Vector Temperature Transport: Gradient and Dot Product AND Forecasting Temperature (Part 2)

Atmospheric Dynamics and Synoptic Meteorology

Calculus 3

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From an earlier session

- We explored how to use the chain rule to determine advection (and the change in temperature at a given location), to support material in Calc 1
- Today we will take that same case and apply it to material you will begin to learn in Calc 3
- This is the same scenario and the principles are the same. However, this time we will formerly introduce you to solving this with multidimensional gradients

Vectors in meteorology

- Vectors are widely used in meteorology
- Vectors provide a convenient notation to describe quantities, such as wind velocity, which have both magnitude and direction.
- By using concepts such as gradients and dot products we can describe the transport of temperature from colder areas to warmer areas by the wind.

Vector Wind

- Wind velocity has speed and direction and may be denoted by a vector. If we consider the horizontal wind, with components in the east, $\hat{i}$, and north, $\hat{j}$,directions, we can write:

$$\vec{V} = u\hat{i} + v\hat{j}$$

where, u=wind component toward the east, and v=wind component toward the north.

The Derivative in Meteorology

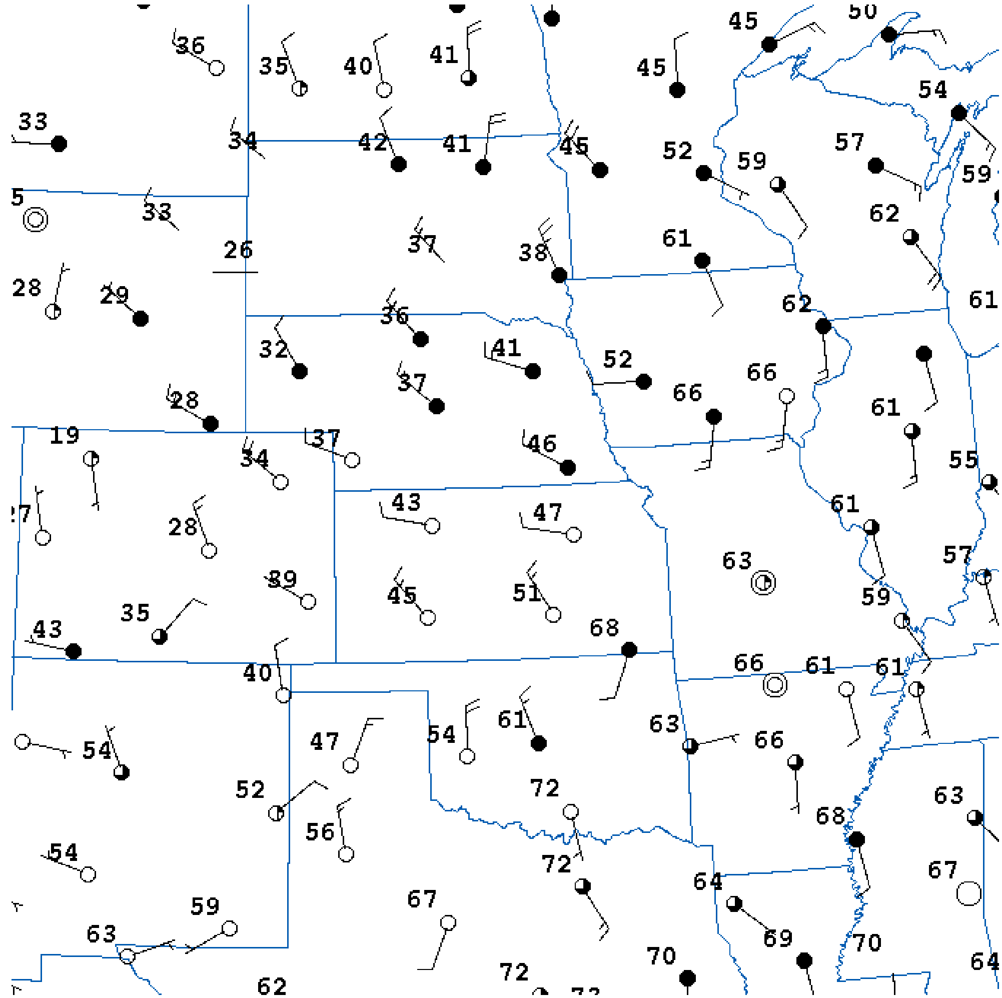
- The Derivative is often used in Meteorology to explain how one feature changes as another feature changes.
- In vector notation, the change of a quantity, such as temperature, with distance can be described by the gradient, $\nabla \mathbf{T}$.

Physical Gradient Redux

- We can easily use the gradient to describe how a parameter (like temperature changes over a given distance.

- Consider the map to the right, for example. It is a ***real*** map of a cold front moving across the plains.

- Temperature (in ºF) is in the upper left-hand corner of each of these “station models” with the dot representing the location of the station and relative cloud cover (the darker the circle the cloudier it is).

