UC Irvine
EECS 241A
Fall 2017

HOMEWORK 3
(due 10/26/2017)

1. Let $\mathbf{f}_1, \mathbf{f}_2, \ldots \mathbf{f}_K$ be N-dimensional orthonormal basis vectors where $K \leq N$. Let $\mathbf{s}$ be an N-dimensional vector. We wish to find $\hat{\mathbf{s}}$, an approximation to $\mathbf{s}$ such that

$$\hat{\mathbf{s}} = \sum_{k=1}^{K} s_k \mathbf{f}_k$$

and the squared norm of $\mathbf{e} = \mathbf{s} - \hat{\mathbf{s}}$ is minimized.

   (a) Express $\|\mathbf{e}\|^2$ as $< \mathbf{s} - \hat{\mathbf{s}}, \mathbf{s} - \hat{\mathbf{s}} >$ where $\hat{\mathbf{s}}$ is as defined above and expand.

   (b) Calculate the coefficients $\{s_k\}_{k=1}^K$ that will minimize $\|\mathbf{e}\|^2$.

   (c) Let

$$\mathbf{f}_1 = \frac{1}{2\sqrt{2}}(1, 1, -1, 1, -1, 1, -1, -1)^T$$
$$\mathbf{f}_2 = \frac{1}{2\sqrt{2}}(-1, 1, -1, 1, 1, -1, 1, -1)^T$$
$$\mathbf{f}_3 = \frac{1}{2\sqrt{2}}(1, 1, 1, -1, 1, -1, -1, -1)^T$$

   and show that $\{\mathbf{f}_k\}_{k=1}^3$ is an orthonormal set.

   (d) Let

$$\mathbf{s} = (2, -2, 2, 2, -2, -2, 2, -2)^T.$$

   Find s_1, s_2, s_3 such that $\hat{\mathbf{s}} = \sum_{k=1}^3 s_k \mathbf{f}_k$ and $\|\mathbf{s} - \hat{\mathbf{s}}\|^2$ is minimized.

   (e) How do you explain the result in part (d) above?

   (f) If you use the Gram-Schmidt Orthonormalization Procedure on $\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3, \mathbf{s}$ (in that order), what would $\mathbf{f}_4$ be?

2. Suppose that $s(t)$ is either a real- or complex-valued signal that is represented as a linear combination of orthonormal functions $\{f_k(t)\}_{k=1}^K$, i.e.,

$$\hat{s}(t) = \sum_{k=1}^{K} s_k f_k(t)$$

where

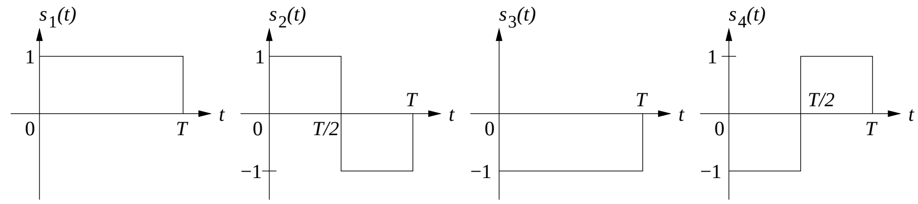
$$\int_{-\infty}^{\infty} f_n(t) f_m^*(t) dt = \begin{cases} 1 & m = n \\ 0 & m \neq n. \end{cases}$$

Determine the expressions for the coefficients $\{s_k\}_{k=1}^K$ in the expansion $\hat{s}(t)$ that minimizes the energy in the error signal

$$\mathcal{E}_e = \int_{-\infty}^{\infty} |s(t) - \hat{s}(t)|^2 dt$$

and the corresponding value of error energy $\mathcal{E}_e$.

