A Structural Model of Liquidity in Over-the-Counter Markets

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December 2022

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Market Liquidity

Corporate bonds:

- Key **source of financing** for firms in the real economy.
- Key **investment** for financial institutions.
- Trade in **over-the-counter markets**.

Market liquidity: firms can trade quickly and at low cost.

Illiquidity in corporate bond markets:

1. Undermines firms' abilities to **issue debt**.
2. Threatens **financial stability**.

Market Liquidity

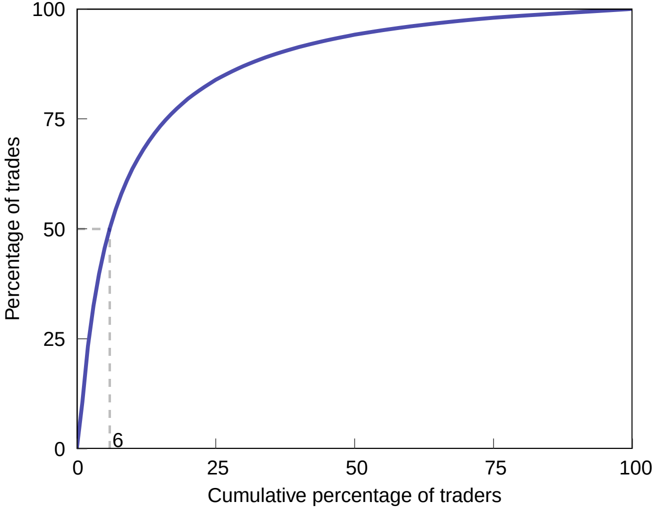
Liquidity is an equilibrium outcome which is shaped by:

1. Financial conditions.
2. Trading technologies.
3. Regulatory policy.

This Paper

Combine theory with unique dataset on **secondary market for sterling corporate bonds** to study liquidity and its determinants.

Firm Heterogeneity



Research Questions

Firm heterogeneity: a small subset of frequent traders do the bulk of trading.

Q1: To what extent is **liquidity** reliant on a small subset of frequent traders?

Q2: How does this heterogeneity interact with:

1. Financial shocks?
2. Trading platforms?
3. Banking regulation?

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2. **Trading platforms?**
3. Banking regulation?

Trading Platforms

Sterling corporate bond market is **over-the-counter, bilateral, and phone-based**.

Electronic trading platforms offer a modern alternative.
$\rightarrow$ Multilateral system where all members can contact each other.

Historically, platforms have struggled to gain traction in these markets. **Why?**

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3. **Banking regulation?**

Banking Regulation

Banking regulation strengthened after global financial crisis.

Bank capital regulation:

- Introduced to enhance stability of banking system.
- Concerns this has harmed liquidity, but evidence is mixed.

How does bank capital regulation impact liquidity, in normal times and in stress?

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Approach

1. Data on **all firms' trading** in sterling corporate bonds.

2. Empirical facts on trader heterogeneity, intermediation, and trading activity.

3. Search & matching model of trading informed by the data.
   $\rightarrow$ **Endogenous search intensity**.

   $\rightarrow$ **Endogenous intermediation**.

   $\rightarrow$ **Heterogeneous search costs** across traders.

4. Structural estimation.
   $\rightarrow$ Pin down search cost distribution.

5. Counterfactual analyses.

Findings

Q1: Is liquidity reliant on a small subset of traders? 8% most frequent traders supply as much liquidity as the remaining 92%.

Q2: How does this heterogeneity interact with:

1. Financial shocks? Market highly vulnerable to shocks to frequent traders.
2. Trading platforms? Platforms efficiency-enhancing, but harm frequent traders → will resist their introduction.
3. Bank capital regulation? Reduces liquidity, but market responses reduce costs by 30%. Costs increase in a stress.

Literature & Contribution

OTC Market liquidity and its determinants.

Market features. Di Maggio, Kermani & Song (2017), Li & Schürhoff (2019), Fontaine & Walton (2020).

Post-crisis regulation. Adrian, Fleming, Shachar & Vogt (2017), Bao, O'Hara & Zhou (2018), Bessembinder, Jacobsen, Maxwell & Venkataraman (2018), Choi & Huh (2021), Schultz (2017).

Financial shocks. Eisfeldt, Herskovic, Rajan & Siriwardane (2021).

Trading mechanisms. Allen & Wittwer (2021), Barclay, Hendershott & Madhavan (2015), Plante (2018).

Innovation:

1. Structurally estimate model of liquidity.

Contribution:

1. Explain mechanisms.
2. Counterfactual scenarios.

Literature & Contribution

Search models of financial markets.

Duffie, Gârleanu & Pedersen (2005, 2007), Afonso & Lagos (2015); Lagos & Rochetau (2009), Vayanos & Weill (2008), Uslu (2019). Neklyudov (2019), Farboodi, Jarosch & Shimer (2021), Liu (2020). Brancaccio, Li & Schürhoff (2020), Allen, Clark & Houde (2019), Gavazza (2016).

Innovation:

1. Structural estimation.
2. Endogenous search & unconstrained holdings.

Contribution:

1. Quantitative importance.
2. Traders vary search intensity to manage balance sheets and respond to shocks.

Data & Setting

The Secondary Market for Sterling Corporate Bonds

Issuers: British Petroleum, Manchester United.

Traders: Banks, asset managers, hedge funds & insurers.

Trading is **bilateral** and typically **phone-based**.

Rough distinction between **dealers**—who intermediate/supply liquidity—and **customers**—who demand it.

Bond market **dealers tend to be banks**.

Data

Trade data

Bank of England transaction-level data on secondary market for **sterling corporate bonds**.

