Translating from Math to Plain English

For pretty much for any ATM Course that has a math prerequisite

Calculus 1

Calculus 2 (a good place to start looking at this lecture)

Calculus 3

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What lies ahead

- In ATM courses from Synoptic through Atmospheric Physics, you will be facing a lot of equations featuring integrals, deriviatives, and vector operations.

- These equations, while often appearing to be abstract and difficult to tie to the real world, are often presented to actually facilitate real world application.

Today’s goal

- We will explore (or “chew”) these two equations
  - First: The Eq. for Perceptible Water

$$W = \int_{0}^{z_{top}} \rho_a q \, dz$$

  - Second: Second the equation of motion for the east-west wind.

$$\frac{\partial u}{\partial t} = -\vec{U} \bullet \overrightarrow{\mathrm{grad}\, u} - \frac{1}{\rho}\frac{\partial p}{\partial x} + fv - F_x$$

“Chewing” or “Concretizing”

- When I refer to “chewing” I refer to taking a complex equation and breaking it down.
- Concretizing refers to talking a seemingly abstract equation and binding it to physical properties that you can measure or otherwise represent so that they are real.

Quick Aside

- The Equation of Motion features Dr. Capehart's preferred notation. Namely the use of the "text" expression for Gradient the gradient of the east-west wind (u).
- Dr Hjelmfelt prefers the "Del Operator" so that the equation is written as:

$$\frac{\partial u}{\partial t} = -\vec{U} \bullet \vec{\nabla} u - \frac{1}{\rho}\frac{\partial p}{\partial x} + fv - F_x$$

- Both tell the same story

Quick Aside

- Technically Dr. Hjelmfelt's notation is more appropriate.
  - He is also your Dynamics Prof.
- Dr. Capehart, however, has very bad chalkboard handwriting and often misses a dot or a cross or a vector symbol and it drives students crazy.
  - And the text version, is more obscure (typically for cheap typesetting) but is more explicit as to the physical meaning of the vector glyphs we use in dynamics. This is somewhat helpful in map interpretation and also for students who are not co-requing Synoptic and Dynamics
- Always use the proper notation for your professor.
  - When in doubt. Use the glyphs <u>not the text.</u>

$$\frac{\partial u}{\partial t} = -\vec{U} \bullet \vec{\nabla} u - \frac{1}{\rho}\frac{\partial p}{\partial x} + fv - F_x \quad \frac{\partial u}{\partial t} = -\vec{U} \bullet \overrightarrow{\mathrm{grad}\, u} - \frac{1}{\rho}\frac{\partial p}{\partial x} + fv - F_x$$

When in doubt

- With Either Equation we can summarize as follows
  - First: We're adding stuff up (below)

$$W = \int_0^{z_{top}} \rho_a q \, dz$$

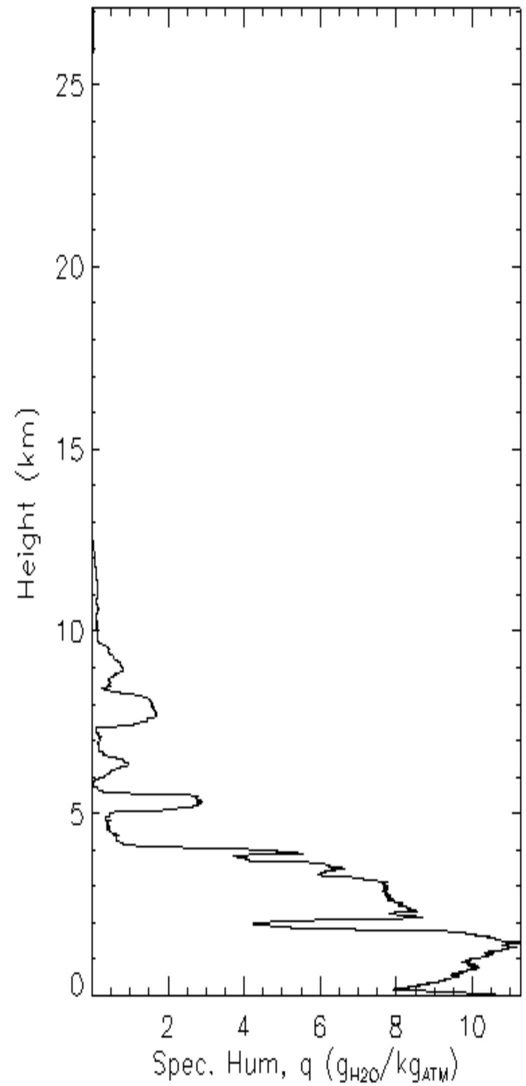
  - Second, We are working with several physical processes (that hopefully we can find on a map!)

