Epidemic Modeling Sandbox
(Prereq Differential Equations)

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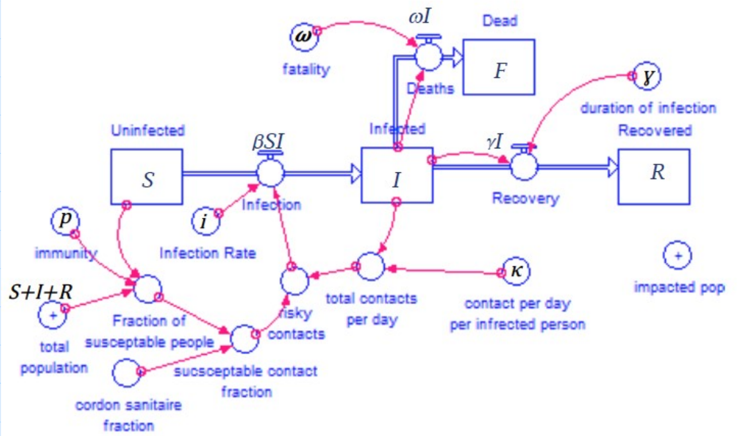
Consider the following System of Susceptable (S), Infected (I), Recovered (R) and Dead (F) populations being exposed to a given contagion.



$i := \frac{1}{4}$ Infection Rate (chance of infection upon exposure per unit time) one in four in this example

$\gamma := \frac{1}{2}$ Recovery Rate (chance of recovery upon exposure per unit time) also 2 days typically needed to recover

$p := 0$ Fracton of people who know more about immunity than Jenny McCarthy and Andrew Wakefield and got their %$&#@ Flu Shot!

$\kappa := 6$ Number of contacts that risk infection per infected person per day

$\omega := 0.1$ Death Rate (fraction of infected people who expire per day)

$R_o := \frac{i \cdot \kappa}{\gamma + \omega} = 2.5$ The R-naught factor: the number of people that one sick person effects

$p_o := 1 - \frac{1}{R_o} = 0.6$ Nominal Herd Immunity Fraction to contain an outbreak

This system combines to create a classic equation in epidemiology called the Kermack-McKendrick Equations

$$dS(t,S,I,R) := -\frac{\kappa \cdot i \cdot (1-p)}{S(t)+I(t)+R(t)} \cdot S(t) \cdot I(t)$$ Uninfected People (lots of unit-Fu here)

$$\beta = \frac{\kappa \cdot i \cdot (1-p)}{S(t)+I(t)+R(t)}$$ This is Beta from the classic K-McK equations

$$dI(t,S,I,R) := \frac{\kappa \cdot i \cdot (1-p)}{S(t)+I(t)+R(t)} \cdot S(t) \cdot I(t) - \gamma \cdot I(t) - \omega \cdot I(t)$$ Infected People

$$dR(t,S,I,R) := \gamma \cdot I(t)$$ Recovered (and Aquired-Immune) people

$$dF(t,S,I,R) := \gamma \cdot \omega \cdot I(t)$$ Fatalities

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This setup requries a "short" game rather than a long multigenerational simulation

$t_0 := 0 \qquad t_f := 30$

Let's use a solve-block for this

Guess Values

$this\_space\_is\_blank := 0$

Constraints

$$\frac{d}{dt}S(t) + \frac{\kappa \cdot i \cdot (1-p)}{S(t)+I(t)+R(t)} \cdot S(t) \cdot I(t) = 0$$

$$\frac{d}{dt}I(t) - \left(\frac{\kappa \cdot i \cdot (1-p)}{S(t)+I(t)+R(t)} \cdot S(t) \cdot I(t) - \gamma \cdot I(t)\right) = 0$$

$$\frac{d}{dt}R(t) - \gamma \cdot I(t) = 0 \qquad \frac{d}{dt}F(t) - \omega \cdot I(t) = 0$$

$S(0) = 998$
$I(0) = 2$
$R(0) = 0$
$F(0) = 0$

Solver

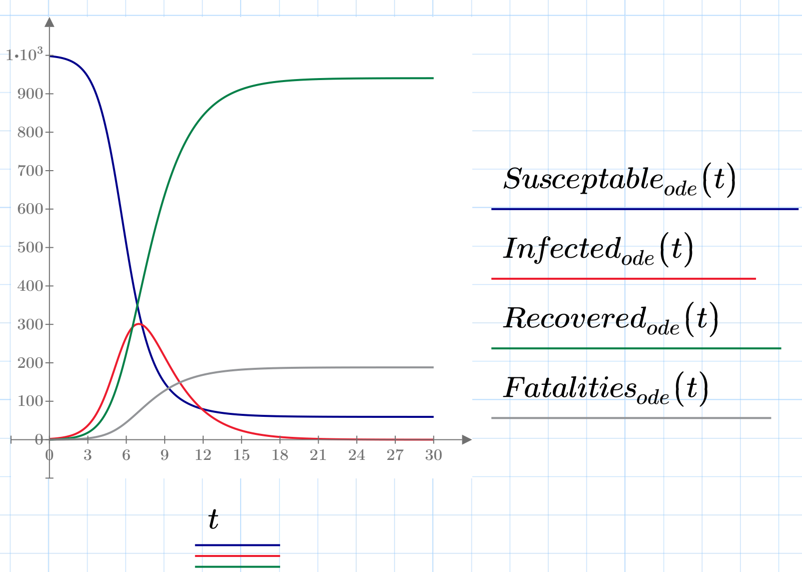
$$SIRF_{odesolve} := \mathbf{odesolve}\left(\begin{bmatrix} S(t) \\ I(t) \\ R(t) \\ F(t) \end{bmatrix}, t_f\right)$$

Extract our participating variables

$Susceptable_{ode} := SIRF_{odesolve_0} \qquad Recovered_{ode} := SIRF_{odesolve_2}$

$Infected_{ode} := SIRF_{odesolve_1} \qquad Fatalities_{ode} := SIRF_{odesolve_3}$

Here is a the plot of the system



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If you want to try a Runge-Kutta solution you must pu thte equations in an array system...

$$SIRF := \begin{bmatrix} 998 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$ Initialziation of our four categories

$$D_{SIRF}(t, SIRF) := \begin{bmatrix} -\dfrac{\kappa \cdot i \cdot (1-p)}{SIRF_0 + SIRF_1 + SIRF_2} \cdot \left(SIRF_0 \; SIRF_1\right) \\ \dfrac{\kappa \cdot i \cdot (1-p)}{SIRF_0 + SIRF_1 + SIRF_2} \cdot \left(SIRF_0 \; SIRF_1\right) - \gamma \cdot SIRF_1 - \omega \cdot SIRF_1 \\ \gamma \cdot SIRF_1 \\ \omega \cdot SIRF_1 \end{bmatrix}$$

We'll need a time step $\Delta t := 0.2$ Which gives us a total number of time steps (N) $N := \dfrac{30}{\Delta t} = 150$

$SIRF_{rk} := \mathrm{rkfixed}\left(SIRF, t_0, t_f, N, D_{SIRF}\right)$ Here is the Runge-Kutta solver

And now we extract our variables (time is the first one)

$$time := SIRF_{rk}{}^{\langle 0 \rangle}$$

$Susceptable_{RK} := SIRF_{rk}{}^{\langle 1 \rangle}$ $Recovered_{RK} := SIRF_{rk}{}^{\langle 3 \rangle}$

$Infected_{RK} := SIRF_{rk}{}^{\langle 2 \rangle}$ $Fatalities_{RK} := SIRF_{rk}{}^{\langle 4 \rangle}$