Advanced Communication Theory Notebook

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1 70045 – Advanced Communication Theory

1.1 Wireless System Overview

A wireless system can be partitioned into three main parts:

- **TX** (a “source” that sends/transmits some information)
- **Wireless channel** (the physical propagation paths)
- **RX** (a “sink” that receives the transmitted waves)

The objective, in general, is to increase the communication speed without sacrificing the quality of service.

`tx_rx_channel_diagram.png`

Classification:

- SISO
- SIMO
- MISO
- MIMO

where:

- S: Single

- M: Multiple
- I: Input
- O: Output

Array Antenna:

array_antenna_diagram.png

1.2 Revisiting the Electromagnetic Fields

$$\boldsymbol{E}(\boldsymbol{r},t) = \boldsymbol{E}_0 \exp\left(j2\pi f_c t - j\frac{2\pi}{\lambda}\boldsymbol{u}^T\boldsymbol{r}\right)$$

$$\boldsymbol{H}(\boldsymbol{r},t) = \boldsymbol{H}_0 \exp\left(j2\pi f_c t - j\frac{2\pi}{\lambda}\boldsymbol{u}^T\boldsymbol{r}\right)$$

where:

- $\boldsymbol{r} = [x, y, z]^T$
- $\|\boldsymbol{u}\| = 1$
- $\boldsymbol{u}^T\boldsymbol{r} \in \mathbb{R}$

1.3 Electrical Field at the kth Antenna

At the RX-array's reference point (Cartesian origin $\mathbf{0}$), the electromagnetic wave becomes:

$$\boldsymbol{E}(\boldsymbol{r},t)\big|_{\boldsymbol{r}=\mathbf{0}} = \boldsymbol{E}(\mathbf{0},t) = \boldsymbol{E}_0 \exp(j2\pi f_c t)$$

Considering the kth antenna displaced by $\boldsymbol{r}_k$:

$$\Delta t_k = \frac{\Delta \boldsymbol{r}_k}{c} = \frac{1}{c}\boldsymbol{u}(\theta,\varphi)^T\boldsymbol{r}_k$$

Thus:

$$E(\boldsymbol{r},t)\big|_{\boldsymbol{r}=\boldsymbol{r}_k} = \boldsymbol{E}_0(\mathbf{0},t)\exp\left(-j2\pi\frac{\boldsymbol{u}^T\boldsymbol{r}_k}{\lambda}\right)$$

Assuming $E(\mathbf{0}, t) = 1$, the kth antenna response is:

$$\text{Antenna Response}_k = \exp\left(-j2\pi \frac{\boldsymbol{u}^T \boldsymbol{r}_k}{\lambda}\right)$$

For all antennas $k = 1, 2, \dots, N$:

$$\begin{bmatrix} \exp\left(-j2\pi \frac{\boldsymbol{u}^T \boldsymbol{r}_1}{\lambda}\right) \\ \exp\left(-j2\pi \frac{\boldsymbol{u}^T \boldsymbol{r}_2}{\lambda}\right) \\ \vdots \\ \exp\left(-j2\pi \frac{\boldsymbol{u}^T \boldsymbol{r}_N}{\lambda}\right) \end{bmatrix} = \exp\left(-j\boldsymbol{k}(\theta, \varphi)^T \begin{bmatrix} \boldsymbol{r}_1 \\ \boldsymbol{r}_2 \\ \vdots \\ \boldsymbol{r}_N \end{bmatrix}\right)$$

where $\boldsymbol{k}(\theta, \varphi)$ is the wavevector.
The column vector $\boldsymbol{S}(\theta, \varphi) \in \mathbb{C}^N$ is called the **array manifold vector**.

