A Structural Model of Interbank Network Formation and Contagion*

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The interbank network, in which banks compete with each other to supply and demand financial products, creates surplus but may also result in risk propagation. We examine this trade-off by setting out a model in which banks form interbank network links endogenously, taking into account the effect of links on default risk. We estimate this model based on novel, granular data on aggregate exposures between banks. We find that the decentralised interbank network is not efficient, primarily because banks do not fully internalise a network externality in which their interbank links affect the default risk of other banks. A social planner would be able to improve surplus on the interbank market without harming bank default risk, or vice versa. We propose two novel regulatory interventions (caps on aggregate exposures and pairwise capital requirements) that result in efficiency gains.

Keywords: Contagion, systemic risk, interbank network, network formation.
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1 Introduction

Direct interconnections between banks are important in two ways. First, these interconnections fulfill a function, in that there are gains to trade. The interconnection could, for example, involve providing liquidity or acting as the other party in a hedging transaction, which may result in surplus on both sides of the trade. Second, interconnections can open up at least one side of the transaction to counterparty risk: a lender, for example, runs the risk that the borrowing bank will not pay it back. Both sides of this trade-off were important during the financial crisis and remain important today, and consequently there is significant debate about optimal regulation in this context (Yellen, 2013).

How do banks form these interconnections, which we term the interbank network,[1] given the effect of such exposures on their risk? What inefficiencies exist in network formation? The answers to these economic questions then lead us to questions about regulation. Given equilibrium responses by banks, is regulation effective in reducing default risk? If it does reduce default risk, does it do so efficiently in a way that preserves interbank surplus?

We answer these questions by estimating a structural equilibrium model in which banks form the interbank network endogenously, taking into account the effect of their choices on their default risk. The key mechanism in this model is that when a bank takes on an exposure through the interbank network it earns a return, but it may also become riskier, which endogenously increases its funding costs. We estimate this model based on novel, rich Bank of England data on aggregate interbank exposures including lending and derivatives. This allows us to (1) quantify the inefficiency of interbank network formation, (2) examine the equilibrium effects of regulation, and propose alternative regulation that is more efficient, and (3) show how standard measures of bank systemic importance can be biased.

The starting point for our work is Bank of England data on interbank exposures. These data are collected by the Bank of England through periodic regulatory surveys of 18 global banks from 2012 to 2018, in which they report the exposures they have to their most important banking counterparties. The data are novel, relative to the data commonly used in this literature, in two ways that are important for our context: (1) the data include a broad range of instruments, making them the best available proxy for a bank's *total* exposure to another bank[2] and (2) the data contain rich detail on the types and characteristics of the

[1] The "interbank market" is often used to describe short-term (often overnight) lending between banks. We use the "interbank network" more generally to cover any form of direct interconnection between banks.

[2] We discuss the remaining dimensions in which the data is incomplete in the data section below.

instruments that make up each exposure. We set out various empirical facts about the network that inform our work, the most important of which is that there is significant variation in the size of exposures between banks, but not much variation in the presence of exposures: in other words, the network is dense but heterogeneous.

The features of our data and the empirical facts we observe guide our modelling choices in the following ways. First, the breadth of the data allows us to specify and estimate an empirical model of the effect of exposures on default risk, in a way which would not be feasible if we only observed exposures relating to a single instrument that is only a small subset of total exposures. Second, the fact that we observe a dense, heterogeneous network leads us to consider heterogeneity in marginal cost, in contrast to those parts of the empirical networks literature that seek to explain sparse network structures using fixed costs (Craig and Ma, 2022). Third, the granularity of our data allows us to specify and then estimate a rich model of network formation, with a focus on allowing for as much observed and unobserved heterogeneity as possible.

With this general guidance in mind, we set out a model consisting of two interrelated parts: a default risk process that relates the default risk of a bank to that of other banks and the exposures network, and a network formation game in which banks demand and supply financial products given the effect on their default risk.

We model the default risk process as being spatially autocorrelated, such that bank i's default risk depends on its fundamentals and on its interbank exposures. These interbank exposures can have a hedging effect that reduces default risk, but also a spillover effect that increases default risk, where the net effect depends on the characteristics of the exposure and the counterparties involved. We generalise a standard spatially autocorrelated regression by allowing the strength of the network spillover to vary across bank pairs: in other words, some links are inherently more risky than others, holding all other things constant. We refer to this as *heterogeneous contagion intensity* in what follows.[3]

Regarding network formation, banks supplying financial products receive a return, but also incur a cost because regulatory capital requirements mandate that they raise a certain amount of capital for the exposure that they take on when they supply. The key mechanism

[3]We do not directly micro-found such heterogeneity, instead we parameterise and estimate it. There are various reasons why such heterogeneity could exist, both at the bank and the bank-pair level, including jurisdictional or geographic differences across banks, heterogeneity in the types of instrument being traded, and heterogeneity across bank pairs in the covariance in their bank fundamentals (representing their ability to share risk). We explore these possible reasons post-estimation.

is that a bank's cost of capital is an increasing function of its default risk. This default risk, per the default risk process we describe above, is a function of the bank's exposures, meaning that a bank supplying financial products endogenously changes its cost of capital when it does so. Heterogeneous contagion intensity means that this marginal cost varies across pairs: inherently risky links involve higher marginal cost. Banks demand interbank financial products to maximise profits from heterogeneous technologies that take these differentiated interbank products as inputs.

Equilibrium trades and prices depend in an intuitive way on the key parameters of the model: (1) heterogeneity in contagion intensity is a key driver of link formation: inherently safe links are less costly and therefore more likely to be large, (2) risky banks pay more to be supplied financial products because contagion means it is more costly to supply them and (3) risky banks supply less, as their funding costs are higher. The source of market failure is network externalities, in which banks take the default risk of other banks as given and so do not fully internalise the effect that their exposure choices have on the risk (and therefore the funding cost) of their counterparties. We show that our model is consistent with the key empirical facts in our data, as well as additional stylised facts from the financial crisis.

We estimate our model by matching two groups of moments: moments related to data on bank default risk and moments related to data on interbank exposures. To represent bank fundamentals we use variation in regional equity indices: for example, we take a shock to a Japanese equity index as a shock that affects Japanese banks more than European banks. We use these fundamentals to identify the key parts of our network formation model and the default risk process. In particular, we show how heterogeneous contagion intensities can be inferred from the observed network even with a broad suite of fixed effects.

Our results imply that contagion through the interbank network is material and heterogeneous across banks and pairs of banks: some exposures are virtually costless in terms of contagion, whereas some are very costly. We explore how this variation is correlated with banks' characteristics, and find weak evidence that geographic co-location, the variance of a bank's fundamentals, and the covariance between banks' fundamentals all matter.

We then use our estimated results to answer the key questions set out above. We first show that heterogeneity in contagion intensity matters for the identification of systemically important banks. There are various measures of systemic importance (or, equivalently in our context, network centrality), but in general terms a bank is deemed systemically important if it has large exposures to other systemically important banks. Heterogeneous contagion intensity and endogenous network formation together show why this approach is likely to be biased: *some links are large because they are inherently safe.* We propose an alternative measure of systemic importance based on network data that is weighted by heterogeneous contagion intensity: an inherently risky (safe) link is scaled up (down). This weighted centrality measure implies materially different centrality rankings among banks: the bank that is most systemically important in our sample based on the unweighted network is only the 3rd most important based on our alternative risk-weighted centrality measure.

We then consider the efficiency of the decentralised interbank network. An important complication is that bank default risk might affect surplus in the broader economy outside of the interbank market, through, for example, financial crises or lending spillovers. We do not model or measure this relationship formally, but instead make a simple directional assumption that (at least locally) outside surplus is decreasing in bank default risk. This implies that, whilst we cannot fully characterise the social planner's preferences, we can derive an efficient frontier that shows the optimal trade-off between interbank surplus and bank default risk. We find that the decentralised interbank network is not on the frontier: a social planner would be able to increase interbank surplus by 79% without increasing mean bank default risk or decrease mean bank default risk by 51% without decreasing interbank surplus. This result is driven by the fact that our empirical results indicate that network externalities are significant. The social planner internalises this externality by (1) reducing aggregate exposures and (2) increasing concentration among exposures, by reducing inherently risky exposures by relatively more than inherently safe exposures.

We then use our model to simulate the equilibrium effects of various forms of regulation, including a cap on individual exposures (Basel Committee, 2014, 2018b) and an increase in regulatory capital requirements (Basel Committee, 2018a). We find that a cap on individual links is relatively ineffective: it has only a small effect on mean bank default risk, as in equilibrium banks shift their supply to uncapped links. We instead propose capping aggregate exposures held by each bank, rather than individual exposures: an aggregate cap is more effective (because it prevents a bank moving capped supply to another bank) and more efficient (because in equilibrium banks respond to a cap on aggregate exposures by reducing relatively risky exposures by more than less risky exposures). This aggregate cap would improve both interbank surplus and bank default risk, such that we can conclude that the social planner would strictly prefer it to a cap on individual exposures, even without fully knowing the social planner's preferences.

We find that a general increase in capital requirements that applies equally across exposures to all banks is effective but inefficient: it decreases mean bank default risk, but at the cost of reduced interbank surplus. We instead propose a mean-preserving spread of capital requirements based on their heterogeneous contagion intensity: we give links that are inherently risky (inherently safe) greater (lower) capital requirements. This targets regulatory intervention more closely at the network externalities that are the key driver of inefficiency in our model. Our results suggest that a social planner would strictly prefer our proposed pairwise capital requirement to a homogeneous capital requirement.

We discuss related literature below. In Section 2, we introduce the institutional setting and describe our data. In Section 3, we set out our model. In Section 4, we describe empirical approach. In Section 5, we set out our results. In Section 6, we undertake counterfactual analyses. In Section 7, we conclude.

1.1 Related literature

Our work is related to three strands of literature: (i) the effects and formation of financial networks, (ii) structural estimation of financial markets and (iii) optimal regulation in financial markets.

There is an extensive literature on the effect of network structure on outcomes in financial markets, both theoretical (Acemoglu et al., 2015; Ballester et al., 2006; Eisfeldt et al., 2023; Elliott et al., 2014; Galeotti and Ghiglino, 2021; Gofman, 2011; Nier et al., 2007) and empirical (Denbee et al., 2021; Eisfeldt et al., 2023; Iyer and Peydro, 2011). There is also a large theoretical literature on network formation in financial markets (Acharya and Bisin, 2014; Babus, 2016; Cabrales et al., 2017; Chang and Zhang, 2018; Farboodi, 2021; Shu, 2022), and a smaller empirical literature (Blasques et al., 2018; Cohen-Cole et al., 2010; De Paula, 2020; Ellul and Kim, 2021).[4] Our paper contributes to this literature by examining the role of heterogeneous contagion intensities in a model in which banks directly trade-off gains to trade against increased risk.[5] We show that this heterogeneity is an important driver of outcomes, and has implications for how bank systemic importance is measured. We are able

[4]Many network formation papers explain sparse endogenous networks through fixed costs to bilateral link formation, followed by a trading game on the realised network (see De Paula (2020) for a summary). Our setting is quite different, in that the empirical network in aggregate exposures that we seek to match is dense, but heterogeneous in the size of links. Our network formation model thus focuses on the role of marginal costs in bank trading.

[5]Our model and empirics is focussed on direct spillovers, rather than informational spillovers (such as those studied by Acharya and Yorulmazer (2008) and Goldsmith-Pinkham and Yorulmazer (2010).

to include such heterogeneity because our novel data and empirical approach allow us to estimate them in a structural context.

There is a growing literature on structural estimation in financial markets, and in particular on the role of banks: Allen and Wittwer (2021), Brancaccio and Kang (2022), Coen and Coen (2022) and Pinter and Uslu (2022) in search-based models of financial markets and Benetton (2021) and Benetton et al. (2021) on the effects of regulation in mortgage markets. There are two particularly important papers on bank lending in financial networks: Craig and Ma (2022) estimate a model of interbank lending and systemic risk in Germany based on monitoring costs, and Gofman (2017) estimates a random network formation model with an efficiency-stability trade-off for the US Fed Funds market. Our contribution is to focus on aggregate exposures and their direct effect on bank risk, using the identifying information contained in both CDS-implied bank default risk and the observed network.