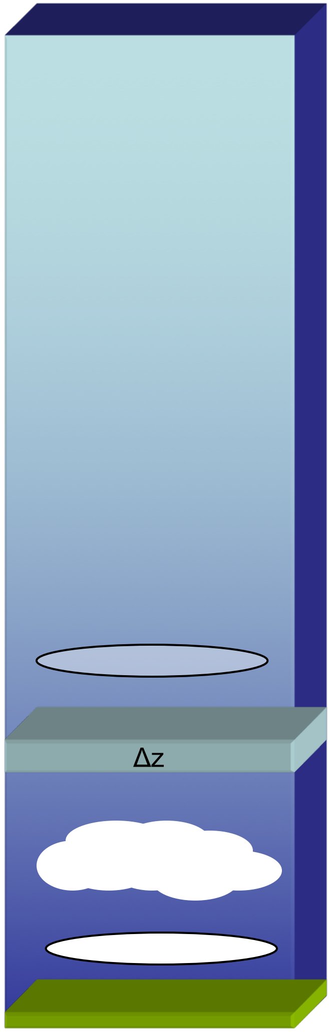
$$\frac{\partial u}{\partial t} = -\vec{U} \bullet \overrightarrow{\mathrm{grad}\, u} - \frac{1}{\rho}\frac{\partial p}{\partial x} + fv - F_x$$

How to proceed?

- Let's start with the perceptible water formula
- You may already have seen this example in another the Integral an Perceptible Water Module