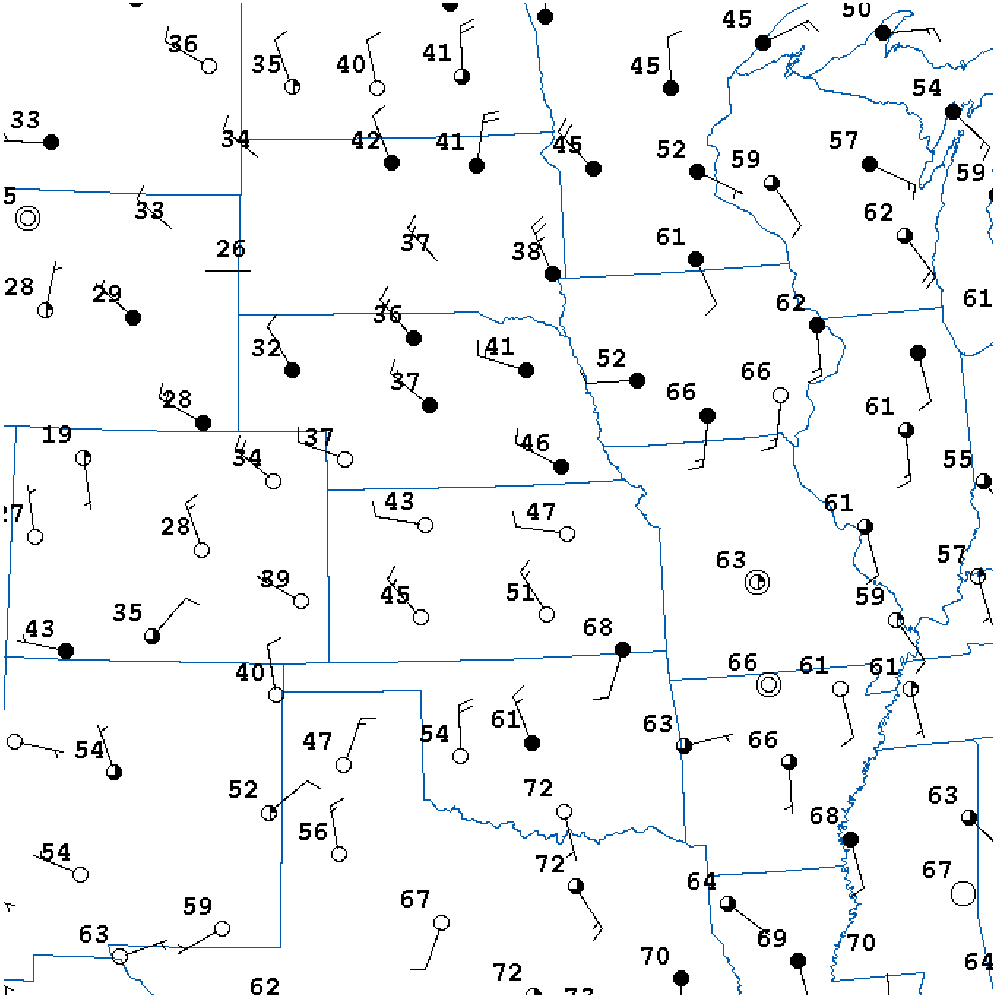


Vector Wind and Advection

- The arrows show the wind direction, and the number of feathers at the back of the arrow indicate the speed.

- Physically the wind is blowing the air (and its properties, such as temperature) from one point to another.

- In meteorology, we call this "***Advection***"



Advection as a Physical Process

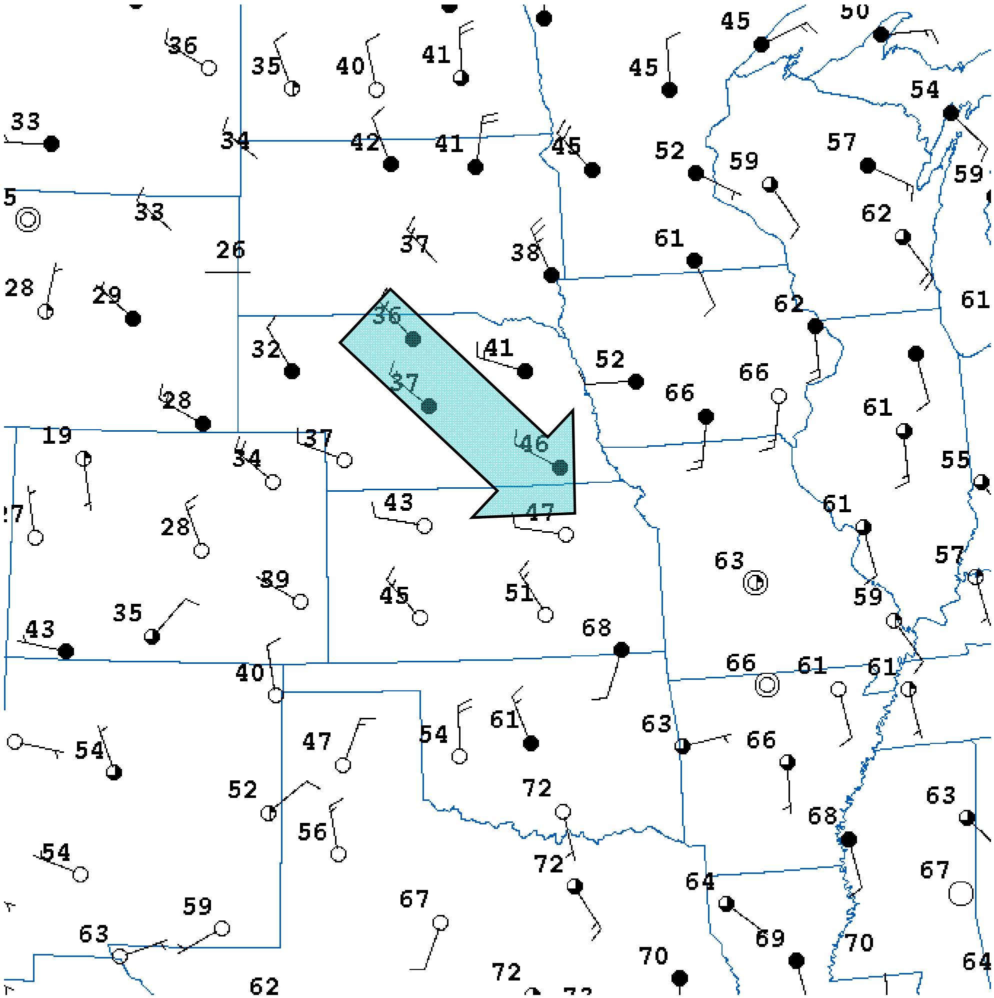
- With this map, and a little applied calculus, we can use the temperature values and wind directions and speeds to estimate how the wind is transporting warm or cold air.

