Using Temperature Transport to Describe the Derivative and Chain Rule… AND Forecasting Temperature (Part 1)

Atmospheric Dynamics and Synoptic Meteorology

Calculus 1

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The Derivative in Meteorology

- The Derivative is often used in Meteorology to explain how one feature changes as another feature changes.

- For example: The force on the atmosphere induced by the earth's rotation (the curious force or "f" is a function of the latitude or "φ") so that. From there, we can discuss how this force increase or decreases as you go northward.

  - …But that's another story…

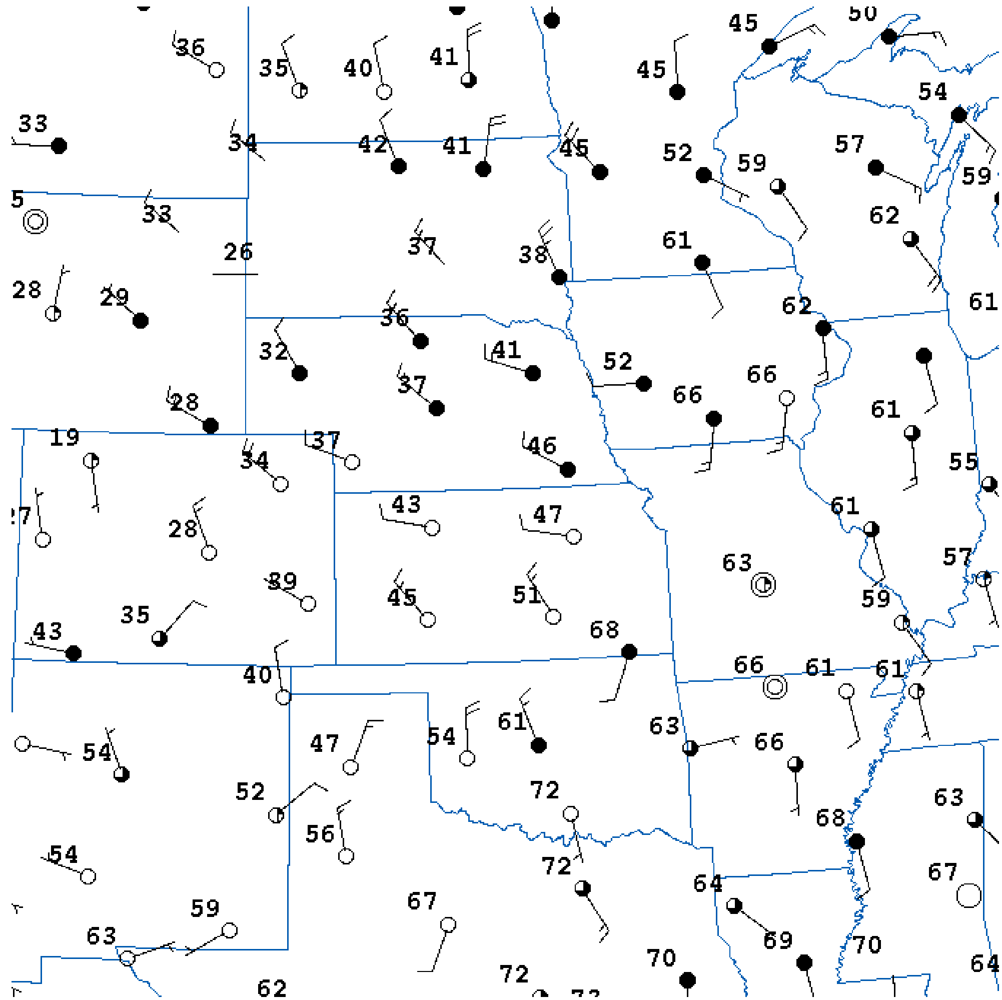
The Derivative as a Physical Gradient

- We can easily use the derivative to describe how a parameter (like temperature) changes over a given distance.

- Consider the map to the right, for example. It is a ***real*** map of a cold front moving across the plains.

- In ATM 301 and ATM 450, you should be able to find the front.

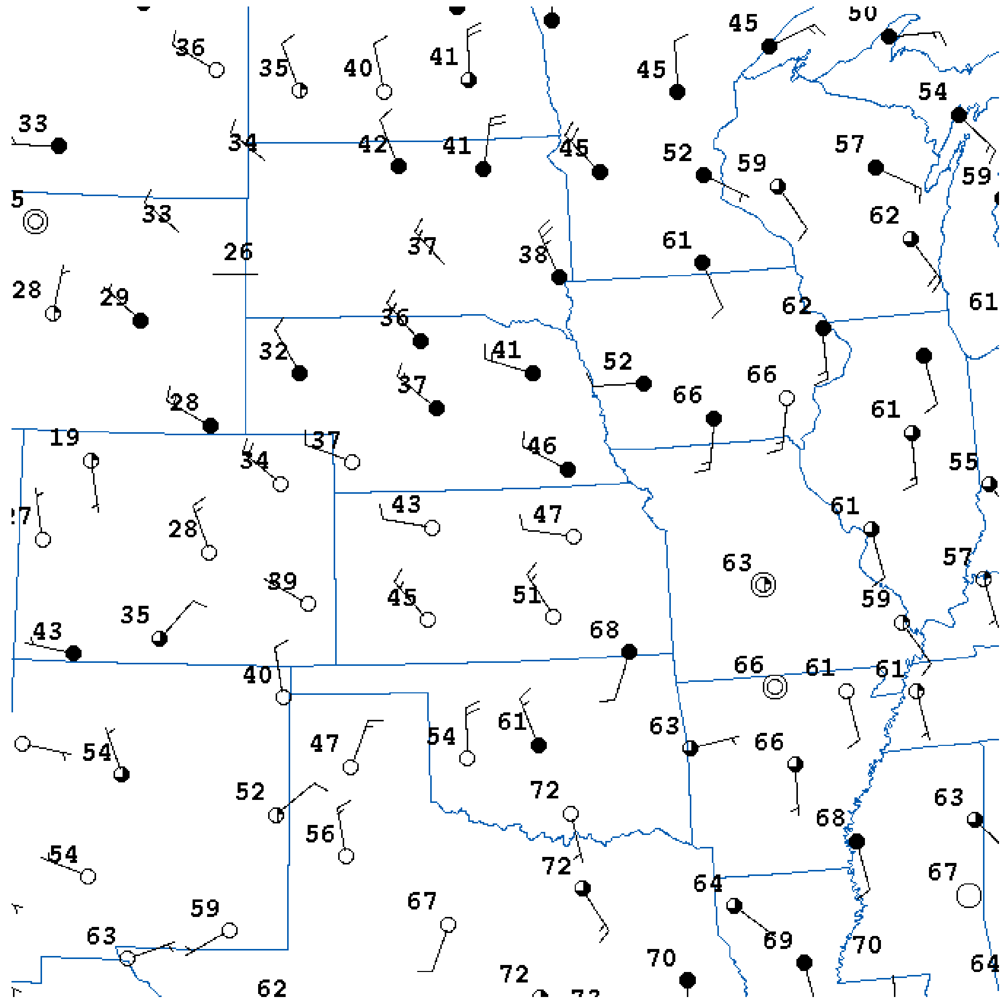
- Temperature (in ºF) is in the upper left-hand corner of each of these “station models” with the dot representing the location of the station and relative cloud cover (the darker the circle the cloudier it is).



