The Integral, Riemann Summing and Atmospheric Precipitable Water

Calc 1 and Atmospheric Physics

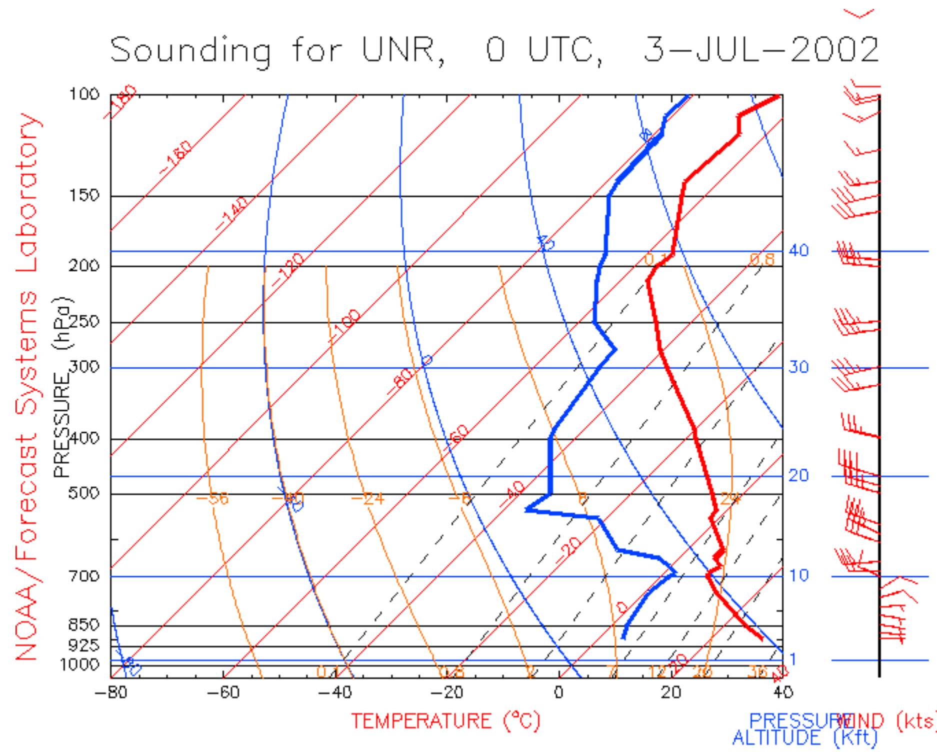
Dr. Bill Capehart
Institute of Atmospheric Sciences
SD School of Mines

Water in the Atmosphere

- Water plays an important role in the atmosphere.
- Obviously it's correlation with rain, snow and clouds is an important factor.
- But it also plays an important role in the buoyancy of air and other processes that indirectly lead to rainfall and storm development.
- It is also an important greenhouse gas
- We also use the presence of water to view atmospheric motion with satellites.

Water in the Atmosphere

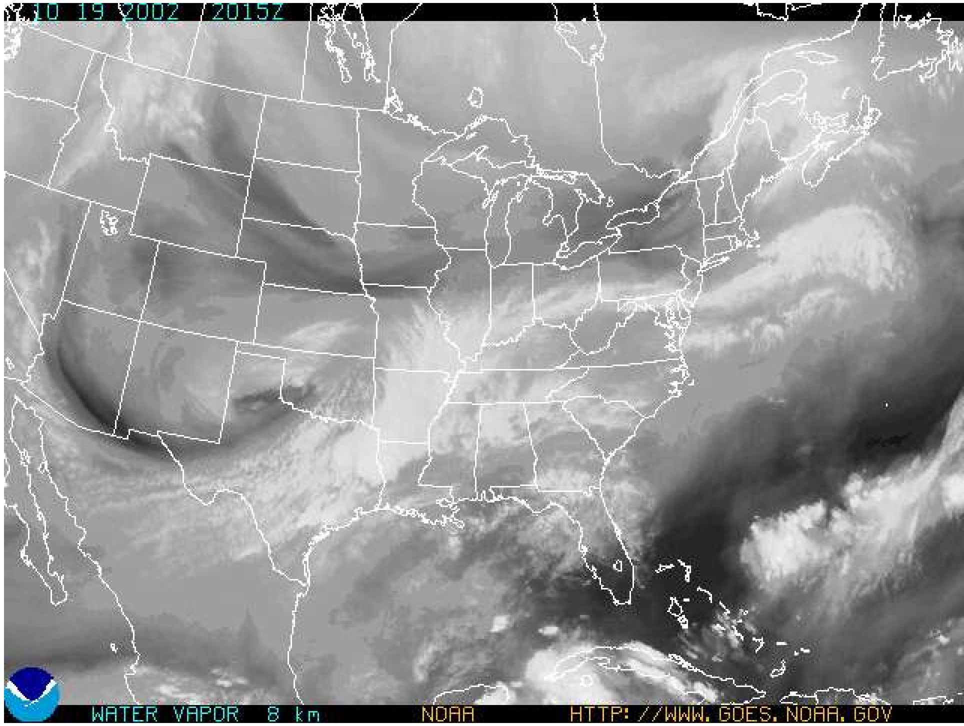
- Atmospheric Water can be measured by weather balloons, satellites that can see the emission of heat from the atmosphere (where water vapor is a key greenhouse gas) and surface instruments that look up into the atmosphere.