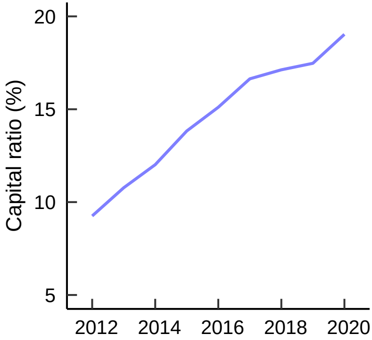
- Any transaction by UK-regulated firm or EU subsidiary of UK-regulated firm.
- 2012-2017.
- **Fields:** price, quantity, time of trade, trader identities.

Bond characteristics data

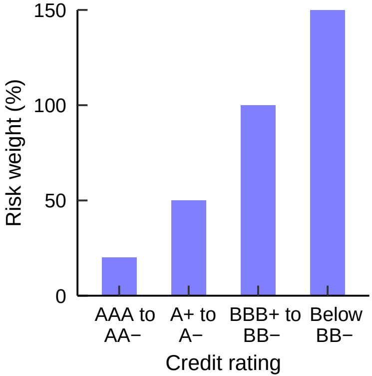
Thomson Reuters data on primary issuance and bond characteristics.

Comparative advantage: Identity of all traders. Holdings

Bank Capital Regulation



UK bank capital ratios



Capital requirements by asset

Capital regulation (a) has become more stringent and (b) varies across bonds.

Summary Statistics

3.3mn trades across 1,000 monthly traders in 7,000 bonds.

Infrequent trading: median bond trades once a month.

Price dispersion within and across bonds.

Heterogeneity across bonds & traders. Summary stats

Empirical Facts

Empirical Facts

1. Trading is concentrated in a small subset of frequent traders.
2. Frequent traders intermediate.
3. Both dealers & customers demand and supply liquidity.
4. Traders vary trading frequency to manage balance sheets.
5. Dealer trading behaviour varies with capital regulation.

Frequency Intermediation

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Frequency Intermediation

Trading by Firm Type

Buyer\ Seller	Customer	Dealer
Customer	23	30
Dealer	33	14

All traders face incentives to demand and supply liquidity.

Firms that are not traditional ‘dealers’ seek to make money by supplying liquidity (BlackRock, 2015; Choi & Huh, 2021).

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Frequency Intermediation

Endogenous Trading Frequency

	Probability (%)	
	Theory	Data
Paired trades	6.9	44
Offsetting trades	3.5	31

Paired: firm trades same bond more than once on same day.

Offsetting: firm buys and sells same bond on same day. Model

Empirical Facts

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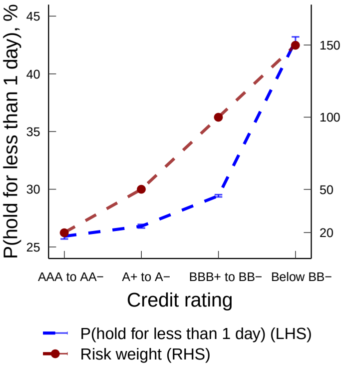
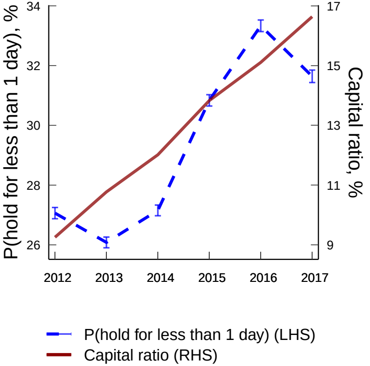
Frequency Intermediation

Dealer Trading and Capital Regulation



By risk weight



Through time

Traders condition their trading behaviour on regulation.

Model

Overview

Search and matching framework in the spirit of Duffie, Gârleanu & Pedersen (2005) and Uslu (2019).

- Continuous time, infinite horizon.
- Single tradable asset with supply a.
- Continuum of traders with measure 1.
- Shocks to traders' valuations $\rightarrow$ gains to trade.
- Search frictions $\rightarrow$ illiquidity.
- Random search $\rightarrow$ meetings $\rightarrow$ bilateral bargaining.

Flow Value and Asset Holdings

Flow utility:

$$u(\beta, h) = \beta h - \frac{1}{2}\kappa h^2$$

where holdings $h \in \mathbb{R}$, valuation $\beta \sim G(\beta)$ and $\kappa \geq 0$.

Lifetime utility: PV of expected utility flows, net of payments for transactions.

Shocks: with independent Poisson arrival rate $\eta > 0$ trader draws a new value from $G(\beta) \to$ motive for trade.

Reduced form of more fundamental setting with CARA utility, where $\kappa \propto$ risk aversion and β interpreted as funding shocks.

Search, Matching & Trading

Search

Trader with **endogenous search intensity** γ pays search cost $s(z, \gamma)$ where search cost $z \sim F(z)$.

Meetings

Traders with search intensity γ and γ' meet at rate $m(\gamma, \gamma')$, where $m()$ is a matching function that is:

- Symmetric.
- Linearly increasing in both arguments.

Trading

When two traders meet they **Nash bargain** over quantity and price, with equal bargaining weights.

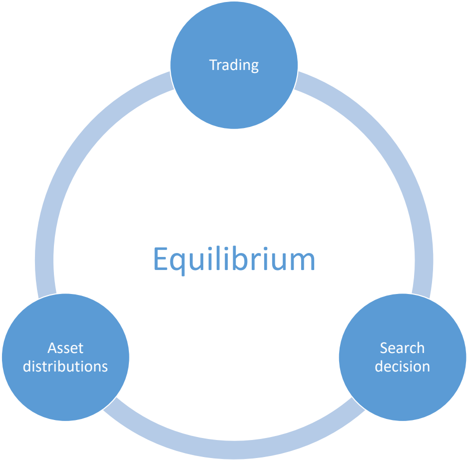
The Model and the Data

Trading is concentrated in small set of firms.
$\rightarrow$ **heterogeneous costs** of searching.

Firms adjust trading frequency to manage balance sheets.
$\rightarrow$ **endogenous search** intensity.

Dealers & customers demand and supply liquidity.
$\rightarrow$ **endogenous intermediation**.

