Lecture 21

Intro to Attacking Non-Linear Equations

Lecture 21

Introduction to Roots

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Introduction to Zeros

Lecture 21

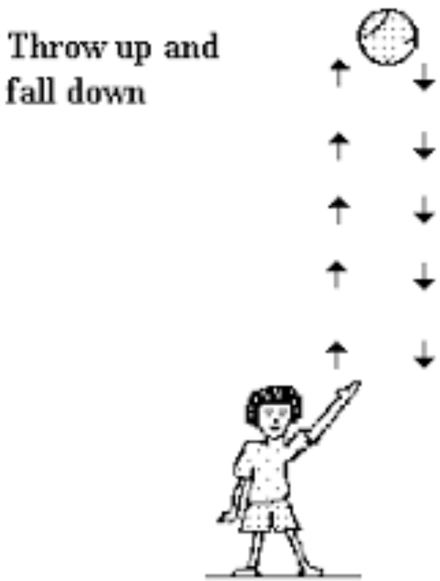
Introduction to Dr C not-so-sneakily getting you to program…

Episode 1

Repressed False Memory Therapy from Middle School

Conversation Starter

- Classic Physics…
  - Toss a brick!
    - Initial Velocity, v0
    - Initial Height, z0
    - When do you need to get out of the way?
  - The governing equation



$$z(t, g, z_o, v_o) := z_o + v_o \cdot t - \frac{1}{2}\, g \cdot t^2$$

How to solve?

- If you take stock in this equation and make note of the knowns and unknowns & plug things in…

$$z(t, g, z_o, v_o) := z_o + v_o \cdot t - \frac{1}{2} g \cdot t^2$$ From Mr McKeon's Physics Class

Accelleration due to gravity...

$$v_o := 50 \frac{ft}{s} \quad g = 32.174 \frac{ft}{s^2}$$ Initial upward velicity...

$$z_o := 4\ ft$$ Initial height of rock...

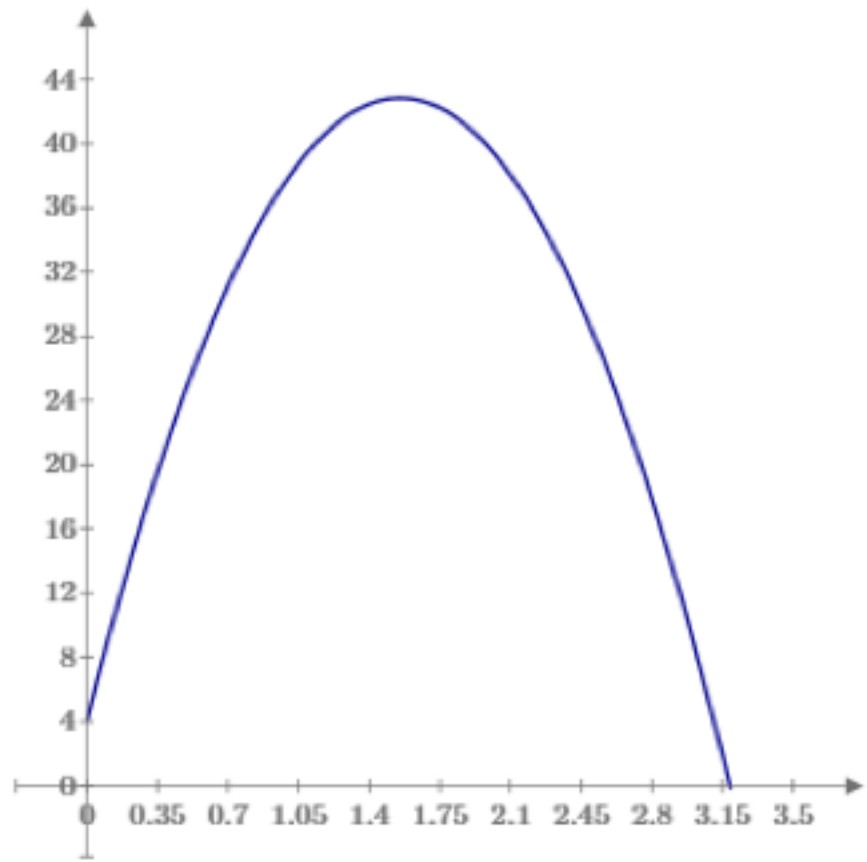
$$z_{head} := 5\ ft + 8\ in = 5.667\ ft$$ Your prof's height [without heels]
This is the answer on the LHS
you're tryng to match

$$5.67 = 4 + 50 \cdot t + 16.1\ t^2$$ ←**You get this!**

A good starting point…

- Always a good place to start is to create a plot of your equation…
- This gives you a perspective of your problem that will help you later on.
- In this case we have a basic curve.