1.4 Notations

Important notations:

- N: Number of RX array elements
- θ: Azimuth angle ()
- φ: Elevation angle ()
- f_c: Carrier frequency
- λ: Wavelength
- k: Wavenumber
- $\boldsymbol{k}$: Wavevector
- $\boldsymbol{u}$: Unit vector, $\|\boldsymbol{u}\| = 1$

1.5 Definition: Array Aperture

The array aperture is defined as:

$$\text{Array Aperture} = \max_{i,j} \|\boldsymbol{r}_i - \boldsymbol{r}_j\|$$

1.6 Definition: Manifold Vector

The $(N \times 1)$ complex vector $\boldsymbol{S}$:

$$\boldsymbol{S}(\theta, \varphi) = \exp\left(-j \begin{bmatrix} \boldsymbol{r}_1 \\ \boldsymbol{r}_2 \\ \vdots \\ \boldsymbol{r}_N \end{bmatrix}^T \boldsymbol{k}(\theta, \varphi)\right)$$

where:

$$\boldsymbol{k}(\theta, \varphi) = \frac{2\pi}{\lambda} \boldsymbol{u}(\theta, \varphi)$$

and $\boldsymbol{u}(\theta, \varphi)$ is the real unit vector in the direction (θ, φ).

1.7 Differential Geometry and Projection Matrix

In physics, Albert Einstein (Nobel 1921) used differential geometry to express his general theory of relativity. Differential geometry goes hand-in-hand with SIMO, MISO, and MIMO systems.

1.8 Projection Matrix

Given a subspace $\mathbb{R}^N \to \mathbb{R}^M$ $(N > M)$:

Matrix $B = [\boldsymbol{b}_1, \boldsymbol{b}_2, \ldots, \boldsymbol{b}_M]$.

Projection onto subspace:

$$\boldsymbol{b} = \sum \lambda_i \boldsymbol{b}_i \quad \text{such that} \quad \langle \boldsymbol{a} - \boldsymbol{b}, \boldsymbol{b}_i \rangle = 0$$

where:

$$\lambda = (B^T B)^{-1} B^T \boldsymbol{a} \quad \boldsymbol{b} = B(B^T B)^{-1} B^T \boldsymbol{a}$$

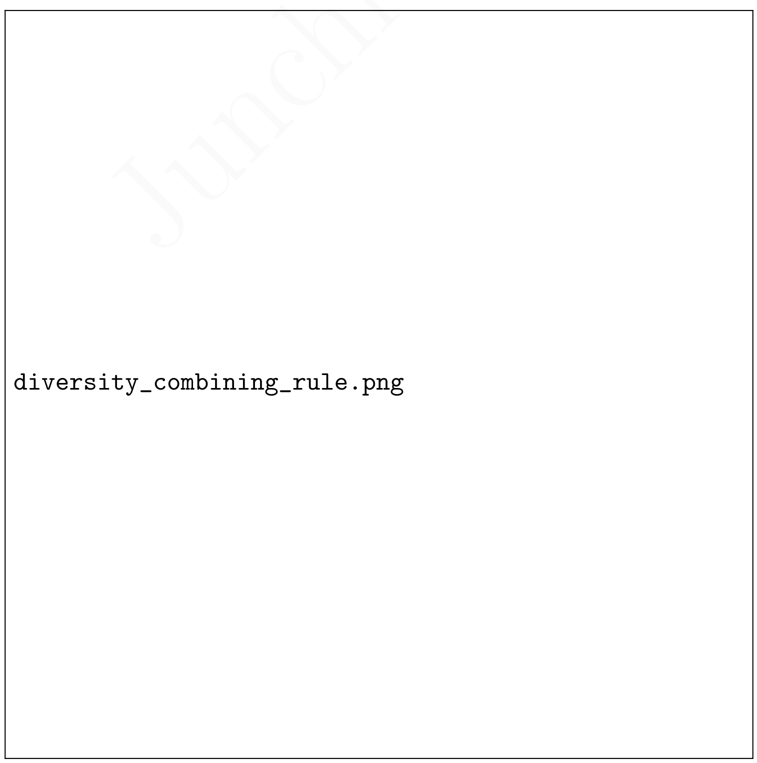
Thus, the projection matrix is:

$$P = B(B^T B)^{-1} B^T$$

2 Diversity Theory

2.1 Concept of Diversity

The "diversity combining" or briefly "diversity" concept: Diversity is defined as a technique that utilizes two or more copies of a signal with varying degrees of noise/interference effects to achieve a higher degree of message-recovery performance than achievable by any one of the copies individually.