The Derivative as a Physical Gradient

- We can easily use the derivative to describe how a parameter (like temperature) changes over a given distance.

- Consider the map to the right, for example.
  - Temperature (in °F) is in the upper left-hand corner of each of these "station models" with the dot representing the location of the station and relative cloud cover (the darker the circle the cloudier it is).
  - The "barbs" leading form the circles indicate the speed of the wind (the more pips and flags the stronger the wind) and direction from which the wind is blowing

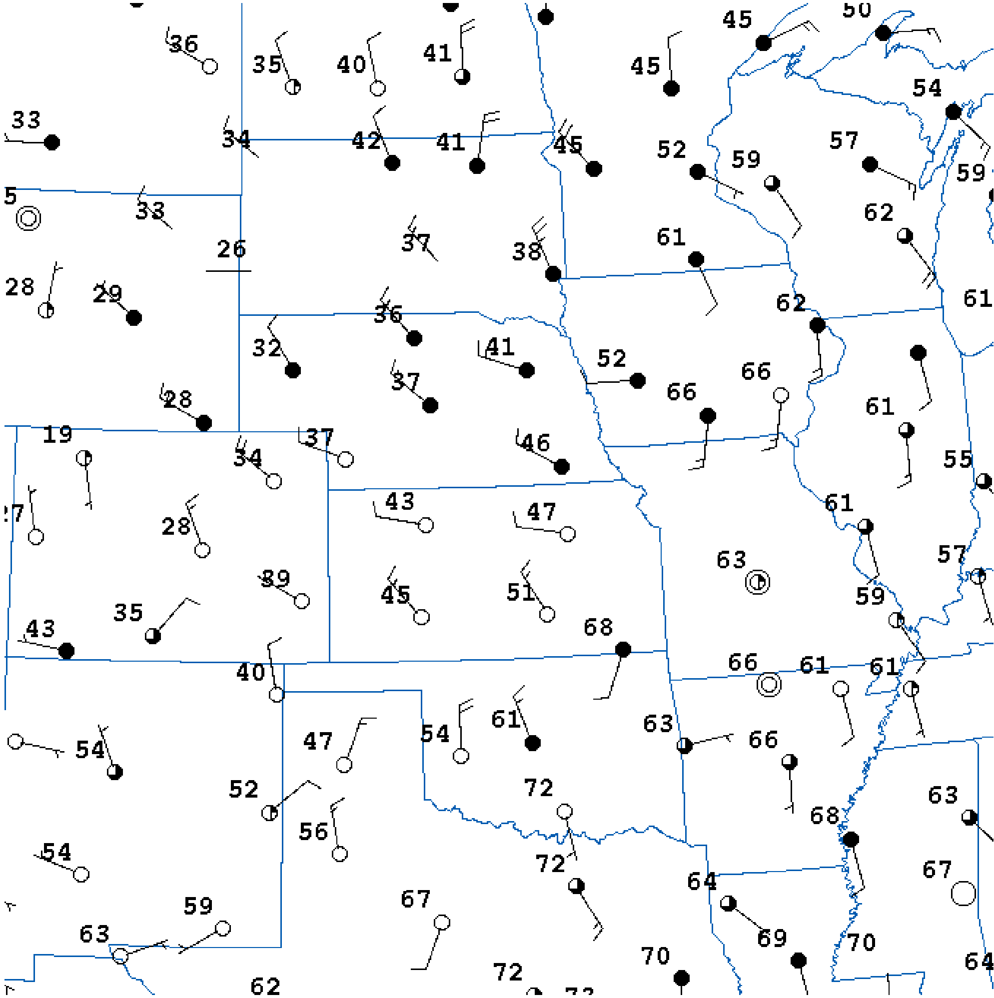


The Derivative as a Physical Gradient

- Physically you may already be able to guess what is happening -- The wind is blowing the air (and its properties, such as temperature) from one point to another.

- In meteorology, we call this "***Advection***"

- In the generic sciences this term is what you learned as "convection." We use that term to specifically mean the *vertical* motion of air.



Advection as a Physical Process

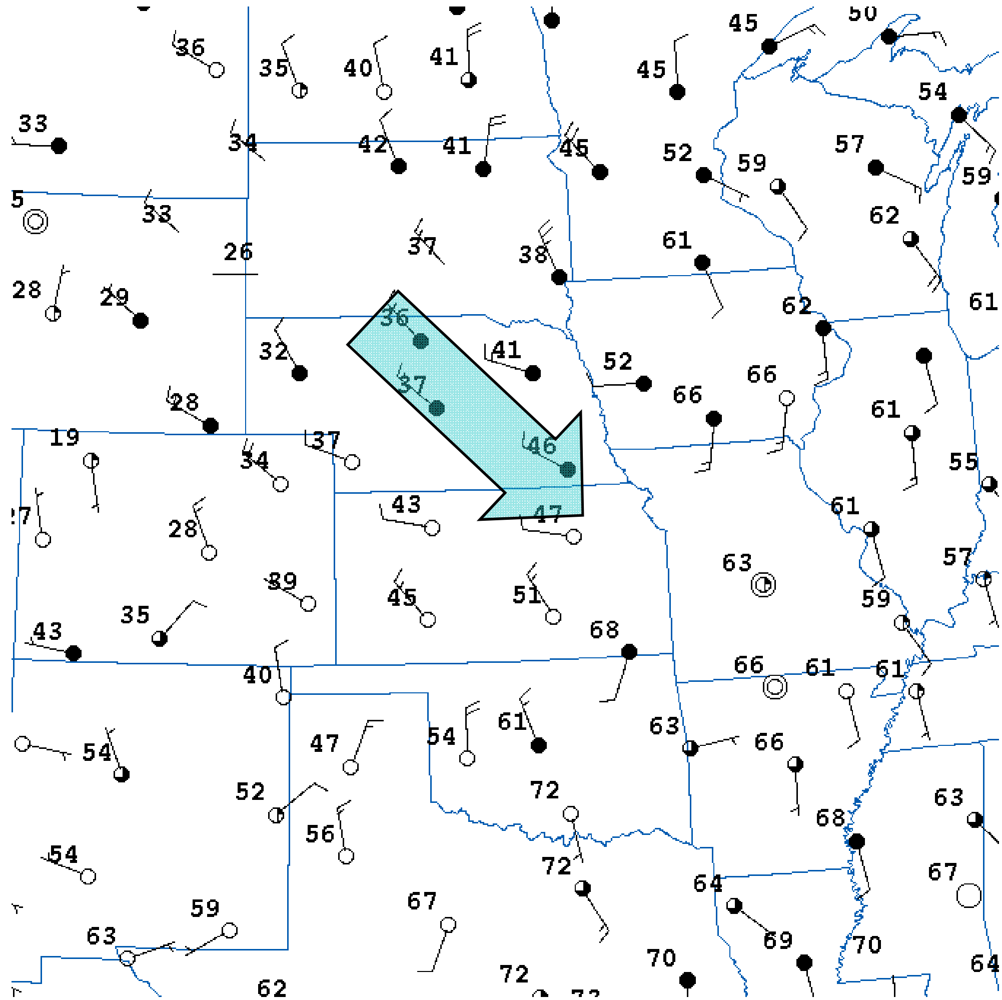
- With this map, and a little applied calculus, we can use the temperature values and wind speeds to estimate how the wind is transporting warm or cold air.