Solving the Model



Components of Equilibrium

Trader type $(z, \beta, h) \equiv \Delta$:

- Search cost z.
- Valuation β.
- Holdings h.

Equilibrium outcomes

- Quantity $q(\Delta, \Delta')$.
- Per-unit price $p(\Delta, \Delta')$.
- Search intensity $\gamma(\Delta)$.
- Value function: $V(\Delta)$.
- Distributions: $\Phi(\Delta)$.

Trading Decisions

Nash bargaining: price $p(\Delta, \Delta')$ & quantity $q(\Delta, \Delta')$ maximise product of traders' surpluses.

Quantity maximises the surplus. Details
$\rightarrow$ Δ sells more to Δ' when $\beta < \beta'$ and/or $h > h'$.

Price splits the surplus. Details
$\rightarrow$ Price is higher when β & β' are higher and/or h & h' are lower.

Let $S(\Delta, \Delta')$ denote surplus at the optimal quantity.

Value Function

Equation governing traders' optimal behaviour:

$$r \underbrace{V(z,\beta,h)}_{\text{Value}} = \underbrace{u(\beta,h) - s(z,\gamma(z,\beta,h))}_{\substack{\text{Flow value \&} \\ \text{search costs}}} +$$

$$\underbrace{\eta \int (V(z,\beta',h) - V(z,\beta,h)) G(d\beta')}_{\text{Switch type}} +$$

$$\frac{1}{2} \iiint \underbrace{m(\gamma(z,\beta,h),\gamma(z',\beta',h'))}_{\substack{\text{Meeting} \\ \text{probability}}} \underbrace{S((z,\beta,h),(z',\beta',h'))}_{\text{Surplus}} \Phi(dz',d\beta',dh')$$

Endogenous Search

FOC of value function:

$$\underbrace{s_2(z,\gamma(\Delta))}_{\substack{\text{Marginal cost}\\ \text{of search}}} = \frac{1}{2}\int \underbrace{\frac{\partial m(\gamma(\Delta),\gamma(\Delta'))}{\partial \gamma(\Delta)}}_{\substack{\text{Increase}\\ \text{in meetings}}} \underbrace{S(\Delta,\Delta')}_{\substack{\text{Surplus from}\\ \text{meeting}}} \Phi(d\Delta')$$

Traders search harder:

1. if they have **low search cost**.
2. when **gains to trade are higher.**

Asset Distributions

Steady state: zero net inflows into all types.
$\rightarrow$ **Trading flows** perfectly balance **shock flows**. Equation

Market clearing:

$$\iiint h\Phi(dz, d\beta, dh) = a$$

Equilibrium

Steady state such that:

- Value functions solve the trader problem.
- Prices and quantities are determined by Nash bargaining.
- Search is chosen optimally.
- The market clears.

Formal Definition

Characterising the Equilibrium

Values, Trading & Distributions

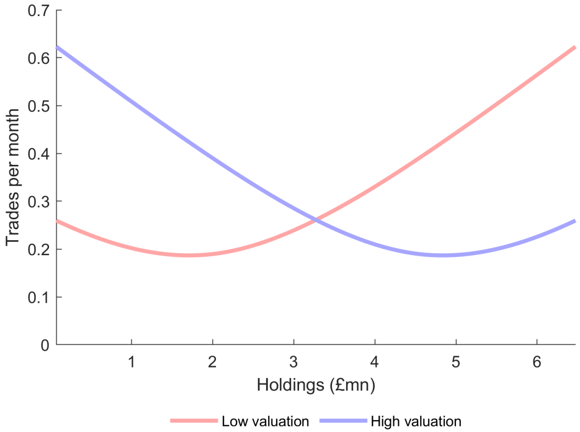
Values $V(z, \beta, h)$ are concave in holdings $\rightarrow$ traders have a **target holding** that depends on their search cost and valuation.

Traders with **low valuations and high holdings sell** the asset.

Traders with **high valuations hold more** of the asset.

Values Trading Distributions

Search in Equilibrium



Traders vary search to manage holdings around a target:

1. Respond to shocks.
2. Offset trades. Empirical fact By search cost Distributions

Liquidity

Require a summary measure of market liquidity.

Market Depth: maximum amount that could be sold per unit time without depressing price by more than a given amount.

- **Extensive margin**: how frequently can I trade?
- **Intensive margin**: how costly is it for me to trade?

Definition

Estimation & Results

Estimation

Parametric assumptions:

- Uniform shock distribution: $\beta \sim U(\mu_\beta, \sigma_\beta)$.
- Search costs: $s(z, \gamma) = (\gamma - z)^2$.
- Parameter $z \sim \text{Gamma}(k_z, \theta_z)$.

Parameter vector $\psi = \{\mu_\beta, \sigma_\beta, \kappa, \eta, k_z, \theta_z\}$.

GMM: match moments $m(\psi)$ to empirical counterparts m_0:

$$\hat{\psi} = \arg\min_{\psi} (m(\psi) - m_0)' \Omega^{-1} (m(\psi) - m_0)$$

where $\Omega = m_0 m_0'$.

Moments summarising the joint distribution of holdings, trading frequency, price and quantity. Search costs Matching Moments

Identification: Search Costs

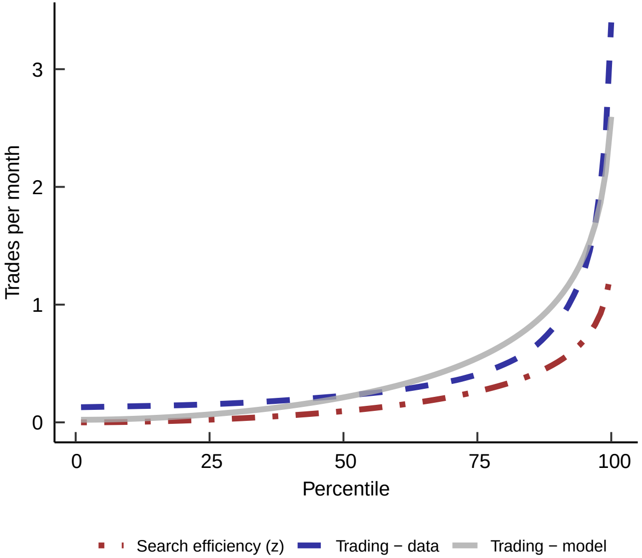
Parameter	Moment
$\mathbb{E}(z) = k_z \theta_z$	[+] $\mathbb{E}(n)$
$\mathbb{V}(z) = k_z \theta_z^2$	[+] $SD^A(n)$

z =search efficiency, n =trade frequency, h =holdings, $SD^A()$ =across-trader std. dev.

