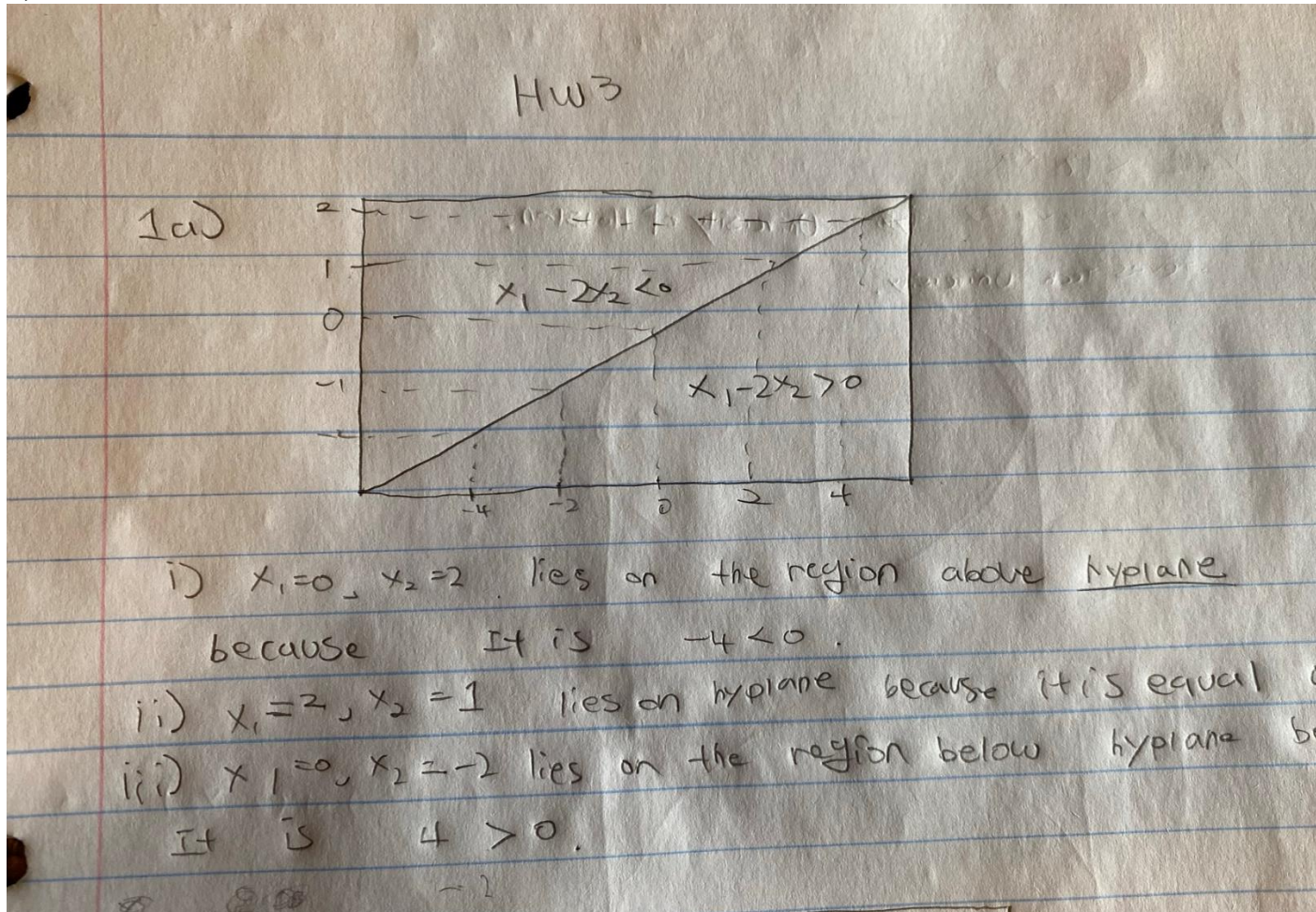


MATH 4323, Fall 2023, Homework # 3

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1)