$$z(t, g, z_o, v_o) := 4 + 50 \cdot t + 16.1\ t^2$$

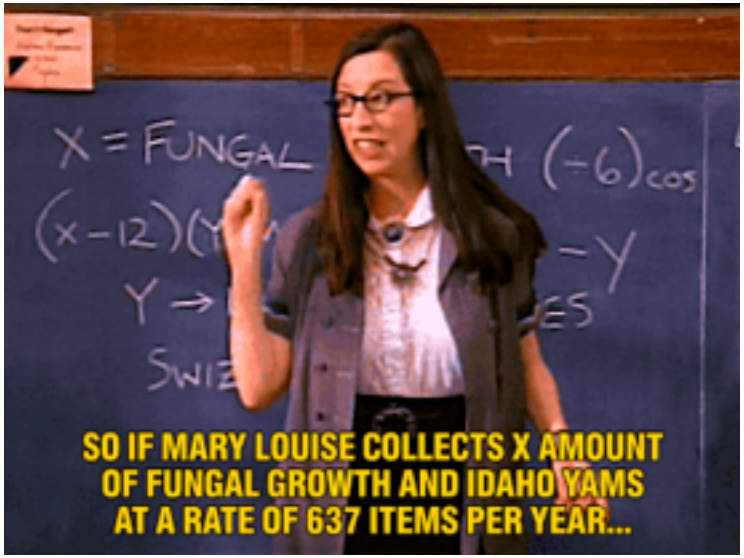


$t\ (s)$

Moving forward…

$$5.67 = 4 + 50 \cdot t + 16.1\ t^2$$

- Now put our specific problem with all of our players in the equation (including the LHS).
- So as we saw before this breaks down to a simple "solve for ~~x~~ *t*" kind if problem from Mrs. Mercer's class…
- This is also the form of a "root" equation that we call <u>"Algebraic"</u> in nature.
- In these cases the "unknown" has positive integers as the powers and no other distinct or intimidating feature. It's a straight polynomial.



(Still not really Mrs. Mercer)

The "One Size Fits All" attack strategy for Root Problems…

- Now for the next step. And it's the most important step in assaulting these problems.
- We move EVERTYTHING to one side of the equal sign… leaving one side with just a zero.

$$f(t) := 0 = 4 + 50 \cdot t + 16.1\ t^2 - 5.67$$

- (This is why some people call these problems "zero" problems)
- Technically this gives us a new equation that we typically model as $f(x)$

The "One Size Fits All" attack strategy for Root Problems…

- Now for the next step. And it's the most important step in assaulting these problems.
- We move EVERTYTHING to one side of the equal sign… leaving one side with just a zero.