Optimal search $\rightarrow$ trading frequency n is a monotonic function of search efficiency z.

Search Equation Utility Parameters Full table

Trading Fit



Trades per month
3
2
1
0
0
25
50
75
100
Percentile
Search efficiency (z)
Trading − data
Trading − model

Model Fit

Moment	Data	Model
Expectations		
Price, %	1.09	1.09
Trade size, £mn	0.67	0.55
Trading frequency, per month	0.44	0.43
Holdings, £mn	3.27	3.27
Across traders		
Std. dev. trading frequency, per month	0.55	0.55
Within traders		
Std. dev. price, %	0.04	0.04
Std. dev. holdings, £mn	1.23	1.31
Correlation inventory & trading frequency	0.08	0.09
Correlation holdings & quantity sold	0.33	0.30

Interpreting Parameter Values

Highly **skewed distribution of search costs** $\rightarrow$ large technological advantage for frequent traders.

Trading more frequent than shocks.

- Quantity rationing.
- Liquidity supply.

Value function | Identification | Estimates

Counterfactuals

Research Questions

Q1: To what extent is liquidity reliant on a small subset of traders?

Q2: How does firm heterogeneity interact with:

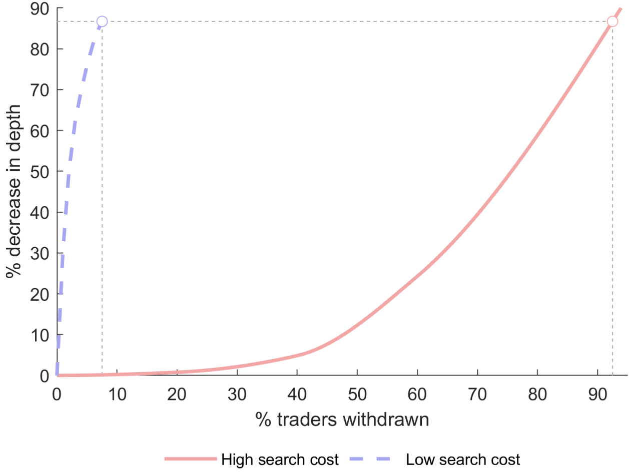
1. Financial shocks?
2. Trading platforms?
3. Banking regulation?

Research Questions: Heterogeneity

Question: To what extent is liquidity reliant on a small subset of traders?

Exercise: Withdraw sets of traders and compute impact on liquidity.

Contributions to Liquidity



8% of traders supply as much liquidity as the remaining 92%.

Research Questions: Vulnerability

Question: How does firm heterogeneity interact with **financial shocks**?

Exercise: Withdraw frequent traders and compute changes in market quantities, liquidity & welfare.

Vulnerability to Shocks

	% change in counterfactual
Market depth	-62
Holdings variance	-9
Mean trading frequency	-17
Price variance	140
Aggregate utility	-5

Withdrawing 4% most frequent traders causes large deterioration in market functioning.

Research Questions: Platforms

Question: How does firm heterogeneity interact with **trading platforms**?

Exercise: Simulate impact of trading platforms on liquidity & welfare.

Trading Platforms

Market historically **over-the-counter**: bilateral and phone-based.

Slow increase in **platform-based trading**:

- Multilateral electronic trading platforms.
- Bids posted to all platform members.

Success requires sufficient uptake.

Two types of counterfactual.

1. Platforms as **reduction & homogenisation of search costs**:
   $\rightarrow$ set all search costs to those of the lowest-cost trader.
2. Platforms as **more efficient trading mechanisms**:
   $\rightarrow$ Walrasian equilibrium.

Trading Platforms: Results

	Baseline		Homog.	Walras.
	Low cost	High cost	Agg.	Agg
Spreads, bps	223	-223	0	0
Utility	16.3	13.3	14.5	14.5
Aggregate				
Price variance, bps	378		90	0
Utility	13.8		14.5	14.5

Platforms improve liquidity and welfare, but **frequent traders worse off**.

Trading Platforms: Drivers

Trading platforms a trade-off for most efficient traders:

- Pro: improved search technology.
- Con: loss of competitive advantage.

Estimation:

- Frequent traders can trade more frequently than shocked.
  $\rightarrow$ **Benefits are small**.
- Skewed search cost distribution $\rightarrow$ large competitive advantage.
  $\rightarrow$ **Costs are large**.

Frequent traders will resist introduction of platforms.

Trading Platforms: Implications

How do frequent traders block platforms?

- Intermediaries' balance sheet capacity required on platform.
- Intermediaries hold more of bond at issuance.
- Intermediaries generate as well as satisfy trading volume.

Could policy help?

- Mandate platforms?
- Favorable regulatory treatment for bonds traded on platforms?

Can traders find a way around this?

- Recent collaborations between platform companies and investment banks.

Research Questions: Banking Regulation

Question: How does firm heterogeneity interact with **bank capital regulation**?

Exercise: Simulate impact of capital regulation on liquidity & welfare.

Capital Regulation

Concerns tighter capital regulation for dealer-banks has harmed liquidity (Duffie, 2018).