Input signal:

$$\begin{aligned} x_1 &= \beta_1 Sd + n_1 \\ x_2 &= \beta_2 Sd + n_2 \\ &\vdots \\ x_N &= \beta_N Sd + n_N \end{aligned} \qquad \text{(Distortion)}$$

Output signal:

$$\begin{aligned} S_{div} &= W_1^* x_1 + W_2^* x_2 + \cdots + W_N^* x_N \quad \text{(recovery)} \\ &= W^H x \\ &= W^H(\beta Sd + n) \\ &= W^H \beta Sd + W^H n \quad \text{(desired + noise)} \end{aligned}$$

2.2 Signal and Noise Power

The power of the desired signal:

$$P_{\text{desired}} = \mathbb{E}\left[\left(W^H \beta Sd\right)^2\right] = W^H \mathbb{E}\left[\beta Sd \beta^H Sd^H\right] W = P_d W^H R_{\beta\beta} W$$

where $R_{\beta\beta}$ is the covariance matrix.
The power of noise:

$$P_{\text{noise}} = \mathbb{E}\left[\left(W^H n\right)^2\right] = W^H R_{nn} W$$

Thus, the Signal-to-Noise Ratio (SNR) is:

$$\text{SNR} = \frac{P_{\text{desired}}}{P_{\text{noise}}} = \frac{P_d W^H R_{\beta\beta} W}{W^H R_{nn} W}$$

where:

$$\begin{aligned} R_{\beta\beta} &= \text{cov}(\beta) = \sum \beta_i \beta_i^H \\ R_{nn} &= \text{cov}(n) = \sum n_i n_i^H \end{aligned}$$

2.3 Diversity Combining Rules (Strategies)

2.3.1 1. Maximal Ratio Combining (MRC)

$$W_{\text{MRC}} = \arg\max_W \text{SNR}(S_{\text{out,div}})$$

An optimization problem.

2.3.2 2. Selection Combining (SC)

$$W_k = \begin{cases} 1, & \text{if } \text{SNR}_k > \text{SNR}_i \quad \forall i \\ 0, & \text{otherwise} \end{cases}$$

Choose the best diversity branch.

2.3.3 3. Equal Gain Combining (EGC)

All weights are equal:

$$w_1 = w_2 = \cdots = w_N$$

2.3.4 4. Scanning Combining (SCC)

If $\text{SNR}_k >$ threshold, then:

$$W_k = 1, \quad W_j = 0 \quad \forall j \neq k$$

If $\text{SNR}_k <$ threshold, then move to $k+1$ and repeat.

2.4 Classification of Diversity

- Multi-path diversity
- Time diversity

2.5 Types of Diversity

2.5.1 Frequency Diversity

frequency_diversity.png

2.5.2 Space Diversity

- **TX diversity**
- **RX diversity**
- **TX-RX diversity**

space_diversity.png

2.5.3 Polarization Diversity

polarization_diversity.png

2.6 Multipath Diversity and Delay Spread

Multipath diversity is used to recover signals.

Impulse response:

$$h(t) = \sum \beta_i \delta(t - \tau_i)$$

where τ_i represents delays.