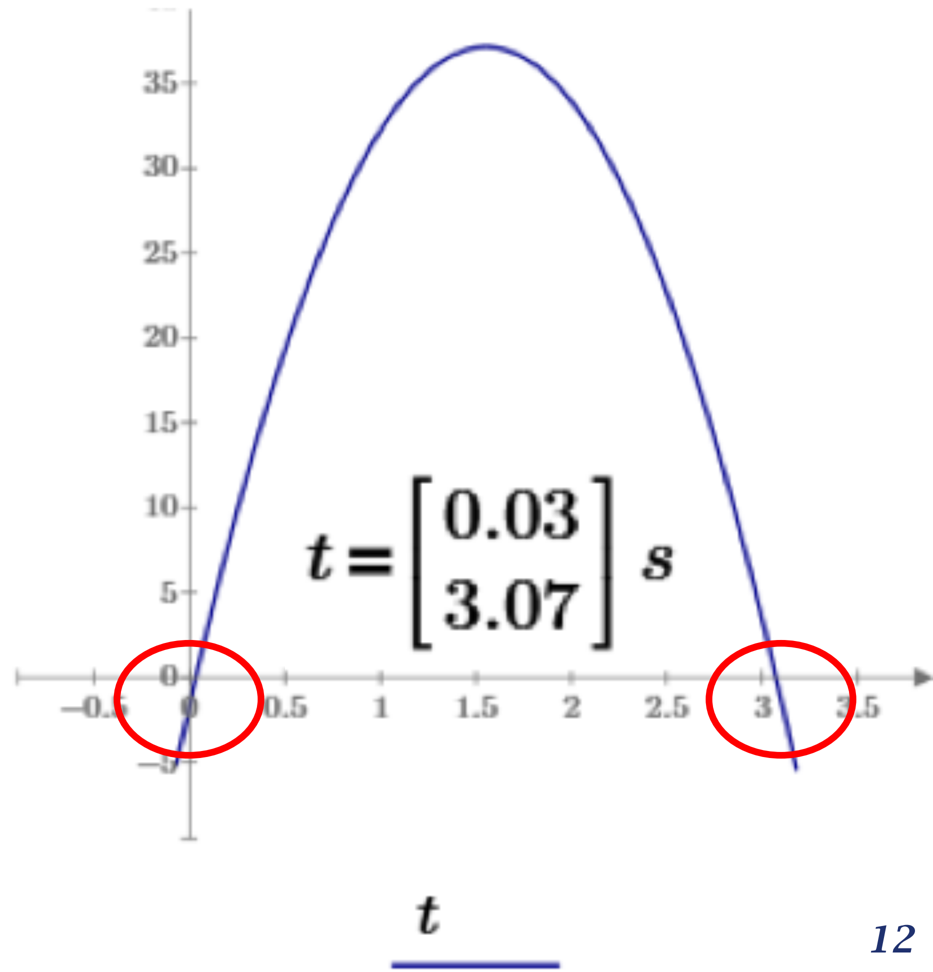
$$f(t) := -16.1\ t^2 + 50 \cdot t - 1.67$$

- (This is why some people call these problems "zero" problems)
- Technically this gives us a new equation that we typically model as *f*(*x*)

The "One Size Fits All" attack strategy for Root Problems…

- Now let's go back to our first step (graphing it out…)
- When we graph out our $f(t)$ function our answer will be where our line crosses the x axis… i.e., where $f(t) = 0$

$$f(t) := -16.1\ t^2 + 50 \cdot t - 1.67$$



$$t = \begin{bmatrix} 0.03 \\ 3.07 \end{bmatrix} s$$

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For quadratics…

- … Getting the roots is classic old school.

$$f(t) := -16.1\ t^2 + 50 \cdot t - 1.67$$

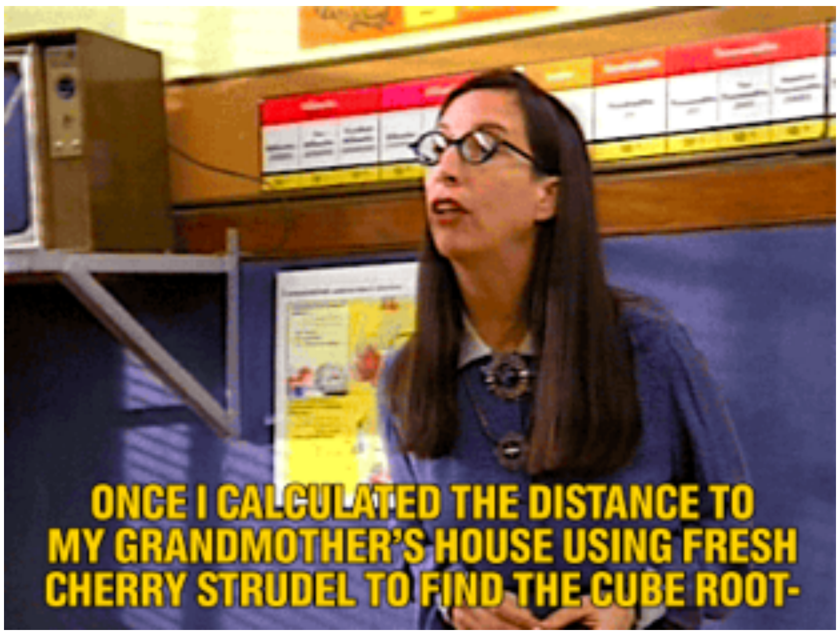
a b c

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t := \begin{bmatrix} \dfrac{-b + \sqrt{b^2 - 4 \cdot a \cdot c}}{2\ a} \\ \dfrac{-b - \sqrt{b^2 - 4 \cdot a \cdot c}}{2\ a} \end{bmatrix} = \begin{bmatrix} 0.034 \\ 3.072 \end{bmatrix}$$