- To begin, lets consider the flow across Nebraska and Kansas.

- We can tell that air from the north west is blowing into the area, which should lower the temperatures at a given point.

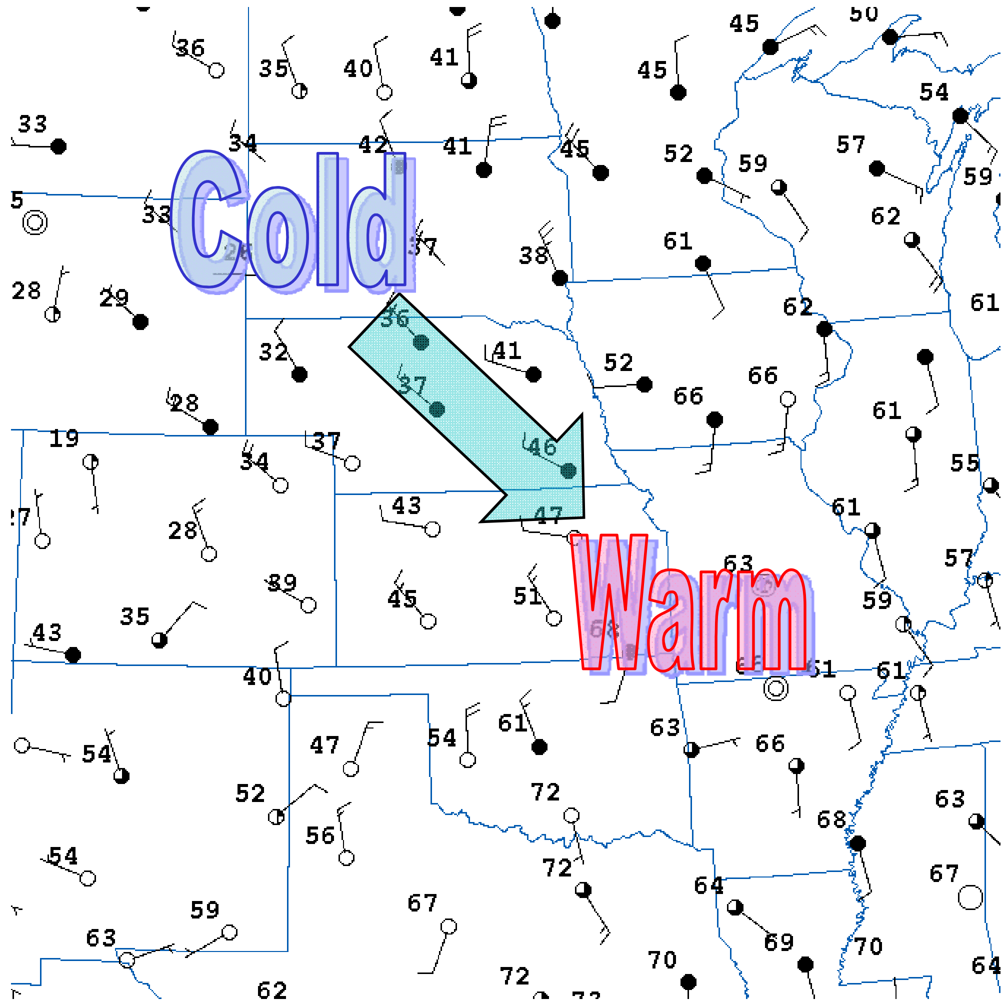
- We call this, appropriately,

  COLD ADVECTION



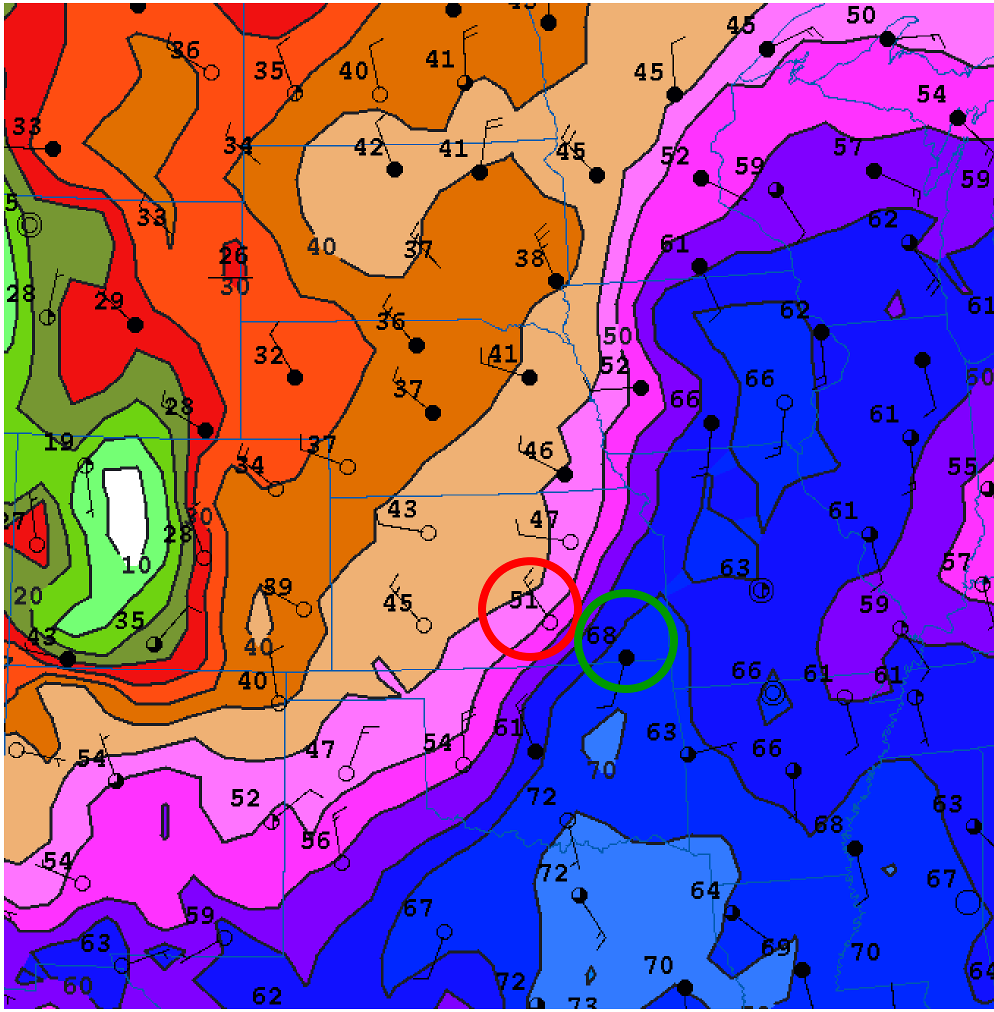
Advection as a Physical Process

- Now, let’s look more closely at the situation. We have cold air near Rapid City, warmer air near Topeka, and a wind blowing from one point to another.
- To better visualize the change of properties such as temperature over space, we often use contour maps like this…
- ***<u>Contours are lines of equal value and are made so that we can easily view physical patterns</u>***