The delay spread is a measure of multipath richness.
Resolving multipaths is critical for modern wireless systems. The pulse duration:

$$\text{Pulse Duration} = \frac{1}{\text{Bandwidth}}$$

Thus:

$$\text{Number of resolvable paths} = \left\lfloor \frac{\text{delay spread}}{\text{pulse duration}} \right\rfloor + 1$$

2.7 Refresher: Modern Communication Systems

Modern systems require:

- High data rates
- Reliable communication links

Data rate:

$$C = B \log_2(1 + \text{SNR})$$

Ways to increase data rate:

- Increase bandwidth
- Increase transmit power

2.8 Important Wireless Channel Parameters

$$\begin{aligned} C &= \text{Channel Capacity} \\ B &= \text{Channel Bandwidth (Hz)} \\ B_{\text{coh}} &= \text{Coherence Bandwidth (Hz)} \\ B_{\text{dop}} &= \text{Doppler Spread (Hz)} \end{aligned}$$

2.9 Timestamps and Fading Classifications

Definitions:

$$\begin{aligned} T_{\text{cs}} &= \text{Duration of a channel symbol (sec)} \\ T_{\text{spread}} &= \text{Delay spread (sec)} \\ T_{\text{coh}} &= \text{Coherence time (sec)} \end{aligned}$$

where:

$$\begin{aligned} B &= \frac{1}{T_{\text{cs}}} \\ B_{\text{coh}} &= \frac{1}{T_{\text{spread}}} \\ B_{\text{dop}} &= \frac{1}{T_{\text{coh}}} \end{aligned}$$

fading_classification.png

Fading types:

- Flat, slow fading
- Flat, fast fading
- Frequency-selective, slow fading
- Frequency-selective, fast fading

2.10 Doppler Effect

The Doppler effect describes the change of frequency in relation to movement:

$$f' = f \times \frac{v \pm v_0}{v \pm v_s}$$

3 Modeling of SIMO, MISO & MIMO Antenna Array

3.1 Notations

- $\boldsymbol{a}$, $\boldsymbol{A}$: Column vector
- A or (A) : Matrix
- I_N : $N \times N$ identity matrix
- 1_N : Vector of N ones

- 0_N : Vector of N zeros
- $0_{N,M}$: $N \times M$ zero matrix
- $(\cdot)^T$: Transpose
- $(\cdot)^H$: Hermitian transpose
- $\circ, \oslash$: Hadamard product/division (element-wise)
- $\otimes$: Kronecker product
- $\exp(A)$, $\exp(\boldsymbol{A})$: Element-wise exponential
- $\mathcal{L}\{A\}$: Linear space/subspace spanned by columns of A
- $\mathcal{L}^{\perp}\{A\}$: Complement space of $\mathcal{L}\{A\}$
- $P_{\mathcal{L}\{A\}}$: Projection operator onto subspace $\mathcal{L}\{A\}$
- $P_{\mathcal{L}^{\perp}\{A\}}$: Projection operator onto subspace $\mathcal{L}^{\perp}\{A\}$

Matrix A structure:

$$A = \begin{bmatrix} \boldsymbol{b}_1 & \boldsymbol{b}_2 & \cdots & \boldsymbol{b}_M \end{bmatrix} \quad \text{where} \quad A \in \mathbb{C}^{N \times M}$$

Any vector $\boldsymbol{x} \in \mathcal{L}\{A\}$ can be written as a linear combination:

$$\boldsymbol{x} = \boldsymbol{b}_1\lambda_1 + \boldsymbol{b}_2\lambda_2 + \cdots + \boldsymbol{b}_M\lambda_M = A\boldsymbol{\lambda}$$

3.2 Projection Operators

The projection operator onto $\mathcal{L}\{A\}$ is:

$$P_{\mathcal{L}\{A\}} = A(A^H A)^{-1} A^H$$

thus:

$$\boldsymbol{a} = P_{\mathcal{L}\{A\}}\boldsymbol{b}$$

Complement space:

$$\dim \mathcal{L}\{A\} = M, \quad \dim \mathcal{L}^{\perp}\{A\} = N - M$$

The projection onto the orthogonal complement is:

$$P_{\mathcal{L}^{\perp}\{A\}} = I_N - P_{\mathcal{L}\{A\}}$$

3.3 Space Selective Fading

$$T_{\text{spread}} \leftrightarrow \text{frequency-selective fading}$$

$$B_{\text{dop}} \leftrightarrow \text{fast/slow fading}$$

$$B_{\text{spread}} \leftrightarrow \text{space-selective fading}$$

Spatial coherence:

- Space-selectivity if the transfer function varies over space.
- Spatial coherence if it does *not* vary over a coherence distance D_{coh}.