What goes up

May cause a concussion



(Too subversively cool to be Mrs. Mercer…)

- In principle getting higher order polynomial roots is doable…

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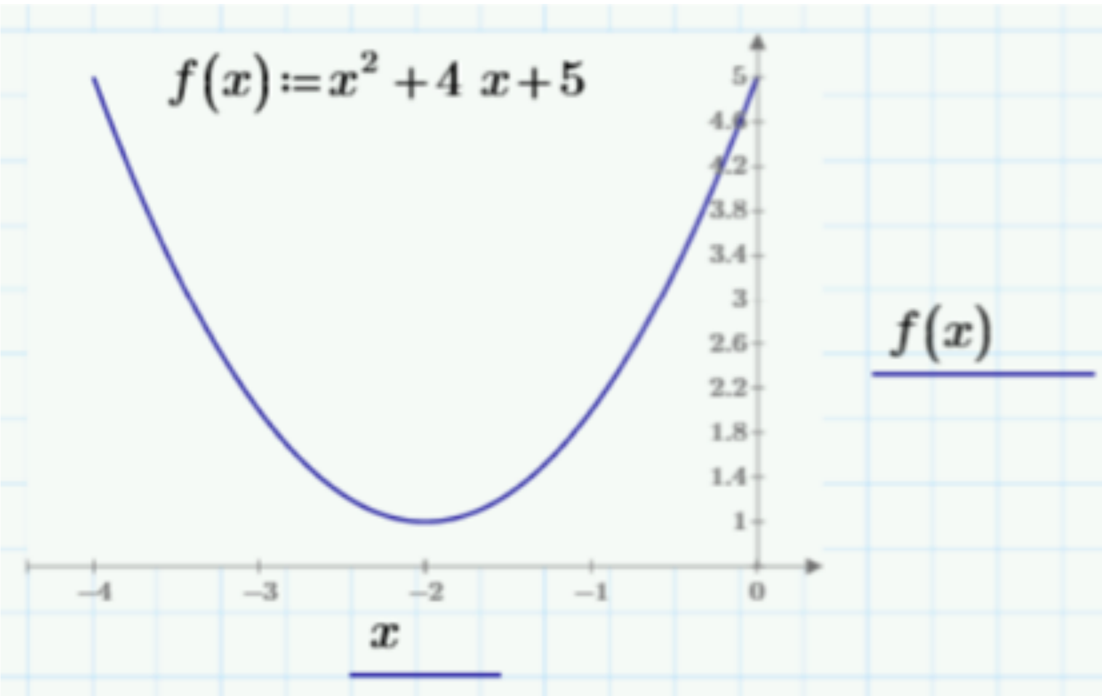
Things to watch out for…



- Imaginary Roots do not represent a Real ($\mathbb{R}$) solution in the *x*-*f*(*x*) plane…

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \begin{bmatrix} -2 + 1i \\ -2 - 1i \end{bmatrix}$$