We also contribute to the literature regarding optimal regulation in financial markets (Baker and Wurgler, 2015; Batiz-Zuk et al., 2016; Duffie, 2017; Greenwood et al., 2017) and policy on networks in two ways. First, we directly model the impact of regulation on both bank surplus and bank default risk. Numerous papers consider the effect of bank regulation on surplus in specific markets, but without considering the effect on bank default risk (which was arguably the primary focus of much recent banking regulation) it is difficult to draw any conclusions about whether regulation is optimal. Our work is thus part of a growing effort in the literature to take a more holistic view of global banks across their multiple activities (Benetton et al., 2022; Diamond et al., 2020). Second, we allow bank default risk to have broader externalities on the real economy. We do not model or measure the scale of these externalities, which means we remain largely agnostic in the debate over whether banks should be regulated more tightly (Admati, 2016; Hellmann et al., 2000). Nevertheless, we show how with a simple directional assumption about these externalities it is still possible to design regulation that improves welfare, by focusing not on the average *level* of regulation but on its *design* and *heterogeneity* across banks.

2 Institutional setting and data

We first describe the institutional setting of our work, including the relevant regulation. We then describe our data.[6] We then use this data to set out some empirical facts that will guide our approach to modelling.

[6]For further details on the data and regulatory setting, see the internet appendix.

2.1 Institutional setting

Direct connections between banks fulfill an important function: "*there is little doubt that some degree of interconnectedness is vital to the functioning of our financial system*" (Yellen, 2013). Debt and securities financing transactions between banks are an important part of liquidity management, and derivatives transactions play a role in hedging. There is, however, widespread consensus that direct connections can also increase counterparty risk, with implications for the risk of the system as a whole (see, for example, Acemoglu et al. (2015)). The importance of both sides of this trade-off is such that direct interconnections between banks are the subject of extensive regulatory and policy-making scrutiny, whose aim is to: "*preserve the benefits of interconnectedness in financial markets while managing the potentially harmful side effects*" (Yellen, 2013).

After the 2008 financial crisis, a broad range of regulation was imposed on these markets. In this paper, we focus on two in particular that are particularly relevant to this trade-off: (1) caps on large exposures and (2) increases in capital requirements.

2.2 Data

2.2.1 Exposures

We define in general terms the exposure of bank i to bank j at time t as the immediate loss that i would bear if j were to default, as estimated at time t. The way in which this is calculated varies from instrument to instrument, but in general terms this can be thought of as (1) the value of the instrument, (2) less collateral, (3) less any regulatory adjustments intended to represent variations in value or collateral in case of default (for example, regulation typically requires a "haircut" to collateral when calculating exposures, as in the event of default assets provided as collateral are likely to be worth less).

We use regulatory data on bilateral interbank exposures, collected by the Bank of England. The dataset offers a unique combination of breadth and detail in measuring exposures. Much of the existing literature (such as Denbee et al. (2021)) on empirical banking networks relies on data from payment systems. This is only a small portion of the activities that banks undertake with each other and is unlikely to adequately reflect the extent of interbank activity or the risk this entails.

18 of the largest global banks operating in the UK report their top 20 exposures to banks over the period 2011 to 2018. Banks in our sample report their exposures every six months

from 2011 to 2014, and quarterly thereafter. They report exposures across debt instruments, securities financing transactions and derivatives. The data are censored: we only see each bank's top 20 exposures, and only if they exceed £10 million. The data include granular breakdowns of each exposure: by type (e.g. they break down derivatives into interest rate derivatives, credit derivatives etc.), currency, maturity and, where relevant, seniority.

We use this dataset to construct a series of snapshots of the interbank network between these 18 banks. We calculate the total exposure of bank i to bank j at time t, which we denote C_{ijt}, as the sum of exposures across all types of instrument in our sample. The result is a panel of $N = 18$ banks over $T = 21$ periods from 2011 to 2018 Q2, resulting in $N(N-1)T = 6,426$ observations. For each C_{ijt}, we use the granular breakdowns to calculate underlying characteristics that summarise the type of financial instrument that make up the total exposure. These 13 characteristics, which we denote d_{ijt}, relate to exposure type, currency, maturity and collateral type.

Although the dataset includes most of the world's largest banks, it omits banks that do not have a subsidiary in the UK.[7] Furthermore, for the non-UK banks that are included in our dataset, we observe only the exposures of the local sub-unit, and not the group. For non-European banks, this sub-unit is typically the European trading business. We consider this partial observation directly in our structural estimation below.

2.2.2 Other data

To measure banks' default risk we use the spreads on publicly traded credit default swaps (obtained from Bloomberg), which as shown by Hull et al. (2009) and Allen et al. (2021) can be used to compute the (risk-neutral) probability of default. We supplement our core data with data from Bloomberg on the geographic sources of revenues for each bank in our sample. Bloomberg summarises information from banks' financial statements about the proportion of their revenues that come from particular geographies, typically by continent, but in some cases by country. We also use S&P regional equity indices for the US, Canada, the UK, Europe, Japan, Asia and Latin America.

[7] This is particularly relevant for some major European investment banks, who operate branches rather than subsidiaries in the UK, and hence do not appear in our dataset.

2.3 Summary statistics

The data reveal certain empirical observations about exposures and how they vary in our sample: (1) exposures are large, (2) the network is dense and reciprocal and (3) network links are heterogeneous in intensity and characteristics. We discuss below how we use these empirical observations to guide our modelling.

Empirical fact 1: Exposures are large

The primary advantage of our data is that it is intended to capture a bank's *total* exposures. The largest exposure in our sample is £$7,682$m, the largest total exposures to other banks in a given period is £$26,367$m. The mean exposure is £285m and the mean total exposure to other banks in a given period is £$4,850$m.

In this respect, our data has two important advantages over many of the data used in the literature. First, our dataset is the closest available representation of *total* exposures, when most other empirical assessments of interbank connections rely on a single instrument, such as CDS (Eisfeldt et al., 2023) or overnight loans (Denbee et al., 2021). Second, our data are on exposures, rather than simply market value, in that when banks report their exposures they account for collateral and regulatory adjustments. Data based solely on market value is a representation of bank activity, rather than counterparty risk.

Empirical fact 2: The network is dense and reciprocal

Figure 1 shows the network of exposures between banks in 2015 Q2. Our sample is limited to the core of the banking network, and does not include its periphery. Our observed network is, therefore, dense: of the $N(N-1)T$ links we observe in total, only approximately 30% are 0. One implication of the density of the network is that it is reciprocal: of the $N(N-1)T/2$ possible bilateral relationships in our sample, 55% are reciprocal, in that they involve a strictly positive exposure in each direction (that is, bank i has an exposure to bank j and bank j has an exposure to bank i).

Empirical fact 3: The network is heterogeneous in intensity and characteristics

Although the network is dense and so not particularly heterogeneous in terms of the presence of links, it is heterogeneous in the intensity of those links (that is, the size of the exposure), as shown in Figure 1. We further demonstrate this in Table 1, which contains the results of a regression of our observed exposures C on fixed effects. The R^2 from a regression on

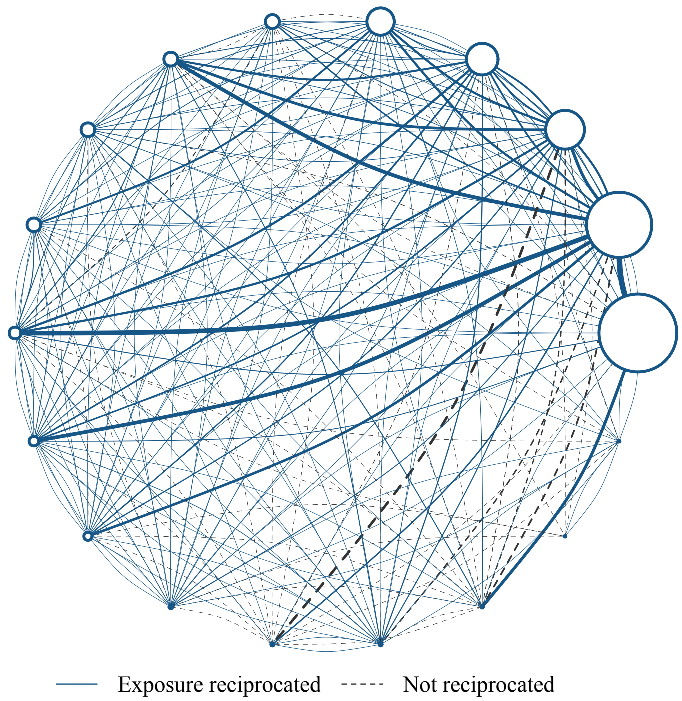


Figure 1: The aggregate network in H1 2015

Note: This is the network of aggregate exposures between banks in H1 2015. Each node is a bank in our sample. A solid line between two nodes shows a reciprocated exposure (each bank has an exposure to the other) and a dashed lines shows an unreciprocated exposure (that goes in one direction only). The line width is proportional to the size of the exposure. The size of the node is proportional to its total outgoings.

it fixed effects is 0.39: if all of bank i's exposures in a given time period were the same, then this would be 1.00, indicating that there is significant cross-sectional variation in the size of exposures. There is also significant persistence in exposures, as set out in Table 1, in which we show that the R^2 for a regression of C_{ijt} on pairwise ij fixed effects is 0.61: a large proportion of the variation in exposures is between pairs rather than across time.

There is significant variation in product characteristics across banks, in that the average product supplied by each bank varies according to currency, maturity and type. For example, between 60% and 80% of the exposures held by most banks in our sample relate to derivatives. For one bank, however, this figure is 95%, and for another it is 15%.

Additional facts

Our sample runs from 2011 to 2018, and therefore earlier periods feature the end of the European debt crisis. Bank default risk has broadly reduced across all banks, as we set out in Figure 2. Importantly, though, there is cross-sectional variation across banks, and inter-temporal variation in that cross-sectional variation. We show this in Figure 2, in which we highlight the default risk of two specific banks. Bank 1's default risk was significantly higher than bank 2's in 2011, but by 2018 this had reversed.

Table 1: Variation and persistence in network

	Dependent variable: Exposure C_{ijt}						
Dummy Variables	i	j	t	it	jt	it+jt	ij
R-squared	0.27	0.12	0.01	0.39	0.15	0.54	0.61
No. obs	6426	6426	6426	6426	6426	6426	6426

Note: This table decomposes variation in exposures C_{ijt} across demanding bank i, supplying bank j, and time t. The table shows R^2 for regressions of exposures on dummies given at the top of each column.

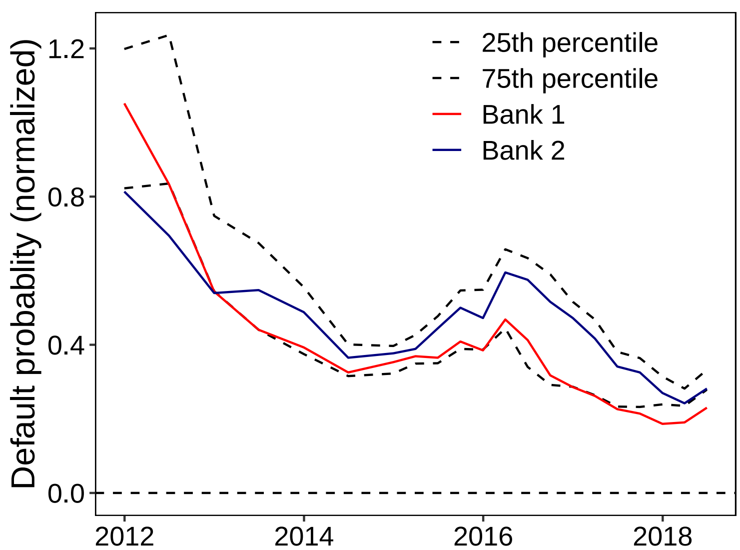


Figure 2: Inter-temporal and cross-sectional variation in default risk

Note: The black dashed lines show the 25$^{\text{th}}$ and 75$^{\text{th}}$ percentiles of bank default risk over time. The solid lines show two banks' default probabilities, normalized by the median overall default probability.

Our sample starts in 2011, it does not feature the financial crisis that began in 2008. We note three features that were observed on the interbank network during the 2008 crisis, on the basis that a good model of interbank network formation should be able to replicate what happened during the crisis. First, risky banks were not supplied; in other words, they experienced lockout (Welfens, 2011). Second, risky banks did not supply (Gale and Yorulmazer, 2013). Third, in the worst periods of the financial crisis there was effectively market shutdown in markets for certain instruments, in that very few banks were supplied anything on the interbank network (Afonso et al., 2011; Allen et al., 2009).