- To begin, lets consider the flow across Nebraska and Kansas.

- We can tell that air from the north west is blowing into the area, which should lower the temperatures at a given point.

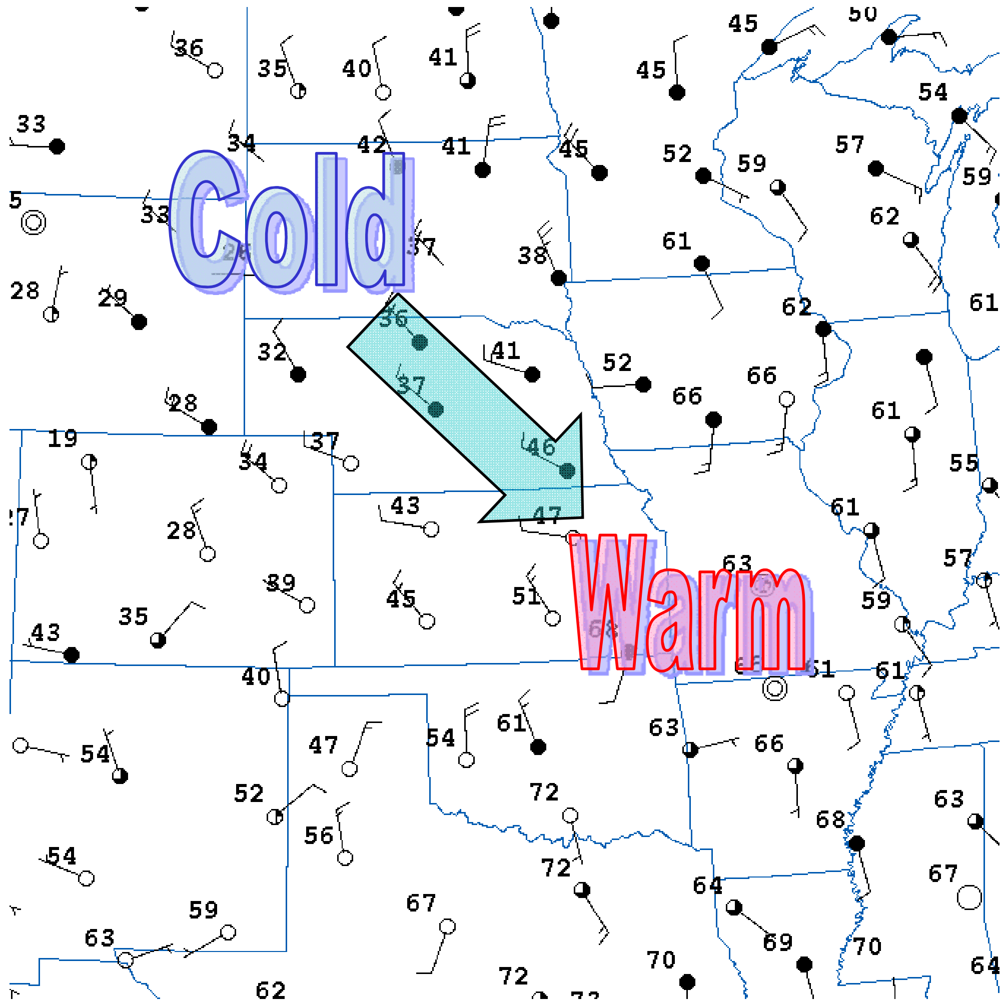
- We call this, appropriately,

  COLD ADVECTION



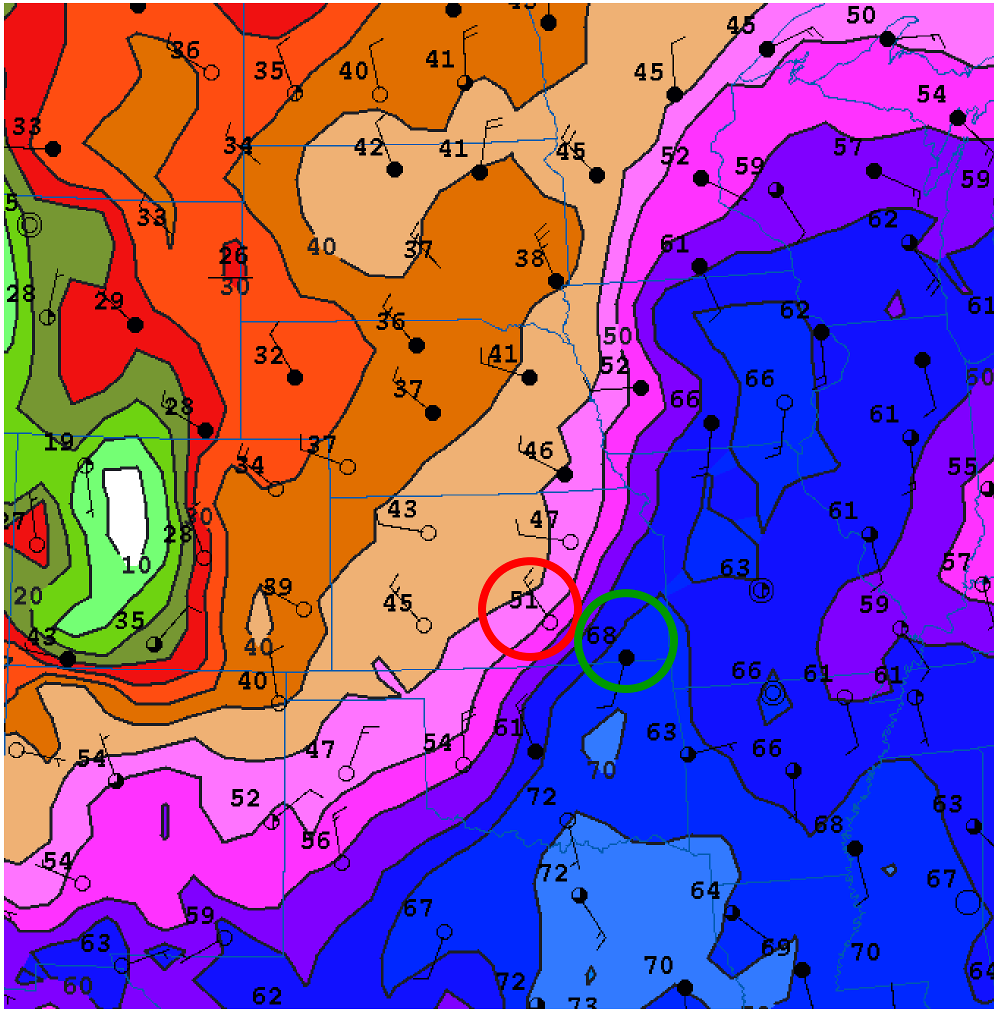
Advection as a Physical Process

- Now, let's look more closely at the situation. We have cold air near Rapid City, warmer air near Topeka, and a wind blowing from one point to another.