A vertical graph of temperature and dewpoint taken from a Rapid City Weather Service weather balloon http://raob.fsl.noaa.gov/
You'll see these charts in Synoptic and Thermodynamics

Water in the Atmosphere

- Atmospheric Water can be measured by weather balloons, satellites that can see the emission of heat from the atmosphere (where water vapor is a key greenhouse gas), and surface instruments that look up into the atmosphere.
- You'll learn about the significance of the bright and dark swirls in Synoptic Meteorology



A image form the NOAA GOES weather satellite – available at http://www.goes.noaa.gov/
You'll see these images pretty much a lot of the time!

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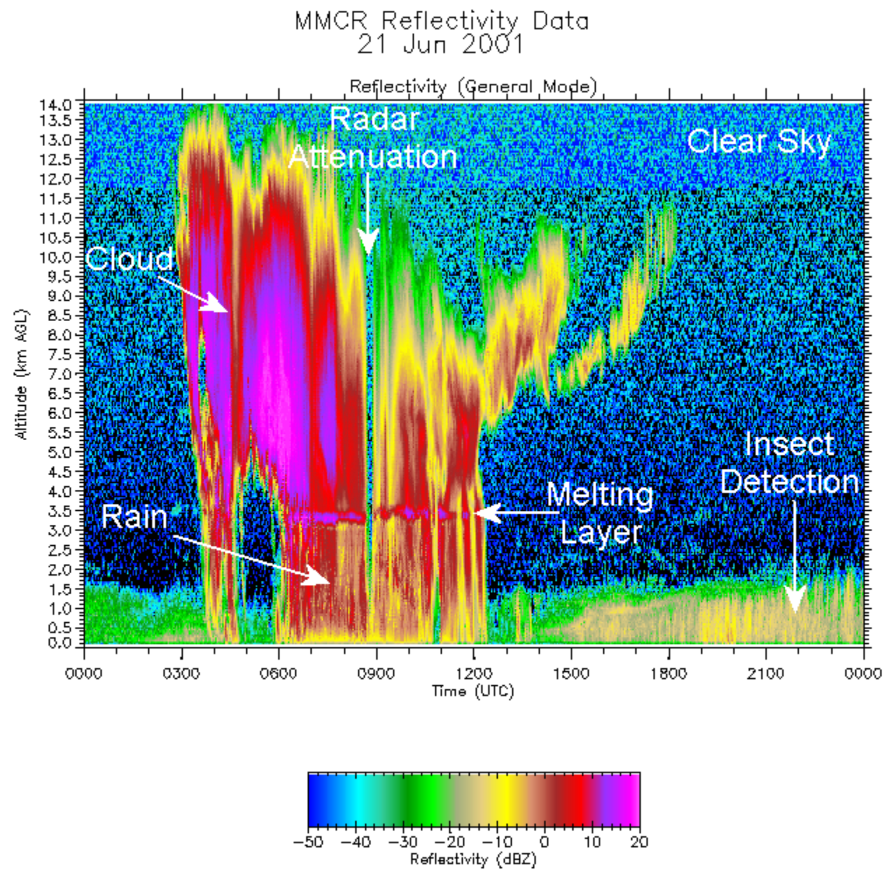
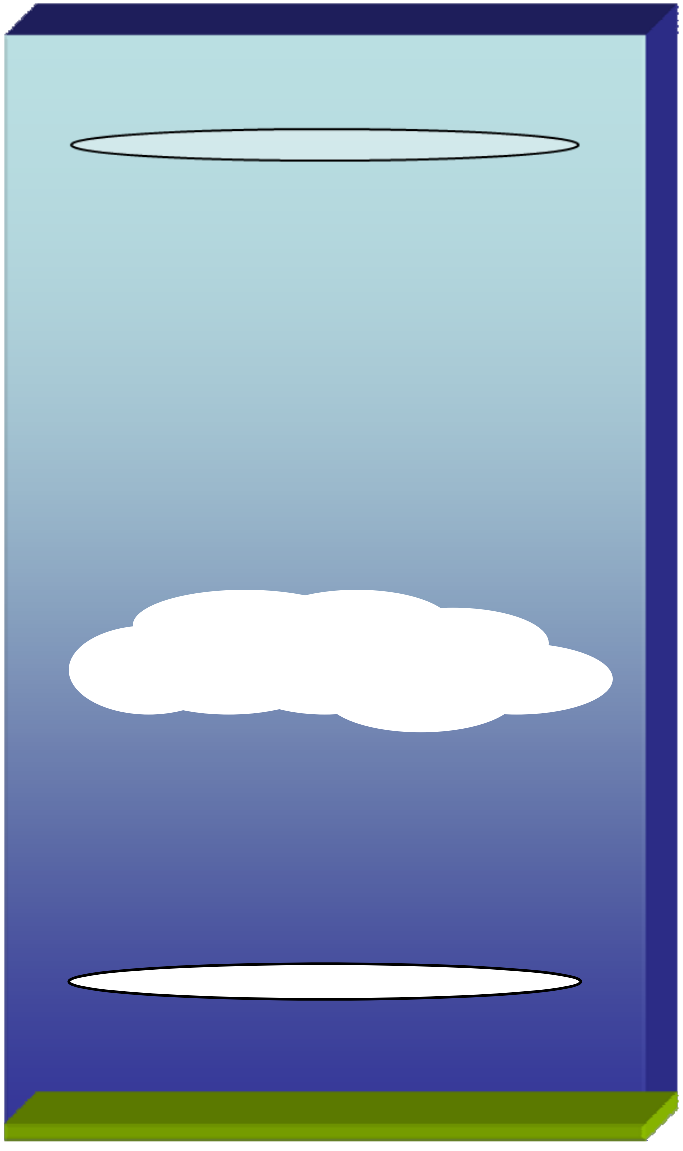


Data from an upward-looking cloud radar from the Dept of Energy's atmospheric observation in Kansas and Oklahoma
http://www.nsdl.arm.gov/Visualization/mmcr/frame.htm

Precipitable Water

- One important way to describe water in the atmosphere is through "Precipitable Water"
- Put simply, the total amount of water (in inches or cm or mm) in a column in the atmosphere.
- This includes not only ice and water in rain or clouds, but also the water vapor in the atmosphere.