3 Model

We first introduce the setup of the model and notation. We then describe the default risk process and the network formation game. Finally, we consider the implications of this model for optimal networks.

Before this, we briefly discuss our motivation for the model in broad terms. Our objective is to focus on the fundamental trade-off between risk propagation and surplus creation, but in a way that is tractable, consistent with our empirical facts (and in particular the clear heterogeneity among banks) and estimable. We do this by including a rich set of parameters that we can estimate given our granular data.

3.1 Setup and notation

There are N banks. At time t, the interbank network consists of an $N \times N$ directed adjacency matrix of total exposures, $\mathbf{C_t}$. C_{ijt} is the element in row i and column j of $\mathbf{C_t}$, and indicates the total exposure of bank i to bank j at time t. $\mathbf{C_t}$ is directed in that it is not symmetric: bank i can have an exposure to bank j, and bank j can have a (different) exposure to bank i. For each bank i, $\mathbf{d_i}$ is an $L \times 1$ vector of product characteristics for the exposures that it supplies, including maturity profile, currency profile and instrument type.

$\mathbf{p_t}$ is an $N \times 1$ vector of bank default risks: the element in position i is the probability of default of bank i. $\mathbf{p_t}$ is a function of $\mathbf{C_t}$ and an $N \times K$ matrix of bank fundamentals, which we denote $\mathbf{X_t}$, and which update over time according to some exogenous process. This function is the default risk process, and the effect of $\mathbf{C_t}$ on $\mathbf{p_t}$ represents contagion, as we will define more formally below.

C_{ijt} results in profits to bank i (we term this supply of exposures) and to bank j (demand for exposures). These profits depend on bank default risk, in a way we will formalise below. The equilibrium interbank network $\mathbf{C_t}$ is formed endogenously based on the supply- and demand-sides, such that markets clear. Banks choose their supply and demand decisions simultaneously. For simplicity, there is no friction between changes in bank fundamentals and the formation of the network: once fundamentals change, the equilibrium network changes immediately, such that are model is focussed on long-run effects.

3.2 Default risk process

By the default risk process, we mean the process by which bank fundamentals and the network of exposures combine to result in bank default risk. In our approach we are guided by the size and completeness of our data on exposures (empirical fact 1), which means that the exposures could reasonably have an impact on the default risk of the banks that hold these exposures, in contrast to papers in the literature that observe exposures relating to a single instrument type (Denbee et al., 2021; Gofman, 2017). As a result, we consider financial contagion of default risk directly through these exposures.

We model a bank's default risk process as the sum of the following components:

$$\underbrace{\mathbf{p_t}}_{\substack{\text{Default}\\\text{risk}}} = \underbrace{\boldsymbol{X_t\beta}}_{\substack{\text{Funda-}\\\text{mentals}}} - \underbrace{\omega\,\mathbf{C}_\mathbf{t}'\,\boldsymbol{\iota}}_{\text{Hedging}} + \underbrace{(\boldsymbol{\Gamma}\circ\mathbf{C_t})\mathbf{p_t}}_{\substack{\text{Counterparty}\\\text{risk}}} + \mathbf{e}_\mathbf{t}^\mathbf{p} \tag{1}$$

where default risk $\mathbf{p_t}$, network $\mathbf{C_t}$ and fundamentals $\mathbf{X_t}$ are defined in the previous subsection, $\boldsymbol{\beta}$ is a $K \times 1$ vector of coefficients on bank fundamentals, $\boldsymbol{\iota}$ is a $N \times 1$ vector of ones, $\omega > 0$ is a scalar parameter that determines the effect of exposures on default risk through hedging, $\boldsymbol{\Gamma} > \mathbf{0}$ is a $N \times N$ matrix of parameters that determine the effect of exposures on default risk through counterparty risk and $\circ$ signifies the Hadamard product.

In broad terms, in this model a bank's default risk depends on its fundamentals and on its interbank network exposures, where the interbank network can increase or decrease bank default risk. More specifically, this is a spatially autocorrelated regression, as is commonly used in network econometrics (De Paula, 2017) and specifically in finance (Herskovic et al., 2020; Ozdagli and Weber, 2017) with two amendments.

The first departure from a standard spatially autocorrelated regression is that the parameter governing the size of the network spillover (counterparty risk in our context), Γ_{ij}, is allowed to be heterogeneous across bank pairs. Γ can thus be thought of as *contagion intensity* in that $\Gamma_{ik} > \Gamma_{im}$ implies that $\frac{\partial p_{it}}{\partial p_{kt}} > \frac{\partial p_{it}}{\partial p_{mt}}$ for any common $C_{ikt} = C_{imt}$. That is, bank i's default risk is more sensitive to exposures to bank k than to bank m, holding exposures and fundamentals constant. We refer to links with relatively low contagion intensity as "inherently safe" and links with relatively high contagion intensity as "inherently risky".

This heterogeneity could come from various sources. At the bank-level, some banks could be particularly risky as counterparties because of their jurisdiction or the volatility of their fundamentals. At the bank-pair-level, it could come from three sources. First, it could be a result of risk sharing, which in our context means the correlation in the underlying fundamentals across pairs: if bank i and k have fundamentals that are positively correlated then exposure C_{ik} is particularly harmful. Second, it could be a result of variations in product characteristics, as described above. This difference across products could be modelled using a richer default risk process that separately includes exposures matrices for each instrument type with differing contagion intensities, but this would introduce an infeasible number of parameters to take to data. Third, it could be a result of some other relevant pairwise variation that is unrelated to fundamentals or product, such as geographic location. It could be, for example, that recovery rates in the event of default are lower if bank i and bank j are headquartered in different jurisdictions, making cross-border exposures riskier than within-border exposures. We do not impose or test any of these motivations in estimation, but explore them once we've estimated these heterogeneous contagion intensities.

The second departure from a standard spatial auto-regression is via the parameter ω, which we label hedging in Equation 1. The exposures we are seeking to match in the data include lending contracts and derivative contracts that could help banks manage their risk, thus it does not follow that bank risk is necessarily minimised when network links are zero. We only include this hedging effect for the bank in whom the exposure is held, and not the bank holding the exposure, because we interpret Γ_{ij} as being net of any hedging benefit.

As well as resulting in contagion, the interbank network can reduce default risk by allowing banks to hedge. The partial equilibrium net effect of an exposure C_{ijt} is as follows:

$$\frac{\partial p_{it}}{\partial C_{ijt}} = -\Gamma_{ij} p_{jt}, \quad \frac{\partial p_{it}}{\partial C_{jit}} = \omega$$

An exposure C_{ijt} is more likely to increase the default risk of bank i if hedging is less important (because ω is small), the counterparty is particularly risky (p_{jt} is large) or the link is particularly risky (Γ_{ij} is large).

To find equilibrium default risk we solve for a fixed point in $\mathbf{p_t}$. Subject to standard regularity conditions on $\mathbf{\Gamma}$ and $\mathbf{C}$ this spatially autocorrelated process can be inverted and expanded as a Neumann series as follows:

$$\mathbf{p_t} = (\mathbf{I} - \mathbf{\Gamma} \circ \mathbf{C_t})^{-1}(\mathbf{X_t}\boldsymbol{\beta} - \omega\mathbf{C_t'}\boldsymbol{\iota} + \mathbf{e_t^p}) = \sum_{s=0}^{\infty}(\mathbf{\Gamma} \circ \mathbf{C_t})^s(\mathbf{X_t}\boldsymbol{\beta} - \omega\mathbf{C_t'}\boldsymbol{\iota} + \mathbf{e_t^p}) \tag{2}$$

3.3 Network formation

In our approach to modelling network formation we are guided by two empirical facts. First, the network we are seeking to model is dense with heterogeneous intensities (empirical facts 2 and 3). Much of the literature focuses on explaining sparse core-periphery structures, which are often rationalised by *fixed* costs to link formation (Craig and Ma (2022), for example, have a fixed cost of link formation relating to monitoring costs). Variation in fixed cost cannot explain heterogeneity in link intensity, however, so this empirical observations leads us to focus on heterogeneity in *marginal* cost instead.

Second, product characteristics are heterogeneous across banks (empirical fact 3). In other words, banks are supplying and demanding different financial products. This heterogeneity has implications for the specificity with which we model the payoffs to demanding financial products, in that we cannot use a standard model of, for example, debt or CDS exposures. Instead, we need to model the demand-side in a general way that is applicable across the range of financial products that feature in our data. This heterogeneity also has implications for how we model competition between banks. In particular, this heterogeneity means we need to consider the extent to which exposures supplied by one bank are substitutable for those supplied by another bank (product differentiation, in other words).

Bank i supplies exposures C_{ijt} and demands C_{kit}. On the supply-side, bank i incurs a cost from supplying and in return receives an interest rate from the demanding bank:

$$\Pi_{it}^S = \sum_j C_{ijt}(r_{ijt} - puc_{ijt})$$

where puc_{ijt} is the per-unit-cost of the exposure of i to j and r_{ijt} is the interest rate paid by j to i. Note that we could include a non-monetary payoff to bank i from having the exposure C_{ijt}, analogous to technology parameter ζ_{ijt} on the demand-side. We choose not to do this because separate demand- and supply-side parameters could not be identified from the network data, and in any case all that matters for our analysis is the aggregate returns to trade, not how those returns are split between the two parties.

Assumption 1. : The effect of bank risk on bank cost : *We assume that the per-unit cost is the cost of the equity that capital regulation requires the bank raise. We parameterise the cost of equity as a linear function of the bank's default risk. The riskier a bank is, the higher the cost of raising equity.*

$$\underbrace{puc_{ijt}}_{\textit{Per-unit cost}} = \underbrace{\lambda_{ijt}}_{\textit{Reg'n}} \underbrace{c^{e}_{it}}_{\textit{Cost of K}} = \lambda_{ijt}\,\phi p_{it}$$

where λ_{ijt} is the capital requirement applied to bank i's exposure to bank j at time t and ϕ is a parameter governing the intensity of the relationship between risk and the cost of equity. This assumption serves two purposes. First, it provides a role for capital regulation in network formation: the regulator can affect the cost function. Second, it links the two parts of the model: banks care about their default risk because it affects their costs. We formalise the way in which banks take this into account in the following assumption.

Assumption 2. : Banks take systemic risk as given : *Bank i internalises the effect of its network choices on p_{it}, but takes all other bank default risks $p_{k\neq i,t}$ as given.*

Turning to the payoff that j receives when it is supplied exposures, we model it as a quadratic in $\mathbf{C_t}$:

$$\Pi^{D}_{jt} = \sum_{i} \zeta_{ijt}C_{ijt} - \frac{1}{2}\left(\sum_{i} C^{2}_{ijt} + 2\sum_{i}\sum_{k\neq i} \theta_{ik}C_{ijt}C_{kjt}\right) - \sum_{i} r_{ijt}C_{ijt}$$

where ζ_{ijt} represents heterogeneity in the sensitivity of the bank j's technology to product i and θ_{ik} governs the substitutability of product i and k. In effect, this is a reduced-form in which the gains to trade are left fixed and encoded in ζ_{ijt}. When we take the model to data, we relate substitutability to exposure characteristics, but for now we leave it general.

The problem of bank i is to choose its exposure supply $\{C_{ijt}\}_j$ and exposure demand $\{C_{kit}\}_k$ to maximise its profits, taking $p_{k\neq i}$ and all interest rates as given, and subject to non-negativity constraint $C_{ijt} \geq 0$ and $C_{kit} \geq 0$:

$$\begin{aligned} max_{\{C_{ijt}\}_j,\{C_{kit}\}_k} \quad & \sum_{i} \zeta_{kit}C_{kit} - \frac{1}{2}\left(\sum_{k} C^{2}_{kit} + 2\sum_{k}\sum_{m\neq k} \theta_{km}C_{mit}C_{kit}\right) \\ & - \sum_{k} r_{kit}C_{kit} + \sum_{j} C_{ijt}(r_{ijt} - \lambda_{ijt}\phi p_{it}(\mathbf{C_t})) \end{aligned}$$

where we emphasise that bank i's risk and thus cost of equity are functions of the network $\mathbf{C_t}$, following Equation 2 and Assumption 1. Taking the first order condition for bank i demand from bank k, C_{kit}, and re-arranging:

$$r_{kit} = \underbrace{\zeta_{kit}}_{\text{Technology}} - \underbrace{C_{kit}}_{\text{Own-effect}} - \underbrace{\sum_{l \neq k} \theta_{il} C_{lit}}_{\text{Cross-effect}} + \underbrace{\omega \sum_{m} C_{imt} \lambda_{imt} \phi}_{\text{Hedging benefit}} \tag{3}$$

In other words, our functional form assumptions imply that the bank demanding exposures has linear inverse demand. The final component of demand, labelled hedging benefit, represents the cost saving that bank i enjoys on the exposures it is supplied.