- To better visualize the change of properties such as temperature over space, we often contour maps like this.



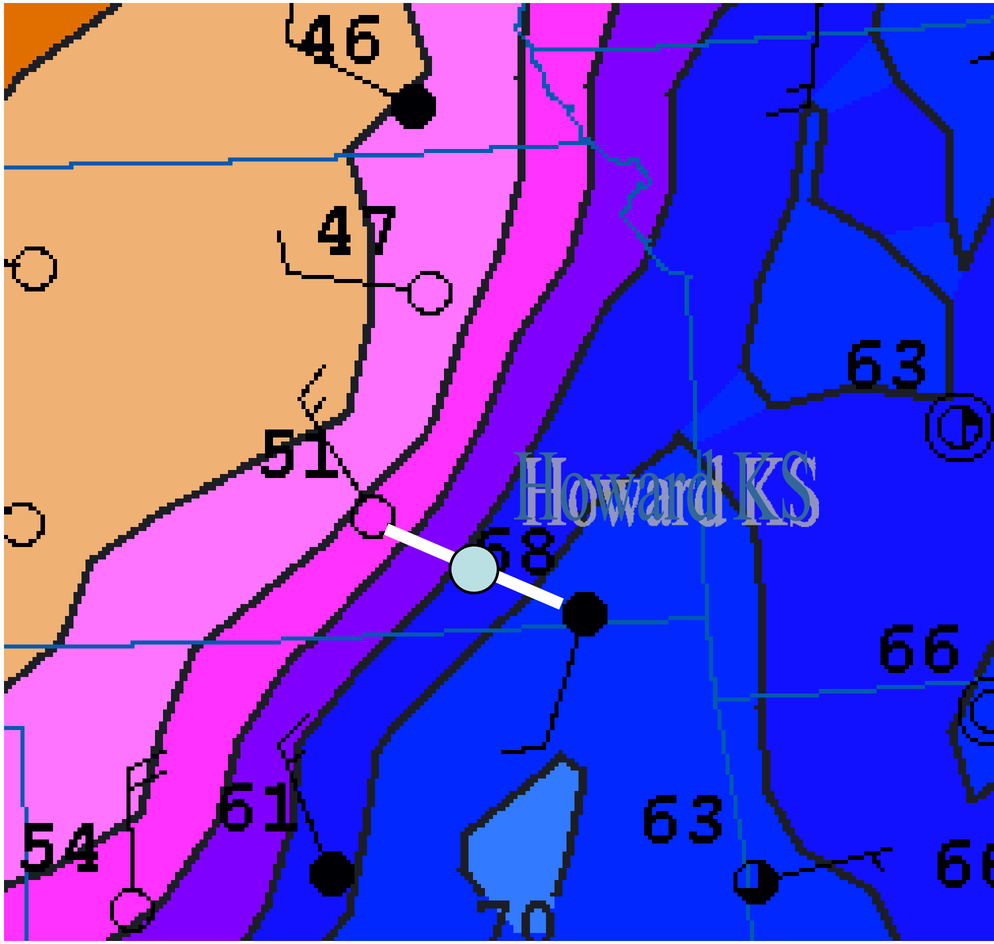
Visualizing the Temperature Gradient—Another case example

- Red is Cold, Blue is Warm.
- Lines connect points with the same temperature.
- Let's look closely at the temperature gradient between Wichita and Coffeyville.