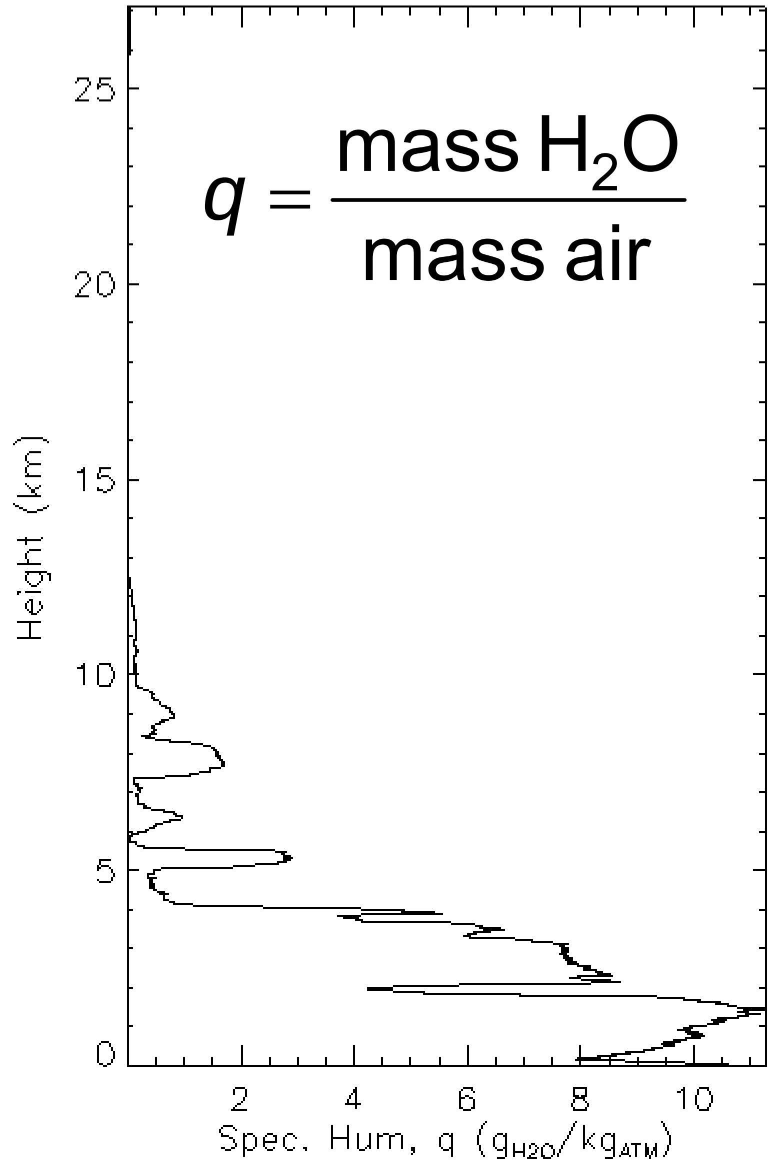


Precipitable Water

- In this example, we are going to use a very detailed sounding from the DOE's Kansas and Oklahoma network to show how basic calculus, the integral and Riemann summing can help you estimate the total amount of water in a column.