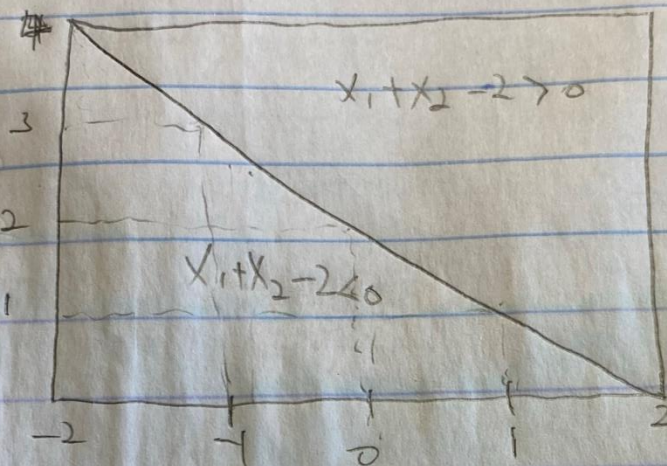


i)  $x_1 = 0, x_2 = -2$  lies on the region below hyperplane because

It is  $4 > 0$ .

~~ii)  $x_1 = 0, x_2 = -2$  lies on the region below hyperplane because~~

1b)



i)  $x_1 = 1, x_2 = 1$  lies on hyperplane because It is equal

ii)  $x_1 = -1, x_2 = -1$  lies on the region below hyperplane because

It is  $-4 < 0$ ,

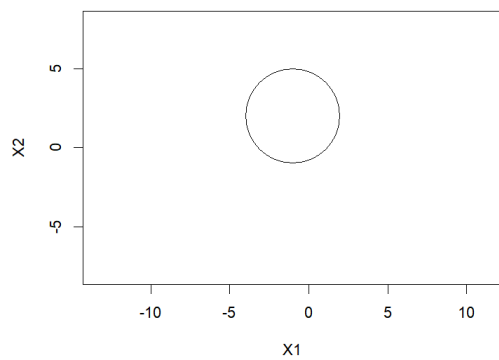
iii)  $x_1 = 1, x_2 = 1$  lies on hyperplane because It is equal

2A)

1.  $(1 + x_1)^2 + (2 - x_2)^2 = 9$ .

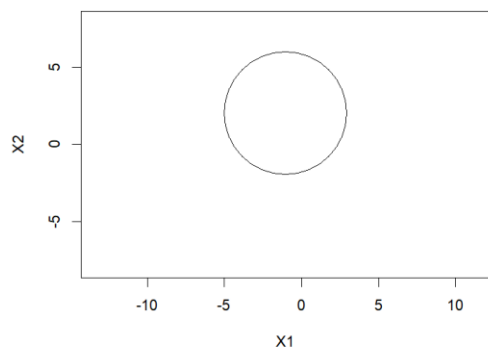
```
> plot(NA, NA, type = "n", xlim = c(-9, 7), ylim = c(-8, 8), asp = 1, xlab = "x1", ylab = "x2")
```

```
> symbols(c(-1), c(2), circles = c(3), add = TRUE, inches = FALSE)
```

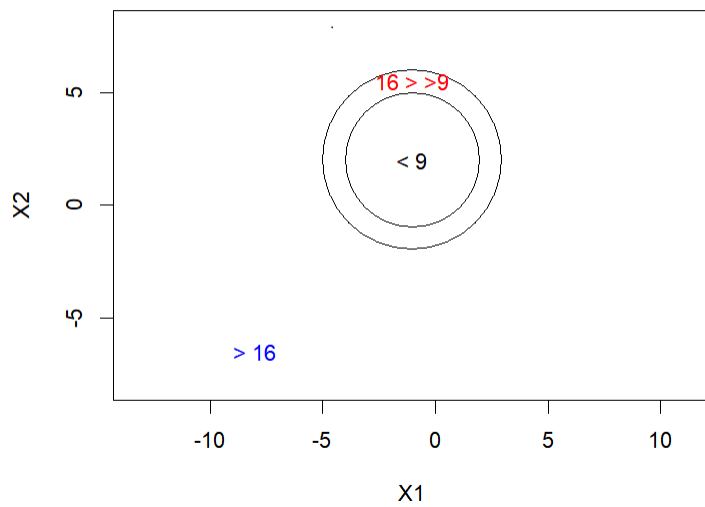


2.  $(1 + X_1)^2 + (2 - X_2)^2 = 16.$

```
> plot(NA, NA, type = "n", xlim = c(-9, 7), ylim = c(-8, 8), asp = 1, xlab = "x1", ylab = "x2")
> symbols(c(-1), c(2), circles = c(4), add = TRUE, inches = FALSE)
```