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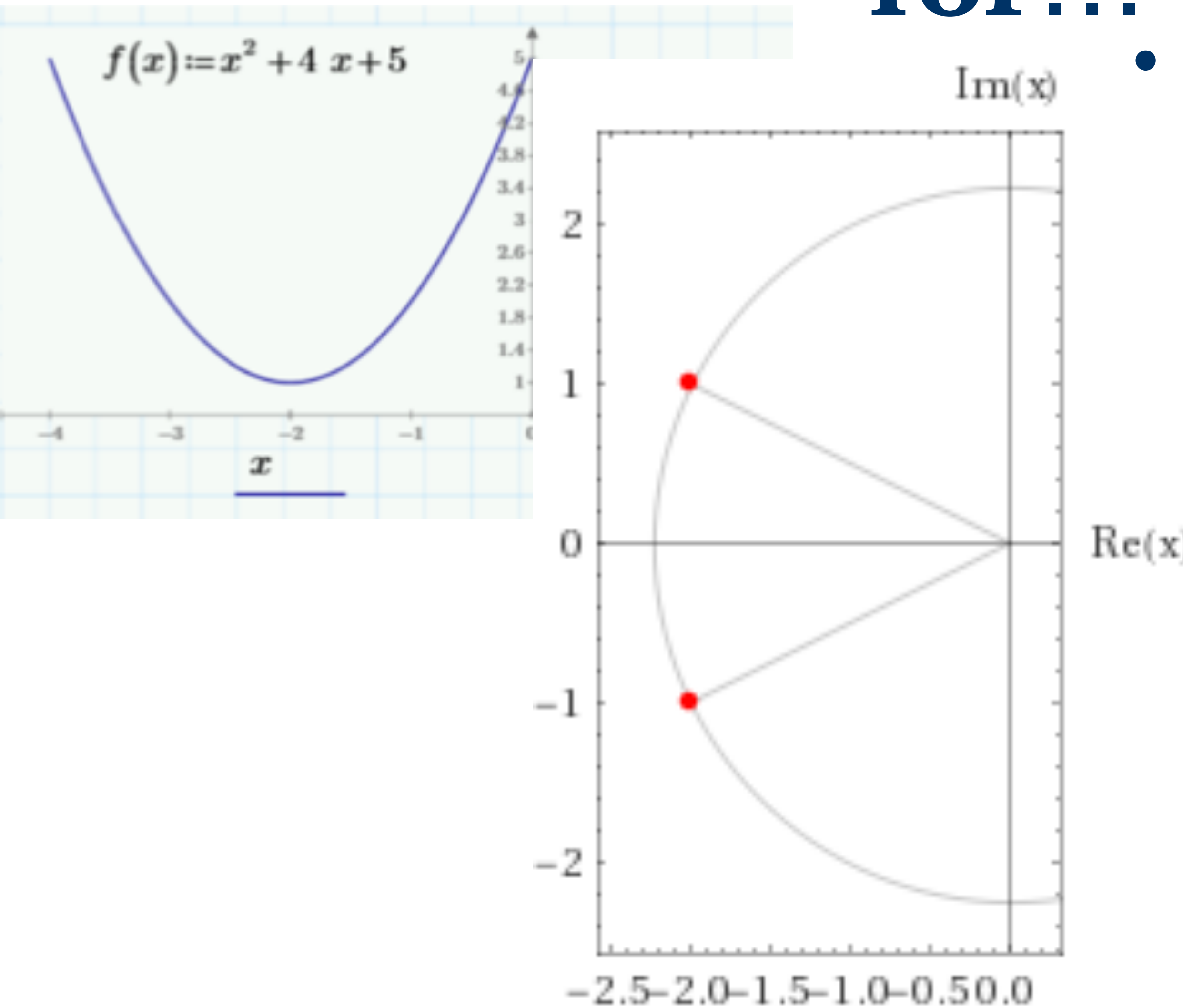
Things to watch out for…