3.4 Small-scale and Large-scale Fading

- **Small-scale fading**: Displacement $< D_{\text{coh}}$ (multipath effects).
- **Large-scale fading**: Displacement $\gg D_{\text{coh}}$ (path loss over long distance).

Scattering function:

$$H(t, f, r) \xrightarrow{\text{AutoCorrelation}} \psi_H(\Delta t, \Delta f, \Delta r) \xrightarrow{\text{FT}} S(t, f, k)$$

3.5 Angle Spectrum

Angle spectrum represents the average power versus Direction of Arrival (DoA) (θ, φ).

- θ: Azimuth angle
- φ: Elevation angle

3.6 Local Area and Slow Fading

For slow fading:

$$T_{\text{cs}} \approx T_{\text{coh}}$$

Local area:

$$d = T_{\text{coh}} \times c = \frac{c}{B}$$

Trade-off: Higher $d \Rightarrow$ lower $B \Rightarrow$ higher reliability, lower data rate.
In the local area, electromagnetic waves can be regarded as **homogeneous plane waves**:

$$H(r) \propto \frac{1}{|r|}$$

Example:

$$f_c = 2.4 \text{ GHz}, \quad B = 50 \text{ MHz} \quad \Rightarrow \quad d = \frac{c}{B} = 60 \text{ m}$$

3.7 Dependency Relationships

Summary:

$$\begin{aligned} \Delta\tau \leftrightarrow \Delta f &\quad \text{(Time shift} \leftrightarrow \text{Frequency shift)} \\ \Delta t \leftrightarrow \Delta f &\quad \text{(Time shift} \leftrightarrow \text{Doppler frequency)} \\ \Delta r \leftrightarrow \Delta k &\quad \text{(Space shift} \leftrightarrow \text{Wavevector shift)} \end{aligned}$$

Using Fourier Transform (FT) mappings.

3.8 Wireless SIMO Channel (Refresher)

An array system is a collection of $N > 1$ sensors (transducing elements, receivers, antennas, etc.) distributed in 3-D Cartesian space with a common reference point.
The locations are given by:

$$\boldsymbol{r}_k = [x_k, y_k, z_k]^T, \quad k = 1, 2, \ldots, N$$

Array aperture:

$$\text{Aperture} = \max_{i,j} \|\boldsymbol{r}_i - \boldsymbol{r}_j\|$$

Array manifold vector:

$$S(\theta, \varphi) = \exp\left(-j \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ \vdots & \vdots & \vdots \\ x_N & y_N & z_N \end{bmatrix} \boldsymbol{k}(\theta, \varphi)\right)$$

where:

$$\boldsymbol{k}(\theta, \varphi) = \frac{2\pi}{\lambda} \boldsymbol{u}(\theta, \varphi)$$
$$\boldsymbol{u}(\theta, \varphi) = [\cos\theta \cos\varphi, \sin\theta \cos\varphi, \sin\varphi]^T$$

3.9 Single Path Channel Model

Propagation delay:

$$\tau = \frac{d + \boldsymbol{r}_k^T \boldsymbol{u}(\theta, \varphi)}{c}$$

Transmit signal: $S(t) \exp(j2\pi f_c t)$.
Received signal at antenna k:

$$r_k(t) = \left(\frac{k}{d}\right)^2 \exp(j\varphi) \exp\left[j2\pi f_c (t - \tau)\right] S(t - \tau)$$

where φ is random phase noise due to path loss and scattering.
Simplified expression:

$$r_k(t) = \beta \exp\left[-j2\pi \frac{\boldsymbol{u}^T(\theta, \varphi) \boldsymbol{r}_k}{\lambda}\right] S(t - d/c)$$