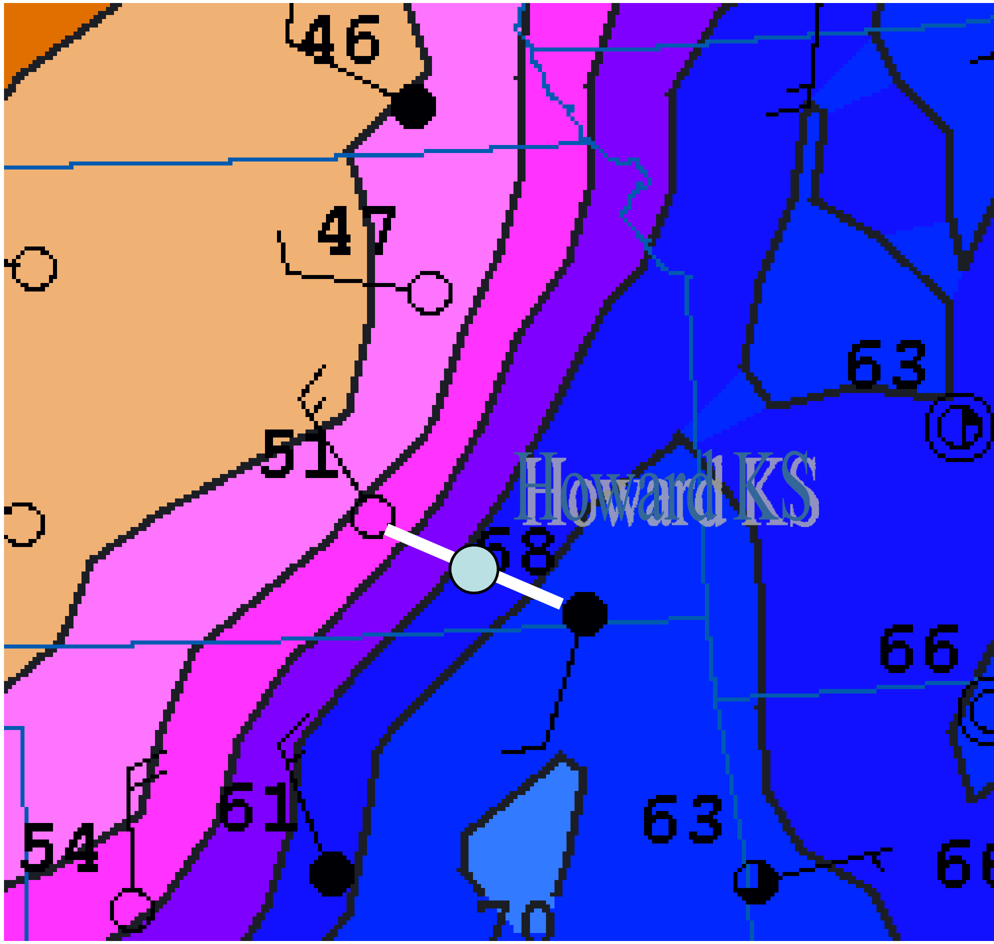
Visualizing the Temperature Gradient

- Red is Cold, Blue is Warm.
- (Also the contours joining areas of equal temperature are from another product, not the observations seen here).
- However, this image is good enough for our application here today.
- Let’s look closely at the temperature gradient between Wichita and Coffeyville.



Visualizing the Temperature Gradient

- Let's estimate that it's about 90 miles between these two stations. (In class, remember to use kilometers!)
- Now let's ask ourselves how quickly is the air cooling at a point in between these two cities (near Howard, Kansas).