Visualizing the Temperature Gradient

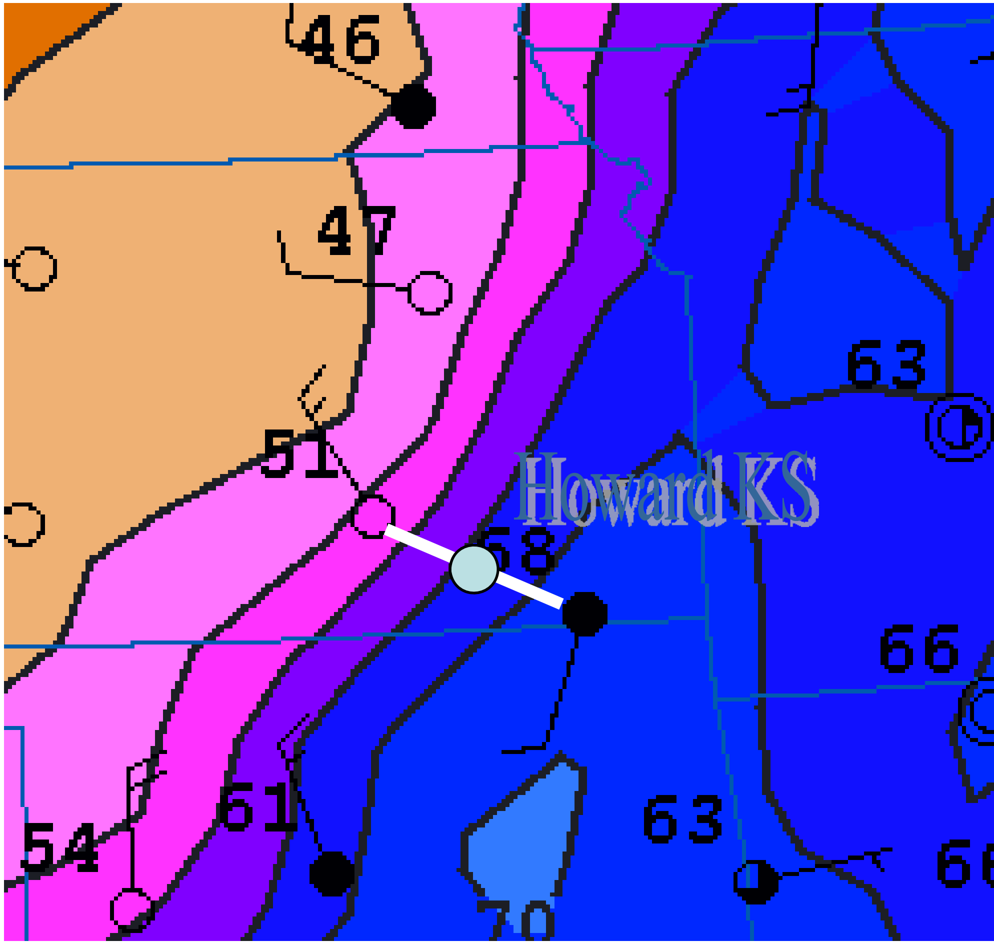
- For simplicity, let's assume that it's about 90 miles to the East between these two stations.
- Now lets ask ourselves how quickly is the air cooling at a point in between these two cities (near Howard, Kansas).



Estimating Temperature Change

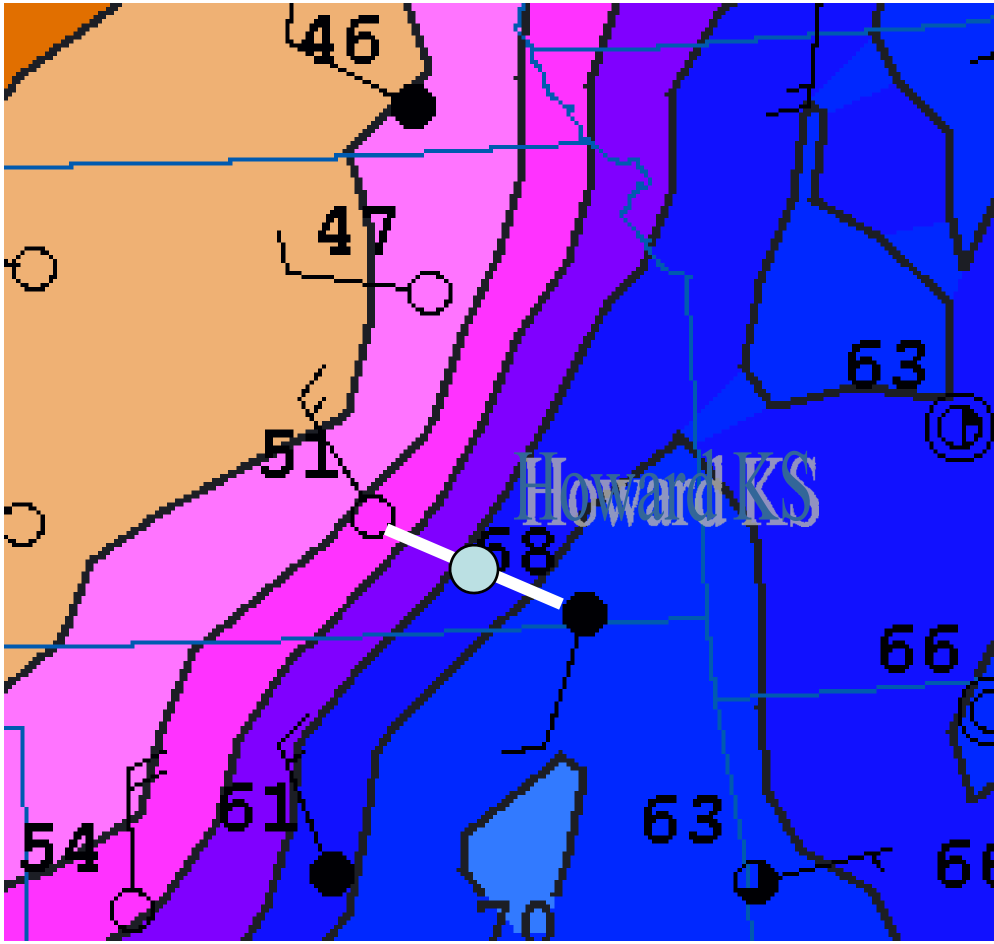
- We have two factors controlling the temperature change.
  - Temperature and its gradient
  - Wind Velocity (speed and direction).