3.10 Channel Impulse Response

Impulse response at each antenna:

$$h_k(t) = \beta S(\theta, \varphi) \delta\left(t - \frac{d}{c}\right)$$

3.11 Multipath Channel

For multipath with L paths:

$$h(t) = \sum_{l=1}^{L} \beta_l S(\theta_l, \varphi_l) \delta(t - \tau_l)$$

Received signal:

$$x(t) = h(t) * m(t) + n(t)$$

where:

$$m(t) = \begin{bmatrix} \beta_1 m(t - \tau_1) \\ \beta_2 m(t - \tau_2) \\ \vdots \\ \beta_L m(t - \tau_L) \end{bmatrix}$$

3.12 Wireless MISO Channel (Reciprocity)

Using antenna reciprocity, the MISO channel is modeled similarly.
Impulse response (single path):

$$h(t) = \beta S^H(\theta, \varphi)\delta\left(t - \frac{d}{c}\right)$$

Multipath case:

$$h(t) = \sum_{l=1}^{L} \beta_l S^H(\theta_l, \varphi_l)\delta(t - \tau_l)$$

3.13 Modeling of RX Scalar Signal

3.13.1 Case 1: Demultiplexed Signals

The signal is demultiplexed into $\bar{N}$ separate signals:

$$x(t) = h(t) * (\bar{W}m(t)) + n(t)$$

3.13.2 Case 2: Same Signal Transmission

All TX-array elements transmit the same $m(t)$:

$$x(t) = h(t) * (\bar{W}m(t)) + n(t)$$

where $\bar{W}$ is a weight matrix for beamforming.

3.14 Transmit Diversity

- Transmit diversity with feedback
- Transmit diversity without feedback

Transmit pilot signals → receiver estimates CSI → transmitter adjusts weights.
Example:

$$w_1 = \frac{h_1^*}{|h_1|}, \quad w_2 = \frac{h_2^*}{|h_2|}$$

$$\boldsymbol{w} = \frac{\boldsymbol{h}^*}{\|\boldsymbol{h}\|}$$

3.15 Open-loop Space Time Block Coding (STBC)

No CSI at the transmitter; symbols (m_1, m_2) encoded as:

$$\begin{bmatrix} m_1 & m_2 \\ -m_2^* & m_1^* \end{bmatrix}$$

Received signals:

$$x = \boldsymbol{H} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

where $\boldsymbol{H}$ is the channel matrix.
Detection:

$$g = \boldsymbol{H}^H x$$

3.16 Wireless MIMO Channels

Modeling a single path from TX-array to RX-array:

$$h(t) = \beta S(\theta, \varphi) S^H(\bar{\theta}, \bar{\varphi}) \delta \left(t - \frac{d}{c} \right)$$

Multipath MIMO channel:

$$h(t) = \sum_{l=1}^{L} \beta_l S(\theta_l, \varphi_l) S^H(\bar{\theta}_l, \bar{\varphi}_l) \delta(t - \tau_l)$$

3.17 Modeling of RX Vector Signal

3.17.1 Case 1: Demultiplexed Signals

$$x(t) = h(t) * (m(t) \otimes w(t)) + n(t)$$

3.17.2 Case 2: Same Signal Transmission

$$x(t) = h(t) * (m(t) \otimes w(t)) + n(t)$$

where $\otimes$ denotes Kronecker product.

4 ACT 4: Array Receivers SIMO MIMO

4.1 Problem Formulation

Assume M sources in SIMO channels:

$$x(t) = S\, m(t) + n(t)$$

where:

$$S = [S_1 \quad S_2 \quad \dots \quad S_M], \quad m(t) = \begin{bmatrix} \beta_1 m_1(t) \\ \beta_2 m_2(t) \\ \vdots \\ \beta_M m_M(t) \end{bmatrix}$$

Three main problems:

1. **Detection Problem**: Find M?
2. **Estimation Problem**: Estimate signal and channel parameters (e.g., DOA, P_{m_i}).
3. **Reception Problem**: Receive one desired signal while suppressing $M-1$ interferences.