Taking the first order condition for bank i supply to bank j, C_{ijt}, and re-arranging:

$$r_{ijt} = \underbrace{\lambda_{ijt} \phi p_{it}}_{\text{Per-unit cost}} + \underbrace{\Gamma_{ij} p_{jt} \sum_{m} C_{imt} \lambda_{imt} \phi}_{\text{Change in total cost}} \tag{4}$$

Bank i, when choosing to supply C_{ijt}, therefore balances the return it gets from supplying against the effect of its supply against its marginal cost, where this marginal cost depends on its per-unit cost and the marginal change in its total cost through the effect of its supply on its default risk, via the default risk process described above.

This simple parameterisation has three important implications. First, p_{it} is endogenously dependent on bank i's supply decisions, via the default risk process that we define above. In other words, when bank i supplies bank j, it takes into account the fact that doing so makes it riskier and so makes it costlier to raise capital. Second, p_{it} is endogenously dependent on the supply decisions of *other* banks, via the default risk process that we define above. In other words, there are network cost externalities. Third, p_{it} is endogenously dependent on regulation λ_{ijt} through the default risk process described above. In other words, an increase in λ_{ijt} has two effects on the total cost of capital for firm i: it increases the amount of capital that the i bank needs to raise, but makes the bank safer and so makes the cost of a given unit of capital lower.

3.4 Equilibrium

Definition 1. : Equilibrium : *We define a competitive equilibrium in each period t as an $N \times N$ matrix of exposures $\mathbf{C_t^*}$ and $N \times 1$ vector of default risks $\mathbf{p_t^*}$ such that markets clear* *and every bank chooses its exposures optimally given the actions of other banks.*

For interior solutions where $C_{ijt} > 0$, market clearing simply equates Equations 3 and 4 for the supply and demand of C_{ijt}, which when re-arranged implies:

$$C_{ijt} = \zeta_{ijt} - \sum_{k \neq i} \theta_{ik} C_{kjt} - \lambda_{ijt} \phi p_{it} - \Gamma_{ij} p_{jt} \sum_{m} C_{imt} \lambda_{imt} \phi + \omega \sum_{l} C_{jlt} \lambda_{jlt} \phi \tag{5}$$

This implies that C_{ijt} depends on the need of j for i's product, the extent to which similar products exist, the cost to i of supplying, and the effect of the transaction on the risk (and thus cost) of both parties. The functional form assumptions, together with Assumption 2, implies that this interior condition is linear in $\mathbf{C_t}$ for given $\mathbf{p_t}$. C_{ijt} depends on three other parts of the network matrix: (1) other exposures held by i, (2) other exposures held by j and (3) other exposures held to j. This Equation 5, together with Equation 2 of the default risk process, pins down equilibrium network exposures.

3.4.1 Solving for an equilibrium

Combining Equations 5 and 2 to substitute out $\mathbf{p}$ gives a system of equations in $\mathbf{C}^*$. It is a system of infinite-length series of polynomials, such that in general no analytical solution exists. Instead, we solve these equilibrium conditions numerically. We make no general claims about uniqueness or existence, but confirm numerically that our estimated results are an equilibrium that is, based on numerical simulations, unique. In practice, we use the following algorithm:

1. Start with initial guess of $\mathbf{C_t}$ in which all of its elements are equal to 0.
2. Given this guess of $\mathbf{C_t}$, calculate $\mathbf{p_t}$ using Equation 2.
3. Given this $\mathbf{p_t}$, calculate $\mathbf{C_t}$ using Equation 5.
4. Repeat steps 2 and 3 until convergence.

This replaces a full non-linear search over $\mathbf{C_t}$ with a series of linear inversions, so it is quick and reliable, and performs consistently well for any reasonable parameterisation.

3.4.2 Characterising the equilibrium

We set out certain empirical facts above that we used to guide our modelling. In this subsection, we illustrate the main mechanisms in the model by explaining how exactly the model is consistent with these empirical facts.

First, our empirical network is heterogeneous in the intensity of links. The model implies heterogeneity in C_{ijt} from three sources: (i) firms have heterogeneous technologies ζ_{ij} that require differing inputs from other firms, (ii) contagion intensity Γ_{ij} is heterogeneous, such that some links are intense because they are less risky and (iii) firms have heterogeneous fundamentals X_{it}, such that some links are intense because the banks involved have good fundamentals. Expanding on the last of these: a given equilibrium link C_{ijt} is decreasing in the risk of the parties, as can be seen from Equation 5 the marginal cost of supply is increasing in the risk of the supplier i and of the demander j. The riskiest banks need not supply or be supplied at all, and if all banks are very risky then the equilibrium network may be empty (replicating anecdotal evidence regarding what happened to interbank interactions during the financial crisis). This is true holding fixed the demanding bank's need for liquidity, ζ_{ijt}, when in practice this may be correlated with bank fundamentals.

Second, our empirical network is persistent over time. Our model is static, but intertemporal persistence in any of the sources of heterogeneity discussed above would lead to persistence in the equilibrium network. This is an empirical question, but a priori there is good reason to expect that bank technology ζ_{ijt}, contagion intensity Γ_{ijt} (particularly with a risk sharing interpretation) and bank fundamentals X_{it} would be persistent.

3.5 Optimal networks

We consider two inefficiencies in our model: one within the interbank market and one outside the interbank market.

The inefficiency within the interbank market comes from Assumption 2 that banks take systemic risk as given. This implies a network externality in which when banks transact bilaterally they do not take into account the impact of their decision on systemic risk via the impact on other banks in the network. This means that (excluding uninteresting cases in which the parties have no other direct or indirect connections apart from C_{ijt}) the equilibrium network can be too large or too small relative to the social optimum, depending on the relative sizes of the hedging benefit ω and contagion $\mathbf{\Gamma}$. These inefficiencies mean that aggregate interbank surplus may not be maximised in equilibrium, where we define aggregate interbank surplus as the sum of aggregate surplus on the demand-side and aggregate surplus on the supply-side across all N banks.

In this context, however, it is insufficient to consider aggregate surplus within the interbank network. A bank's default risk can impact agents outside of the interbank network, such as its depositors, debtors and various other forms of counterparty. A crisis in the interbank network could, in principle, lead to a wider crisis with implications for the "real" economy. In other words, a social planner would not set exposures and default risk solely to maximise surplus in the interbank network, but instead to maximise total surplus in the economy, including aggregate interbank surplus and real surplus, which we define as follows.

Definition 2. : Real surplus : *We define "real surplus" as surplus in the economy outside of the interbank network, and denote it by R_t.*

The relationship between bank default risk and real surplus is important, as if there is such a relationship then it reveals an inefficiency outside the model, in that banks do not take into account the effect of their network formation decisions on real surplus. Characterising the relationship between real surplus and default risk, or estimating it empirically, is not straightforward. We do not model or estimate this relationship, but only make the following directional assumption:

Assumption 3. : Real surplus and bank risk : *Suppose real surplus R_t is a function of the mean default risk of banks $\bar{p}_t$: $R_t = r(\bar{p}_t)$. We assume that R_t is weakly decreasing in $\bar{p}_t$.*

This assumption is clearly an approximation of what is likely to be a complex relationship between real surplus and bank default risk. It may not always hold; it may be, for example, that when bank default risk is very low, some additional bank default risk increases real surplus. It could even be that there is no inefficiency outside the market, in that the banks' effect on real surplus is fully internalised via its cost of raising equity. It could also be that *mean* bank default risk is not the only thing that is important, but also some measure of dispersion or the minimum or maximum. Nevertheless, we think that this assumption reasonably represents the fundamental, local trade-off that regulators face when intervening in these markets: the trade-off between default risk and surplus in the market.

In particular, this assumption allows us to think about optimal default risk and interbank surplus in the sense of Pareto-optimality. That is, denote total surplus in the interbank network by TS_I (where the I subscript emphasises that this is total surplus in the interbank network only) and mean default probability by $\bar{p}$. Suppose $TS_I^H > TS_I^L$ and $\bar{p}^H > \bar{p}^L$. Assumption 1 implies that $(TS_I^H, \bar{p}^L) \succ^{SP} (TS_I^L, \bar{p}^H)$, where $\succ^{SP}$ denotes the social planner's preferences, but it does not allow us to rank $(TS_I^H, \bar{p}^H)$ and $(TS_I^L, \bar{p}^L)$, as we illustrate in Figure 3.

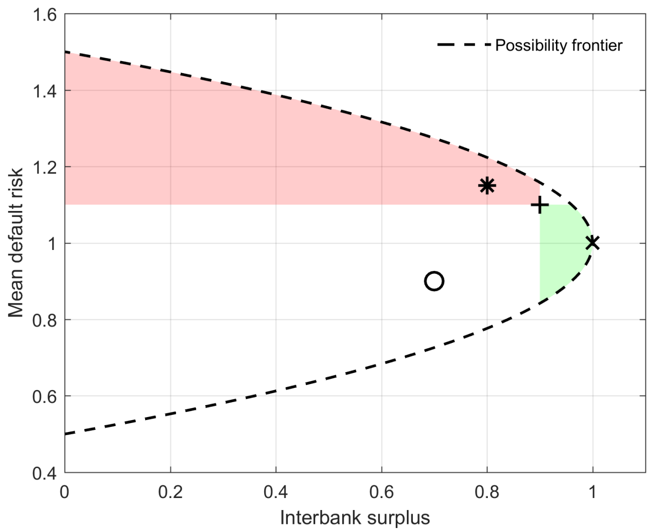


Figure 3: Stylised example: Interbank surplus and default risk

Note: Point $+$ dominates any point in the red area but is dominated by any point in the green area. For example, $\times \succ^{SP} + \succ^{SP} *$, but we cannot rank $\circ$ relative to the other points. We cannot even rank $\circ$ relative to $\times$ despite $\times$ being on the efficient frontier: the social planner's preferences over $\times$ and $\circ$ depend on the scale of externalities outside of the interbank network, which we leave open. The extent of inefficiency of point $\circ$ can be expressed as the vertical distance south to the efficient frontier and the horizontal distance east to the frontier.

It is helpful to think about the trade-off between TS_I and $\bar{p}$ in terms of constrained maximisation of interbank surplus subject to a default risk constraint.

Definition 3. : Efficient frontier : *For an arbitrary, exogenous value of mean default risk, $\bar{p}^F$, define $TS_I^F = max_{\mathbf{C}}\ TS_I(\mathbf{C})\ st\ \bar{p}(\mathbf{C}) = \bar{p}^F$. We define the efficient frontier as the locus traced out in $(\bar{p}^F, TS_I^F)$ space as $\bar{p}^F$ is varied.*

In other words, the efficient frontier is agnostic about the scale of externalities outside of the interbank network. It requires only that there is no feasible alternative $(TS_I^A, \bar{p}^A)$ that is a Pareto-improvement in the sense that (i) $TS_I^A > TS_I^F$ and $\bar{p}^F \leq \bar{p}^A$ or (ii) $TS_I^A \geq TS_I^F$ and $\bar{p}^F < \bar{p}^A$. The extent to which a given point is inefficient can then be characterised by its vertical or horizontal distance from the frontier, as we set out in the definitions below. Figure 3 shows the frontier and illustrates what conclusions we can draw using this model about different outcomes.