- If we know how the temperature changes with distance and how fast the air is moving between the stations, *we can estimate how quickly it's cooling in Howard!*



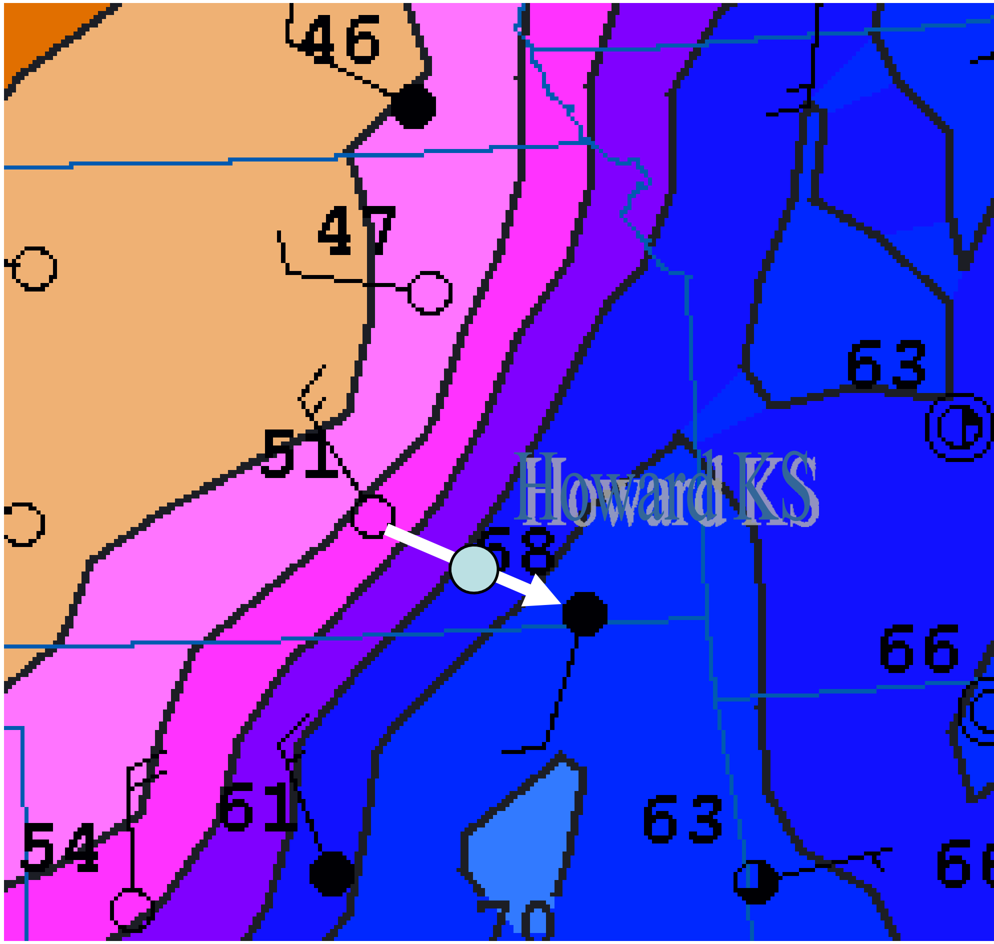
Estimating Temperature Change

- Let's assume two things from this map.
  - First, the wind is blowing from Wichita to Coffeyville at about 10 miles per hour from the Northwest.
  - Second, the temperature difference between Wichita to Coffeyville is about 17°F



Bringing in the Math (and Calc)

- So…
- Delta-T = 17°F
- Delta-d = 90 miles to the East
- Wind Speed = 10 miles per hour from the North west.



Bringing in the Math (and Calc)