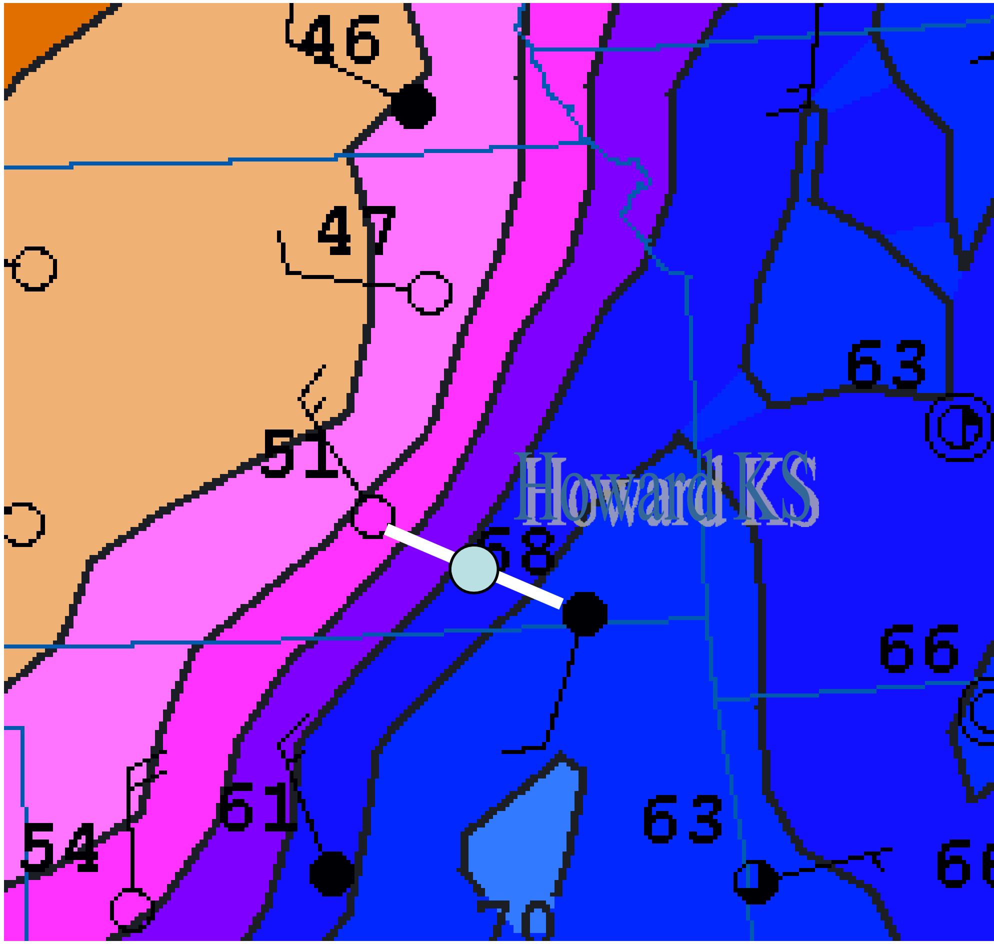


- How would we do this?

Estimating Temperature Change

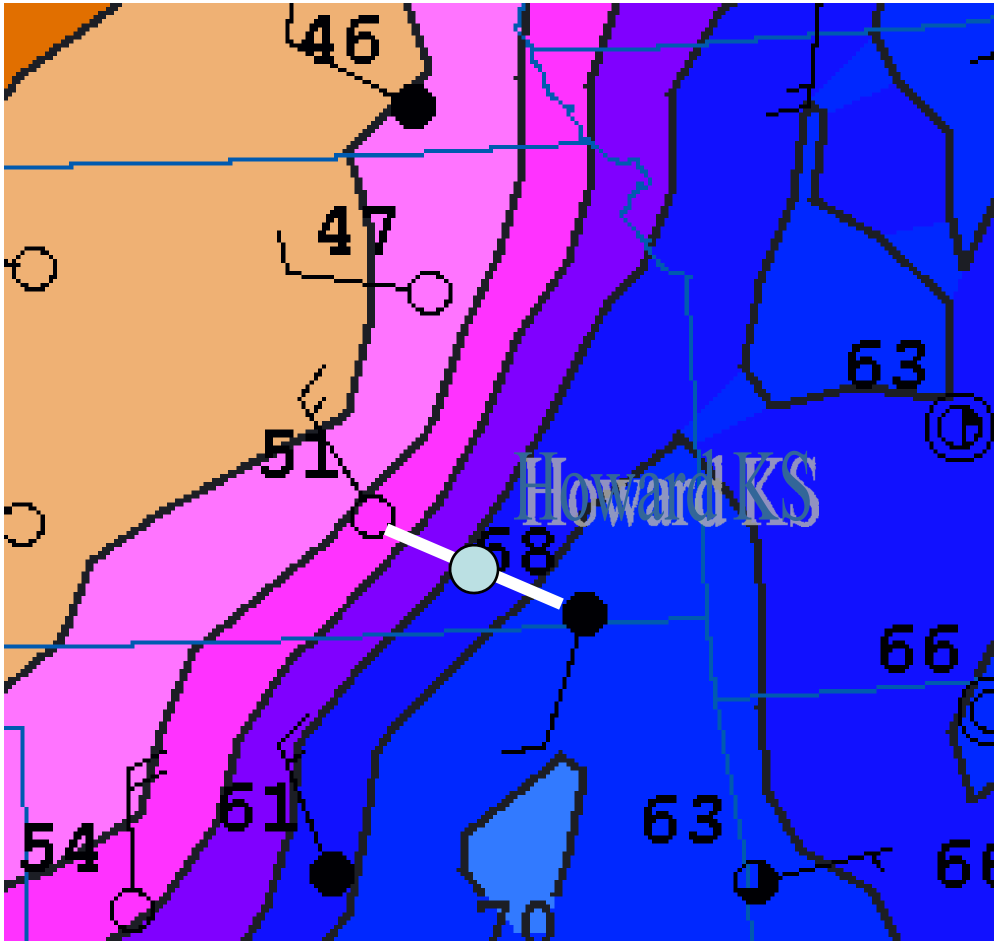
- We have two features controlling the temperature change by our reckoning in this exercise.
  - Temperature and Wind Speed (and direction).

- If we understand how the temperature changes with distance and how fast the air is moving between the stations, *__we can estimate how quickly it's cooling in Howard!__*