Definition 4. : p inefficiency : *The default risk inefficiency of some allocation $(TS_I, \bar{p})$ is* *the percentage decrease in $\bar{p}$ that could be obtained without decreasing TS_I. In other words, it is the vertical distance in percentage terms from the frontier.*

Definition 5. : TS inefficiency : *The total surplus inefficiency of some allocation $(TS_I, \bar{p})$ is the percentage increase in TS_I that could be obtained without increasing $\bar{p}$. In other words, it is the horizontal distance in percentage terms from the frontier.*

3.5.1 Sources of inefficiency

In our counterfactuals we calculate the efficient frontier and the implied inefficiency numerically. Here, we illustrate the mechanisms at play in a simplified version of our model. Let $\omega = 0$, $\theta_{ij} = 0$, $\lambda_{ijt} = 1$ and $\phi = 1$ for all i and j, and drop all time subscripts for ease of exposition as this is a cross-sectional example. Equation 2 for risk reduces to the following:

$$\mathbf{p} = \mathbf{X}\boldsymbol{\beta} + (\boldsymbol{\Gamma} \circ \mathbf{C})\mathbf{p} \tag{6}$$

We are interested in the effect of a single network link C_{ij} on each bank's risk, as represented by the $N \times 1$ vector $\frac{\mathbf{dp}}{\mathbf{dC_{ij}}}$, whose k'th element is $\frac{dp_k}{dC_{ij}}$. Differentiating Equation 6 and re-arranging:

$$\frac{\mathbf{dp}}{\mathbf{dC_{ij}}} = \mathbf{M}(\boldsymbol{\Gamma} \circ \mathbf{C})\, \tilde{\mathbf{z}}_{\mathbf{ij}} = \tilde{\mathbf{z}}_{\mathbf{ij}} + \sum_{s=1}^{\infty} (\boldsymbol{\Gamma} \circ \mathbf{C})^s\, \tilde{\mathbf{z}}_{\mathbf{ij}} \tag{7}$$

where $\mathbf{M}(\boldsymbol{\Gamma} \circ \mathbf{C}) = [\mathbf{I} - \boldsymbol{\Gamma} \circ \mathbf{C}]^{-1}$ is the standard $N \times N$ matrix network multiplier and $\tilde{\mathbf{z}}_{\mathbf{ij}}$ is a $N \times 1$ vector with $\Gamma_{ij} p_j$ in position i and zeros elsewhere. The right-hand-side of Equation 7 demonstrates how there is a direct effect on bank i, followed by an indirect effect on all banks (including i) as the direct effect propagates via their counterparties, their counterparties' counterparties, and so on. In what follows it will prove convenient to define the vector $\mathbf{w}_{\mathbf{ij}} = \sum_{s=1}^{\infty} (\boldsymbol{\Gamma} \circ \mathbf{C})^s\, \tilde{\mathbf{z}}_{\mathbf{ij}}$, representing the indirect effects through the network.

In this example, suppose the social planner's problem is to choose the full network to maximise aggregate interbank surplus, without regard for real surplus outside the interbank network. Transfers net out, such that all that remains is the cost incurred by the bank holding the exposure and the payoff gained by the bank in home the exposure is held: $\sum_i \sum_j \zeta_{ij} C_{ij} - 0.5 C_{ij}^2 - C_{ij} p_i(\mathbf{C})$. The first order condition for interior solutions with respect to C_{ij} for the social planner is:

$$C_{ij} = \zeta_{ij} - p_i - \Gamma_{ij} p_j \sum_m C_{im} - w^i_{ij} \sum_m C_{im} - \sum_{k \neq i} \sum_l w^k_{ij} C_{kl} \tag{8}$$

In comparison, the corresponding decentralized first order condition under these parameteric assumptions follows from Equation 5:

$$C_{ij} = \zeta_{ij} - p_i - \Gamma_{ij} p_j \sum_m C_{im} \tag{9}$$

The two final terms in Equation 8 constitute wedges between the decentralized outcome and the socially efficient outcome. The penultimate term represents the effect on bank i of not fully internalizing the effect of its network choices on other banks' risk and thus on its own risk. The final term represents the effect of banks i and j's choices on the payoffs of other banks.

These wedges are themselves functions of endogenous $\mathbf{C}$. The fact that they are closely related to the standard network multiplier means they are useful for providing intuition about when the decentralized outcome is far from the social optimum: the wedges for a given link C_{ij} are large when C_{ki} is large, as k is thus more exposed to the risk of i, or similarly when Γ_{ki} is large, as k is more sensitive to the risk of i. Relating this to the exogenous variables in the model, this means that wedges are particularly large when (i) technology ζ is such that link ij is central (if, for example, ζ_{ki} is large for all k, indicating many banks are exposed to bank i's risk taking) and (ii) when bank j's fundamentals X_j are bad.

We quantify the difference between actual exposures and optimal exposures for our estimated model in the section on counterfactuals below, as well as showing comparative statics on how the key parameters affect the extent of inefficiency.

4 Empirical approach

We first describe parameterizations that we make when we take this model to data and our approach to bank fundamentals. We then describe the structure of our estimation in general, before discussing network formation and default risk in turn.

4.1 Parameterisations

We impose four parameterisations to feasibly take this model to our data. The first parameterisation we make is with respect to Γ_{ij}. General symmetric Γ_{ij} consists of $N(N-1)/2 = 153$ elements. These are individually identifiable, as we will show below, but because the length of our panel is limited we cannot estimate them with reasonable power. For this reason, our baseline estimation approach imposes the following structure on Γ_{ij}:

$$\Gamma_{ij} = \tilde{\Gamma}_i + \tilde{\Gamma}_j$$

where $\tilde{\mathbf{\Gamma}}$ is an $N \times 1$ vector of parameters. This parameterisation is significantly more parsimonious but retains variation at the ij level. It does result in some loss of generality, in that loosely speaking it implies that if Γ_{12} and Γ_{23} are high, then Γ_{13} must also be high. This kind of structure is broadly consistent with each of the three motivations for heterogeneous Γ_{ij} that we introduce above.

The second parameterisation we make relates to θ_{ik}, which governs the extent to which the products supplied by bank i are substitutes for those supplied by bank k. General θ_{ik} cannot be reasonably estimated from our dataset; instead we parameterise it as function of certain product characteristics, including maturity, currency and instrument-type.

$$\theta_{im} = 0.5 - \sum_l \tilde{\theta}_l (d_{il} - d_{im})^2$$

where $d_{i,l}$ denotes the value for characteristic l of bank i and $\tilde{\theta}_l > 0$ is a parameter that determines the importance of characteristic l to the substitutability of different products. If banks i and k have very different product characteristics, then θ_{ik} is small and the two are not close substitutes. If, on the other hand, banks i and k have very similar product characteristics then θ_{ik} is large and the two are close substitutes. This parameterisation replaces θ_{ik} (which across all pairs has dimension $N(N-1)$) with θ_l (which has dimension L, the number of characteristics). We use 13 characteristics that break down a bank's fixed income exposures by currency, seniority and maturity; its securities financing transaction exposures by maturity; and its total exposures by type.

The third parameterisation we make relates to the structure of our data, and in particular the fact that, as described in Section 2, for non-British banks we only observe local-unit-to-group exposures, under-estimating their total exposure. We assume that:

$$C_{ijt} = a_i \, \tilde{C}_{ijt}$$

where we denote local-unit-to-group exposures by $\tilde{C}_{ijt}$ and group-to-group (that is, total) exposures by C_{ijt}, and a_i are bank-specific parameters that we estimate. In principle, a finer disaggregation (such as a_{it}) is identifiable in this way, but we restrict variation to a_i to preserve degrees of freedom. This gives us a tractable, estimable approach to dealing with the incompleteness of our data, but at the cost of some flexibility.

The final parameterisation relates to the technology parameters ζ_{ijt}, which we disaggregate into fixed effects and a random component:

$$\zeta_{ijt} = FE_{ij} + FE_{it} + e^{C}_{ijt} \tag{10}$$

The fixed pair-wise component represents, for example, the fact that banks headquartered in the same jurisdiction might naturally have greater exposures to each other. The bank-time varying component represents changes in bank scale or funding needs over time. Finally, the random pairwise component represents idiosyncratic funding needs.

4.2 Modelling fundamentals

We capture bank-specific fundamentals as a revenue-weighted average of equity indices. We start with S&P indices for the US, Canada, the UK, Europe, Japan, Asia and Latin America. We then calculate the proportion of each bank's revenues that come from that region, based on corporate account data from Bloomberg, and use those proportions as weightings. For example, suppose that at time t bank k obtained 70% of its revenues from the US and the remaining 30% from Japan. In this case, $X_{kt} = 0.7 \times S\&P500_t + 0.3 \times S\&PJapan_t$. Absolute index values are not meaningful, so we normalise each S&P index by its value on 1 June 2019. Although this is clearly an imperfect measure of the bank's fundamentals, we argue it has informative value: this bank k would plausibly be more affected by a slowdown in Japan than some other bank with no Japanese revenues.

4.3 Estimation structure

The parameters we seek to estimate are $\mathbf{\Theta} = (\tilde{\mathbf{\Gamma}}, \omega, \beta, \boldsymbol{\delta}, \boldsymbol{\zeta}, \tilde{\boldsymbol{\theta}})$; respectively, contagion intensities $\tilde{\mathbf{\Gamma}}$, hedging effect ω, fundamental loadings β, technology importance $\boldsymbol{\zeta}$, characteristic-based product differentiation $\tilde{\boldsymbol{\theta}}$ and the relationship between cost and risk ϕ.

We use our data on banks' exposures and CDS spreads (both normalised by their largest value) as our data for C_{ijt} and p_{it}. We estimate ϕ outside of the model through a linear regression of Bloomberg-estimated cost of equity on p_{it}, which yields a value of 0.585 for ϕ.

For the remaining parameters, as estimating equations we use the equilibrium conditions for interior solutions from our network formation game and the default risk process. We fit these estimating equations using non-linear GMM. For a given parameter guess $\hat{\boldsymbol{\Theta}}$ we calculate the vector of errors $\mathbf{e^C}(\hat{\boldsymbol{\Theta}})$ and $\mathbf{e^p}(\hat{\boldsymbol{\Theta}})$, and interact them with a matrix of instruments Z^C and Z^p which we specify below. This gives us two sets of moment conditions, which we weight evenly. We search numerically over $\hat{\boldsymbol{\Theta}}$ to match empirical analogues of these moments, concentrating out the linear parameters such as the fixed effects. We obtain standard errors by block bootstrapping individual time periods.

We discuss network formation and default risk in turn below, but before doing so we summarise in high level terms how our estimation maps into our research question. Our primary research question is about how banks form their exposures network given the effect of such exposures on their default risk. In our model, we encode this in the contagion intensity parameters, Γ_{ij}.

The focus of our estimation is thus on the identification of these key parameters. On the network formation side, we infer that a given link has high contagion intensity when the bank holding the exposure i is particularly responsive to the fundamentals of its counterparty j. We include fixed effects at the pair and the bank-time levels, so this responsiveness should be understood in relative terms. On the default risk side, we infer that a given link has high contagion intensity when the default risk of bank i is particularly sensitive to the fundamentals of bank j, given model-implied instruments for the endogenous network.

4.4 Network formation

Applying our parameterizations to Equation 5:

$$\tilde{C}_{ijt} = -\Gamma_{ij} p_{jt} \sum_m \tilde{C}_{imt} \lambda_{imt} \phi - \sum_{k \neq i} \frac{\theta_{ik}}{a_i} a_k \tilde{C}_{kjt} + \frac{\omega}{a_i} \sum_l a_j \tilde{C}_{jlt} \lambda_{jlt} \phi + FE_{ij} + FE_{it} + e^C_{ijt} \quad (11)$$

Each of the three terms of interest in estimating equation 11 are endogenous, in that equilibrium networks links and default risk depend on unobserved shocks to $\tilde{C}_{ijt}$. We demonstrate identification in two steps. First, we show the instruments implied by our model. Second, we show how $\mathbf{a}$ and $\mathbf{\Gamma}$ are separately recoverable, given our parameterisation and the network structure of the data.

Our model implies that equilibrium outcomes $\mathbf{C}$ and $\mathbf{p}$ are non-linear functions of the underlying fundamentals $\mathbf{X}$: this follows immediately from the fact that p_{jt} is linear in $\mathbf{X}$ and enters multiplicatively into equation 11. One approach would be to solve for equilibrium $\mathbf{p}$ and $\mathbf{C}$ as non-linear functions of fundamentals, and then construct moments using these equilibrium outcomes. Instead, for tractability, we simply use linear and non-linear functions of these fundamentals as exogenous shifters for each of the endogenous terms in equation 11, interacted with each other to give us the pairwise variation we need.

$$Z^C_{ijt} = [\ X_{jt}\ , X^2_{jt}\ , X_{it}/X_{jt}\ ] \tag{12}$$

For each element z^C_{ijt} of Z^C_{ijt} we construct sample analogues of the implied moment conditions within bank pair, giving the variation we need to identify heterogeneous $\tilde{\Gamma}_i$:

$$\sum_t z^C_{ijt} e^C_{ijt}(\hat{\mathbf{\Theta}}) = 0 \tag{13}$$

Consider, for example, an improvement in the fundamentals of bank i and a worsening of those of bank j. This is a negative cost shock to any exposure held by i and a positive cost shock to any exposure held to j. This *increases* the first term in equation 11, as bank j becomes riskier, and bank i becomes less risky and so increases its exposures. On the other hand, this *decreases* the second term, as other banks reduce their exposures to bank j now that it is riskier and now that they face increased competition from i. In summary, provided there is reasonable cross-sectional variation in bank fundamentals (Figure 2), then that variation has differing exogenous implications for each of the network pairs.

