ECE 405: Communication Systems

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Outline

F.T. Properties

[allowframebreaks]

1. Linearity: $\mathcal{F}\{c_1 f_1(t) + c_2 f_2(t)\} = c_1 F_1(f) + c_2 F_2(f)$
2. Duality: If $\mathcal{F}\{f(t)\} = F(f)$, then $\mathcal{F}\{F(t)\} = f(-f)$.
3. Even conjugate property: For all real signals $f(t)$, $F(-f) = F^*(f)$.
4. Time scaling: If $\mathcal{F}\{f(t)\} = F(f)$, then $\mathcal{F}\{f(at)\} = \frac{1}{|a|} F(\frac{f}{a})$
5. Delay: If $\mathcal{F}\{f(t)\} = F(f)$, then $\mathcal{F}\{f(t-d)\} = \exp(-j2\pi fd)F(f)$.
6. Modulation 1: If $\mathcal{F}\{f(t)\} = F(f)$, then $\mathcal{F}\{f(t)\exp(j2\pi f_c t)\} = F(f - f_c)$
7. Modulation 2: If $\mathcal{F}\{f(t)\} = F(f)$, then $\mathcal{F}\{f(t)cos(2\pi f_c t)\} = \frac{1}{2}[F(f - f_c) + F(f + f_c)]$
8. Modulation 3: If $\mathcal{F}\{f(t)\} = F(f)$, then $\mathcal{F}\{f(t)sin(2\pi f_c t)\} = \frac{1}{2j}[F(f - f_c) - F(f + f_c)]$
9. Convolution: $\mathcal{F}\{\int_{-\infty}^{+\infty} f(u)g(t-u)du\} = F(f)G(f)$.
10. Product: $\mathcal{F}\{f(t)g(t)\} = \int_{-\infty}^{+\infty} F(u)G(f-u)du$

F.T. Properties (cont'd)

Additional properties:

1. Parseval 1: $\int_{-\infty}^{+\infty} f(t)g^*(t)dt = \int_{-\infty}^{+\infty} F(f)G^*(f)df$
2. Parseval 2: $\int_{-\infty}^{+\infty} |f(t)|^2(t)dt = \int_{-\infty}^{+\infty} |F(f)|^2 df$
3. Periodic Signals: If $f(t)$ is periodic, period T, then

$$F(f) = f_0 \sum_{n=0}^{+\infty} F_T(nf_0)\delta(f - nf_0)$$

   where $f_0 = 1/T$ and $F_T(nf_0) = \int_T f(t)\exp(-j2\pi nf_0 t)dt$
4. Differentiation (be careful!): $\mathcal{F}\{\frac{d}{dt}f(t)\} = j2\pi fF(f)$
5. Integration (be careful!): $\mathcal{F}\{\int_{-\infty}^{t} f(u)du\} = \frac{1}{j2\pi f}F(f)$