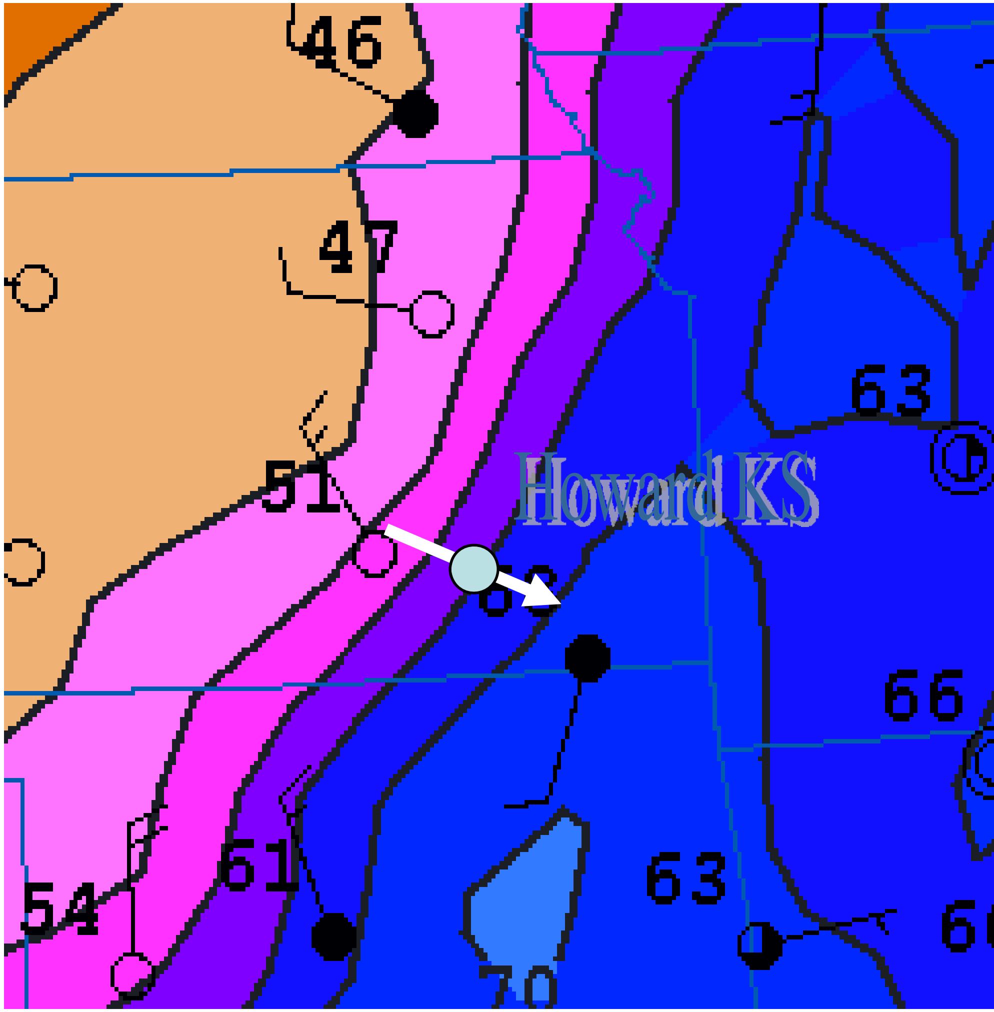
- This is where it helps to express things as vectors:
- First, lets do the temperature gradient.

  $\Delta T = 17°F$

  $\Delta X = 90$ miles

$$\nabla T = \frac{\Delta T}{\Delta x} = \frac{17°F}{90\text{mi}} = .1\overline{8}\,\frac{°F}{\text{mi}}\,\hat{i}$$

So the temperature increases 0.18 F/mi to the East



Bringing in the Math (and Calc)

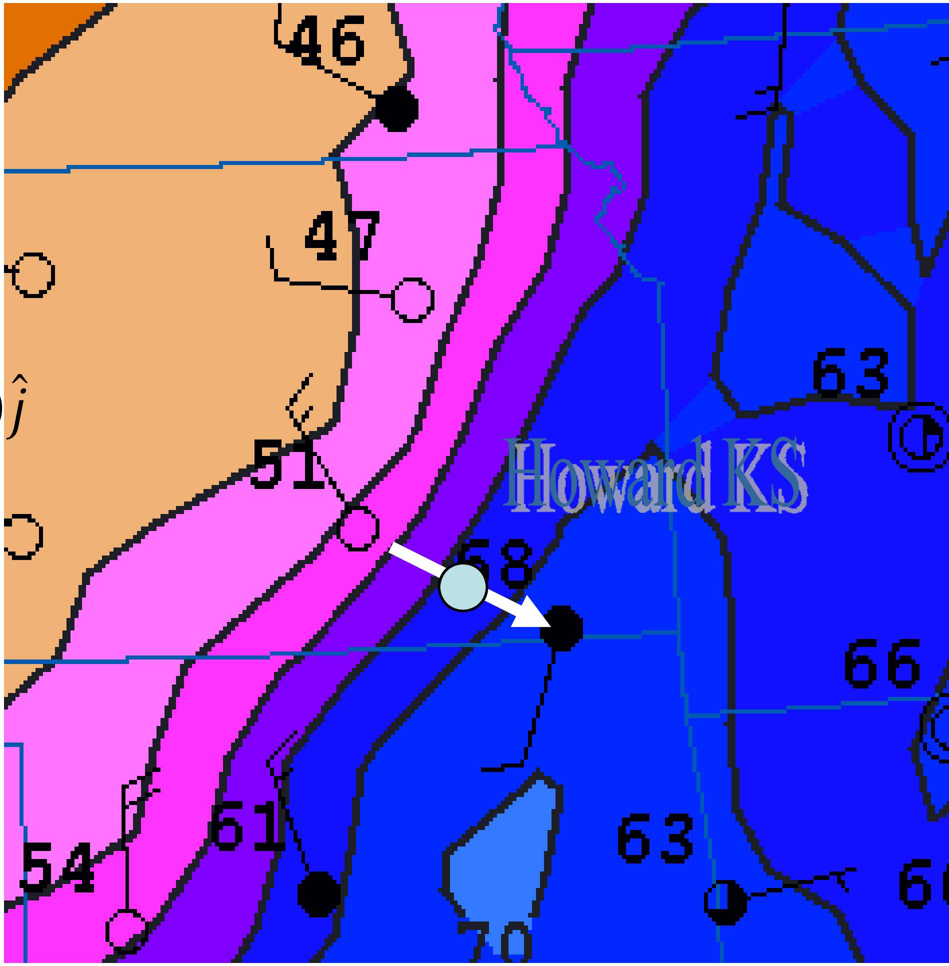
- Now, what about the wind:
- V= 10 mi/hr from the Northwest.
- By components we have 7.07 mi/hr from the North and 7.07 mi/hr from the West.
- Thus we may write the wind vector, **V** as

$$\vec{V} = u\hat{i} + v\hat{j} = (7.07 mi/hr)\hat{i} + (7.07 mi/hr)\hat{j}$$

- Alternatively, this may be written as

$$\vec{V} = u\hat{i} + v\hat{j} = \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 7.07 \\ 7.07 \end{bmatrix} mi/hr$$

- Dr Hjelmfelt likes the former notation and it is the more formal notation, Dr. Capehart prefers the latter when teaching.
- And you may want to use SI units!
  - This is a simple demo.