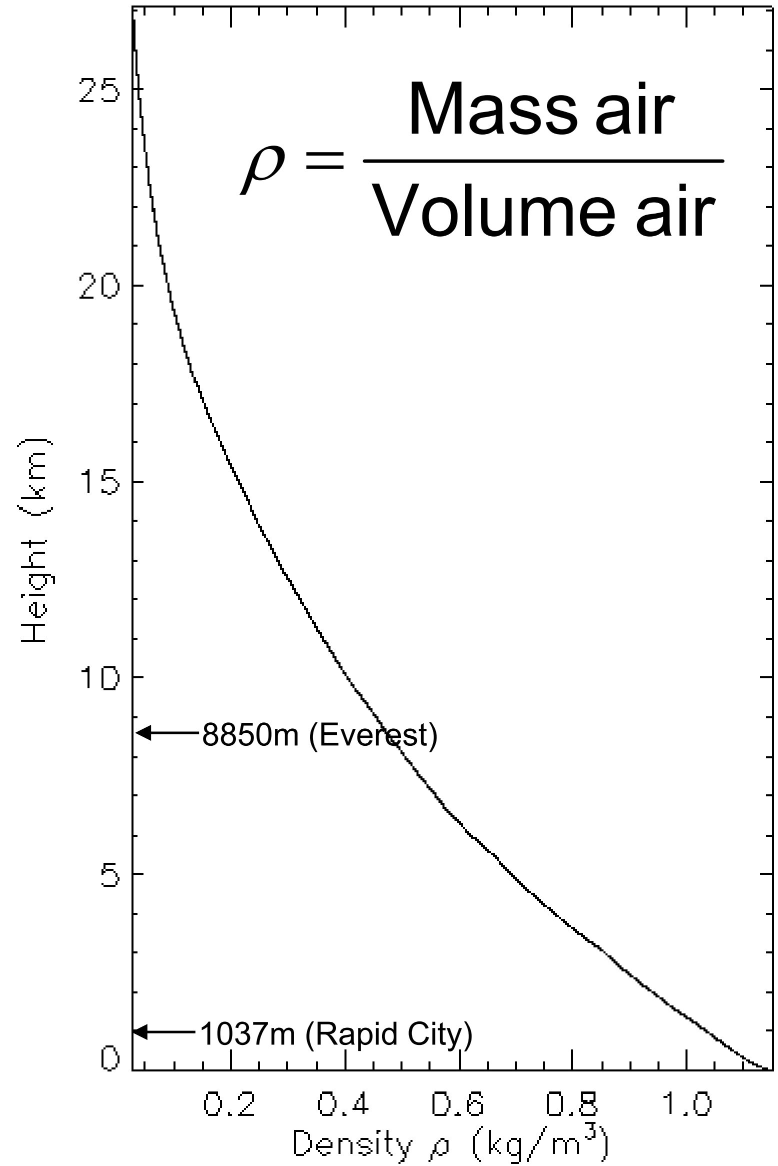
Humidity Profile

- You're probably accustomed to looking at humidity in terms of relative humidity or, perhaps, dew point.
- Here, we use a parameter called "absolute humidity" (q) which is defined as the ratio of total mass of water vapor to the total mass of air for a given layer.
- This will give us a more direct way to estimate the total amount of water vapor a column
- Notice that there is much less water in a column than total air (even in a cloud). As such we use a ratio of grams of water per kilogram of atmosphere



Density Profile

- It also helps if you have a profile of atmospheric density (ρ).
- Notice that the profile of density decreases exponentially with height.
- (Which is why people who climb Everest and other high mountains, need oxygen tanks)
- For more about how this graph is calculated read the natural log and exponential decay module



Precipitable Water and Integration

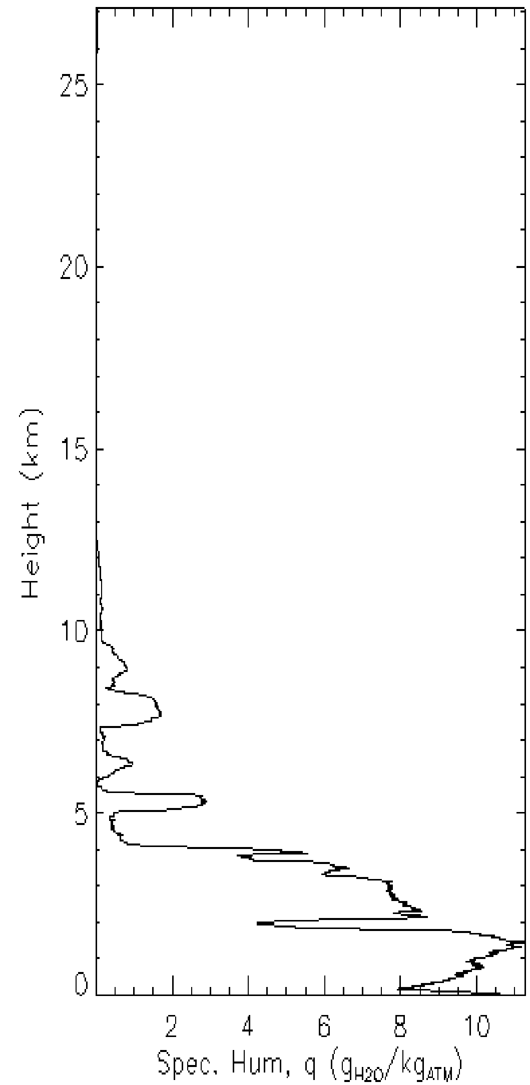
- Precipitable Water is effectively the *integrated* columnar water. Not the emphasis of "integrated."
- Indeed, the definition is

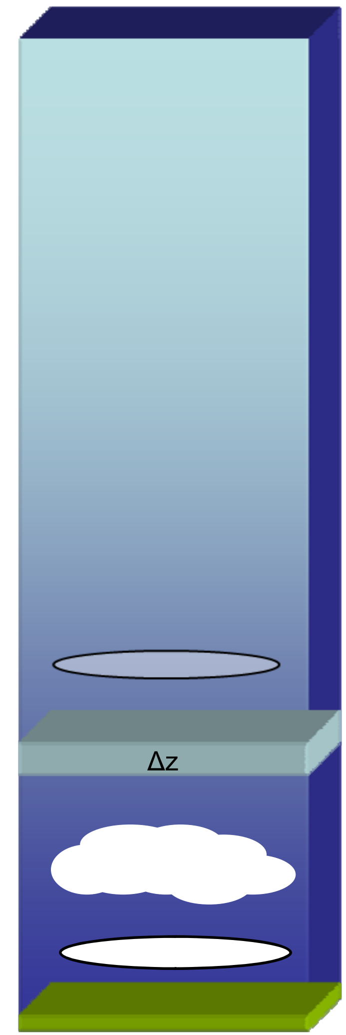
$$W = \int_0^{z_{top}} \rho_a q \, dz$$

- By looking at the formula we can see why the formula is structured as it is. Let's look at the units…

$$W\left(\frac{kg_{water}}{m^2_{space}}\right) = \int_0^{z_{top}} \rho_a \left(\frac{kg_{air}}{m^3_{air}}\right) q\left(\frac{kg_{water}}{kg_{air}}\right) dz\left(m_{air}\right)$$

- Unit integrity is very important in these types of calculations.
  - Notice how we've not used km or g – only "MKS" units