Treating bank fundamentals as exogenous assumes that a bank's revenue distribution and the equity indices themselves are independent of the structural errors in the interbank network. The fact that we are able to include *it* fixed effects makes this assumption reasonable. HSBC, for example, which has deep roots in Asia, would not shift its geographic revenue base in response to pair-specific shocks in the interbank network. Similarly, we think it is a reasonable assumption that the equity indices that form the basis of our bank-specific fundamentals are independent of pair-specific shocks in the interbank network.

We treat product characteristics as exogenous, in keeping with the literature on demand estimation in characteristic space. We treat λ_{it}, regulatory capital requirements, as exogenous, in keeping with various papers that use capital requirements as cost shifters (Benetton, 2021; Robles-Garcia, 2018).

Finally, we need to show that the **a** parameters are separately identifiable from the other parameters. Consider a hypothetical regression of Equation 11 undertaken *within bank pairs*. Given that $a_i = 1$ for any UK bank in our sample, we can recover ω from such a regression within pairs of UK banks, and then a_l from pairs involving a single UK bank. In other words, the network structure and the fact that we know some a parameters mean that we can separately identify the relevant parameters.

4.5 Default risk process

Applying our parameterizations to Equation 2, and including time fixed effects:

$$p_{it} = X_{it}\beta - \omega \sum_k a_k C_{kit} + \sum_j \Gamma_{ij} a_i C_{ijt} p_{jt} + FE_t + e^p_{it} \tag{14}$$

There are two sources of endogeneity in equation 14: first, p_{jt} is a function of e^p_{it} by construction. Kelejian and Prucha (1998) and Kelejian and Prucha (1999) instrument for p_{jt} using bank fundamentals interacted with the network. Our network formation model, however, implies that these network links C_{imt} are also correlated with unobserved shocks to bank risk.

The advantage of explicitly considering network formation is that we can account for this endogeneity of the network, by using similar non-linear functions of fundamentals as instruments as described above. Define $X_{ot} = \frac{1}{N-1}\sum_{j\neq i} X_{it}$, the mean bank fundamental excluding i. We use instruments:

$$Z^p_{it} = [X^2_{it},\ X^2_{ot}\ ] \tag{15}$$

For each element z^p_{it} of Z^p_{it} we construct sample analogues of the implied moment conditions within bank:

$$\sum_t z^p_{it} e^p_{it}(\hat{\mathbf{\Theta}}) = 0 \tag{16}$$

The justification for these instruments is analogous to that described in the preceding sub-section for the observed network: our model implies that a bank's network choices vary non-linearly with its fundamentals and those of the other banks. Given our parameterization of Γ_{ij}, applying such instruments within bank then gives us the necessary number of instruments. That is, instead of constructing instruments based on bank fundamentals interacted with an exogenous network, as in Kelejian and Prucha (1998) and Kelejian and Prucha (1999), we construct similar instruments where non-linear fundamentals proxy for the endogenous network.

It is worth emphasising the advantages of using network formation to identify heterogeneous contagion intensities, relative to a typical estimation procedures that solely rely on outcomes on the network (in our case the default risk process). First, there is simply more information in network formation, as we have $N(N-1)T$ network links but only NT default risks. Heterogeneous Γ_{ij} is in principle identifiable using only variation in bank default risk, but not with any reasonable power. Second, the network structure affords us additional variation within which we can more cleanly identify Γ_{ij} by leveraging *within pair* variation in network formation. Furthermore, assuming bank fundamentals are orthogonal to unobserved shocks to bank default risk is more restrictive than in the case of the network formation data, as we cannot include *it* fixed effects. Finally, $\tilde{\Gamma}_i$ is not separately identifiable from a_i for the non-UK banks using only information in default risk.

5 Results

We set out our results in Table 2. We draw the following immediate conclusions for contagion intensity from our results.

First, contagion is heterogeneous: there is substantial pairwise variation in contagion intensity Γ_{ij}, in that some links involve close to no contagion and some links involve much more material levels of contagion. We plot the estimated distribution of Γ_{ij} in Figure 4a.

Second, contagion is material: our results imply that if there were no interbank exposures, then mean bank default risk would be 50% lower. Note that this is true even though some links are almost entirely costless in terms of contagion, indicating that banks form exposures even where they are costly. The hedging parameter, ω is immaterial.

Third, contagion is time-varying. In Figure 5 we show how contagion – given by $(\mathbf{\Gamma} \circ \mathbf{C_t})\mathbf{p_t}$ – varies through time. Contagion varies as the level of exposures and the level of banks' fundamentals change. Contagion has shrunk through time, largely driven by improvements in banks' fundamentals since the start of our sample.

Our results also have implications for the form of competition between banks. We plot the distribution of our estimated competition parameters θ_{ij} in Figure 4b. There is significant differentiation between banks based on product characteristics. Exposure type, currency, maturity and seniority all play significant roles in determining substitutability.

Table 2: Parameter estimates

	Min	Mean	Max
Contagion $\tilde{\Gamma}_i$	0.00	0.71	5.83
	[0,0]	[0.27,0.85]	[1.09,7.63]
Characteristics $\tilde{\theta}_l$	0.00	0.41	2.24
	[0,0]	[0.35,0.45]	[1.5,2.75]
Scaling a_i	1.00	3.09	8.83
	[1,1]	[2.31,4.02]	[5.1,10]
Hedging ω		0.00	
		[0,0.02]	
Fundamentals β_1		-0.09	
		[-0.12,-0.04]	
Network			
Fixed effects		it, ij	
Observations		6426	
Default risk			
Fixed effects		t	
Observations		378	

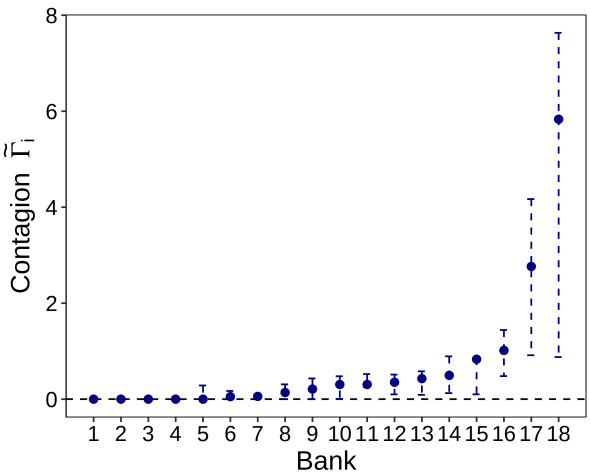
Note: This table summarises the estimates of the model's key parameters. Figures in square brackets are 95% bootstrap confidence intervals based on 100 bootstrap samples. The roles of the parameters in the model's equations are set out in Section 4.

5.1 Implications of our results

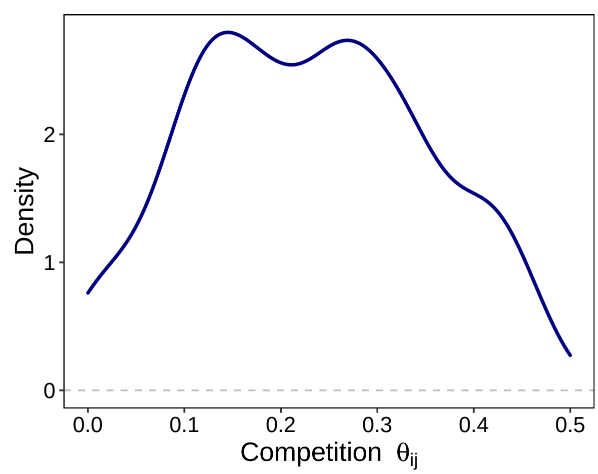
5.1.1 The sources of heterogeneity in contagion

We set out in Section 3 various motivations for why contagion intensity Γ_{ij} could be heterogeneous, at the bank level and at the bank-pair level. We provide suggestive evidence on this by estimating completely heterogeneous Γ_{ij} across pairs (although obviously with much less precision) and relating these to bank and bank pair characteristics.

In Table 3, we regress these estimates on various combinations of fixed effects. A regression on both i and j fixed effects (which represents the structure of our baseline parameter-



(a) Estimated contagion parameters



(b) Estimated competition parameters

Figure 4: Results: contagion & competition

Note: The left panel summarizes the estimated distribution of $\tilde{\Gamma}_i$ across banks, where contagion between banks i and j is governed by $\Gamma_{ij} = \tilde{\Gamma}_i + \tilde{\Gamma}_j$. The right panel summarizes the estimated distribution of the competition parameter θ_{ij} across pairs. A higher θ_{ij} signifies greater substitutability between the exposures of banks i and j.

isation of Γ_{ij}) produces R^2 of around 0.3, indicating that there is material variation at the bank-pair level, not just at the bank level.

Table 3: Variation in contagion parameters

	Dependent variable: Contagion Γ_{ij}		
Dummy Variables	i	j	i+j
R-squared	0.09	0.19	0.28
No. obs	306	306	306

Note: This table decomposes variation in our estimated, unparameterized contagion parameters Γ_{ij} across demanding bank i and supplying bank j. The table shows R-squareds for regressions of exposures Γ_{ij} on dummy variables given at the top of each column.

In Table 4, we show how our estimates vary with characteristics. We show that contagion intensity is greater when the demanding bank has volatile fundamentals, when the two banks are on different continents, and when their fundamentals co-move closely (supportive of a risk-sharing motive). This evidence is suggestive only, because we cannot estimate fully heterogeneous contagion intensities with any reasonable power, such that none of the patterns

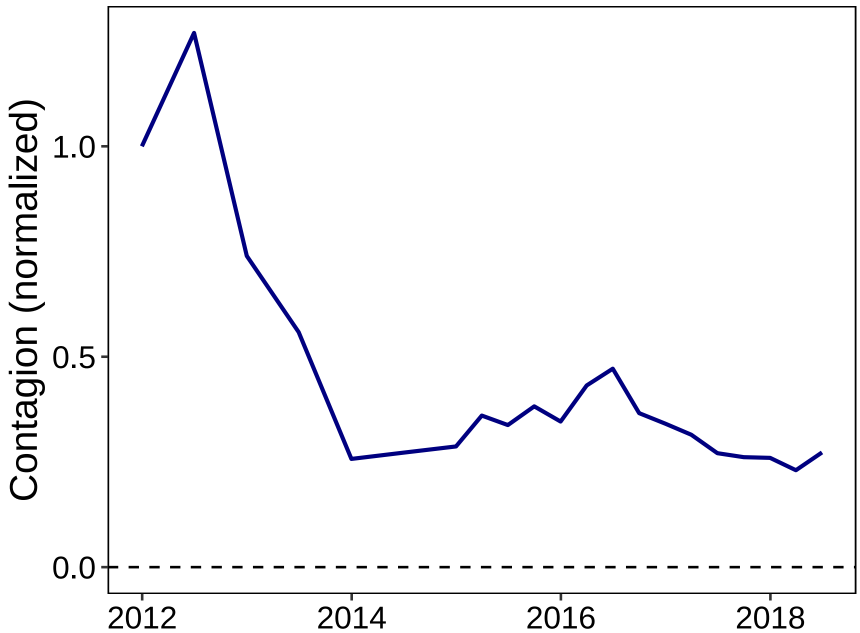


Figure 5: Contagion through time

Note: This figures summarizes how the contribution of contagion to banking sector risk has changed through time. Contagion is measured as the weighted average $(\mathbf{\Gamma} \circ \mathbf{C_t})\mathbf{p_t}$ across banks in each period, where weights are given by banks' total exposures in the data and the first period is normalized to 1.

in Table 4 is statistically significant. Nevertheless, it provides weak motivating evidence for the heterogeneity in contagion intensity.