Covariance matrix:

$$R_{xx} = \mathbb{E}[x(t)x(t)^H]$$

4.2 Covariance Matrix Derivation

Expanding:

$$R_{xx} = \mathbb{E}[(S\, m(t) + n(t))(S\, m(t) + n(t))^H] = S\, R_{mm} S^H + R_{nn}$$

Noise covariance:

$$R_{nn} = \mathbb{E}[n(t)n(t)^H] = I_N \sigma^2$$

4.3 Hypothesis Testing

Maximum likelihood:

$$\max_K \{LF^{(K)}\}$$

where LF denotes the likelihood function.
Assume $m_i(t)$ are uncorrelated:

$$R_{\text{signal}} = \mathbb{E}[S\, m(t)m(t)^H S^H] = \sum_{i=1}^{M} P_i S_i S_i^H$$

Rank of R_{signal} is M.

4.4 Detection Problem Summary

If $L = \infty$ (theoretical):

$$R_{xx} = \sum x(t)x(t)^H$$

then:

$$M = N - (\text{multiplicity of minimum eigenvalues})$$

If L is finite (practical), use AIC or MDL criteria.
Noise variance estimate:

$$\hat{\sigma}_n^2 = \frac{1}{N-M} \sum_{i=M+1}^{N} \lambda_i$$

4.5 AIC and MDL Criteria

AIC:

$$\min_K \{AIC(K)\}$$

$$AIC(K) = -2 \ln \left(\max_K LF^{(K)} \right) + 2K$$

MDL:

$$\min_K \{MDL(K)\}$$

$$MDL(K) = -\ln \left(\max_K LF^{(K)} \right) + \frac{1}{2} K \ln L$$

4.6 The Subspace Approach

Subspace decomposition:

$$\mathcal{L}\{S\} = \mathcal{L}\{E_S\}$$

$$\mathcal{L}\{E_N\} = \mathcal{L}^{\perp}\{S\}$$

Here, E_S is the signal subspace, E_N is the noise subspace.
Dimensionalities:

$$\dim(\mathcal{L}\{E_S\}) = M, \quad \dim(\mathcal{L}\{E_N\}) = N - M$$

4.7 The MUSIC Algorithm

MUSIC = MUltiple SIgnal Classification.

Basic idea:

$$z(p) = P_{E_N} S(p) \to 0$$

Cost function:

$$\|z(p)\|^2 = S(p)^H P_{E_N} S(p)$$

Objective: find p where $\|z(p)\|$ is minimized (nulls).

4.8 Signal Power and Cross-Correlation Estimation

Estimate:

$$R_{xx} = S R_{mm} S^H + R_{nn}$$

$$R_{nn} = S^{\#}(R_{xx} - R_{nn})(S^{\#})^H$$

where $S^{\#}$ is the pseudo-inverse of S.

4.9 Reception Problem: Array Pattern / Beamforming

With array weights $\boldsymbol{w}$:

$$y = \boldsymbol{w}^H x$$

Array pattern:

$$g(\theta) = \boldsymbol{w}^H S(\theta)$$

Largest lobe: main lobe. Remaining lobes: sidelobes.

Beamwidth:

$$\text{Beamwidth} = 2 \sin^{-1}\left(\frac{\lambda}{Nd}\right) \times \frac{180}{\pi}$$

Beam steering:

$$\boldsymbol{w} = S(\theta_{\text{main-lobe}})$$

4.10 Beamforming Methods

4.10.1 Wiener-Hopf Beamforming

$$\boldsymbol{w} = c R_{xx}^{-1} p$$

where:

$$p = \mathbb{E}[m_k(t) x(t)]$$

Objective: maximize SNR.

4.10.2 Superresolution Beamforming

Suppress interference completely:

$$\boldsymbol{w} = P_{S_{\text{desired}}}$$

where $P_{S_{\text{desired}}}$ is the projection operator onto the desired signal.