3. Consider the signals $\{s_m(t)\}_{m=1}^4$ below. At every T seconds, one of $\{s_m(t)\}_{m=1}^4$ is transmitted.



   (a) Using the Gram-Schmidt Orthonormalization Procedure, generate the orthonormal basis set $\{f_k(t)\}_{k=1}^K$ that spans $\{s_m(t)\}_{m=1}^4$. What is K?

   (b) Draw the signal constellation.

   (c) Calculate $\{s_{mk}\}_{k=1}^K$ in the orthonormal series expansion

$$s_m(t) = \sum_{k=1}^{K} s_{mk} f_k(t).$$

4. Let four signals be given as

$$\begin{aligned} s_1(t) &= \cos(2\pi f_0 t) \\ s_2(t) &= \sin(2\pi f_0 t) \\ s_3(t) &= \cos(2\pi(f_0 + \Delta f)t) \\ s_4(t) &= \sin(2\pi(f_0 + \Delta f)t) \end{aligned}$$

   over a period of $[0, T]$. All four signals are equal to 0 outside this interval. Let $\Delta f = 1/T$ and $T = n/f_0$ for an integer n.

   (a) Express T in terms of the inverse of Δf.

   (b) Note T is given as n/f_0.

   (c) Similarly, express T in terms of each of $2f_0$, $2f_0 + \Delta f$, and $2f_0 + 2\Delta f$.

   (d) Use the Gram-Schmidt orthonormalization procedure to calculate a set of orthonormal basis signals for the space spanned by $\{s_i(t)\}_{i=1}^4$. (*Hint:* You will show that these signals are all orthogonal (six orthogonality conditions). After multiplying two signals, use the corresponding trigonometric identity and employ the fact that the integral of a sinusoid over an integer multiple of its period is zero. There will be two such sinusoids for each orthogonality condition. To see T is an integer multiple of the period of a sinusoid use (a)-(c) above.)

5. Let four signals be given as

$$\begin{aligned} s_1(t) &= \cos\left(\frac{2\pi m}{T} t\right) \qquad s_2(t) = \cos\left(\frac{2\pi n}{T} t\right) \\ s_3(t) &= -\cos\left(\frac{2\pi m}{T} t\right) \qquad s_4(t) = -\cos\left(\frac{2\pi n}{T} t\right) \end{aligned}$$

   over a period of $[0, T]$ for two integers $m \neq n$. All four signals are equal to 0 outside this interval.

(a) Calculate

$$\int_0^T \cos\left(\frac{2\pi m}{T}t\right)\cos\left(\frac{2\pi n}{T}t\right)dt.$$

(b) Calculate a set of orthonormal basis functions $\{f_n(t)\}_{n=1}^N$ that span $\{s_m(t)\}_{m=1}^4$. What is the dimensionality N of this space?

(c) Draw the signal constellation.

6. Let six signals be given as

$$\begin{array}{lll} s_1(t) = \sqrt{\frac{2\mathcal{E}}{T}}\cos\left(\frac{2\pi n_1}{T}t\right) & s_2(t) = \sqrt{\frac{2\mathcal{E}}{T}}\cos\left(\frac{2\pi n_2}{T}t\right) & s_3(t) = \sqrt{\frac{2\mathcal{E}}{T}}\cos\left(\frac{2\pi n_3}{T}t\right) \\ s_4(t) = -\sqrt{\frac{2\mathcal{E}}{T}}\cos\left(\frac{2\pi n_1}{T}t\right) & s_5(t) = -\sqrt{\frac{2\mathcal{E}}{T}}\cos\left(\frac{2\pi n_2}{T}t\right) & s_6(t) = -\sqrt{\frac{2\mathcal{E}}{T}}\cos\left(\frac{2\pi n_3}{T}t\right) \end{array}$$

over a period of $[0, T]$ for $n_i = n_c + i$ where n_c is an integer and $i = 1, 2, 3$. All six signals are equal to 0 outside this interval.

(a) What is the dimensionality N of $\{s_k(t)\}_{k=1}^6$?

(b) Draw the signal constellation.