5.1.2 The network under financial stress

In Figure 6 below we simulate the effect of a recession on the interbank network and default risk. We do this by simulating an arbitrary increase (deterioration) in all bank fundamentals (represented by the blue line). This deterioration makes exposures more costly, such that aggregate exposures decline convexly to zero (the red line). Mean bank default risk increases due to the deterioration in fundamentals (the black line with the circles), but by materially less than it would increase if the network were held fixed at its initial value (the black line). That is, the network endogenously responds to a deterioration in fundamentals in a way that mitigates the impact on bank default risk. Another way to express this is that the pass-through of fundamentals to bank default risk decreases as fundamentals deteriorate, because of the endogenous scaling back of the network.[8]

Table 4: Determinants of contagion

	High contagion parameter Γ_{ij}			
	(1)	(2)	(3)	(4)
Std. dev. X_{it}	-0.566			-0.668
Std. dev. X_{jt}	0.768			0.666
Cov(X_{it},X_{jt})		0.208		0.338
$\mathbf{1}$(same jurisdiction)			-0.106	-0.110
Observations	306	306	306	306
R^2	0.027	0.0003	0.007	0.034

Note: This table summarizes the relationship between our estimated, unparameterized contagion parameters Γ_{ij} and the fundamentals of i and j. A Γ_{ij} is high if it exceeds the median across banks. X_{it} are the bank-specific fundamentals defined in Section 4, and standard deviations and covariances are computed across time periods. $\mathbf{1}$(same jurisdiction) is a dummy equal to 1 if banks i and j have head offices in the same jurisdiction. Standard errors are omitted as the dependent variable is a generated regressor.

5.1.3 Systemic importance

A recurring issue in the network literature is the identification of "important" nodes. We have an equilibrium process that relates an outcome (bank default risk, in our case) to a network, and it is reasonable to ask which node in the network contributes most to the outcome in which we are interested. Understanding this communicates important information about this equilibrium process, but may also have implications for regulation (as we describe above, large parts of the banking regulatory framework are stricter for banks that are judged to be "systemically important" (Basel Committee, 2014)). Various measures of systemic importance, or centrality, exist, where the most appropriate measure depends on the context and on the way in which nodes interact with each other (Bloch et al., 2023). Our contribution to this literature is not about the most appropriate measure, but instead

[8]Note that we intend this as an illustration of how the model works. For our model to be useful as a regulatory stress test, we would need to understand how the estimated fixed effects change in a stress, which is outside the scope of this project.

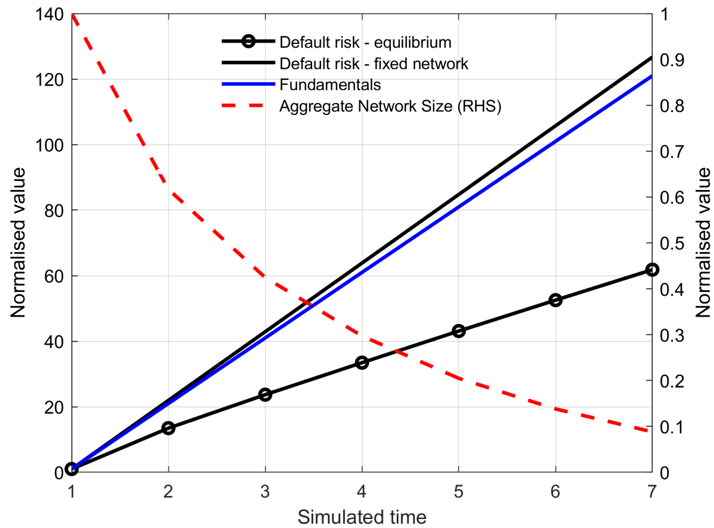


Figure 6: Simulated recession

Note: This figures shows the impact of a recession on banking sector risk and the interbank network. We arbitrarily inflate bank fundamentals (representing a deterioration) by an increasing factor (blue line). The black line shows the impact of risk when the network is held fixed at its initial value and the shock propagates through the network. The black line with circles shows the equilibrium impact on risk when banks respond to the shock by adjusting their network links. The red dashed line shows how banks respond to the shock by shrinking their exposures in equilibrium.

about how any such measure should be calculated: it must account for the heterogeneity in contagion intensity Γ_{ij}.

We illustrate this using one of the simplest measures of centrality: eigenvector centrality. Applying this centrality measure to the network $\boldsymbol{C_t}$ gives a ranking of which banks are most systemically important in driving bank default risk. If contagion intensity is homogenous, $\Gamma_{ij} = \Gamma$, then the level of Γ has no impact on this relative ranking. If, however, contagion intensity is heterogeneous, then accounting for this heterogeneity is important when assessing centrality: a more reasonable measure of centrality would be based on the weighted adjacency matrix $\boldsymbol{\Gamma} \circ \boldsymbol{C_t}$. Importantly, the effect of this weighting on the ranking of systemic importance is not random noise, because the equilibrium network depends on this weighting. More specifically, links C_{ij} where Γ_{ij} is low are inherently safe and so are more likely to be large, all other things being equal. As a result, assessing centrality based on the raw, unweighted exposures matrix is likely to misrepresent the relative centrality of different banks.

In Figure 7, we show that calculating eigenvector centrality based on unweighted $\boldsymbol{C_t}$

and weighted $\boldsymbol{\Gamma} \circ \boldsymbol{C_t}$ lead to quite different rankings of systemic importance. Bank 18, for example, would be identified as the most systemically important node based on the unweighted network. Based on the weighted network, however, 2 other banks are most systemically important than Bank 18: in other words, Bank 18's links are large because its links are relatively safe. Bank 8's centrality, on the other hand, is significantly understated when looking solely at the unweighted network: in other words, Bank 8's links are small because its links are relatively unsafe. We do this for Eigenvector Centrality, but the same point applies to other centrality measures.

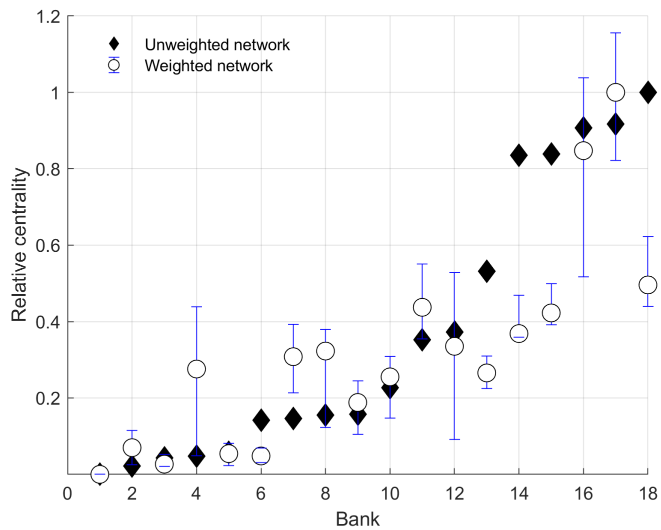


Figure 7: Identifying systemic nodes

Note: This figure plots the relative centrality of each of the 18 banks in our sample using eigenvector centrality. The black diamonds show relative centrality based on the unweighted network of observed exposures: banks with large exposures are more central. The white circles show relative centrality based on observed exposures weighted by their relative contagion intensities: relatively risky links are given a higher weighting. The blue lines show a 95% confidence interval around this weighted measure.

6 Counterfactual Analysis

In our counterfactual analyses, we first consider the social planner's solution, and show what that implies for efficiency. We then consider two broad forms of regulation: caps on exposures and capital ratios.

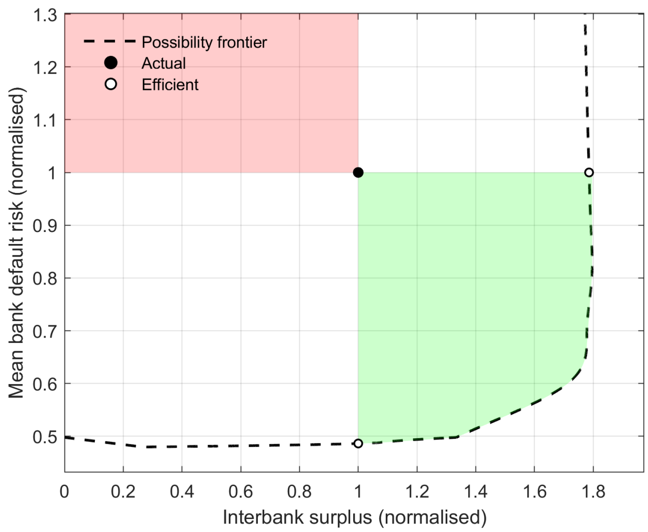


Figure 8: Decentralised inefficiency

Note: This figure shows that the decentralised outcome in the interbank network is inefficient. Point (1,1) shows the mean bank default risk and interbank surplus that our model implies for actual exposures. The dotted line shows the efficient possibility frontier of combinations of surplus and risk.

6.1 Efficiency

We describe above how our model implies a trade-off between mean bank default risk and interbank surplus, and how there is an efficient frontier on which this trade-off is optimised. We use our estimated model to derive this frontier, by choosing $\mathbf{C_t}$ to maximise interbank surplus, subject to mean bank default risk being less than some critical value. We then vary this critical value to trace out the efficient frontier. As described above, we do not know what allocations a social planner that was maximising aggregate surplus would choose, as we do not directly model the relationship between bank default risk and real surplus. We do know that this optimal allocation would be somewhere along the efficient frontier. The distance to the frontier in either direction is in this sense an estimate of inefficiency, as we describe above when we define p inefficiency and TS inefficiency.

We find that the decentralised interbank network is not on the efficient frontier: a social planner would be able to increase interbank surplus by 79% without increasing mean bank default risk or decrease mean bank default risk by 51% without decreasing interbank surplus, as set out in Figure 8. In Table 5 we show how optimal exposures at these two points on the frontier differ from actual exposures in two important dimensions. First, optimal exposures are slightly lower, given the negative externality that a bank imposes on others when it takes on an exposure. Second, optimal exposures are more concentrated, as represented by the HHI over exposures: that is, the social planner loads relatively more on low risk links (that are therefore large in the decentralised equilibrium) than on high risk links. The difference between these two points on the frontier is primarily about the average size of exposures.

Table 5: Frontier vs equilibrium exposures

	Change vs equilibrium (%)	
	Surplus improvement	Risk improvement
Mean exposures	-33	-50
Exposures variance	55	16
HHI: aggregate	166	244
HHI: exposures supply	90	157
HHI: exposures demand	156	131

Note: This table summarises exposures on the efficient frontier relative to exposures in the decentralised equilibrium. The equilibrium is point (1,1) in Figure 8. The *surplus improvement* achieves the maximum surplus without changing mean default risk relative to the equilibrium. The *risk improvement* achieves the minimum risk without changing surplus. HHI is the Herfindahl-Hirschman index. The supply HHI is calculated by computing the HHI for each bank i's exposures before taking a weighted average across banks, and the demand HHI is analogously defined for the exposures *to* bank i.

6.1.1 Comparative statics for efficiency

In Table 6 below we undertake comparative statics on the degree of inefficiency. These comparative statics confirm the intuition set out in Section 3.5.1. p inefficiency is increasing in both the scale of contagion intensity Γ_{ij} and of hedging benefit ω, as they directly affect the scale of the externality one bank's exposures impose on other banks. TS inefficiency is increasing in the variance of Γ_{ij} because of the convexity of the wedges $\mathbf{w_{ij}}$ set out in Section 3.5.1: more dispersed Γ_{ij} means the propagation of a bank's exposure choices through the network is greater. TS inefficiency is decreasing in competition, as represented by the parameters $\tilde{\theta}_l$: price-taking banks respond to being closer competitors by reducing their exposures, thereby mitigating the negative network externality. p inefficiency and TS inefficiency do not always move in the same direction.

6.2 Regulation

We evaluate regulation with only an incomplete understanding of welfare, as represented in Figure 3. Specifically, we do not know the relationship between real surplus and bank default risk, meaning we can only conclude that policy is welfare improving if it weakly improves both bank surplus and bank default risk. We are guided in this by our quantification of optimal exposures on the frontier, in which we show that optimal exposures are smaller and more concentrated than actual exposures, as in Table 5. We thus look for regulatory interventions that achieve this.