2B)



2C)

$f(0,0) \Rightarrow \text{red}$

$f(-1,1) \Rightarrow \text{red}$

$f(2,2) \Rightarrow \text{blue}$

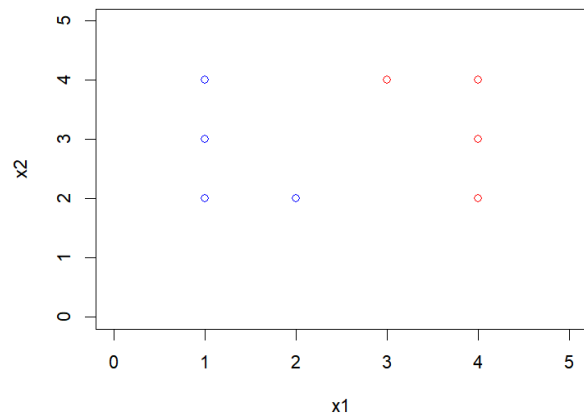
$f(3,4) \Rightarrow \text{red}$

2D)

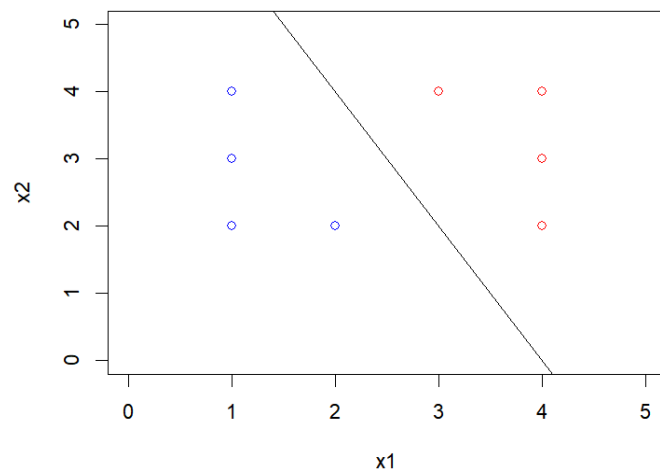
Answer: In terms of  $X_1$  and  $X_2$ , it is not linear because degree is 2. However, in terms of  $X_1, X_1^2, X_2$ , and  $X_2^2$ , it is linear because degree is 1.

3A) We are given  $n = 8$  observations in  $p = 2$  dimensions. For each observation, there is an associated class label.





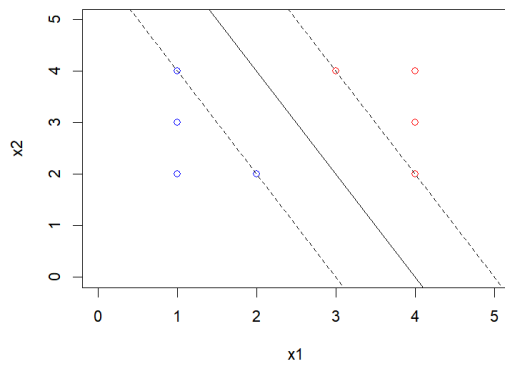
3B)



3C)

Answer: The classification rule is “Classify to red  $-2X_1 - X_2 + 8 < 0$  and classify to Blue otherwise.”

3D)



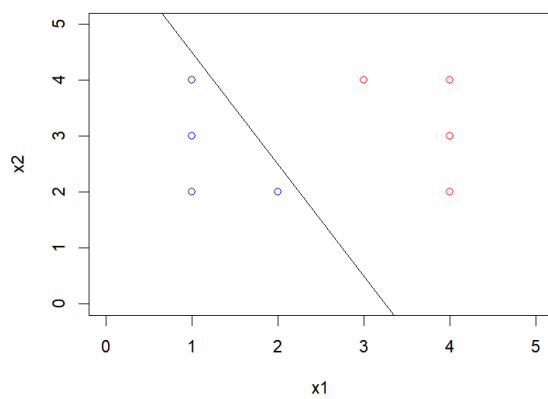
3E)

Answer: Supporting vectors are the points (2,2), (4,2), (1,4), (3,4) .