$$W = \int_0^{z_{top}} \rho_a q \, dz$$

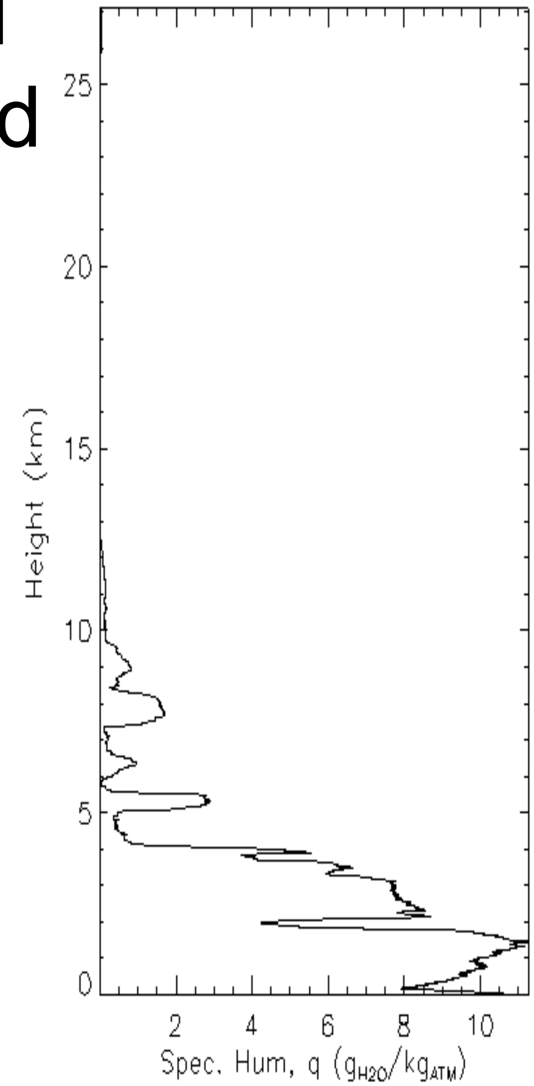


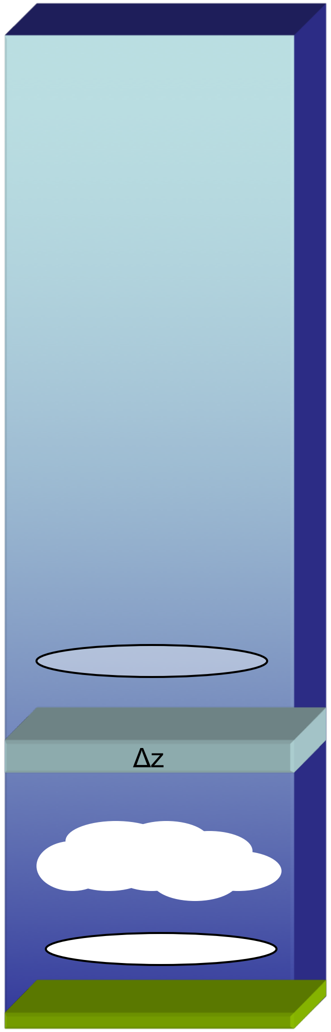


The Integral

- When you first learned about integrals you had it in two perspectives
  1) The opposite of a derivative
  2) Adding Stuff Up Graphically with Riemann Sums

$$W = \int_0^{z_{top}} \rho_a q \, dz$$

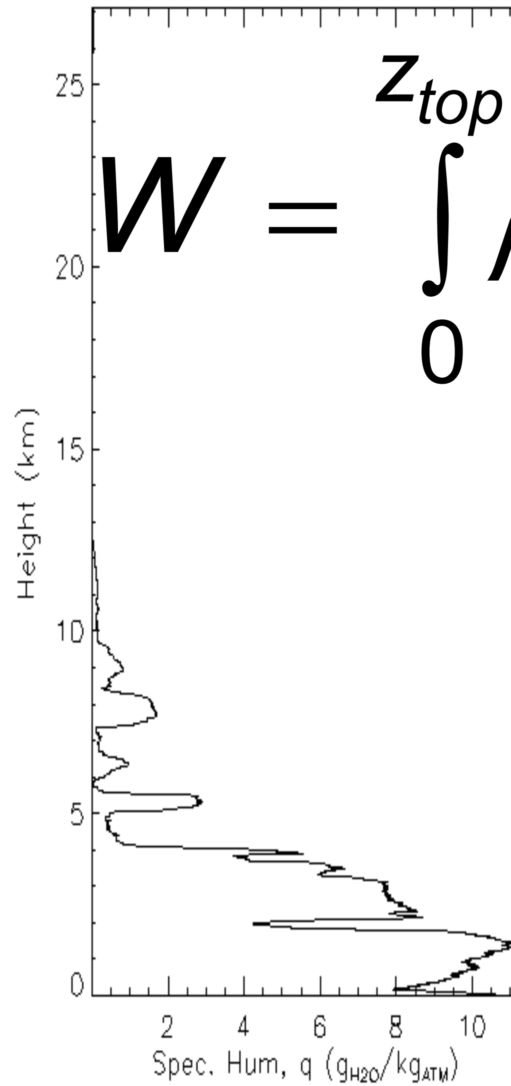


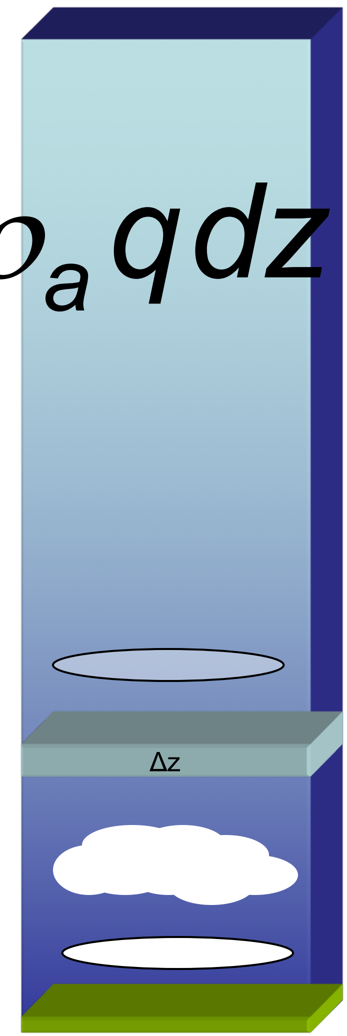


The Integral

- The Good News:
  - We often are referring to some type of summation
    - "We are Integrating over a column"
    - "We are taking into account what happens over a the total path" (This is something you'll see in Calc 3 called "Green's Theorem")
    - "Over X-hours of a physical process at a given rate"
  - When you hear such language as the equations begin to go up on the board, an integral will likely come into play somewhere.