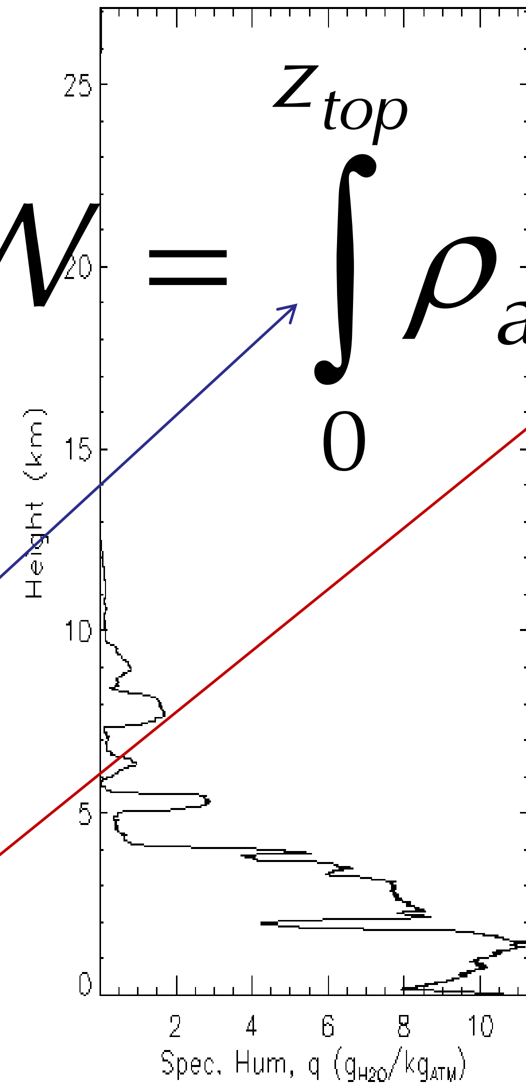


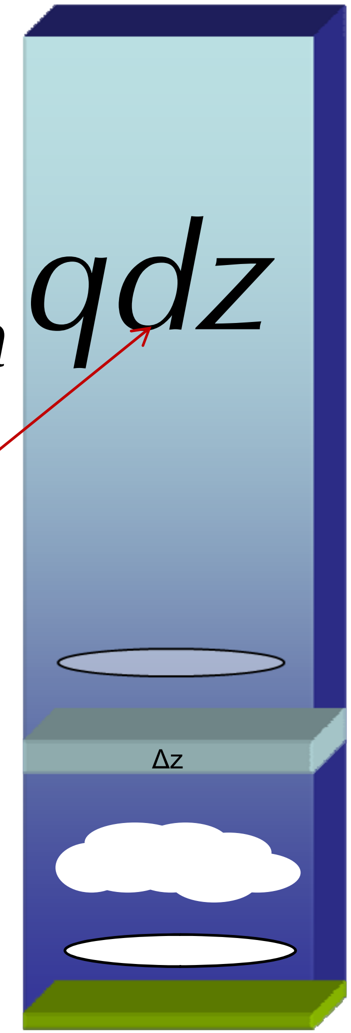
Intermission

- Let's Pause and second look at that Integral

- In Physical Equations, an Integral often implies that we will be adding things up along a path (ds, dx, etc) or over time (dt)
  - Here we are going from the surface (z=0) to a fictional "top" of the atmosphere ($z=z_{top}$)
  - And we are doing this in tiny (infinitesimal) increments of Δz (hence the dz and not the delta).