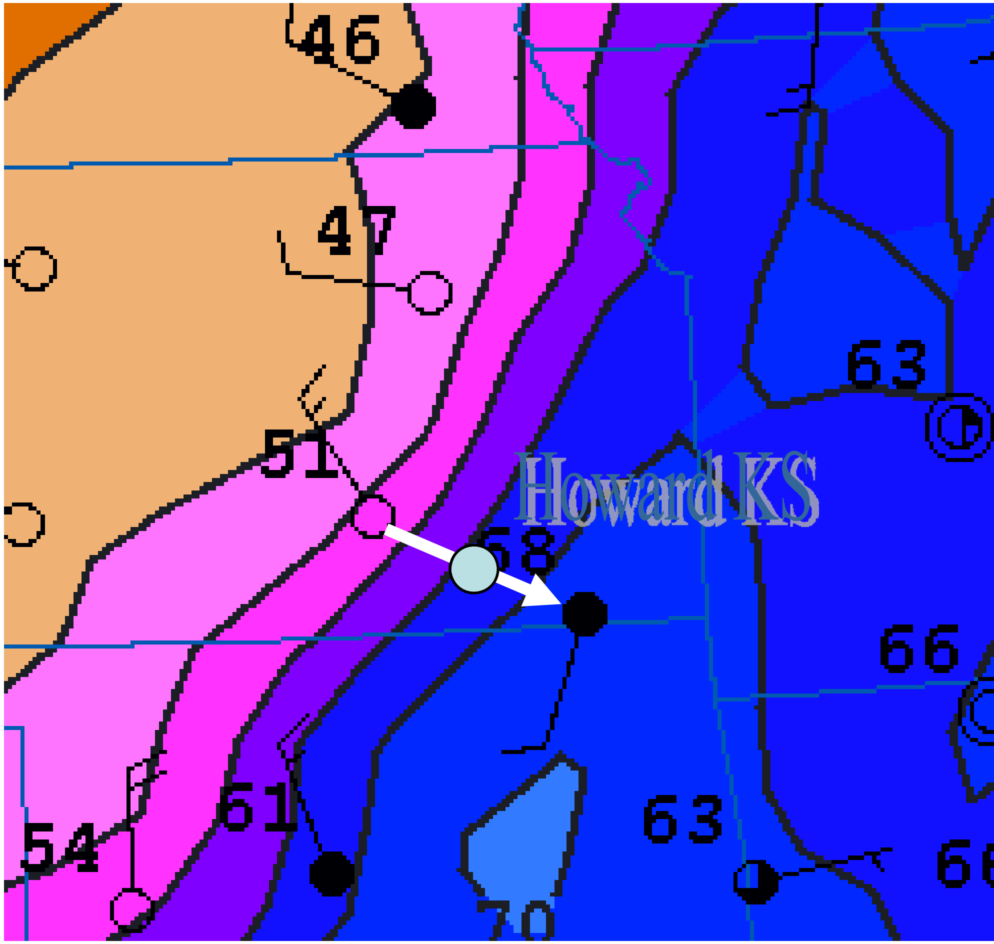
Estimating Temperature Change

- Let's assume two things from this map.
  - First, the wind is blowing from Wichita to Coffeyville at about 10 miles per hour.
  - Second, the temperature difference between Wichita to Coffeyville is about 17°F



Bringing in the Math (and Calc)

- So…
- Delta-T = 17°F
- Delta-X = 90 miles
  - Normally we reserve “x” as the east-west distance but we’re using it here generically
- Wind Speed = 10 miles per hour
- Now we’re stuck… or are we…



Bringing in the Math (and Calc)

- This is where it helps to express things as derivatives.
- First, let's do the temperature gradient.

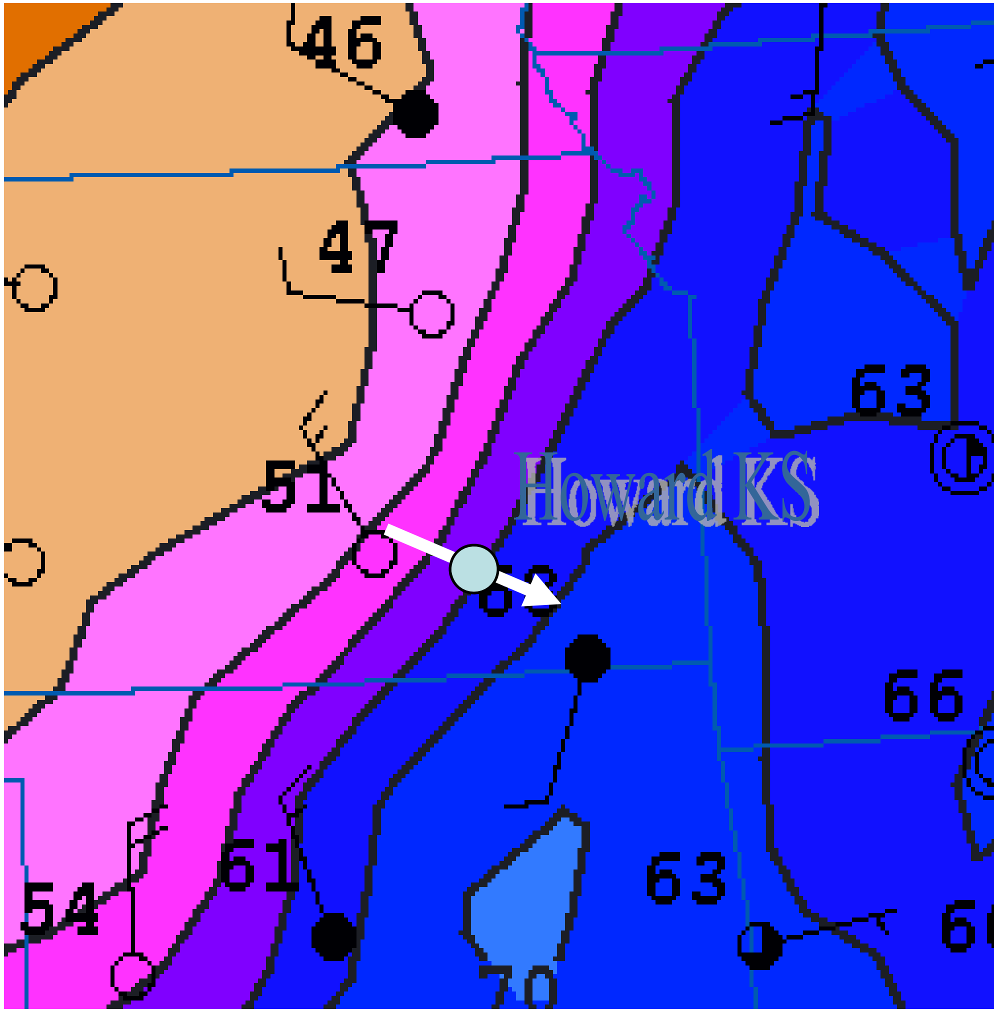
  $\Delta T = 17°F$

  $\Delta X = 90$ miles

  $$\frac{\Delta T}{\Delta x} = \frac{17°F}{90mi} = .1\overline{8}\,\frac{°F}{mi}$$