3F)

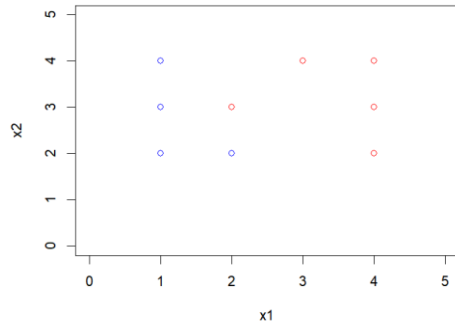
Answer: If we move to the point (1,3), we can't change the maximal marginal hyperplane as it is a support vector.

3G)



Answer: Hyperplane equation is  $-2X_1 - X_2 + 6.5$

3H) Answer: red point(2,3) is added to the plot, two classes are obviously not separable.



4A)

```
> library(MASS)
> data("Boston")
> median_medv <- median(Boston$medv)
> Boston$medv01 <- ifelse(Boston$medv <= median_medv, 0, 1)
> Boston$medv <- NULL
> head(Boston)
```

4B)

Values of K:

1. K = 3(small value)
2. K = 5(moderate value)
3. K = 7(Larger value)

Subsets of Predictors:

1. Full set of all 13 predictors:

- crim
- zn
- indus
- chas
- nox
- rm
- age
- dis
- rad
- tax
- ptratio
- black
- lstat

2. Subset based on logical considerations (a simplified subset for illustration):

- rm (average number of rooms per dwelling)
- lstat (percentage of lower status population)
- dis (weighted distance to employment centers)
- ptratio (pupil-teacher ratio)
- nox (nitrogen oxide concentration)

3. Subset based on correlation analysis (retaining one variable from highly correlated groups):

- rm (average number of rooms per dwelling)
- indus (proportion of non-retail business acres per town)
- tax (property tax rate)
- age (proportion of owner-occupied units built before 1940)
- dis (weighted distance to employment centers)

4C)

Answer: We used k – fold cross-validation with different  $K = 3, 2, 1$

- For  $K=3$ , the combination of predictors "lstat" and "rm" had the lowest test error (0.1442688).
- For  $K=2$ , the combination of predictors "lstat" and "rm" had the lowest test error (0.173913).
- For  $K=1$ , the combination of predictors "lstat" and "rm" had the lowest test error (0.1778656).

Therefore, for each value of  $K$ , the combination of predictors "lstat" and "rm" was the winning model with the lowest test error.

4D)

Answer: No, the variables in the Boston dataset are not on the same scale. We can deal with the problem using loocv.

4E)

Answer: No, it is not same scale compared to part (c).

5A)

```
> library(ISLR)
> data("Auto")
> head(Auto)
> Auto$mpglevel<- as.factor(ifelse(Auto$mpg > median(Auto$mpg), 1, 0))
> Auto$mpg<-NULL
```

