Vertical Temperature Coordinates, Stability and the Natural Log

Calc 2

Atmospheric Physics

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1st Law of Thermodynamics for Adiabatic Processes

(basically, energy conservation)

$$c_v dT + p d\alpha = dh$$

Change in internal energy + work = change in enthalpy

Ideal Gas Equation of State

$$p\alpha = RT$$

or

$$p = \rho RT$$

Pressure = density x gas constant x temperature

This allows the 1st Law to also be expressed as

$$c_p dT - \alpha dp = dh$$

Setting dh = 0 and substituting for α use our 1st Law expression:

$$c_p dT - \alpha dp = 0$$

$$c_p dT = \frac{RT}{p} dp$$

When dh = 0 we have an "adiabatic" process

Re-arrange and integrate:

$$\int \frac{dT}{T} = \frac{R}{c_p} \int \frac{dp}{p}$$

$$\ln\left(\frac{T}{T_0}\right) = \frac{R}{c_p} \ln\left(\frac{p}{p_0}\right)$$

T_0 becomes what we call potential temperature

$$\ln\left(\frac{T}{T_0}\right) = \frac{R}{c_p}\ln\left(\frac{p}{p_0}\right)$$

$$\frac{T_0}{T} = \frac{p_0}{p}^{\frac{R}{cp}}$$

$$T_0 = T\left(\frac{p_0}{p}\right)^{\frac{R}{cp}}$$

In more traditional meteorological notation….

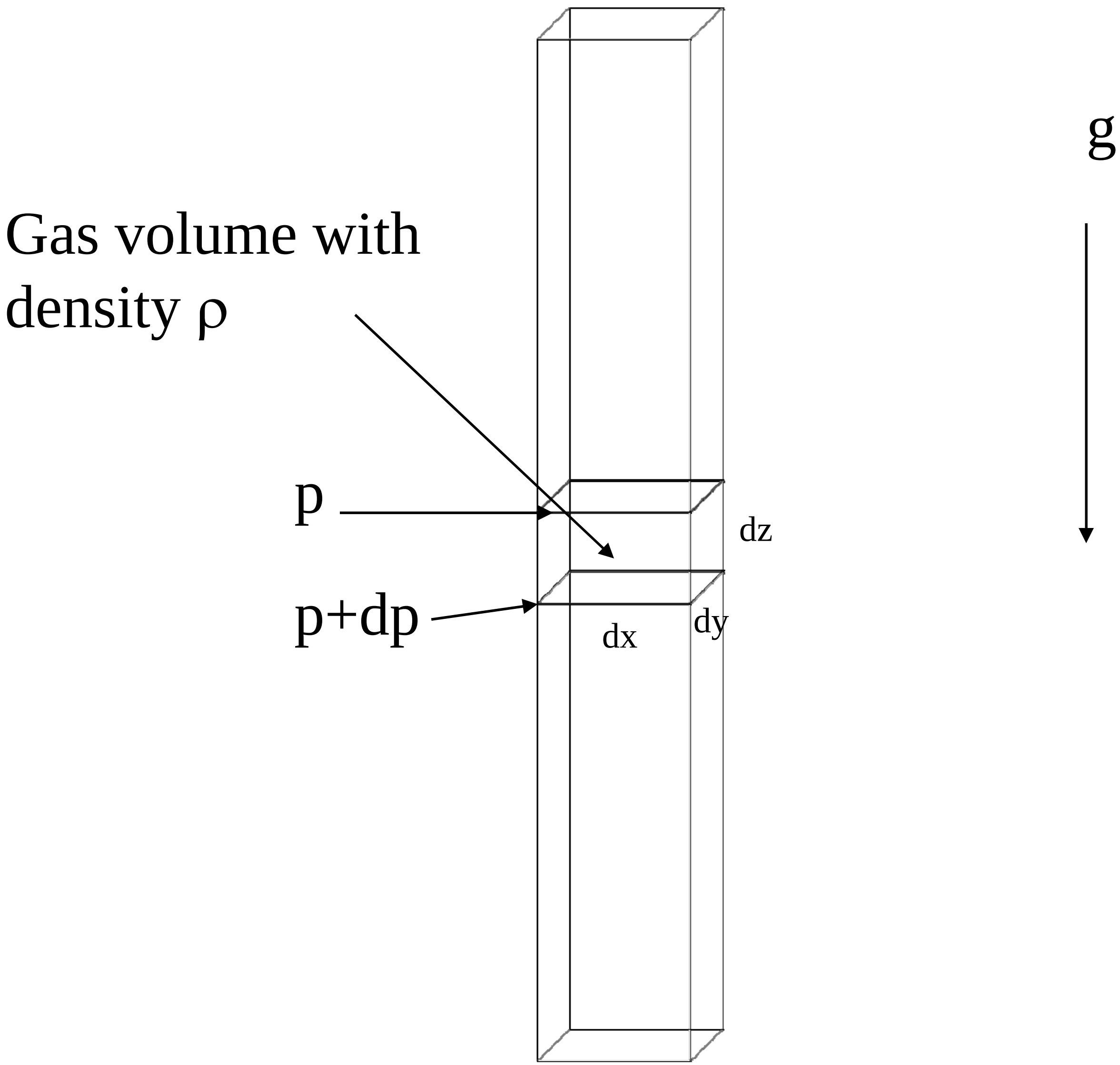
$$T_0 = \theta$$

$$p_0 = 1000$$

$$\theta = T\left(\frac{1000}{p}\right)^{\frac{R}{c_p}}$$

In an *adiabatic* process, θ is constant as T and p vary

Next, let's consider a hydrostatic column. The fluid is not moving. The pressure at any level in the column is solely due to the mass of the fluid above that level acted upon by gravity.



Gas volume with
density ρ
p
p+dp
dz
dx
dy
g

$$p + dp = p + g\,\rho\,\frac{dx\,dy\,dz}{dx\,dy}$$

Where the last term on the RHS is the incremental force per unit area due to the weight of the gas in the element

$$\frac{g\,\rho\,dx\,dy\,dz}{dx\,dy} = \frac{\text{acceleration} \times \text{mass}}{\text{area}}$$

The previous relationship, when simplified, with the assumption that fluid properties are continuous, yields an expression for hydrostatic balance

$$\frac{dp}{dz} = -\rho g$$

or

$$\frac{dp}{dz} = -\frac{g}{\alpha}$$

Take the 1st Law and substitute for dp from the expression for hydrostatic balance:

$$c_p dT - \alpha dp = 0$$

$$c_p dT - g dz = 0$$

If one assumes the fluid and its properties are all continuous, and takes the limit as temperature and vertical distance changes become infinitessimal

$$\frac{dT}{dz} = -\frac{g}{c_p}$$

The adiabatic "lapse rate" in the atmosphere of the earth near the surface:

$g \sim 10 \text{ m s}^{-2}$

$c_p \sim 1000 \text{ J /(kg K)}$

$dT/dz \sim 0.01 \text{ K/m}$

Got Questions

- Go Visit
  - Dr. Helsdon or Dr. Detwiler: Atmospheric Physics and Thermodynamics