Capital counterfactual:

$$u(h) = \beta h - \tau|h| - \frac{1}{2}\kappa h^2$$

for $\tau = 0.01$ for 15% of traders with lowest search costs.

Exercise: suppose capital regulation raises cost of inventory by τ – what is the impact on markets?

Assessing **cost of capital regulation**—impact on markets—but not the benefits—bank resilience.

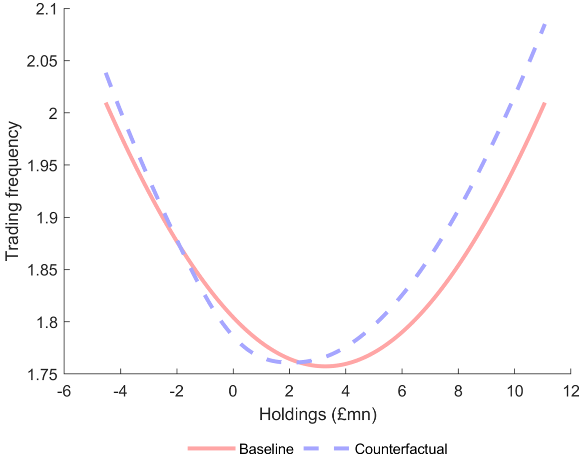
Capital: Impact on Traders

	% change in counterfactual	
	Dealers	Other traders
E(holdings)	-33	6
E(utility)	-37	1
E(spread)	17	17

Bond **shifts from dealers to other traders**, benefiting the latter.

Spreads increase as dealers reluctant to take on large positions.

Capital: Search Response



Dealers shift their target holdings, and search harder when away from target → dealers offset trades more. Empirical fact

Capital: Impact on Markets

	% change in counterfactual
Market depth	-10
Aggregate welfare	-5

Endogenous responses limit costs of regulation
$\rightarrow$ welfare cost without endogenous responses would be 7%.

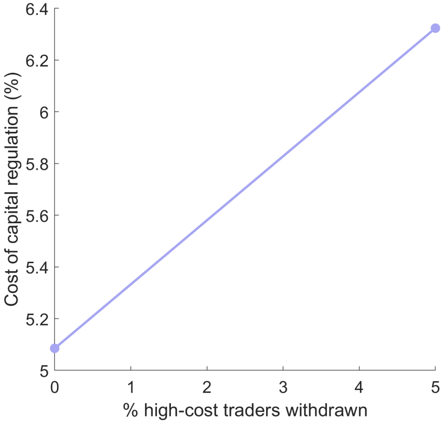
Capital Regulation during Stress

Evidence in literature that **liquidity is worse in a stress** post-crisis.

Market functioning worse when index trackers forced to sell downgraded bonds (Dick-Nielsen & Rossi, 2019).

Exercise: apply **capital regulation alongside sell-offs** by non-dealers.

Capital Regulation in a Sell-Off



Capital regulation limits dealers' willingness to supply liquidity in a sell-off. Prices

Capital Regulation and Bond markets

Rationalises recent trends in markets:

- Capital **regulation reduces liquidity**, but **markets adjust** to reduce the costs.
    - Adjust search to better control balance sheet.
    - Bonds pass to non-dealers.
    - Non-dealers supply liquidity.
- **In sell-offs**, dealers are called upon to buy assets, and are less willing to do so $\rightarrow$ **costs go up**.

Trade-off between reduction in market liquidity and benefits of bank resilience (including for markets).

Conclusion

Combine theory and data to quantitatively study **liquidity in OTC markets**.

$$\text{Data} \xrightarrow[\text{Identification}]{\text{Features}} \text{Model} \xrightarrow{\qquad\qquad} \text{Results}$$

Study implications for:

1. Resilience of liquidity.
2. Trading technologies.
3. Regulatory policy.

Thanks!
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Holdings Data

Portfolio data

Bank of England data on 7 UK **banks'** instrument-level bond portfolios at end-2016 and end-2017.

Morningstar data on 300 **mutual funds'** instrument-level bond portfolios, quarterly from 2012-2017.

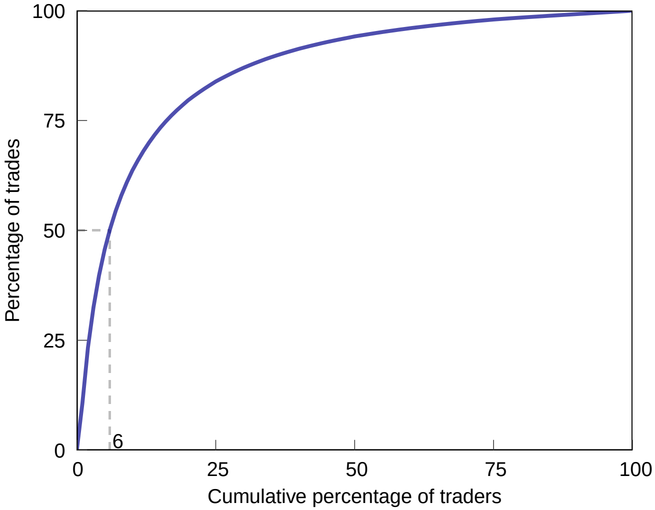
Back

Summary statistics

	Mean	Std. Dev.	Median
Aggregate			
Price (%)	109	16	106
Trade size (£000)	475	991	100
Monthly volume (£bn)	25	5	26
Monthly traders	972	69	975
Instrument-level			
Issuance (£mn)	195	441	40
Trades per month	19	37	5
Number of traders	56	77	15
Trader-level			
Monthly volume (£000)	11,997	118,351	33
Instruments traded	62	199	7
Trades per instrument traded	5	35	2