5B)



```

Parameter tuning of 'svm':
- sampling method: 10-fold cross validation

- best parameters:
  cost
    5

- best performance: 0.08673077

- Detailed performance results:
  cost      error dispersion
1 1e-02 0.56115385 0.04344202
2 1e-01 0.17371795 0.07157577
3 1e+00 0.09942308 0.05029358
4 5e+00 0.08673077 0.05819036
5 1e+01 0.09435897 0.06043406
6 1e+02 0.08679487 0.05158759
7 1e+03 0.08929487 0.04871612

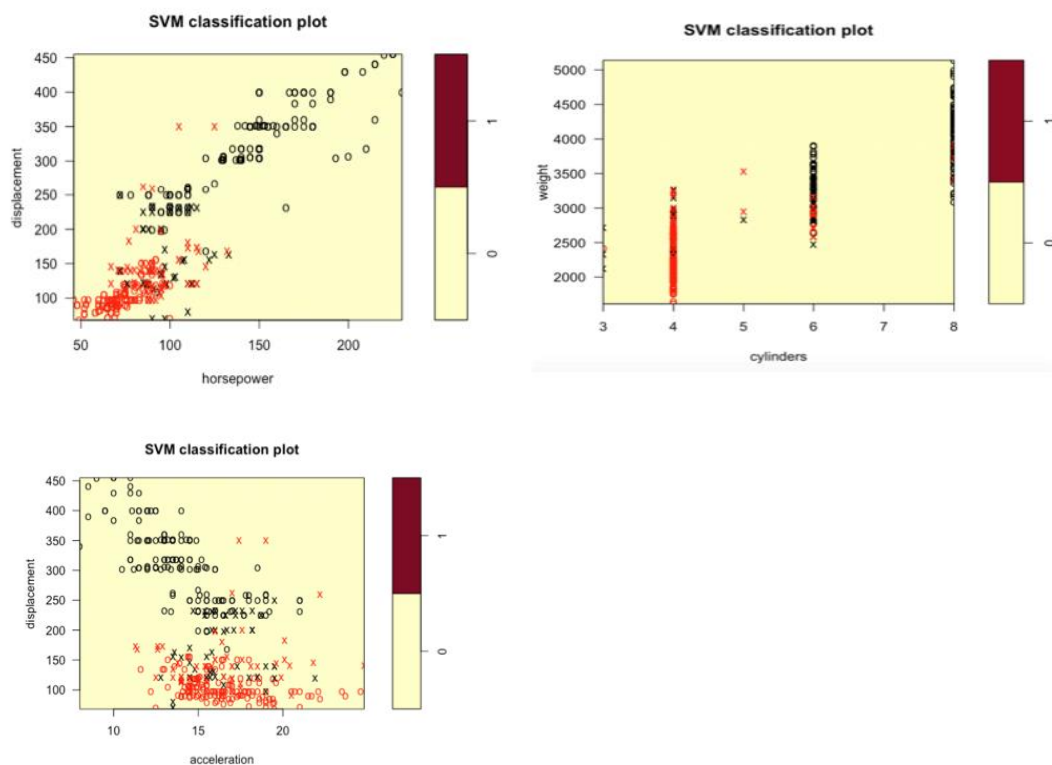
```

5C)

		0	1
predicted	0	51	2
	1	10	55

Answer: For the best option cost = 5, the svm gave 12 misclassification errors as seen in the above confusion matrix.

5D)



6A)

Answer: Programming R code

```
library(ISLR)
```

```
set.seed(1)
```

```
train<- sample(nrow(OJ), 800)
```

```
OJ.train <- OJ[train,]
```

```
OJ.test <- OJ[-train,]
```

6B)

```
R 4.2.2 · ~/
Call:
svm(formula = Purchase ~ ., data = OJ.train, kernel = "linear", cost = 0.01)

Parameters:
  SVM-Type:  C-classification
 SVM-Kernel: linear
      cost:  0.01

Number of Support Vectors:  432

( 215 217 )

Number of Classes:  2

Levels:
CH MM
```

Answer: There are 432 support vectors. 432 are close to hyperplane. Changing the position of these 432 points slightly can affect the classification results. All the points are classified into 2 distinct classes. 215 points are in near class 1, and 217 points are in near class 2.

6C)

```
> trainPredict <- predict(svm.linear, OJ.train)
> testPredict <- predict(svm.linear, OJ.test)
> mean(trainPredict != OJ.train$Purchase)
[1] 0.16625
> mean(testPredict != OJ.test$Purchase)
[1] 0.1814815
```

Answer: The training error is 0.16625 and test error is 0.1814815.

6D)

Answer:

```
> tune_out <- tune(svm, Purchase ~ ., data = OJ.train, ranges = list(cost  
= 10^(-2:1)))  
> best_cost <- tune_out$best.parameters$cost  
> print(best_cost)
```

It is 1.

6E)

```
> svm_model_best <- svm(Purchase ~ ., data = OJ.train, cost = best_cost)  
> train_pred_best <- predict(svm_model_best, train_data)  
Error in predict.svm(svm_model_best, train_data) :  
  object 'train_data' not found  
> train_pred_best <- predict(svm_model_best, OJ.train)  
> test_pred_best <- predict(svm_model_best, OJ.test)  
> mean( train_pred_best != OJ.train$Purchase)  
[1] 0.145  
> mean(test_pred_best != OJ.test$Purchase)  
[1] 0.1703704  
> |
```

The new training error is 0.145.

The new test error is 0.1703704.