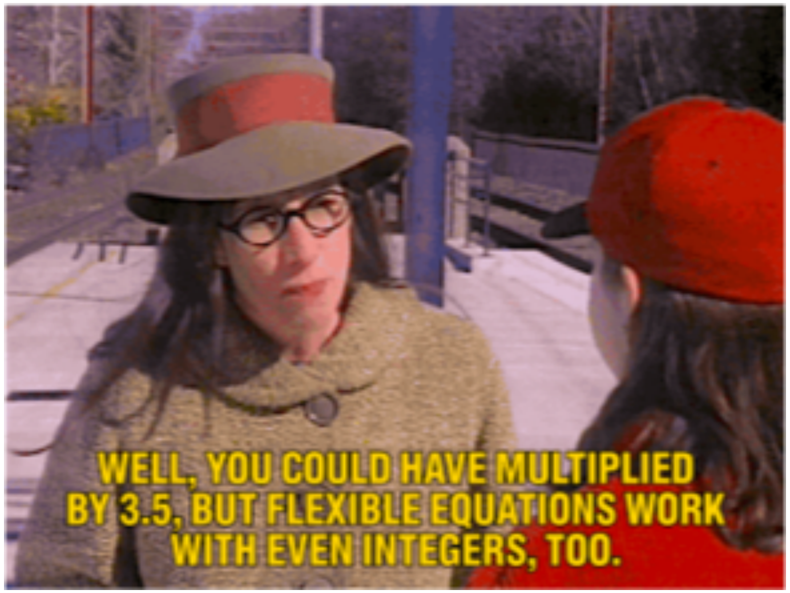


- The solutions are there, they are crossing the $f(x)=0$ in the plane and are in x's Imaginary ($\mathbb{I}$) space

$$x = \begin{bmatrix} -2+1i \\ -2-1i \end{bmatrix}$$

Solving Higher Order Polynomials

- There are other ways attack polynomials (especially cubic and higher-order polynomials)…
- In Mathcad there is the polyroot() function, the Symbolic Arrow and the "Solve Block"



(…Unless Mrs Mercer secretly jammed with the Feelies, REM and Mathew Sweet)

The Symbolic Arrow

$$f(t) := -16.1\ t^2 + 50 \cdot t - 1.67$$

$$f(t) \xrightarrow{solve,\, t} \begin{bmatrix} 0.033767150996630210323 \\ 3.0718229111151710319 \end{bmatrix}$$

DO NOT USE WITH UNITS!!!

$$f(t) := (4\ ft) + \left(40\ \frac{ft}{s}\right) \cdot t - \frac{1}{2}\left(32.174\ \frac{ft}{s^2}\right) \cdot t^2 - (5.667\ ft)$$

$$f(t) \xrightarrow{solve,\, t} \begin{bmatrix} \dfrac{0.062161994156772549263 \cdot s^2 \cdot \left(\dfrac{20.0 \cdot ft}{s} + \dfrac{\sqrt{257.392 \cdot s^2 \cdot ft \cdot ft - 364.660116 \cdot s^2 \cdot ft^2 + 1600.0 \cdot ft^2 \cdot s^2}}{2 \cdot s \cdot s}\right)}{ft} \\ \dfrac{0.062161994156772549263 \cdot s^2 \cdot \left(\dfrac{20.0 \cdot ft}{s} - \dfrac{\sqrt{257.392 \cdot s^2 \cdot ft \cdot ft - 364.660116 \cdot s^2 \cdot ft^2 + 1600.0 \cdot ft^2 \cdot s^2}}{2 \cdot s \cdot s}\right)}{ft} \end{bmatrix}$$

The Polyroot function…

DO NOT USE WITH UNITS!!!

$$f(t) := -16.1\ t^2 + 50 \cdot t - 1.67$$

$$f(t) \xrightarrow{coeffs} \begin{bmatrix} -1.67 \\ 50 \\ -16.1 \end{bmatrix}$$

0th-order coefficient

1th-order coefficient

2nd-order coefficient

… and so on…

$$\text{polyroots}\left(\begin{bmatrix} -1.67 \\ 50 \\ -16.1 \end{bmatrix}\right) = \begin{bmatrix} 0.034 \\ 3.072 \end{bmatrix}$$

The Solve Block

- The Swiss Army Knife of Mathcad
- We'll do this officially later in this section of the class.

Guess Values

$$t := 3.072\ s$$

$$z_{head} := 5.667\ ft \quad v_o := 50\ \frac{ft}{s}$$

$$z_o := 4\ ft$$

Constraints

$$z_{head} = z_o + v_o \cdot t - \frac{1}{2}\ g \cdot t^2$$

Solver

$$\text{find}(t) = 3.074\ s$$

Excel…

- Solver and What If…
- More later in this unit.