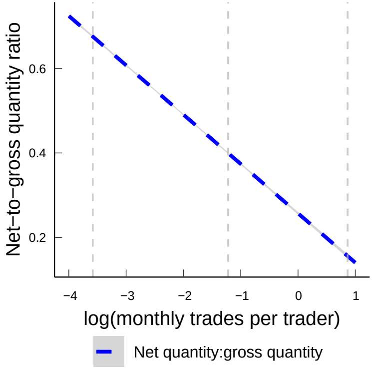
Back

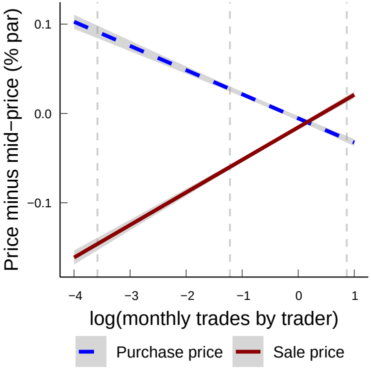
Secondary Market for Sterling Corporate Bonds



Back

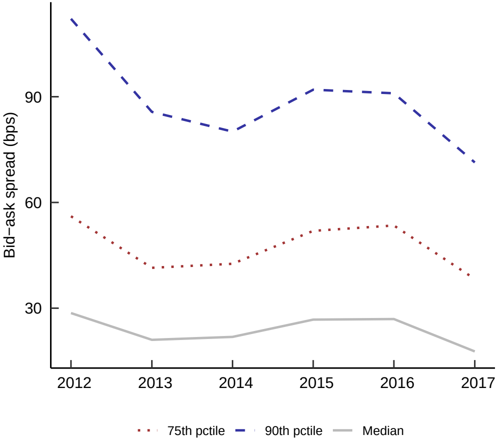
Intermediation & Trading Frequency





Frequent traders earn money by transferring bonds from sellers to buyers. Back

Spreads through time



Despite large increases in capital requirements, spreads have not increased. Back

Trading Quantity

Nash bargaining: price $p(\Delta, \Delta')$ & quantity $q(\Delta, \Delta')$ maximise product of traders' surpluses.

Quantity maximises the trading surplus. Further Details

$$\underbrace{V_3(z, \beta, h - q(\Delta, \Delta'))}_{\Delta \text{ marginal valuation}} = \underbrace{V_3(z', \beta', h' + q(\Delta, \Delta'))}_{\Delta' \text{ marginal valuation}}$$

Δ sells more to Δ' when

1. $\beta < \beta'$.
2. $h > h'$.

Back

Trading Price

Price determined by traders' changes in values.

$$p(\Delta,\Delta') = \frac{1}{2}\left(\overbrace{\frac{V(z',\beta',h'+q(\Delta,\Delta')) - V(\Delta')}{q(\Delta,\Delta')}}^{\approx \text{ slope of } \Delta' \text{ value in } h'} + \underbrace{\frac{V(\Delta) - V(z,\beta,h-q(\Delta,\Delta'))}{q(\Delta,\Delta')}}_{\approx \text{ slope of } \Delta \text{ value in } h} \right)$$

Price is:

- Higher when β & β' are higher.
- More variable when the slope of $V(z,\beta,h)$ is more variable.

Identification Back

Terms of trade

Nash bargaining:

$$\max_{p,q} \quad \overbrace{\left(V(z,\beta,h-q) - V(\Delta) + pq\right)}^{\text{type } \Delta \text{ surplus}} \overbrace{\left(V(z',\beta',h'+q) - V(\Delta') - pq\right)}^{\text{type } \Delta' \text{ surplus}}$$

$$\text{s.t.} \quad V(z,\beta,h-q) - V(\Delta) + pq \geq 0,$$

$$V(z',\beta',h'+q) - V(\Delta') - pq \geq 0.$$

$p(\Delta, \Delta')$ is per-unit price.

$q(\Delta, \Delta')$ is quantity sold from Δ to Δ'.

Back

Steady state

$$\iint_{\underline{\beta}}^{\beta^*}\int_{h^*}^{\infty} m(\gamma(\Delta),\gamma(\Delta'))\phi(\Delta)\phi(\Delta')\mathbb{1}(q(\Delta,\Delta') \geq h-h^*)dhd\beta d\Delta' -$$

$$\iint_{\underline{\beta}}^{\beta^*}\int_{-\infty}^{h^*} m(\gamma(\Delta),\gamma(\Delta'))\phi(\Delta)\phi(\Delta')\mathbb{1}(q(\Delta,\Delta') < h-h^*)dhd\beta d\Delta'$$

$$= \eta(1-G(\beta^*))\int_{\underline{\beta}}^{\beta^*}\int_{-\infty}^{h^*}\phi(\Delta)dhd\beta - \eta G(\beta^*)\int_{\beta^*}^{\bar{\beta}}\int_{-\infty}^{h^*}\phi(\Delta)dhd\beta$$

Back

Equilibrium

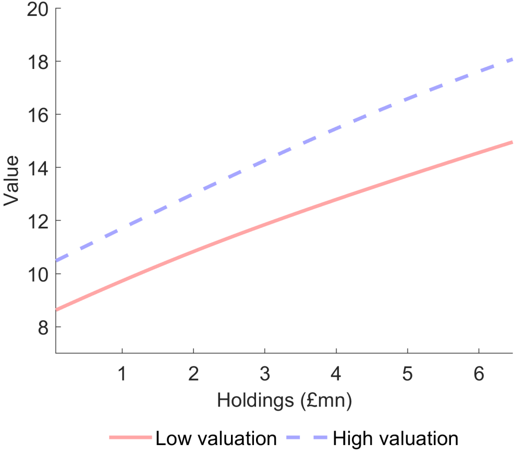
Let $\mathcal{T} \equiv \mathbb{R}^+ \times [\beta_L, \beta_H] \times \mathbb{R}$ be the type space.
An equilibrium is

1. distribution function $\Phi : \mathcal{T} \to [0, 1]$;
2. value function $V : \mathcal{T} \to \mathbb{R}$;
3. search intensity function $\gamma : \mathcal{T} \to \mathbb{R}^+$;
4. pricing $p : \mathcal{T}^2 \to \mathbb{R}^+$ and trade quantity $q : \mathcal{T}^2 \to \mathbb{R}$;

such that

- Value functions solve the trader problem.
- Price and quantities are determined by Nash bargaining.
- Search is chosen optimally.
- The system is in steady state.
- The market clears. Back

