Approximating Pi via Integration (and Programming)

[In this class, a "sandbox" will refer to an environment in Matchad, Excel, MATLAB, etc where we can explore and idea form class using the software (and learning some software along the way). The best way to learn to work with this class's material is to play hard with it and explore.]

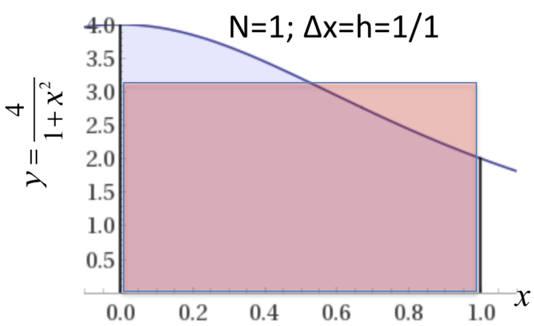
To the side is our Pi Pseudocode Homework:

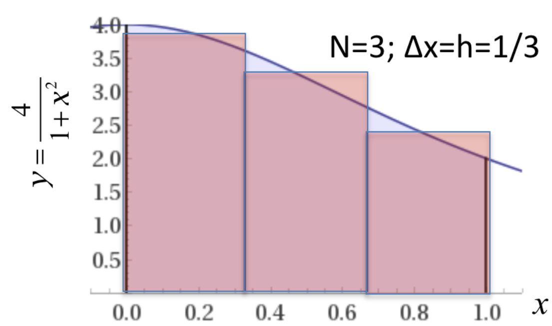
(Let's also remember what this system is really trying to represent). We are emulating an arctangent integral function...

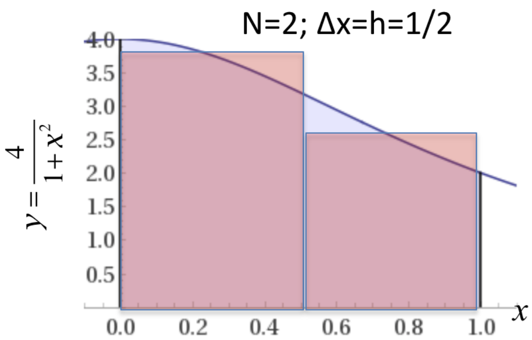
```
Input the value "n" ;  N must be greater than 1.
if n < 1 then
{
    print an error statement
}
else
{
    h = 1 / n
    sum  = 0
    repeat i from 1 to n by 1
    {
        x = h * (i - 0.5)
        sum = sum + 4 / (1 + x²)
    }
    pi = h * sum
    return, pi
}
```

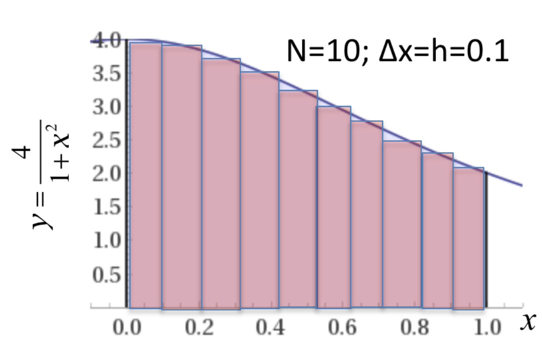
$$\pi = \int_0^1 \frac{4}{1+x^2}dx = 4\arctan(1.0)$$

$$\pi \approx \sum_{i=1}^{N} \frac{4}{1+x_i^2}\Delta x \quad \text{where } x_i = \Delta x\left(i-\frac{1}{2}\right) \ \& \ \Delta x = \frac{1}{N}$$









This case is "simple" so we can acutally do this with the sum operator in Matchad.

$$Pi1(N) := \frac{1}{N}\sum_{i=1}^{N} \frac{4}{\left(1+\left(\frac{1}{N}\left(i-\frac{1}{2}\right)\right)^2\right)}$$

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Our next big step is to do this with programming which happens in the middle of this course.

You can set your value of the number of iterations (N) here -> $N_{test} := 20$

Here is our calculation using the Sum operator

$$Pi1(N) := \frac{1}{N} \sum_{i=1}^{N} \frac{4}{\left(1 + \left(\frac{1}{N}\left(i - \frac{1}{2}\right)\right)^2\right)}$$

$$Pi1(1) = 3.2$$
$$Pi1(2) = 3.1624$$
$$Pi1(3) = 3.1508$$

$$Pi1(N_{test}) = 3.142$$

And here is the calculation using Matchad's programming language.

```
Pi2(N) := | if N < 1
          |   | pi ← "N" "< 1 :-("
          | else
          |   | h ← 1/N
          |   | sum ← 0
          |   | for i ∈ 1..N
          |   |   | x ← h·(i − 1/2)
          |   |   | sum ← sum + 4/(1 + x²)
          |   | pi ← h·sum
          | return (pi)
```

$$Pi2(N_{test}) = 3.142$$

And here is the error for both methods using Matchad's internal value of pi!

$$error1 := Pi1(N_{test}) - \pi = 2.083 \cdot 10^{-4} \quad error2 := Pi2(N_{test}) - \pi = 2.083 \cdot 10^{-4}$$

We also can do this into a percentage of error.

Beware, of putting a percent sign in the formula. Give it a try but it will automatically take it out when it gives the answer and compromise your answer

Sum notation error

$$perc\_error1 := \frac{(Pi1(N_{test}) - \pi) \cdot 100}{\pi} = 0.007$$

code-calculated error

$$perc\_error2 := \frac{(Pi2(N_{test}) - \pi) \cdot 100}{\pi} = 0.007$$

You can also input your functon's argument as a list...

$$perc\_error(N) := \frac{(Pi2(N) - \pi) \cdot 100}{\pi} \qquad n := 1, 2 .. 100$$