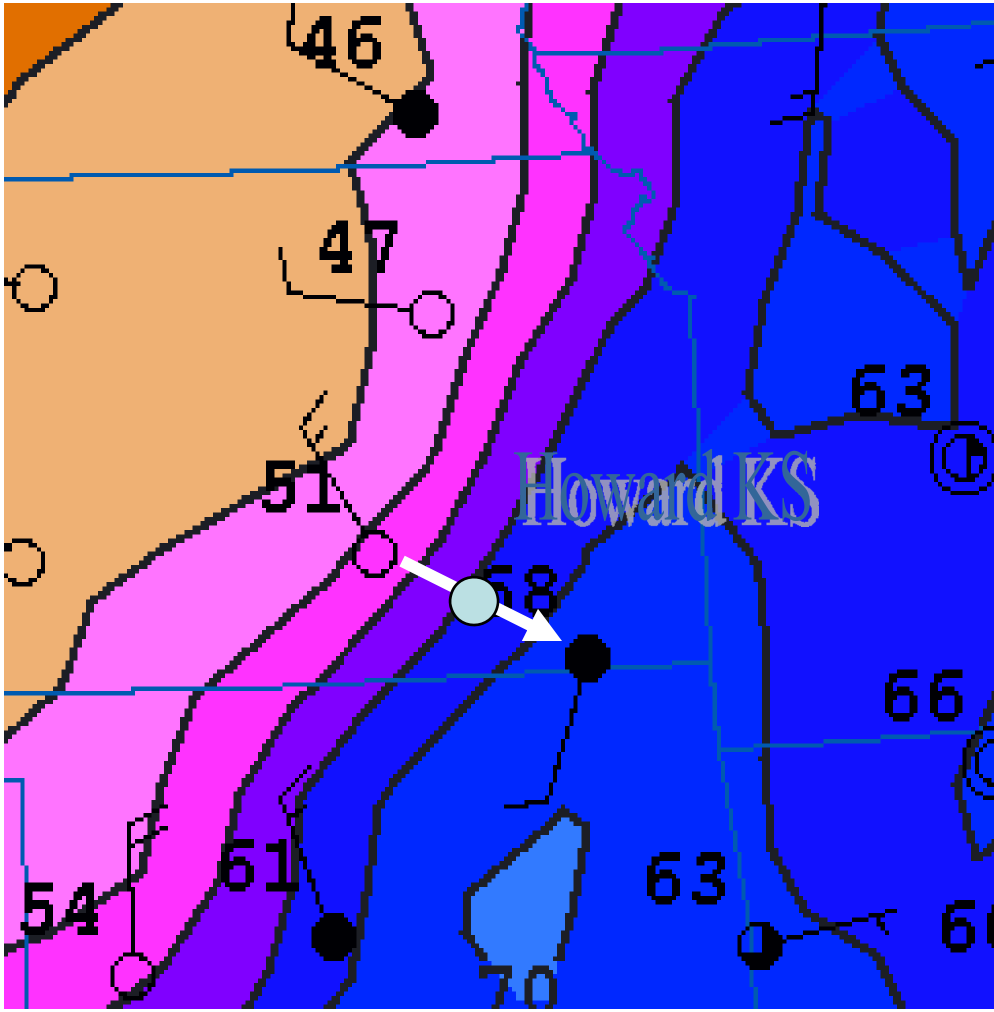
- So if we go to limit theory…

$$\frac{dT}{dx} = 0.1\overline{8}\,\frac{°F}{mi}$$



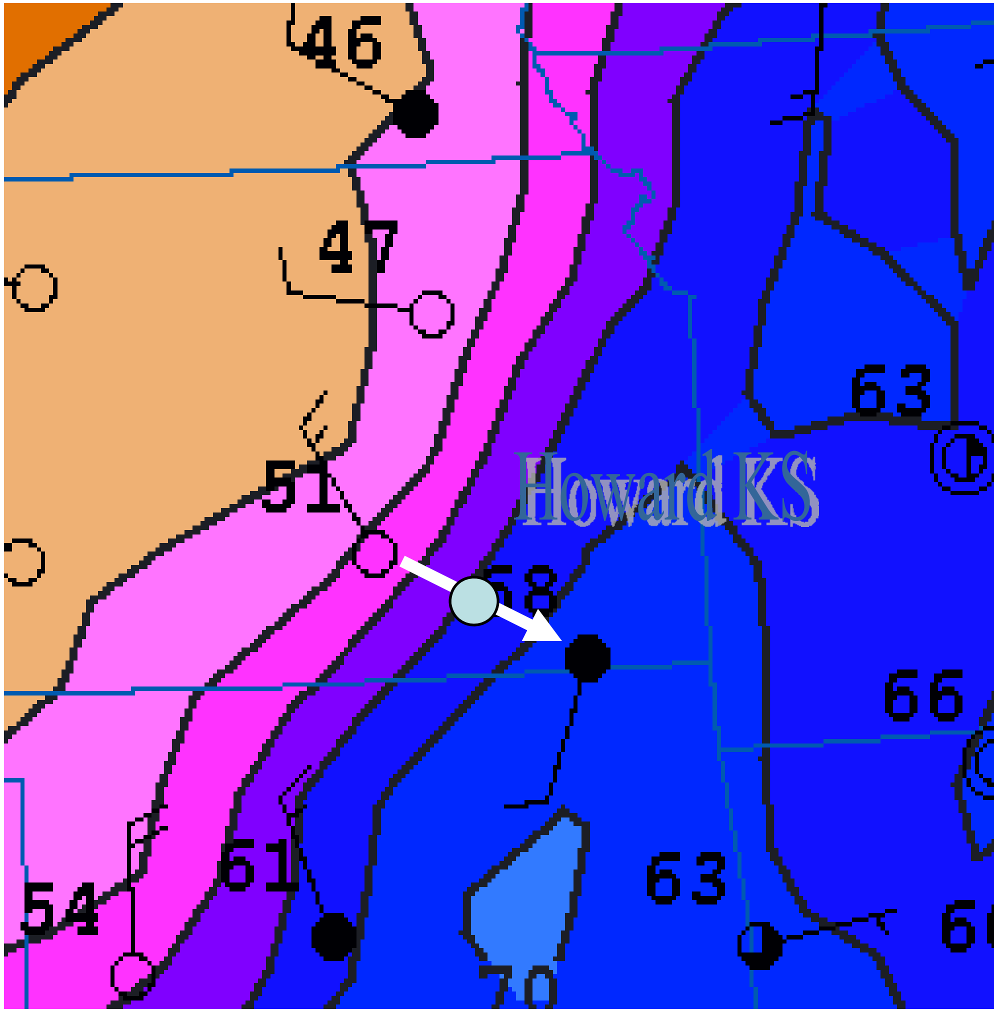
Bringing in the Math (and Calc)

- Now, what about the wind? We're still stuck, right?
- Let's first think about what wind speed (v for now) *is*.
  - Miles/Hour
  - Length/Time
  - $v = \dfrac{dx}{dt} = 10\,\frac{\text{mi}}{\text{hr}}$!!
  - As a quick "patch" we'll add a minus sign to the wind speed to indicate the direction from which the wind is *coming.* When you take Dynamics, you'll see how this "correction" is more formally represented through the use of "The Total Derivative." We'll address that at the close of this module



Bringing in the Math (and Calc)

- Let's recall what we have now.
  - $v = \dfrac{dx}{dt} = 10\,\frac{\text{mi}}{\text{hr}}$
  - $\dfrac{dT}{dx} = 0.19\,\frac{^\circ\text{F}}{\text{mi}}$
  - And change of temperature with time is
  - $\dfrac{dT}{dt}$
  - ("Can you say 'Chain Rule?'")