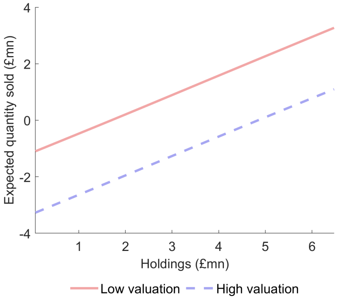
Value in equilibrium



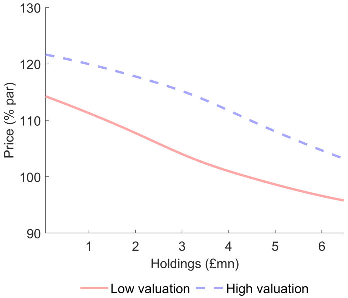
Concave utility: traders target finite holdings.

Utility Parameters Parameters

Trading in equilibrium



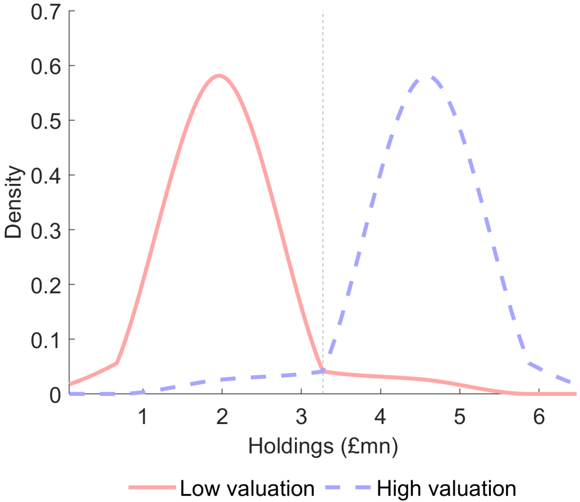
Quantity sold



Trading price

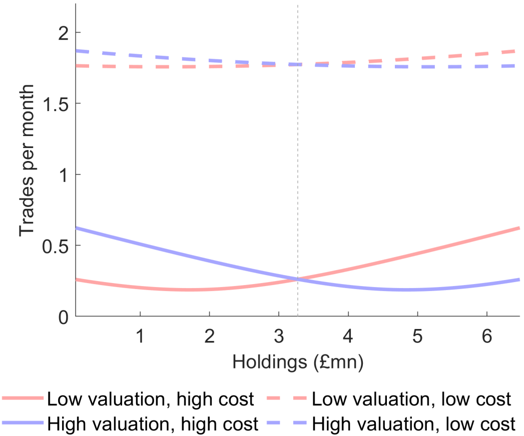
Traders with low values and high holdings offload the asset. Back

Distributions in equilibrium



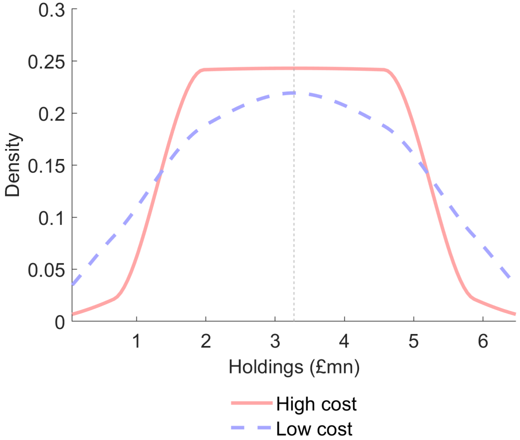
Assets held by high-valuation traders. Back

Search in equilibrium



Low-cost traders search harder and vary search less. Back

Search and distributions



Low-cost traders' holdings are more dispersed. Back

Market Depth

Maximum willingness to pay:

$$\frac{V(z,\beta,h+q)-V(z,\beta,h)}{q} \xrightarrow[q\to 0]{} V_3(z,\beta,h)$$

Maximum $q^\pi(\Delta,D)$ that can be sold to this trader at price discount no greater than D:

$$\underbrace{\frac{V(z,\beta,h+q^\pi(\Delta,D))-V(z,\beta,h)}{q^\pi(\Delta,D)}}_{\text{price for } q^\pi(\Delta,D)} = \underbrace{V_3(z,\beta,h)}_{\text{current price}} - D,$$

Market depth is the trade-weighted average of $q^\pi(\Delta,D)$ times the average frequency of trading 2Γ:

$$\Pi(D) = 2\Gamma \int \frac{\gamma(\Delta)\phi(\Delta)}{\Gamma} q^\pi(\Delta,D)\,d\Delta.$$

Back

Matching function

Linear matching function:

$$m(\gamma, \gamma') = 2\gamma \frac{\gamma'}{\Gamma}$$

where

$$\Gamma = \int \gamma(\Delta)\Phi(d\Delta)$$

$\rightarrow$ Conditional on contact, counterparty chosen randomly with likelihood proportional to γ'. Back

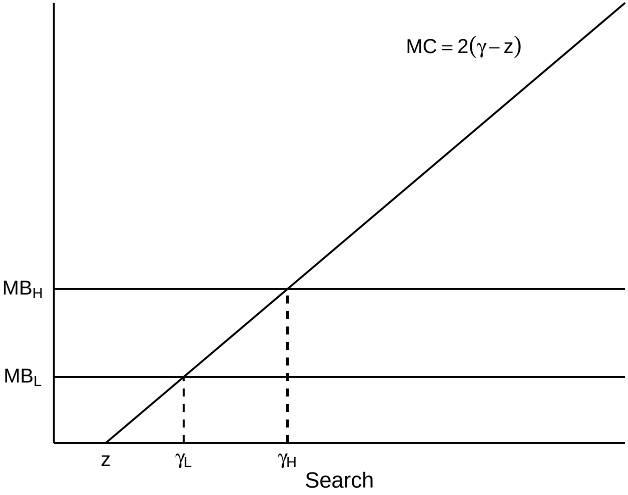
Search costs

Search cost function given by:

$$s(z, \gamma) = (\gamma - z)^2$$

$\rightarrow$ base level of contacts, with constant marginal cost above this base. Back