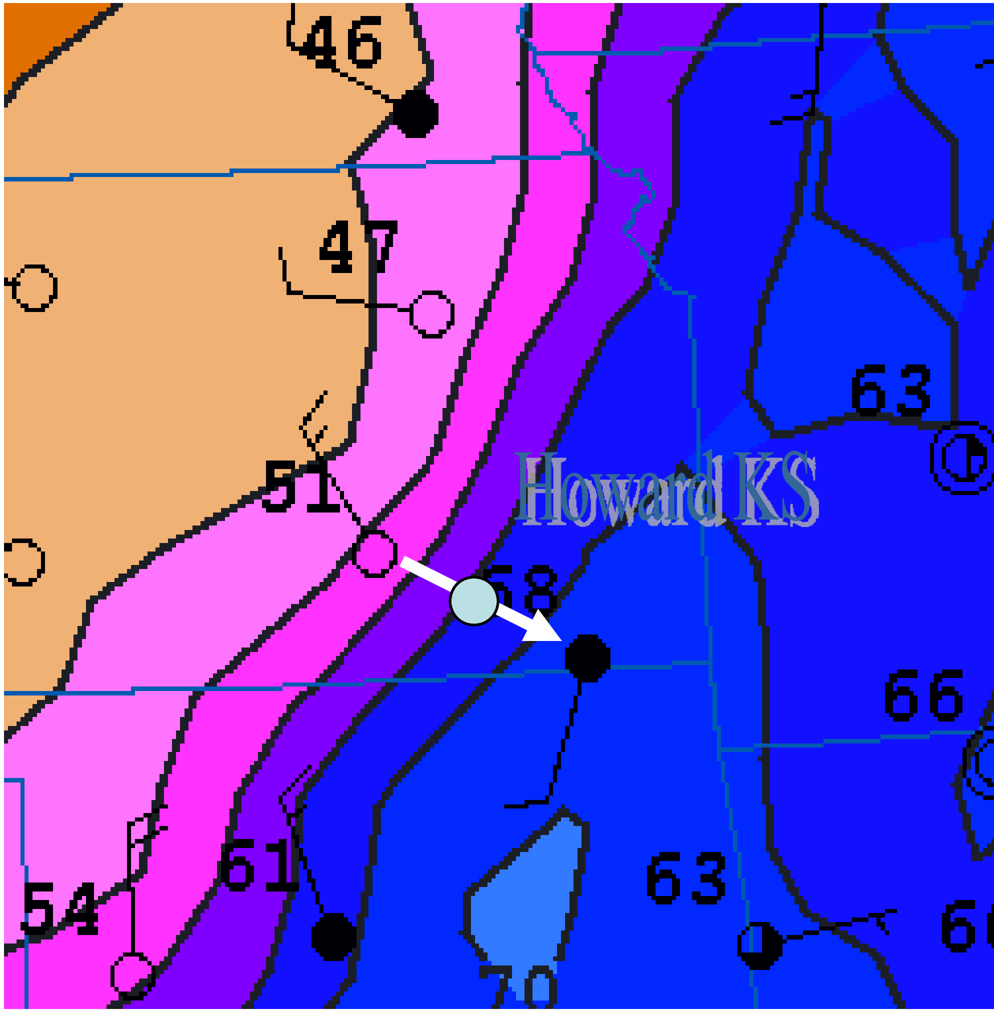


Bringing in the Math (and Calc)

Calculating the advection

- The rate of temperature change resulting from the cold air advection is given by the rate at which the wind is blowing the cold air into the region of warmer air.
- This is just the dot product of the wind velocity and the temperature gradient:

$$\frac{dT}{dt} = \vec{V} \bullet \nabla T$$



Bringing in the Math (and Calc)

So now we can use our dot product:

$$\frac{dT}{dt} = \vec{V} \bullet \nabla T = |V||\nabla T|\cos\theta$$

Or,

$$\vec{V} \bullet \nabla T = |V||\nabla T|\cos\theta$$

$$\frac{dT}{dt} = \vec{V} \bullet \nabla T = (7.07\frac{mi}{hr}\hat{i} + 7.07\frac{mi}{hr}\hat{j}) \bullet (0.18\frac{F}{mi}\hat{i})$$

Which gives a temperature change at Howard due to cold air advection of:

$$\frac{dT}{dt} = 1.27\frac{F}{hr}$$

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In closing

- Quantities having both magnitude and direction are easily represented by *vectors*.
- Changes in quantities over distance can be represented by *gradients.*
- The *dot product* can be used to describe the meteorological process of temperature advection: how the temperature changes as the wind moves air from areas of higher (lower) temperature into areas of lower (higher) temperature.

In closing

- For now, when you see a derivative, it may help you to see that this is representing a change of a property over time, space or in comparison with another variable.   You can think if it as a physical slope (d-height/d-horiziontal distance) not just as a slope on a graph.

Got Questions

- Go See
  - Dr. Hjelmfelt: Dynamics
  - Dr. Capehart: Synoptics