$$W = \int_{0}^{z_{top}} \rho_a q \, dz$$





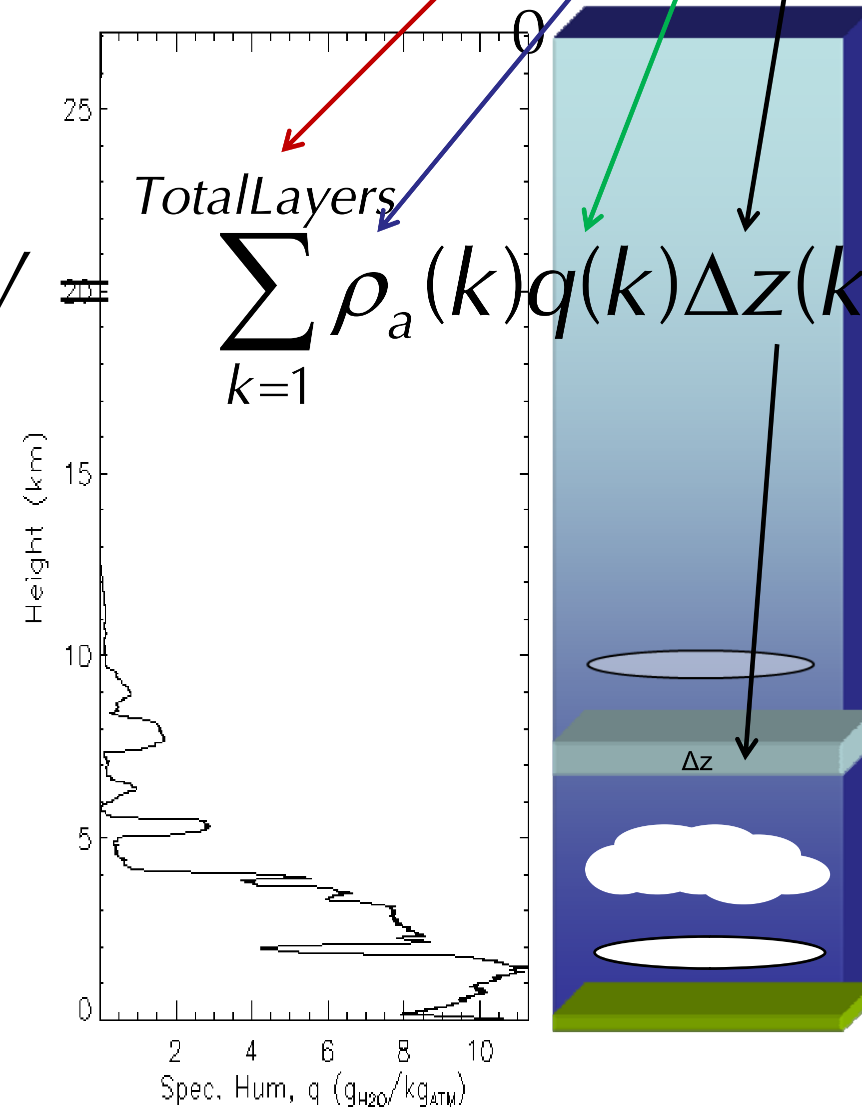
Intermission

$$W = \int_0^{z_{top}} \rho_a q \, dz$$

- Often when we work with integrals in the real world, especially in this example, we do what you do in Calc 1. Break it down into little boxes that we *can* measure.

$$W = \sum_{k=1}^{TotalLayers} \rho_a(k) q(k) \Delta z(k)$$

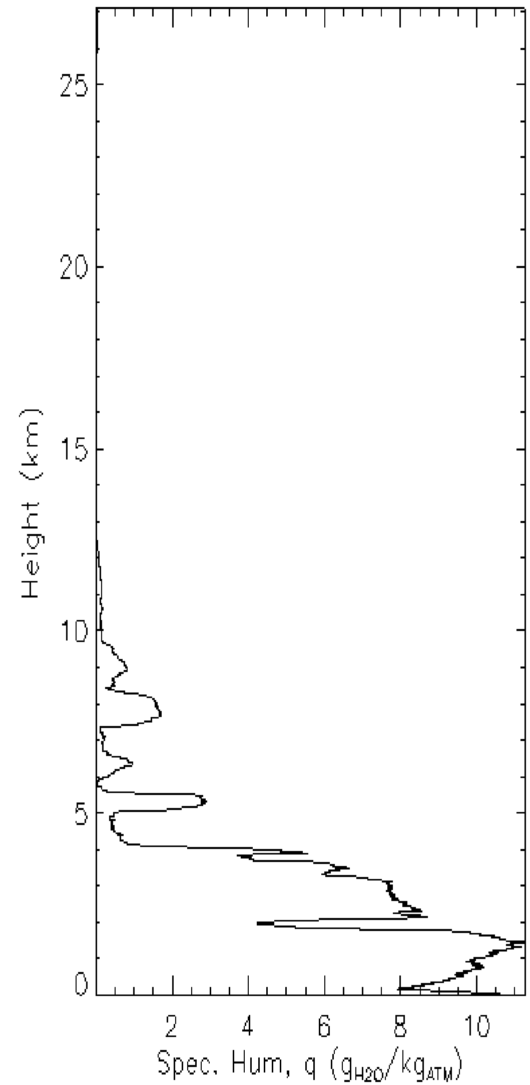
- If you need to physically interpret an equation (and you should do this every time you see one), you can often use the Riemann Sum translation in your head!
    - (here we use “k” as a counter for the height layers)

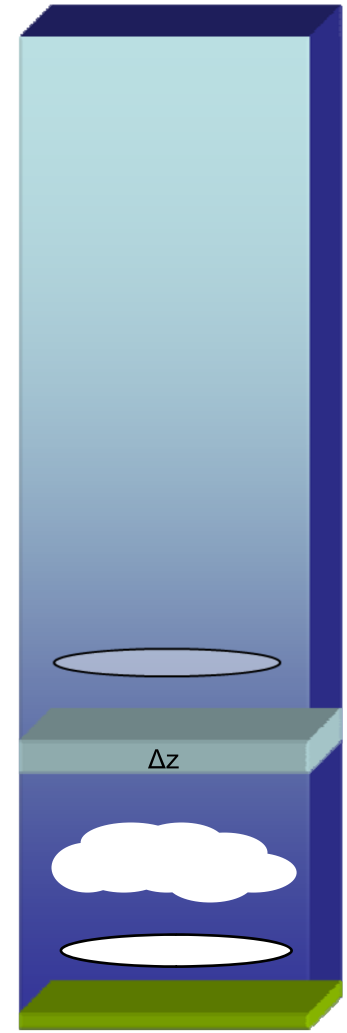


Precipitable Water and Riemann Sums

- When we solve this using real data, we actually don't use anything as elegant as a integrals. Looking at the graph of specific humidity to the right, we can see the complexity in the plot. Getting a simple function of q = f(z) would simply not provide an acceptable result. Therefore, we rely in the precursor to integration: The Riemann Sum and use the integral form to present the idealized principal.

$$W\left(\frac{kg_{water}}{m^2_{space}}\right) = \sum_{k=1}^{TotalLayers} \rho(k)\, q(k)\, \Delta z(k)$$