Bringing in the Math (and Calc)

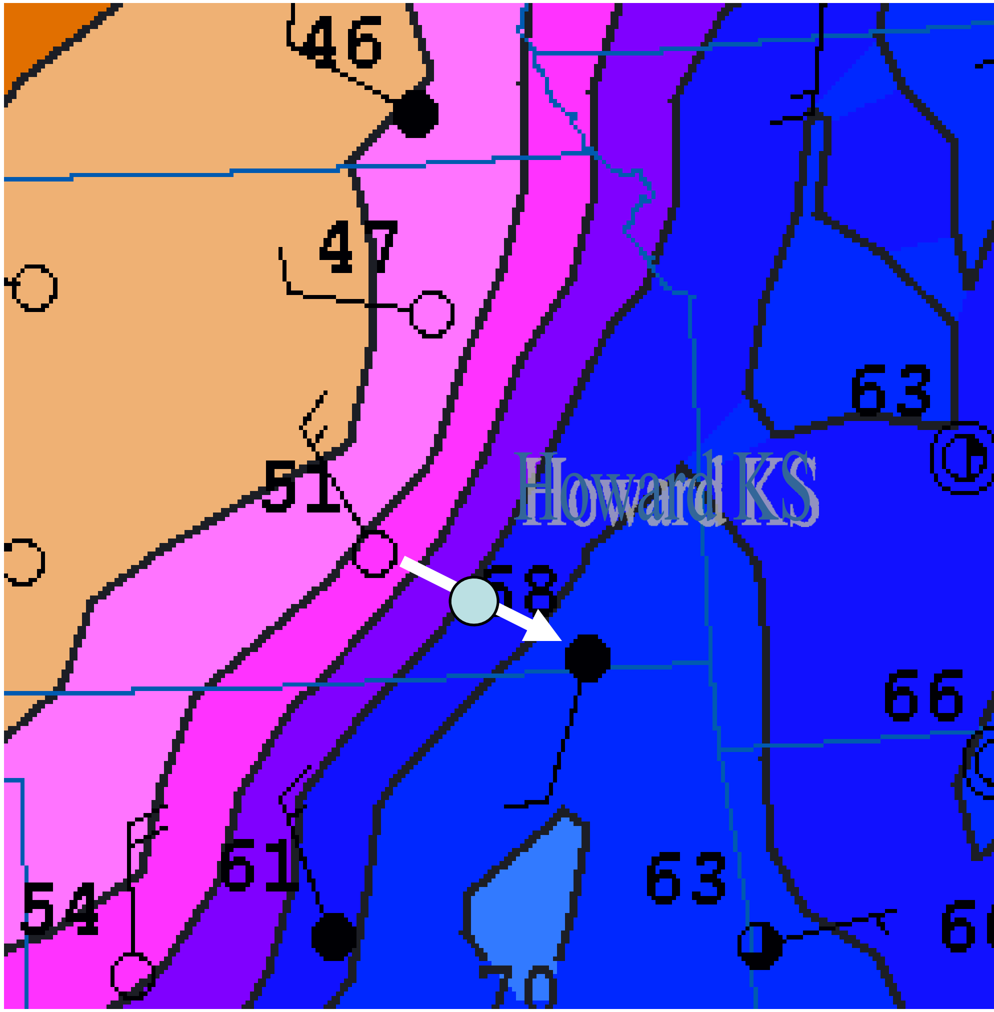
- I knew you could!

$$\frac{dT}{dt} = \frac{dx}{dt}\frac{dT}{dx}$$

$$\frac{dT}{dt} = v\frac{dT}{dx}$$

$$\frac{dT}{dt} = -10\,\tfrac{\text{mi}}{\text{hr}} \cdot 0.19\,\tfrac{^\circ\text{F}}{\text{mi}}$$

$$\frac{dT}{dt} \approx -2\,\tfrac{^\circ\text{F}}{\text{hr}}$$



Orthodoxy Check

- As mentioned earlier we actually use an approach to calculate the change of temperature of at a location using the concept of *The Total Derivative*. However, you can still see how the chain rule can be used to represent it.
- Here we divide the contributions of temperature change by following a single parcel of air.
- We can then, in this simple case, assume that no change is occurring within a moving parcel of air.
  - (if heat is being added to the parcel we can add that heating term to the end of the equation.)

$$\underbrace{\frac{DT}{Dt}}_{\substack{\text{Total}\\\text{change}\\\text{of a}\\\text{moving}\\\text{parcel}\\\text{of air}}} = \underbrace{\frac{\partial T}{\partial t}}_{\substack{\text{Change}\\\text{at a single}\\\text{location}}} + \underbrace{\left(u = \frac{\partial x}{\partial t}\right)\frac{\partial T}{\partial x}}_{\substack{\text{Advections}\\\text{in the}\\\text{east-west}\\\text{direction}}} + \underbrace{\left(v = \frac{\partial y}{\partial t}\right)\frac{\partial T}{\partial y}}_{\substack{\text{Advections}\\\text{in the}\\\text{north-south}\\\text{direction}}} + \underbrace{\left(w = \frac{\partial z}{\partial t}\right)\frac{\partial T}{\partial z}}_{\substack{\text{Advections}\\\text{in the}\\\text{vertical}\\\text{direction}}}$$

$$0 = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}$$

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} - w\frac{\partial T}{\partial z}$$

In closing

- Here, we’ve demonstrated a quick example of how the derivative can be used in forecasting temperature change.
- We also did a clever trick to bring in the chain rule.
- The derivative will be often used to represent gradients and similar properties in the atmosphere.

In closing

- For now, when you see a derivative, it may help you to see that this is representing a change of a property over time, space or in comparison with another variable. You can think if it as a physical slope (d-height/d-horiziontal distance) not just as a slope on a graph.

Questions?

- Talk to
  - Dr Capehart: Synoptic Meteorology
  - Dr Hjelmfelt: Atmospheric Dynamics