Table 6: Comparative statics: efficiency

	Baseline	$\downarrow \tilde{\Gamma}_{ij}$	$\downarrow \tilde{\theta}_l$	$\uparrow \omega$	$\downarrow \mathrm{V}(\tilde{\Gamma}_{ij})$
TS inefficiency	79%	80%	49%	82%	69%
p inefficiency	51%	31%	55%	54%	80%

Note: This table summarises how equilibrium efficiency – defined in Section 3 – changes as parameters change. The first columns hows our estimated results. The second shows results when we decrease the contagion parameters $\tilde{\Gamma}_i$ by 50%. The third shows results when we decrease the parameters $\tilde{\theta}_l$ by 50%, indicating an increase in substitutability between exposures. The fourth shows results when we increase the hedging parameter ω by a factor of 10. The fifth shows results when we move each contagion parameter $\tilde{\Gamma}_i$ 50% closer to its mean value.

6.2.1 Caps on exposures

After the global financial crisis banks were subject to caps on their exposures, whereby no single bilateral exposure can exceed 25% of its capital (15% for exposures between two "globally systemic institutions"). We evaluate the effects of a cap on individual exposures by simulating new equilibrium exposures $\mathbf{C}^{\mathbf{C}}$ under a generic cap at the bank level on their largest exposures, using our estimated parameters and assuming that fundamentals are unchanged:

$$C_{ij}^{C} \leq \kappa \cdot max_j\{C_{ij}\}$$

where $\kappa \in [0, 1]$ is the percentage size of the cap. $\kappa = 0.8$, for example, constrains bank i's largest exposure to 80% of its largest unconstrained exposure. This cap is stylised, in that it is defined relative to observed exposures, rather than relative to its capital. This avoids issues about measuring capital appropriately and measuring total exposures (our exposures do not include every possible financial instrument), while still showing the economic effect of a cap in general. We simulate the effect of this cap in Figure 9 for various levels of κ. We show that the cap improves outcomes, in that default risk goes down and interbank surplus goes up.[9] The initial effect on default risk, however, is modest, primarily because

[9]For sufficiently tight κ this reverses and interbank surplus must go down: for $\kappa = 0$, for example, interbank surplus must be 0.

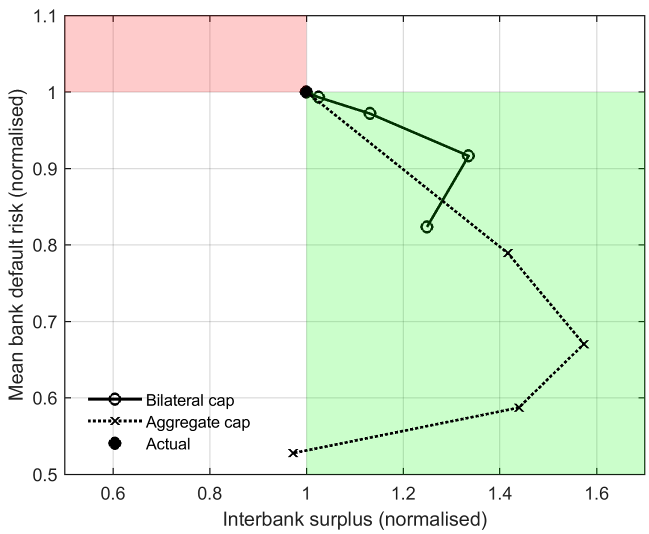


Figure 9: Counterfactual analysis of exposure caps

Note: This table starts with actual default risk and and interbank surplus (the black circle). We then plot the effect of bilateral caps on banks' exposures, such that no bank can have an exposure greater than $(1-x)\%$ of their existing greatest exposure, for $x = 20\%, 40\%, 60\%$ and 80% (the solid line). We then plot the effect of aggregate caps, such that each bank must have total exposures no greater than $(1-x)\%$ of their existing *total exposures*, for the same increments of x (the dashed line). Aggregate caps can both decrease risk and incrase surplus relative to bilateral caps.

of leakage: a cap on individual links creates excess supply and unmet demand that causes other uncapped links in the network to increase. That is, the bank rebalances away from its bigger links and towards its smaller links.

We propose an alternative form of regulation in which total exposures held by bank i are capped, rather than individual exposures:

$$\sum_j C^C_{ijt} \leq \kappa \cdot \sum_j C_{ijt}$$

We simulate the effect of this cap in Figure 9, and show that (i) the aggregate cap has materially larger effects on bank default risk than the individual cap and (ii) for any individual exposure cap there is an aggregate cap that dominates it (that is, at any point on the dashed line there is a point on the black line to the south-east of it). These effects arise because an aggregate cap prevents endogenous leakage, plus banks respond to an aggregate cap by concentrating more on their relatively safe exposure. In other words, a cap on

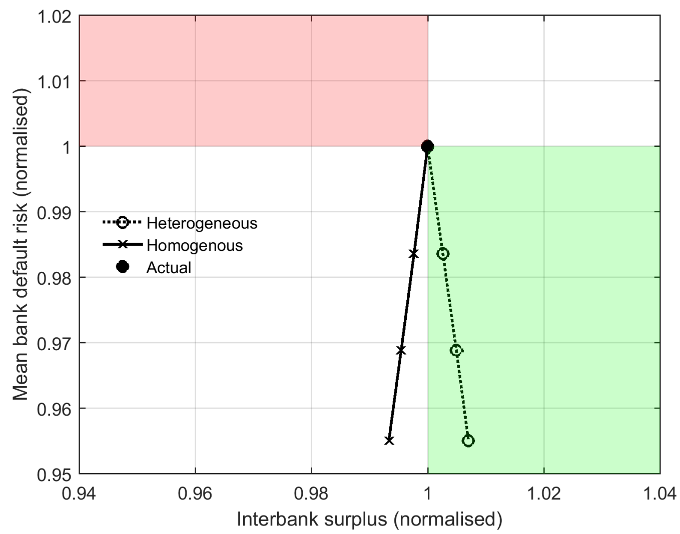


Figure 10: Counterfactual analysis of capital requirements.

Note: This table starts with actual default risk and and interbank surplus (the black circle). We then plot the effect of increasing capital requirements for all banks in increments of 1 percentage point (the solid line). We then plot the effect of risk-dependent changes to capital requirements as described in the text. Heterogeneous capital requirements can achieve the same decrease in risk as homogeneous capital requirements, without reducing surplus.

individual exposures targets inherently safe exposures, whereas a cap on total exposures targets inherently risky exposures.

6.2.2 Capital ratios

The second form of regulation we consider is a minimum capital requirement, as applied by regulators since the crisis. As described in Section 2, there is very little variation in risk-weights for exposures to banks under the standardised approach to risk-weighting. To assess the effect of a stylised risk-insensitive capital requirements, we simulate further increases in capital requirement λ_{it} in intervals of 1 percentage point, as set out in Figure 10. This entails a trade-off: such a policy effectively decreases risk, but at the cost of interbank surplus.

We propose a pairwise adjustment (that is, we allow λ_{ijt} to vary at the pair level) to capital ratios that is more closely targeted at network externalities. In this respect, we are part of a literature on optimal interventions in financial networks (Elliott et al., 2014; Jackson and Pernoud, 2021; Leduc and Thurner, 2017) and in networks more generally (Galeotti et al., 2020).

The key parameter in our model is Γ_{ij}, contagion intensity: links where this is high are particularly costly in terms of their effect on default risk. We propose increasing the capital requirements for any link with $\Gamma_{ij} > median(\mathbf{\Gamma})$ ("high risk links") by some value x (where we increase x in increments of 2 percentage points in Figure 10). For any link where Γ_{ij} is less than the median ("low risk links"), we propose *decreasing* the associated capital requirements by x. Our results suggest a social planner would strictly prefer this targeted median-preserving spread in capital ratios to a risk-insensitive increase in capital ratios.

7 Conclusion

In contrast to much of the literature on financial networks, our model and empirical approach is in the spirit of the wider industrial organisation literature in two ways. First, we model network formation as the interaction of demand for financial products and their supply, with a focus on identifying the relevant underlying cost function. Second, in specifying our model and taking it to data we pay particular attention to the role of unobserved firm- and pair-level heterogeneity. In particular, the core of this paper is heterogeneity in contagion intensity, including (i) why one might reasonably expect contagion intensity to be heterogeneous, (ii) how this heterogeneity can be identified empirically and (iii) what implications this heterogeneity has for strategic interactions between firms and their regulation. The primary message of this paper is that this heterogeneity in contagion intensity has material implications for systemic importance, efficiency and optimal regulation.

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Appendices: For Online Publication

A1 Data Appendix

In this section we explain the steps that we took in cleaning and processing the raw data to be used in estimation.

Throughout our sample period – 2012 to 2018 – a set of the largest banking entities in the UK were required to report to regulators a granular breakdown of their 20 largest exposures to other banks. After different reporting entities from the same banking group and removing entities that rarely reported any large exposures to other banks in our sample, we obtained a sample of 18 large banks reporting their exposures half yearly from 2011 to 2014, and quarterly thereafter.

We took the following steps in cleaning and processing our data. First, we manually searched through each of our 18 banks' reports for reports to other reporting banks, and cut the sample down to these interbank exposures. Each bank reports exposures to their top 20 banking counterparties. In 2014 the reporting requirements were changed in two ways: banks were no longer required to report exposures smaller than £10mn, and were required to report their exposures to eight specific UK banks even if they were not in their top 20 bank counterparties. To ensure consistency in our panel, we apply the minimum threshold ourselves to all data before 2014, and after 2014 delete exposures to banks that are not in the top 20 bank counterparties.

For each pair of banks, we compute an aggregate exposure measure across instruments, which aims to account for credit mitigation techniques – such as netting and collateralisation – and various regulatory adjustments intended to represent counterfactual variations in exposures values or collateral in the case of default. For fixed income exposures, we take current mark-to-market exposure. For derivative exposures we take the *exposure at default* and for securities financing transactions we take the *potential exposure*, each of which incorporate regulatory adjustments designed to capture adverse market movements that can increase exposures in case of default.[10]

In some cases we faced missing data, for a specific bank reporting a specific exposure in a specific period. Our approach requires complete information on the exposures, with no missing data, so in these cases we interpolated the data. In particular, in some cases in a

[10] For, see https://www.newyorkfed.org/medialibrary/media/banking/regrept/IDH_Guidelines.pdf.

specific time period, a bank's report for a given exposure is missing or implausible. In these cases we interpolate between the previous and subsequent periods. This affects a very small number of our exposures. In some periods certain banks do not report potential exposure – a measure of after-collateral exposure – for SFTs. In these cases, we scale up the notional – a measure of pre-collateral exposure – by the average ratio of potential exposure to notional that that bank reports in other periods.

A2 Institutional details

In this section we provide further details on the two key banking regulations which we study in this paper: caps on exposures and capital regulation.

In 2014 the Basel Committee on Banking Supervision (BCBS) set out new standards for the regulatory treatment of banks' large exposures (Basel Committee, 2014, 2018b). The new regulation, which came into force in January 2019, introduces a cap on banks' exposures: a bank can have no single bilateral exposure greater than 25% of its capital (where there the precise definition of capital, in this case "Tier 1 capital", is set out in the regulation (Basel Committee, 2014, 2018b)). For exposures held between two "globally systemic institutions", as defined in the regulation, this cap is 15%. These requirements represent a tightening of previous rules, where they existed. For example, in the EU exposures were previously measured relative to a more generous measure of capital and there was no special rule for systemically important banks (AFME, 2017; European Council, 2018).

Banks are subject to capital requirements, which mandate that their equity (where the precise definition of capital, Common Equity Tier 1, is set out in the regulation) exceeds a given proportion of their risk-weighted assets. In 2013 all banks in our sample faced the same capital requirement per risk-weighted unit, λ_i, which was 3.5%. We use the minimum capital requirements as published by Basel Committee (2011) as the minimum requirements for banks. National supervisors can add discretionary buffers on top of these requirements, which we do not include in our empirical work. Since the financial crisis, regulators have changed capital requirements in three ways. First, and most importantly, the common minimum requirement that applies to all banks has increased significantly. Second, capital requirements vary across banks, as systemically important banks face slightly higher capital requirements than non-systemically important banks. Third, capital requirements vary countercylically, in that in times of financial distress they are slightly lower (Basel Committee, 2018a). The result of these changes is that mean capital requirements for the banks in our sample has increased significantly, from 3.5% to over 9% in 2019. There have also been changes to the definition of capital and the measurement of risk-weighted assets, with the general effect of making capital requirements more conservative.