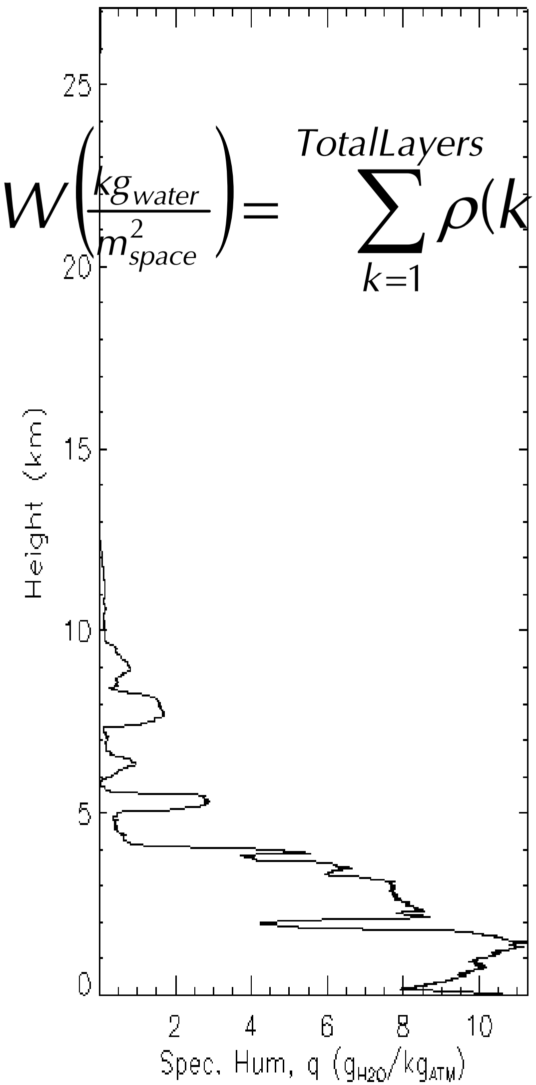


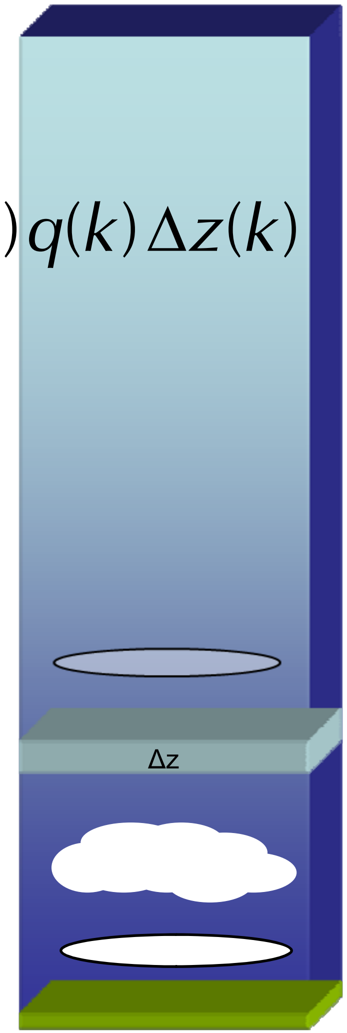


Precipitable Water and Riemann Sums

- To solve this problem, we could use this equation in a spreadsheet, if we don't have that many Δz increments.
- For a large number of Δz's (this case to the side has over 3000 layers – well in excess of conventional weather balloon observations!)
- For this case, which represents a rather wet atmosphere, the result is about 34.46 kg_{water} sitting over a square meter of real estate
  - Prove that units estimate
- However, we often prefer to express this quantity in terms of water depth (not mass).

$$W\left(\frac{kg_{water}}{m^2_{space}}\right) = \sum_{k=1}^{TotalLayers} \rho(k)\, q(k)\, \Delta z(k)$$





One Last Unit Conversion

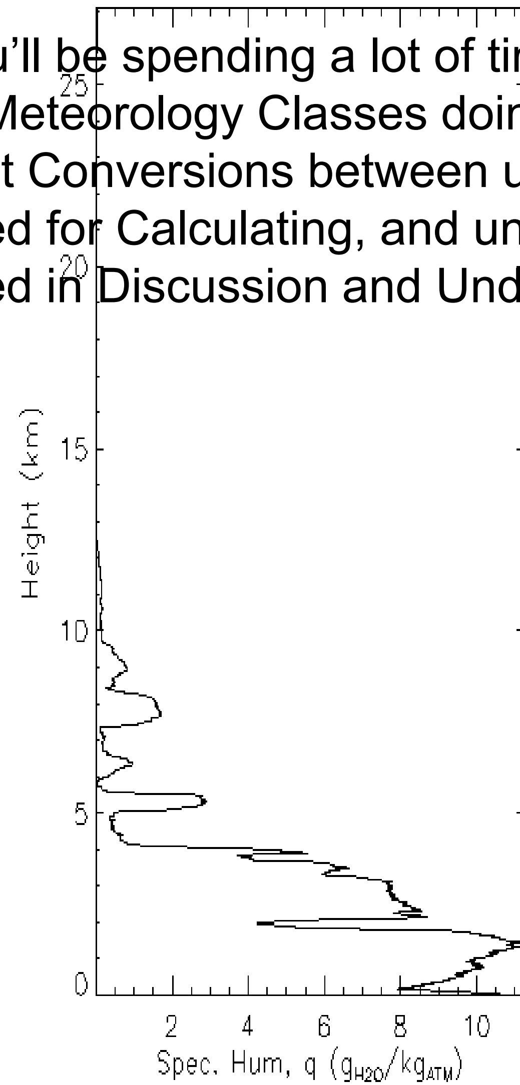
- To convert our 34.46 kg_{water}/m^2_{air} of column moisture into a depth we need one last conversion.

