2.1: Find the energy of $g(t) = 2\exp(-t/2)u(t)$
- The RMS value of $g(t)$ is the square root of P_g.
- Periodic signals repeat with a period T, i.e. $s(t) = s(t+T)$.
- Periodic signals have infinite energy, but finite power. P_g is just the mean power in one period

$$P_g = \frac{1}{T}\int_{-T/2}^{+T/2} |g(t)|^2 dt$$

- Example 2.2 (c): Find the RMS, power of $g(t) = D\exp(j\omega t)$

Classification of Signals

1. Continuous time, discrete time.
2. Analog, digital
3. Periodic, aperiodic
4. Energy, power
5. Deterministic, stochastic

Basic operations

- Delay $g(t) \to g(t - D)$. Plot?
- Time scaling $g(t) \to g(|a|t)$. Plot?
- Time inversion, or folding: $g(t) \to g(-t)$. Plot?
- Combination of above, example: $g(t) \to g(ct - b)$

The Dirac function $\delta(t)$

Definition

$\delta(t)$ is defined through test functions. For each test function (good signal) $f(t)$ we have

$$\int_{-\infty}^{+\infty} f(t)\delta(t)dt = f(0)$$

i.e. the function acts like a sampling device. out of the entire function, only the value at zero makes it out!

Inner Product

Definition

The inner product of signals $f(t),\ g(t)$ is defined as follows

$$f(t) \cdot g(t) = \int_{-\infty}^{+\infty} f(t) g^*(t) dt$$

- This is a generalization of the ordinary inner product.
- It has all required properties of an inner product:

$$\begin{aligned} x \cdot y &= y \cdot x^* \\ (a+b)x \cdot y &= ax \cdot y + bx \cdot y \\ ax \cdot y &= ax \cdot y \\ x \cdot x &\geq 0 \text{ with } 0 \text{ iff } x = 0 \end{aligned}$$

Inner Product (cont'd)

- An inner product always defines a norm through $||x||^2 = x \cdot x$. This means

$$||g(t)||^2 = E_g$$

- Schwartz inequality: VERY USEFUL FOR DSP

$$|\langle f(t), g(t) \rangle| \leq ||f(t)||\ ||g(t)||$$

with equality iff $f(t) = cg^*(t)$

- Can you prove it??

Fourier Transforms

Definition

The F.T. of $f(t)$, or the spectrum of $f(t)$ is defined as

$$F(f) = \int_{-\infty}^{+\infty} f(t) \exp(-j2\pi ft) dt$$

i.e. it gives a complex "score" of $f(t)$ for each frequency f in Hz.

Extremely important properties abound with *extra important ones like the modulation property prevailing for use in ECE 405!!!*