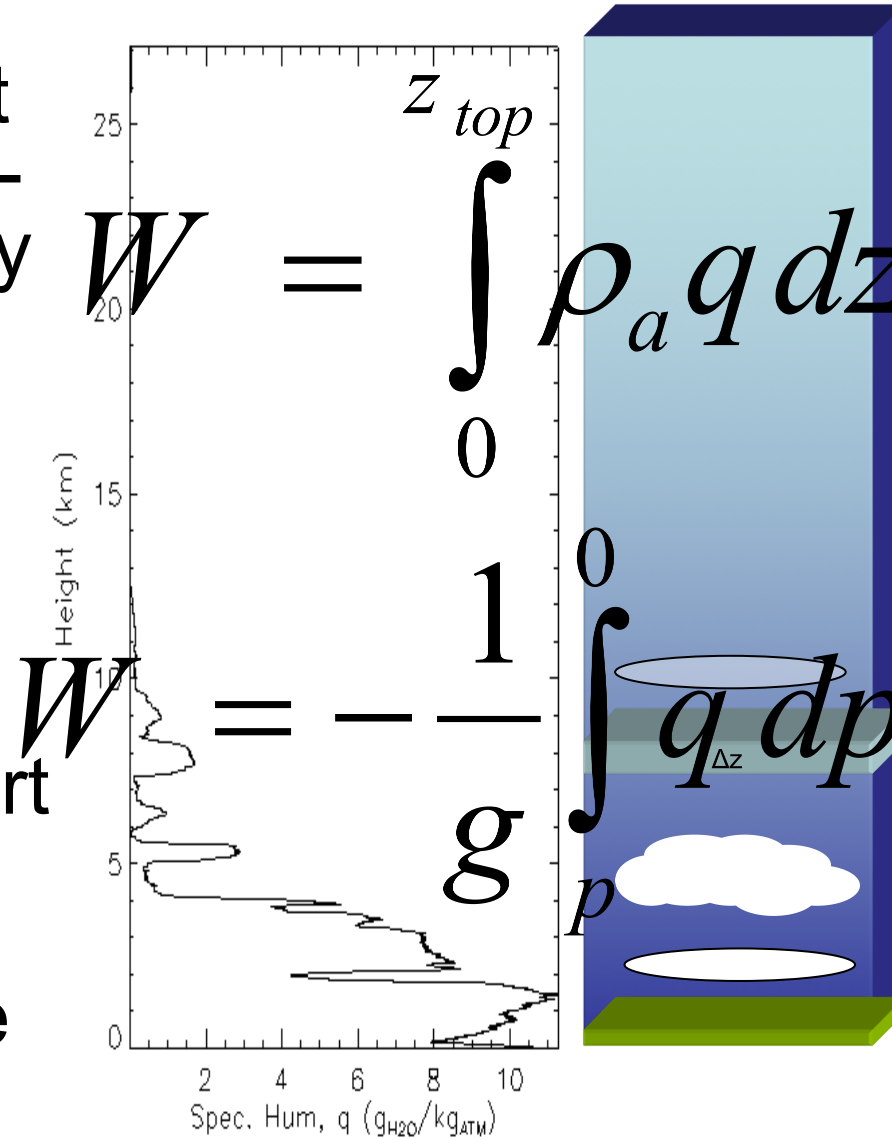
$$W = \int_{0}^{z_{top}} \rho_a \, q \, dz$$





Riemann's and Calc-based Integration

- When doing solutions that require the “Riemann add-em-up Approach” you may also have to do include antiderivitives and integration by substitution or even by parts.   One example of this would be to take the precipitable water example and convert it to a system where pressure is the vertical coordinate (as is often the case in meteorology).



$$W = \int_{0}^{z_{top}} \rho_a q \, dz$$

$$W = -\frac{1}{g}\int_{p}^{0} q \, dp$$