Search technology



Back

Moments

Expectations

1. Average trading frequency n.
2. Average trade size $|q|$.
3. Average price p.

Across traders

4. Standard deviation of trading frequency n.

Within traders

5. Standard deviation of holdings h.
6. Standard deviation of prices p.
7. Correlation between quantity sold q and holdings h.
8. Correlation between absolute inventory $inv \equiv |h - s|$ and trading frequency n. Back

Identification: Search Equation

$$z = \underbrace{\gamma(\Delta)}_{\text{Meetings}} - \underbrace{\int \frac{1}{2}\frac{\gamma(\Delta')}{\Gamma} S(\Delta, \Delta')\Phi(d\Delta')}_{\text{Expected surplus}}$$

Optimal search $\rightarrow$ trading frequency is a monotonic function of search efficiency.

Back

Parameter Estimates

Parameter	Estimate
Search efficiency $z \sim \Gamma(k_z, \theta_z)$	
k_z	0.545
θ_z	0.374
Shock frequency η	
η	0.040
Utility $u(h) = \beta h - 0.5\kappa h^2$; $\beta \sim U(\mu_\beta, \sigma_\beta)$	
μ_β	0.031
σ_β	0.015
κ	0.008

Inference Back

Parameter Estimates

Parameter	Estimate
Search efficiency $z \sim \Gamma(k_z, \theta_z)$	
k_z	0.545
	(0.46; 0.632)
θ_z	0.374
	(0.333; 0.429)
Shock frequency η	
η	0.04
	(0.038; 0.042)
Utility $u(h) = \beta h - 0.5\kappa h^2$; $\beta \sim U(\mu_\beta, \sigma_\beta)$	
μ_β	0.031
	(0.029; 0.034)
σ_β	0.015
	(0.014; 0.016)
κ	0.008
	(0.007; 0.008)

Back

Identification: Utility

Parameter	Moment
σ_β	$\overset{[+]}{SD^W(p)}, \overset{[+]}{\mathbb{E}(\lvert q\rvert)}, \overset{[+]}{SD^W(h)}$
κ	$\overset{[+]}{SD^W(p)}, \overset{[-]}{\mathbb{E}(\lvert q\rvert)}, \overset{[-]}{SD^W(h)}$

p =price, $|q|$ =trade size, h =holdings, $SD^W()$ =within-trader std. dev.

Risk aversion $\rightarrow$ **curvature** of value $V(z, \beta, h)$.

Shock variance $\rightarrow$ **variation** in slope of $V(z, \beta, h)$.

$\uparrow \kappa$ and $\uparrow \sigma_\beta$

- Same effect on price variation.
- Opposite effect on holdings variation.

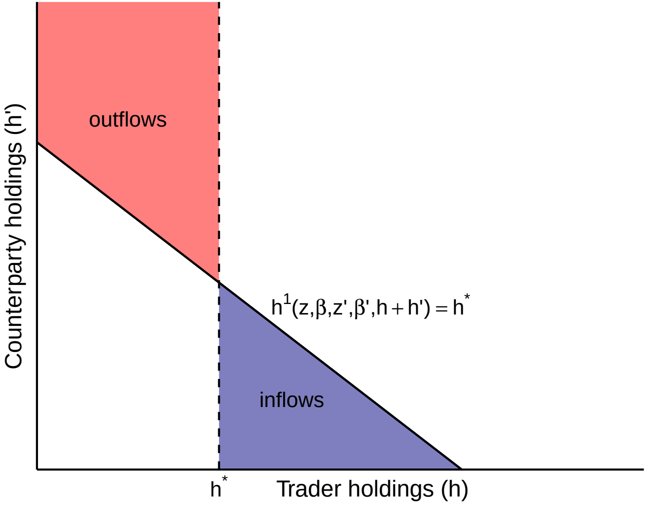
Value function | Full table | Price | Back

Identification

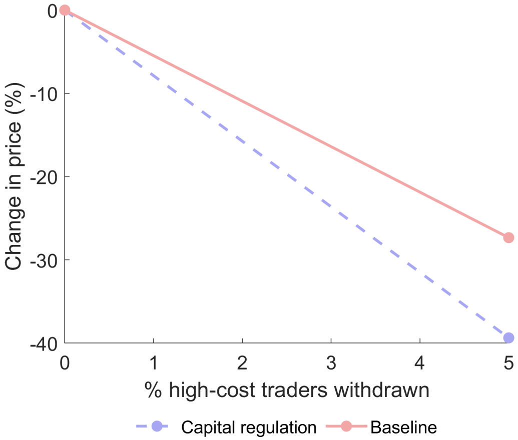
Parameter	Moment
$\mathbb{E}(z) = k_z\theta_z$	$\overset{[+]}{\mathbb{E}(n)}$
$\mathbb{V}(z) = k_z\theta_z^2$	$\overset{[+]}{SD^A(n)}$
μ_β	$\overset{[+]}{\mathbb{E}(p)}$
σ_β	$\overset{[+]}{SD^W(p)}, \overset{[+]}{\mathbb{E}(\|q\|)}, \overset{[+]}{SD^W(h)}$
κ	$\overset{[+]}{SD^W(p)}, \overset{[-]}{\mathbb{E}(\|q\|)}, \overset{[-]}{SD^W(h)}$
η	$\overset{[+]}{corr^W(\|h-s\|, n)}, \overset{[+]}{corr^W(h, q)}$

Back

Trading flows



Capital Regulation in a Sell-Off



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