When Algebra-Fu isn't enough…

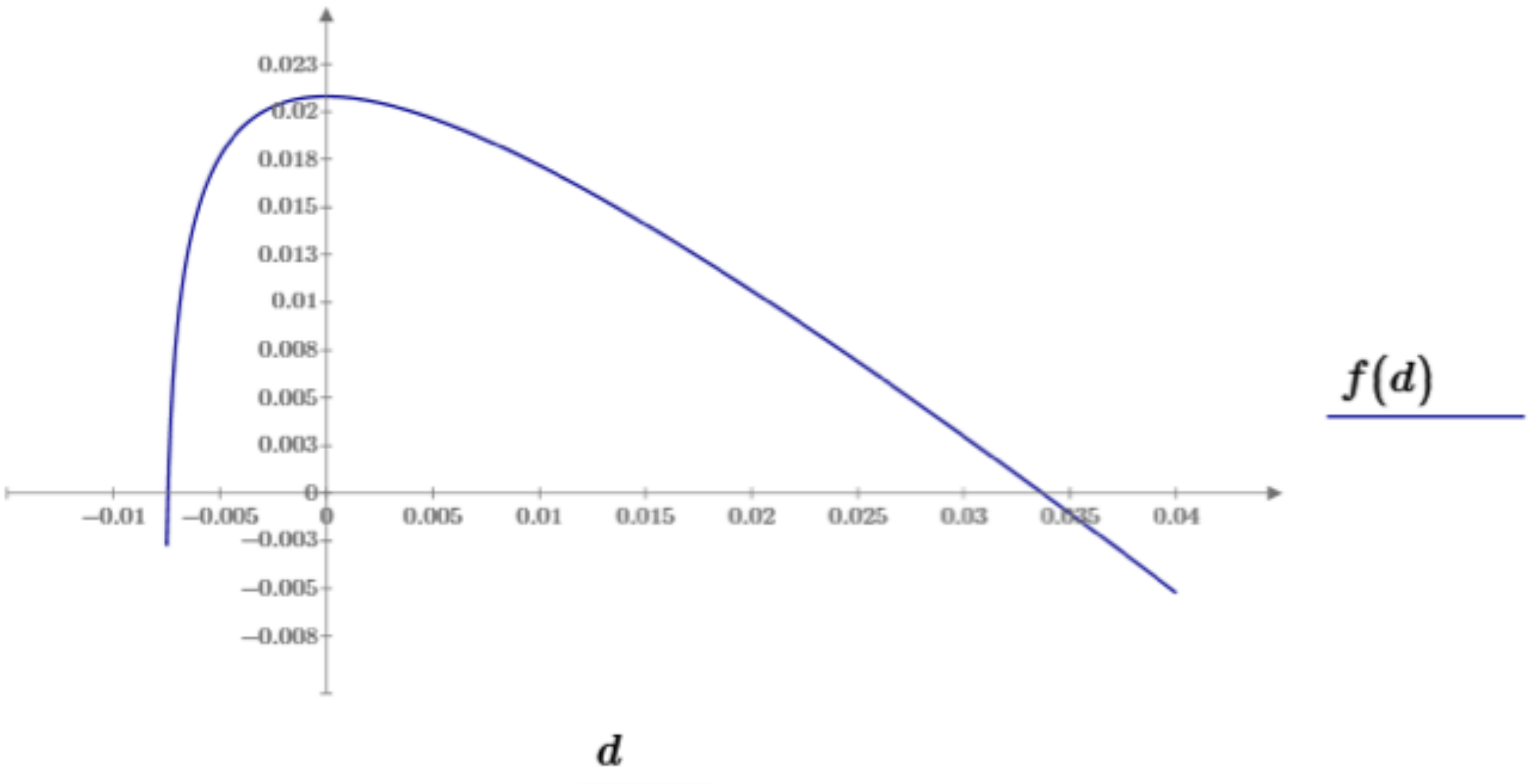
- There are also non-algebraic equations which are not solvable by polynomial approaches…
  - As is shown here where we are solving for the *depth* of rain infiltration into the soil at time *t*…
- This is a "***Transcendental***" equation.

$$depth := K_s \cdot t + \psi_s \cdot \Delta\Theta \cdot \ln\left(1 + \frac{depth}{\psi_s \cdot \Delta\Theta}\right)$$

Our Cliffhanger?

- How to solve?

$$f(d) := K_s \cdot t + \psi_s \cdot \Delta\Theta \cdot \ln\left(1 + \frac{d}{\psi_s \cdot \Delta\Theta}\right) - d$$



Next Time

S.W.A.G.ing Transcendental Equations using Approximation Methods