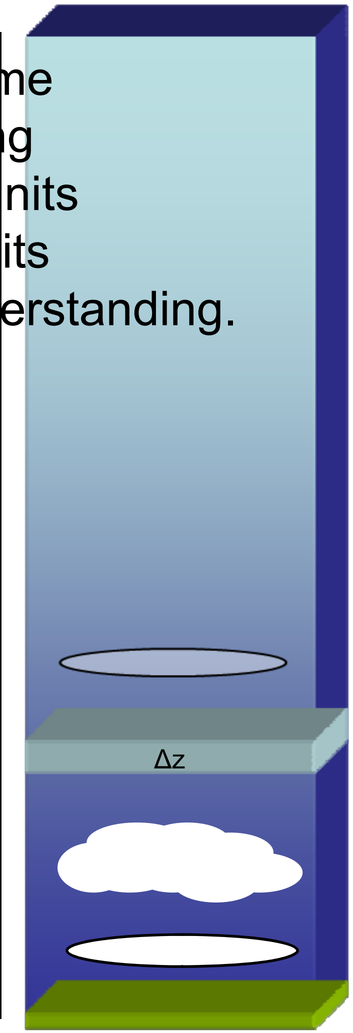
$$W(m_{water}) = W\left(\frac{kg_{water}}{m^2_{space}}\right)\left[\rho_w = \left(\frac{m^3_{space}}{1000\,kg_{water}}\right)\right]$$

$$W(cm_{water}) = W(m_{water})\left(\frac{100cm}{m}\right)$$

- Thus our depth of water in the column is 3.446 cm or (conveniently!) 34.46 mm

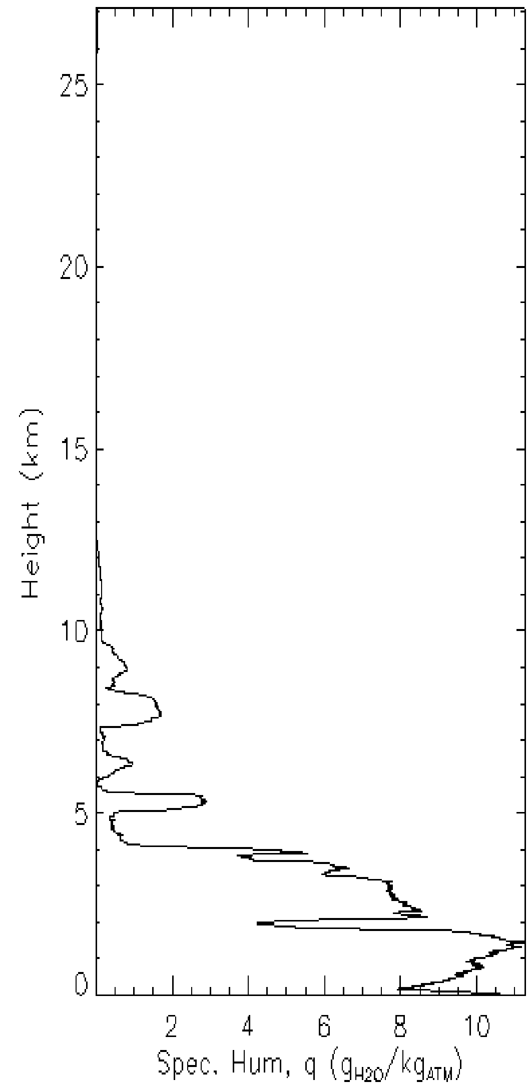
You'll be spending a lot of time in Meteorology Classes doing Unit Conversions between units used for Calculating, and units used in Discussion and Understanding.

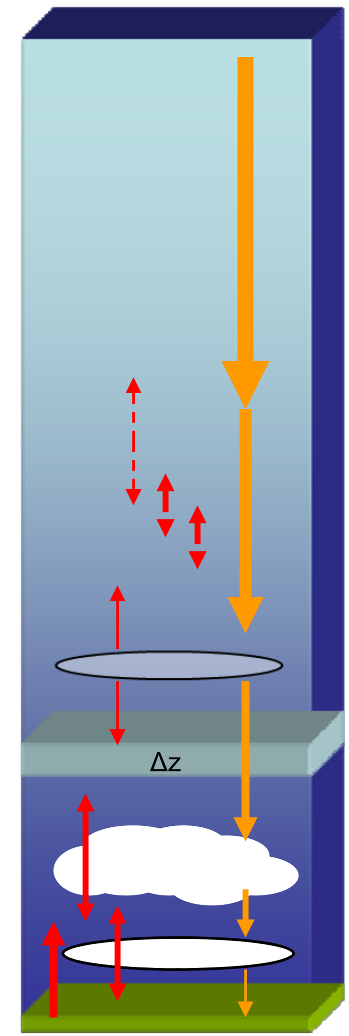




Other Uses of the Integral and Summing

- We also use this application of the integral to add up the contributing components of atmospheric and solar radiation coming from, and traveling through, the atmosphere.
- You'll also see it when we do budgets where we tally up the properties within a volume or region





Bored? Oh! we can fix that!

- When you are at a point where you've covered integration by substitution consider this:
- Instead of Z (height) we often use p (pressure) as a vertical coordinate (this is often the case in meteorology. (Pressure decreases with height so you'll see a minus sign). This relationship is called the hydrostatic approximation. You will get it in thermo, dynamics, synoptics and atmospheric physics.

$$\frac{dp}{dz} = -\rho_a g$$

- Instead of integrating from z=0 to a fictional z="ztop", do it from p=psfc to p=0.
- Hint: this is what you'll get: $W = -\frac{1}{g}\int_{p_{surface}}^{p_{top}=0} q\,dp$

Got Questions

- Talk to
  - Dr Helsdon and Dr Detwiler: Atmospheric Physics and Thermodynamics