4.11 SNR Criterion at Beamformer Output

Output:

$$y(t) = \boldsymbol{w}^H x(t)$$

Signal power:

$$P_{d,\text{out}} = \boldsymbol{w}^H S_{\text{desired}} P_d S_{\text{desired}}^H \boldsymbol{w}$$

Noise power:

$$P_{n,\text{out}} = \boldsymbol{w}^H \sigma_n^2 I \boldsymbol{w}$$

Thus:

$$\text{SNR}_{\text{out}} = \frac{P_{d,\text{out}}}{P_{n,\text{out}}}$$

4.12 Cramér-Rao Bound (CRB)

The uncertainty in estimation:

$$\delta_e = \sqrt{CRB(S)}$$

where:

$$CRB(S) = \frac{1}{2\,\text{SNR}\,R_{xx}\,L\,C}$$

and:

- SNR: Signal-to-noise ratio
- L: Number of snapshots
- $C = 1$ (constant)

5 ACT 5: Spatial-Temporal Massive MIMO and mmWave

5.1 Problem Description

Very dense co-channel interference environment. **Note:** Number of sources/signals $M > N$.

5.2 Solutions

1. Employ **massive MIMO**: increase N so that $M < N$.
2. Employ **spatiotemporal algorithms**: increase the observation space from N to N_{ext} so that $M < N_{\text{ext}}$.
3. Employ both **massive MIMO** and **manifold extenders**.

5.3 Massive MIMO (mMIMO)

Advantages:

- Increased data rate
- Reduction in air latency

Challenges:

- Hardware complexity $\uparrow$
- Computational complexity $\uparrow$
- Channel estimation difficulty $\uparrow$

5.4 Spatiotemporal Wireless Communication

Extended manifold:

$$S \in \mathbb{C}^{N \times 1} \quad \rightarrow \quad h \in \mathbb{C}^{N_{\text{ext}} \times 1}$$

5.5 Basic STAR Manifold

$$h_{\text{STAR}} = S \otimes J_c$$

where:

- $\otimes$ denotes the Kronecker product
- S is the spatial manifold vector
- J_c is the shifting matrix

Kronecker product:

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \cdots \\ a_{21}B & a_{22}B & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

5.6 Examples

Given:

$$J = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

An example STAR manifold is constructed by:

$$S = \begin{bmatrix} -0.917 + 0.408j \\ 1 \\ -0.917 + 0.408j \end{bmatrix} \quad \Rightarrow \quad h_{\text{STAR}} = \text{Kronecker}(S, J)$$

5.7 mmWave Wireless Communications

Definition:

$$\text{mmWave spectrum: } 30\,\text{GHz} \sim 300\,\text{GHz} \quad \Leftrightarrow \quad \text{short wavelength}$$

(1 mm ∼ 10 mm)
Challenges:

- Very high atmospheric attenuation
- Very high path loss (signal strength ↓, range ↓)

Solution: Use highly directional beams.

5.8 ADC/DAC Problems and Solutions

Problem: ADCs and DACs operating at multi-GHz rates are very power consuming.
Solution: Analog/hybrid beamforming (operate at baseband instead of RF).

analog_hybrid_beamforming.png

5.9 Wideband Assumption

If the transmitted baseband wavefront changes across sensors, different sensors receive different parts of the signal.

$$x(t) = \sum_{i=1}^{M} S_i^0 m_i(t) + n(t)$$

Covariance:

$$R_{xx} = \sum S_i^0 R_{mimi} (S_i^0)^H + R_{nn}$$

5.10 Array Capacity for Infinite Bandwidth

In SIMO:

$$C = B \log_2 \left(1 + \frac{NP_s}{P_n + P_i}\right)$$

Assuming:

$$B \to \infty, \quad \frac{NP_s}{P_n + P_i} \to 0$$

Taylor series approximation:

$$\log_2(1 + x) \approx 1.44x$$

thus:

$$C \approx 1.44 \times N \times \frac{P_s}{P_n + P_i}$$